Hindawi
Discrete Dynamics in Nature and Society
Volume 2020, Article ID 9587353, 12 pages
https://doi.org/10.1155/2020/9587353

Research Article

The Interval Parameter Optimization Model Based on Three-Way Decision Space and Its Application on "Green Products Recommendation"

