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Research Article

A Novel Regret Theory-Based Decision-Making Method Combined with the Intuitionistic Fuzzy Canberra Distance

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In practical decision-making, the behavior factors of decision makers often affect the final decision-making results. Regret theory is an important behavioral decision theory. Based on the regret theory, a novel decision-making method is proposed for the multiattribute decision-making problem with incomplete attribute weight information, and the attribute values are expressed by Atanassov intuitionistic fuzzy numbers. At first, a new distance of intuitionistic fuzzy sets is put forward based on the traditional Canberra distance. Then, we utilize it for the definition of the regret value (rejoice) for the attribute value of each alternative with the corresponding values of the positive point (negative point). The objective of this method is to maximize the comprehensive perceived utility of the alternative set by the decision maker. The optimal attribute weight vector is solved, and the optimal comprehensive perceived utility value of each alternative is obtained. Finally, according to the optimal comprehensive perceived utility value, the rank order of all alternatives is concluded.

1. Introduction

Since Professor Zadeh introduced the concept of fuzzy set in 1965, it has been successfully applied in many fields such as intelligent control, military engineering, economic prediction, and decision-making [1–5]. Zadeh's fuzzy sets have been proved to be an effective tool to deal with fuzzy and imprecise problems [6, 7]. However, in the process of solving some decision-making problems, due to the limitation of the time, energy, or incomplete knowledge of decision makers, decision makers often hesitate, which makes the evaluation results show three aspects: affirmation, negation, and hesitation. Traditional fuzzy sets cannot describe such problems very well, so Professor Atanassov [8] extended fuzzy sets in 1986 and introduced the concept of intuitionistic fuzzy set (IFS). By introducing the parameter of nonmembership degree, IFS can express the information of affirmation and negation at the same time; furthermore, it can describe the fuzzy concept of "not this or that," and then it can describe the hesitation and uncertainty of the decision maker's judgment [9, 10]. Because of this, it can depict the fuzzy essence of the objective world more delicately than Zadeh's fuzzy set in the processing mode. In recent years, the research on the IFS theory has attracted great attention of scholars and has been applied to the fields of economic decision-making [11, 12], medical diagnosis [13, 14], image processing [15, 16], pattern recognition [17, 18], fault tree analysis [19, 20], and so on. Some other extensions of Zadeh's fuzzy set such as picture fuzzy set [6] and rough set [21] all have received great attention and have many successful applications in practice.

Most of the existing intuitionistic fuzzy MADM methods are based on the expected utility theory, which assumes that decision makers are completely rational. However, decision makers often have subjective preferences, such as psychological and behavioral factors when making decision. So, it is important to consider the subjective preferences of decision makers in the decision process. As an important behavioral decision-making theory, regret theory was firstly proposed by Bell [22] and Loomes and Sugden [23]. In recent years, the research and application of the regret theory have attracted many scholars' attention [24–27]. Regret theory holds that decision makers are concerned about the possible results if they choose other schemes while considering the results of schemes. If they find that they can get better results by choosing other schemes, they will regret them psychologically. Otherwise, they will be happy. Therefore, when making a decision, the decision maker will estimate the regret or rejoice that the decision may produce in advance and try to avoid choosing the plan of expected regret; that is to say, the decision maker is regret-averse. In this way, the perceived utility value of the decision maker includes two parts: the utility value of the current result and the perceived utility value after comparing with other possible results, "regret-rejoice" value. Chorus [28] and Qu et al. [29] pointed out that the regret theory has some advantages over the cumulative prospect theory in application. For example, in decision-making, reference points need not be given, and few parameters in the calculation formula were involved in decision-making, which makes the calculation simpler [30]. In the decision-making model, regret theory replaces the expected utility theory, and it is in line with the objective reality of human beings [31].

Many information measures are proposed for fuzzy sets, such as entropy, similarity, and distance [32]. In the classical regret theory, the deviation involving two numbers can be directly measured in accordance with the absolute value of the difference between two numbers. To measure the difference between two IFSs, we are required to define the distance between two IFSs. Although many intuitionistic fuzzy distance measures have been constructed, some existing distance measures have counter-intuitionistic special cases, so it is very important to develop new improved intuitionistic fuzzy distance measures. Canberra distance, as a classical distance measure, has been widely applied in image processing, pattern recognition, and other fields based on the exact number. Taking into account this distance measure, this paper develops a novel intuitionistic fuzzy distance based on the Canberra distance and further applies it to develop a new intuitionistic fuzzy decision-making method combined with the regret theory.

The structure of this paper is as follows: Section 2 first introduces the concept of IFS and then provides some preliminaries of the regret theory. Section 3 puts forward a new intuitionistic fuzzy distance measure based on the traditional Canberra distance. Section 4 develops the intuitionistic fuzzy multiattribute decision-making (MADM) method based on the regret theory combined with the proposed intuitionistic fuzzy Canberra distance. Section 5 provides an example, which explains the new method through the example analysis. Finally, Section 6 is the conclusion of this paper.

2. Preliminaries

Some basic concepts and properties of IFSs and regret theory are reviewed in this section.

Definition 1 (see [8]). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universal set. A set U is called an IFS in X if $U = \{ < x_i, \mu_U(x_i), \nu_U(x_i) > | x_i \in X\}$. Here, $\mu_U(x_i)$ and $\nu_U(x_i)$ are the membership degree and nonmembership degree of x_i, respectively. They satisfy $\mu_U(x_i)$, $\nu_U(x_i)$, $\mu_U(x_i) + \nu_U(x_i) \in [0, 1]$ for $\forall x_i \in X$. Let $\pi_U(x_i) = 1 - \mu_U(x_i) - \nu_U(x_i)$; then, $\pi_U(x_i)$ is called the hesitancy degree of x_i. Sometimes, IFS $U = \{ < x_i, \mu_U(x_i), \nu_U(x_i) > | x_i \in X\}$ is also expressed as $U = \{ < x_i, \mu_U(x_i), \nu_U(x_i), \pi_U(x_i) > | x_i \in X\}$.

Remark 1. Grzegorzewski [33] introduced the concept of intuitionistic fuzzy number (IFN) as an extension of the IFS in the continuous case, and intuitionistic fuzzy numbers have many applications in the engineering field [34]. In order to avoid confusion and for convenience, if there is only one element in X, we call U an Atanassov intuitionistic fuzzy number (AIFN). Each AIFN has a physical interpretation, for example, if A=<0.6, 0.2, 0.2>, then $\mu_A = 0.6$, $\nu_A = 0.2$, and $\pi_A = 0.2$, which can be interpreted as "the vote for resolution is 6 in favor, 2 against, and 2 abstention."

In actual decision-making process, most of the decision makers are not so rational, so the decision maker's behavior factors need to be considered when making a decision. The prospect theory and regret theory are put forward in this context. In the increasingly complex, modern, political, and economic environment, decision makers need to consider not only the results obtained after choosing a certain scheme but also the possible decision results after assuming that other alternatives are chosen. In the regret theory, the perceived utility function is composed of two parts: the utility function of current decision-making results and the regret-rejoice function compared with other decision-making results. Let a and b, respectively, represent the results that can be obtained by selecting scheme A and scheme B. Then, the perceived utility of decision makers for scheme A is

$$u(a,b) = v(a) + R(v(a) - v(b)). \quad (1)$$

Among them, $v(\theta)$ represents the utility value of scheme θ and $R(v(a) - v(b))$is called regret-rejoice value. If $R(v(a) - v(b))$ is positive, then it is called a rejoice value, which indicates the extent to which the decision maker is glad to choose the scheme or give up the scheme. If $R(v(a) - v(b))$ is negative, then it is called a regret value, which indicates the extent to which the decision maker regrets to choose the scheme or give up the scheme. Obviously, the regret gratification function $R(\cdot)$ should be monotonically increasing and concave, i.e., it satisfies $R'(\cdot) > 0$, $R''(\cdot) < 0$, and $R(0) = 0$. Loomes and Sugden [23] pointed out that regret-rejoice function $R(\cdot)$can be expressed as follows:

$$R(\Delta v) = 1 - \exp(-\delta \Delta v). \quad (2)$$

Here, $\delta > 0$ is the regret avoidance coefficient of the decision maker, and the greater δ related to, the larger the regret avoidance degree of the decision maker. Δv is the difference between the utility value of any two schemes.

Figure 1 shows the image of the regret-rejoice function with different values.

Let $A_i (i = 1, 2, \ldots, m)$ be i-th alternatives, and a_i is the result of alternative A_i. According to the regret theory, in decision analysis, when the positive ideal point is taken as the reference point, the decision-making evaluation value will not be greater than the positive ideal point, and at this time, the decision maker will regret. When the negative ideal point is taken as the reference point, the decision-making evaluation value will not be less than the negative ideal point, and at this time, the decision maker is happy. Note that x_{ij} is the attribute evaluation value of scheme A_i under the evaluation attribute o_j given by the decision maker; then, according to Loomes and Sugden [23], the regret value of each attribute evaluation value x_{ij} of scheme A_i is related to the corresponding attribute valuex_j^+ of the positive ideal point, and the joy value of each attribute evaluation value x_{ij} of scheme A_i is related to the corresponding attribute value x_j^- of the negative ideal point. The gratification values can be expressed as

$$\begin{aligned} R_{ij}^1 &= 1 - \exp\left(\delta\left|x_{ij} - x_j^+\right|\right), \\ R_{ij}^2 &= 1 - \exp\left(-\delta\left|x_{ij} - x_j^-\right|\right), \end{aligned} \tag{3}$$

where $\delta > 0$ is the regret avoidance coefficient of decision makers.

A large number of psychological studies have shown that regret, as a negative emotion, has a stronger effect on utility than rejoice. Therefore, the decision maker's comprehensive regret-rejoice value for the evaluation value of scheme A_i under the evaluation attribute o_j is

$$R_{ij} = R_{ij}^1 + R_{ij}^2 = 2 - \exp\left(\delta\left|x_{ij} - x_j^+\right|\right) - \exp\left(-\delta\left|x_{ij} - x_j^-\right|\right). \tag{4}$$

According to Loomes and Sugden [23], power function $v(x) = x^\alpha$, $0 < \alpha < 1$, is used as a utility function of the attribute value in this paper. The greater the degree of risk aversion of decision makers is, the smaller α is. α is called the risk aversion coefficient of decision makers. It can be proved that if $\Delta v_0 > 0$, there is $|R(-\delta\Delta v_0)| > R(\Delta v_0)$. This shows that compared with Δv_0, decision maker's psychological perception is more sensitive to $-\Delta v_0$, that is, decision makers are regret-averse.

3. A New Distance Based on the Canberra Distance

In this section, we will propose a new distance measure between two IFSs based on the Canberra distance. The

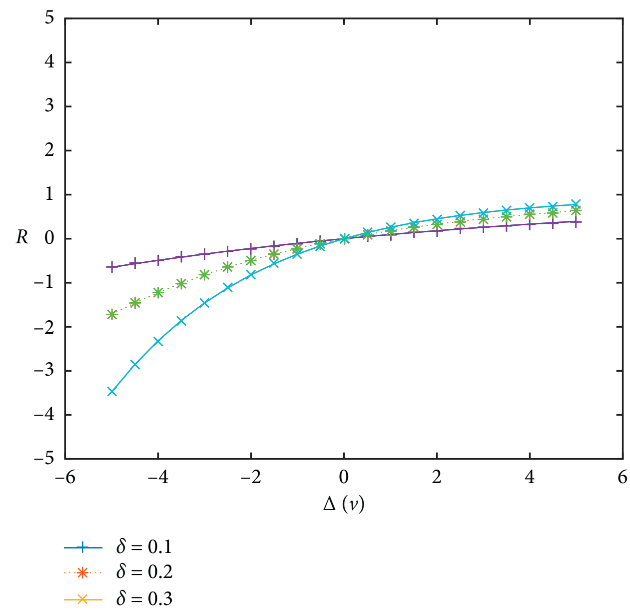

Figure 1: Regret-rejoice function, $R\Delta v$.

Canberra distance of two real vectors $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$is defined as follows (Perlibakas [35]):

$$d(x, y) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{|x_i| + |y_i|}. \tag{5}$$

As an important information measure, Canberra distance has been successfully applied in image processing, medicine, and other fields [36–38]. Due to the fact that the denominator is zero, the numerical value is meaningless. Then, we propose a revised version of the Canberra distance measure as follows:

$$\sum_{i=1}^{n} d(x, y) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{2 + |x_i| + |y_i|}. \tag{6}$$

Note that constant 2 can be changed as any other positive numbers.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universal set. Then, for two given IFSs, $A = \{< x_i, \mu_A(x_i), \nu_A(x_i), \pi_A(x_i) > | x_i \in X\}$and $B = \{< x_i, \mu_B(x_i), \nu_B(x_i), \pi_B(x_i) > | x_i \in X\}$, and the new intuitionistic fuzzy information measure based on Canberra distance $d(A, B)$ is constructed as follows:

$$d(A, B) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{|\mu_A(x_i) - \mu_B(x_i)|}{2 + \mu_A(x_i) - \mu_B(x_i)} + \frac{|\nu_A(x_i) + \nu_B(x_i)|}{2 + \nu_A(x_i) + \nu_B(x_i)} + \frac{|\pi_A(x_i) + \pi_B(x_i)|}{2 + \pi_A(x_i) + \pi_B(x_i)}\right]. \tag{7}$$

Next, we will prove $d(A,B)$ is a valid distance measure. In this section, let R^* be a set of nonnegative real numbers.

Lemma 1. *Let $a,b,c\in R^*$ and $a\le b\le c$. Then,*

$$\begin{aligned}&\text{(i)}\quad \frac{|a-c|}{2+a+c}\ge\frac{|a-b|}{2+a+b},\\ &\text{(ii)}\quad \frac{|a-c|}{2+a+c}\ge\frac{|b-c|}{2+b+c},\\ &\text{(iii)}\quad \frac{|a-c|}{2+a+c}\le\frac{|a-b|}{2+a+b}+\frac{|b-c|}{2+b+c}.\end{aligned}\tag{8}$$

Lemma 2. *Let $a,b,c\in R^*$ and $d_1(a,b)=(|a-b|)/(2+a+b)$; then,*

$$d_1(a,b)\le d_1(b,c)+d_1(a,c).\tag{9}$$

Proof. For the case $0\le a\le b\le c$, according to Lemma 1, we have

(i) $d_1(a,c)\ge d_1(a,b)$; then, $d_1(a,b)\le d_1(a,c)+d_1(b,c)$

(ii) $d_1(a,c)\ge d_1(b,c)$; then, $d_1(b,c)\le d_1(a,c)+d_1(a,b)$

(iii) $d_1(a,c)\le d_1(a,b)+d_1(b,c)$

That is, $d_1(\cdot,\cdot)$ satisfies trigonometric inequality. And we can easily prove that $d_1(\cdot,\cdot)$satisfies trigonometric inequality in other cases using a similar reasoning process. Then, the conclusion is proved. □

Theorem 1. *Let $A=\{<x_i,\mu_A(x_i),\nu_A(x_i)>|\,x_i\in X\}$ and $B=\{<x_i,\mu_B(x_i),\nu_B(x_i)>|\,x_i\in X\}$ be two IFSs in $X=(x_1,x_2,\ldots,x_n)$. Then, $d(A,B)$ defined in (7) is a valid distance measure between A and B. That is, $d(A,B)$ satisfies the following properties:*

(i) $d(A,B)\ge 0$

(ii) $d(A,B)=d(B,A)$

(iii) $d(A,B)\le d(B,C)+d(A,C)$, for any IFSs A,B, and C

Proof

(i) Obviously, $d(A,B)\ge 0$.

(ii) $d(B,A)=(1/n)\sum_{i=1}^{n}[(|\mu_B(x_i)-\mu_A(x_i)|/2+\mu_B(x_i)+\mu_A(x_i))+(|\nu_B(x_i)-\nu_A(x_i)|/2+\nu_B(x_i)+\nu_A(x_i))+(|\pi_B(x_i)-\pi_A(x_i)|/2+\pi_B(x_i)+\pi_A(x_i))]=(1/n)\sum_{i=1}^{n}[(|\mu_A(x_i)-\mu_B(x_i)|/2+\mu_A(x_i)+\mu_B(x_i))+(|\nu_A(x_i)-\nu_B(x_i)|/2+\nu_A(x_i)+\nu_B(x_i))+(|\pi_A(x_i)-\pi_B(x_i)|/2+\pi_A(x_i)+\pi_B(x_i))]=d(A,B)$.

(iii) If A, B, and C are three IFSs, $A=\{<x_i,\ \mu_A(x_i),\nu_A(x_i)>|\,x_i\in X\}$, $B=\{<x_i,\mu_B(x_i),\nu_B(x_i)>|\,x_i\in X\}$, and $C=\{<x_i,\mu_C(x_i),\nu_C(x_i)>|\,x_i\in X\}$, then by (i) of Lemma 2, we have

$$\begin{aligned}d_1(\mu_A(x_i)+\mu_B(x_i))&\le d_1(\mu_B(x_i)+\mu_C(x_i))+d_1(\mu_A(x_i)+\mu_C(x_i)),\\ d_1(\nu_A(x_i)+\nu_B(x_i))&\le d_1(\nu_B(x_i)+\nu_C(x_i))+d_1(\nu_A(x_i)+\nu_C(x_i)),\\ d_1(\pi_A(x_i)+\pi_B(x_i))&\le d_1(\pi_B(x_i)+\pi_C(x_i))+d_1(\pi_A(x_i)+\pi_C(x_i)),\end{aligned}\tag{10}$$

while

$$d(A,B)=\frac{1}{n}\sum_{i=1}^{n}[d_1(\mu_A(x_i)+\mu_B(x_i))+d_1(\nu_A(x_i)+\nu_B(x_i))+d_1(\pi_A(x_i)+\pi_B(x_i))].\tag{11}$$

Consequently, $d(A,B)\le d(B,C)+d(A,C)$

Then, we complete the proof of Theorem 1.

If we consider the important degree of x_i $(i=1,2,\ldots,n)$ and let w_i $(i=1,2,\ldots,n)$ be the important degree of x_i, $(i=1,2,\ldots,n)$, which satisfies $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$, then we can get a weighted distance $d_W(A,B)$ between A and B as follows:

$$d_W(A,B)=\sum_{i=1}^{n}w_i\left[\frac{|\mu_A(x_i)-\mu_B(x_i)|}{2+\mu_A(x_i)-\mu_B(x_i)}+\frac{|\nu_A(x_i)+\nu_B(x_i)|}{2+\nu_A(x_i)+\nu_B(x_i)}+\frac{|\pi_A(x_i)+\pi_B(x_i)|}{2+\pi_A(x_i)+\pi_B(x_i)}\right].\tag{12}$$

□

Remark 2. If $w_i = 1/n$, $(i = 1, 2, \ldots, n)$, then $d_W(A, B) = d(A, B)$. Obviously, $d_W(A, B)$ is also a valid distance, and the proof process is similar to $d(A, B)$ in Theorem 1.

4. A New Regret Theory-Based Decision-Making Method

In this section, we will put forward a new intuitionistic fuzzy MADM method based on the regret theory combined with the above proposed distance. The detail decision process is shown in Figure 2.

For an intuitionistic fuzzy MADM problem, for the convenience of description, the following symbols represent the set or quantity in the decision:

$X = \{x_1, x_2, \ldots, x_n\}$: the set of m alternatives $(m \geq 2)$.

$O = \{o_1, o_2, \ldots, o_n\}$: the set of n attributes $(n \geq 2)$, where o_j represents the jth attribute.

$\mathbf{w} = (w_1, w_2, \ldots, w_n)^T$: the vector of attributes' weights. This is because in the decision-making process, different attributes usually have different importance. Here, w_j is the weight information of attribute o_j $(j = 1, 2, \ldots, n)$, satisfying $w_j \geq 0$ $(j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} w_j = 1$. When the attribute weight information is partially known, the set of mathematical expressions that record the known partial weight information is denoted by H.

$x_{ij} = <\mu_{ij}, v_{ij}>$: the evaluation value of alternative x_i given by the decision maker under attribute o_j. The numbers μ_{ij} and v_{ij} show the degree of satisfaction and dissatisfaction of the decision maker with the value of the alternative x_i under the index o_j, respectively. They satisfy $0 \leq \mu_{ij} \leq 1$, $0 \leq v_{ij} \leq 1$, and $0 \leq \mu_{ij} + v_{ij} \leq 1$.

Now, we can get an intuitionistic fuzzy decision-making matrix $\widetilde{X} = (x_{ij})_{m \times n}$. It is required to determine the order of alternatives and choose the optimal alternative.

Now, we propose a new decision-making method based on the regret theory. The decision maker's comprehensive regret-rejoice value of the evaluation value $x_{ij} = <\mu_{ij}, v_{ij}>$ of scheme x_i under the evaluation attribute o_j is

$$R_{ij} = R_{ij}^1 + R_{ij}^2 = 2 - \exp\left[\delta d\left(x_{ij}, x_j^+\right)\right] - \exp\left[-\delta d\left(x_{ij}, x_j^-\right)\right]. \tag{13}$$

In this paper, function $v_{ij}(x_{ij}) = S(x_{ij})^{\alpha}$, $0 < \alpha < 1$, is used as a utility function of the attribute value, and $S(x)$ is the score function of the AIFN. Then, the decision maker's perception utility function of the corresponding attribute value x_{ij} of scheme x_i can be expressed as

$$\begin{aligned} F_{ij} = v_{ij} + R_{ij} = 2 + S\left(x_{ij}\right)^{\alpha} - \exp\left[\delta d\left(x_{ij}, x_j^+\right)\right] \\ - \exp\left[-\delta d\left(x_{ij}, x_j^-\right)\right]. \end{aligned} \tag{14}$$

Next, we discuss the method to determine the attribute weight of intuitionistic fuzzy MADM. Let H be the set of known weight information. For each scheme x_i, its comprehensive perceived utility function is

$$\begin{aligned} F(x_i) &= \sum_{j=1}^{n} w_j F_{ij} \\ &= \sum_{j=1}^{n} w_j \left\{2 + S\left(x_{ij}\right)^{\alpha} - \exp\left[\delta d\left(x_{ij}, x_j^+\right)\right] \right. \\ &\quad \left. - \exp\left[-\delta d\left(x_{ij}, x_j^-\right)\right]\right\}. \end{aligned} \tag{15}$$

The weight should be determined so that the greater the comprehensive perceived utility, the better the scheme x_i is. Therefore, the following optimization model can be established, and its objective function is

$$\max F = (F(x_1), F(x_2), \ldots, F(x_m)). \tag{16}$$

According to the fact that "the greater the comprehensive perceived utility, the better the scheme" and the fair competition among the schemes aimed at the maximization of the comprehensive perceived utility of the decision maker to the scheme set, the optimization model for solving the attribute weight is established as follows:

$$\begin{aligned} \max V &= \sum_{i=1}^{m} F(x_i) \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n} w_j \left\{2 + S\left(x_{ij}\right)^{\alpha} - \exp\left[\delta d\left(x_{ij}, x_j^+\right)\right] \right. \\ &\quad \left. - \exp\left[-\delta d\left(x_{ij}, x_j^-\right)\right]\right\}, \\ \text{s.t.} & \begin{cases} \mathbf{w} \in \mathbf{H}, \\ \sum_{j=1}^{n} w_j = 1, \\ w_j \geq 0, \quad j = 1, 2, \ldots, n. \end{cases} \end{aligned} \tag{17}$$

The optimal weight vector $\mathbf{w}^* = (w_1^*, w_2^*, \ldots, w_m^*)^T$ can be obtained by solving the above model with MATLAB or LINGO software.

Thus, it can be seen that the optimal comprehensive perceived utility value of the decision maker to scheme x_i is

$$\begin{aligned} H_i = \sum_{j=1}^{n} w_j^* \left\{2 + S\left(x_{ij}\right)^{\alpha} - \exp\left[\delta d\left(x_{ij}, x_j^+\right)\right] \right. \\ \left. - \exp\left[-\delta d\left(x_{ij}, x_j^-\right)\right]\right\}. \end{aligned} \tag{18}$$

Finally, according to the comparison of the optimal comprehensive perceived utility value, the ranking results of all schemes can be obtained. The larger the value H_i is, the better the corresponding alternative x_i is.

Next, the calculation steps of the MADM method based on the regret theory are given as follows:

Step 1: calculate the score $S_{ij} = \mu_{ij} - v_{ij}$ of attribute value $x_{ij} = <u_{ij}, v_{ij}>$, and get the score matrix $S = (S_{ij})_{m \times n}$.

Step 2: determine the positive and negative ideal point.

The positive ideal point is defined as $x^+ = (x_1^+, x_2^+, \ldots, x_n^+)$, where $x_j^+ = <1, 0>$, $j = 1, 2, \ldots, n$

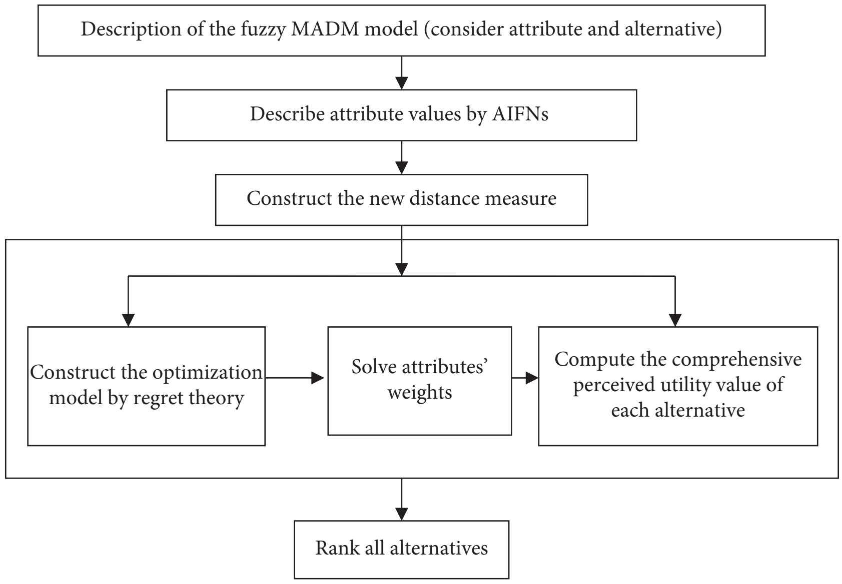

FIGURE 2: The calculation process of the intuitionistic fuzzy MADM model.

The negative ideal point is defined as $x^- = (x_1^-, x_2^-, \ldots, x_n^-)$, where $x_j^- = <1, 0>$, $j = 1, 2, \ldots, n$

Step 3: calculate the distances $d(x_{ij}, x_j^+)$ and $d(x_{ij}, x_j^-)$, where $d(A, B)$ represents the intuitionistic fuzzy Canberra distance between $A = <\mu_A, \nu_A>$ and $B = <\mu_B, \nu_B>$. Then,

$$d\left(x_{ij}, x_j^+\right) = \frac{1-\mu_{ij}}{3+\mu_{ij}} + \frac{\nu_{ij}}{2+\nu_{ij}} + \frac{1-\mu_{ij}-\nu_{ij}}{3-\mu_{ij}-\nu_{ij}},$$
$$d\left(x_{ij}, x_j^-\right) = \frac{\mu_{ij}}{2+\mu_{ij}} + \frac{1-\nu_{ij}}{3+\nu_{ij}} + \frac{1-\mu_{ij}-\nu_{ij}}{3-\mu_{ij}-\nu_{ij}}. \quad (19)$$

Step 4: according to equation (14), calculate the perceived utility function value F_{ij} of attribute value x_{ij} corresponding to each alternative.

Step 5: establish optimization model (17), and calculate the optimal weight vector $\mathbf{w}^*$ with the help of MATLAB software.

Step 6: the optimal weight obtained from Step 5 is substituted into equation (18), and the comprehensive perceived utility value of each alternative is obtained. The merits and demerits of the scheme are determined according to the comprehensive perceived utility value H_i. The higher the value of H_i, the better the corresponding alternative x_i.

5. Numerical Example

The effectiveness and practicability of this method are illustrated by an example of assembly parts' supplier selection in Xu [39]. With the economic globalization and the continuous expansion of enterprise scale, the problem of supplier selection has become an important management decision-making problem that all large enterprises need to seriously consider. Let a manufacturing company prepare to find the best supplier in the world for purchasing the most critical parts in the assembly process.

After the primary selection, there are five alternative suppliers x_i $(i = 1, 2, 3, 4, 5)$. Now, the company will evaluate the suppliers according to the following five evaluation indicators (attributes): product price (o_1), product quality (o_2), service performance (o_3), supplier's situation (o_4), and risk factors (o_5). After experts' discussion, the evaluation values of each attribute of the candidate suppliers are finally obtained. Suppose the manufacturing company invites N experts to make the judgment. They are expected to answer "Yes" or "No" or "I do not know" to the question whether alternative x_i satisfies attribute o_j. Let $Q_Y(i, j)$ and $Q_N(i, j)$ denote the sum of "Yes" and "No," respectively. Then, the degrees to which alternative A_i satisfies and does not satisfy attribute o_j can be calculated as

$$\mu_{ij} = \frac{Q_Y(i, j)}{N},$$
$$\nu_{ij} = \frac{Q_N(i, j)}{N}. \quad (20)$$

Then, the evaluation values are expressed by AIFNs, as shown in Table 1.

It is assumed that the attribute weight information is partially known, and the attribute weight satisfies

$$H = \left\{ w = (w_1, w_2, \ldots, w_5)^T \,\middle|\, w_1 \le 0.3, 0.1 \le w_2 \le 0.2, 0.2 \le w_3 \le 0.5, 0.1 \le w_4 \le 0.3w_3 - w_1 \le 0.1, w_4 \ge w_1, w_5 \le 0.4, w_3 - w_2 \ge w_5 - w_4 \right\}. \quad (22)$$

Table 1: Attribute evaluation value of each alternative supplier o_1.

Suppliers	Evaluation attribute				
	o_1	o_2	o_3	o_4	o_5
x_1	<0.449, 0.370>	<0.565, 0.162>	<0.705, 0.232>	<0.730, 0.170>	<0.646, 0.354>
x_2	<0.719, 0.188>	<0.630, 0.232>	<0.448, 0.378>	<0.557, 0.160>	<0.597, 0.192>
x_3	<0.546, 0.192>	<0.727, 0.182>	<0.641, 0.322>	<0.399, 0.200>	<0.658, 0.192>
x_4	<0.520, 0.337>	<0.630, 0.100>	<0.539, 0.271>	<0.679, 0.188>	<0.708, 0.198>
x_5	<0.727, 0.128>	<0.520, 0.299>	<0.619, 0.318>	<0.618, 0.229>	<0.609, 0.120>

Next, we use the proposed decision-making method to sort the five suppliers and choose the best desirable supplier. These suppliers are sorted according to $H_i\,(i = 1, 2, 3, 4, 5)$ from large to small. The result is $x_4 \succ x_5 \succ x_1 \succ x_3 \succ x_2$, and supplier x_4 is the best choice.

Step 1: calculate the score $S_{ij} = \mu_{ij} - \nu_{ij}$ of attribute $x_{ij} = <\mu_{ij}, \nu_{ij}>$. Then, we get the score matrix $S = (S_{ij})_{5\times 5}$, and the calculation result is shown in Table 2.

Step 2: determine the positive and negative ideal point.

The positive ideal point is defined as

$$x^+ = (x_1^+, x_2^+, \ldots, x_5^+), \tag{23}$$

where $x_j^+ = <1, 0>$, $j = 1, 2, \ldots, 5$.

The negative ideal point is defined as

$$x^- = (x_1^-, x_2^-, \ldots, x_5^-), \tag{24}$$

where $x_j^- = <0, 1>$, $j = 1, 2, \ldots, 5$.

Step 3: calculate the Canberra distances $d(x_{ij}, x_j^+)$ and $d(x_{ij}, x_j^-)$, and the results are shown in Tables 3 and 4.

Step 4: the perceived utility function F_{ij} of attribute x_{ij} corresponding to each alternative is calculated. In this paper, $\alpha = 0.88$ and $\delta = 0.3$ are used for calculation, and the calculation results are shown in Table 5.

Step 5: according to equation (17), the following linear programming model is established:

$$\begin{aligned}
\max V &= 1.8739w_1 + 2.2942w_2 + 1.5653w_3 \\
&\quad + 2.2187w_4 + 2.3540w_5, \\
\text{s.t.} &\begin{cases}
w_1 \le 0.3, \\
0.1 \le w_2 \le 0.2, \\
0.2 \le w_3 \le 0.5, \\
0.1 \le w_4 \le 0.3, \\
w_3 - w_1 \le 0, \\
w_4 \ge w_1, \\
w_5 \le 0.4, \\
w_3 - w_2 \ge w_5 - w_4, \\
w_1 + w_2 + w_3 + w_4 + w_5 = 1, \\
w_1, w_2, \ldots, w_5 \ge 0.
\end{cases}
\end{aligned} \tag{25}$$

Table 2: Score matrix of attribute values of each alternative under each attribute.

Suppliers	Evaluation attribute				
	o_1	o_2	o_3	o_4	o_5
x_1	0.0790	0.4030	0.4730	0.5600	0.2920
x_2	0.5310	0.3980	0.0700	0.3970	0.4050
x_3	0.3540	0.5450	0.3190	0.1990	0.4660
x_4	0.1830	0.5300	0.2680	0.4910	0.5100
x_5	0.5990	0.2210	0.3010	0.3900	0.4890

Table 3: Distance set between attribute values of each scheme and corresponding values of PIS.

	x_1^+	x_2^+	x_3^+	x_4^+	x_5^+
x_1	0.3989	0.3171	0.2141	0.1983	0.2475
x_2	0.2059	0.2704	0.3991	0.3226	0.2951
x_3	0.3314	0.2002	0.2554	0.4347	0.2509
x_4	0.3473	0.2685	0.3364	0.2355	0.2137
x_5	0.2010	0.3494	0.2730	0.2786	0.2843

Table 4: Distance set between attribute values of each scheme and corresponding values of NIS.

	x_1^+	x_2^+	x_3^+	x_4^+	x_5^+
x_1	0.4533	0.6054	0.5288	0.5768	0.4367
x_2	0.5636	0.5417	0.4472	0.6076	0.5784
x_3	0.5834	0.5672	0.4650	0.5833	0.5705
x_4	0.4718	0.6488	0.5219	0.5705	0.5571
x_5	0.6130	0.5018	0.4724	0.5458	0.6348

Table 5: Perceived utility function values of attribute values corresponding to each alternative.

Suppliers	Evaluation attribute				
	o_1	o_2	o_3	o_4	o_5
x_1	0.0739	0.3923	0.5591	0.6703	0.2657
x_2	0.6186	0.4826	0.0060	0.3975	0.4545
x_3	0.3341	0.6254	0.4041	0.1804	0.5028
x_4	0.1167	0.5838	0.3110	0.6002	0.5441
x_5	0.7307	0.2101	0.2850	0.3702	0.5868

Solving model (25), we get the optimal attribute weight vector $\mathbf{w}^* = (0.1, 0.1, 0.2, 0.25, 0.35)^T$.

Step 6: calculate the comprehensive perceived utility value of each alternative, and get

$$
\begin{aligned}
H_1 &= 0.4190, \\
H_2 &= 0.3698, \\
H_3 &= 0.3979, \\
H_4 &= 0.4728, \\
H_5 &= 0.4490.
\end{aligned}
\tag{26}
$$

According to the result of Xu [39], the ranking order is $x_5 \succ x_4 \succ x_1 \succ x_2 \succ x_3$, which is different from the result of this paper. This is because the optimization model established by the score function in Xu [39] does not consider the influence of the degree of hesitation on the ranking of IFSs.

Most of the existing intuitionistic fuzzy MADM methods are based on the expected utility theory assuming that decision makers are completely rational. However, in the actual MADM process, decision makers often have subjective risk preferences, such as psychological and behavioral factors for alternatives. So, it is important to consider the risk attitude of decision makers in the decision process. In this article, a new regret theory-based decision-making method is proposed for the MADM problem in which attribute values are expressed by AIFNs.

The advantage and limitation of the proposed decision-making method can be summarized as follows:

(1) The comprehensive perceived utility value constructed in this paper not only considers the score function but also considers the decision maker's regret gratification value; therefore, it is more in line with the objective reality

(2) How to determine the most suitable values of parameters in the regret theory is its limitation

The novelty of this paper is the proposition of a new decision-making method based on the regret theory, which can better reflect the psychological and behavioral factors of the decision maker than many existing decision-making methods. This paper also develops a new weighting method based on the intuitionistic fuzzy Canberra distance.

6. Conclusion

For the MADM problem in which attribute values are expressed by AIFNs, this paper develops a new decision-making method based on the regret theory combined with an extension of the Canberra distance measure. The main contributions of this article are as follows:

(1) Regret theory can describe humans' psychological behavior under uncertain conditions more truthfully, and it can explain the phenomena that expected utility theory cannot. Our decision-making method considers the psychological factors of decision makers based on the regret theory, which can be more in line with the reality.

(2) This article first constructs a new distance of IFSs based on the traditional Canberra distance. Then, a new weighting method is put forward by establishing an optimization model, which is a model of the maximum optimal comprehensive perceived utility value under given weighting information. The new method enriches and develops the weight attribute determination method.

(3) A numerical example of supplier selection is utilized to show the effectiveness and feasibility of the proposed method. The proposed MADM method based on the regret theory has advantages of simple calculation process and easy software implementation.

In future, we will apply the proposed MADM method to solve other decision-making problems, such as the risk evaluation, system optimization, and material selection. Furthermore, we will develop the new intuitionistic fuzzy distance for the application of clustering analysis and image processing.

Data Availability

The data supporting this numerical example are from [39] which has been cited.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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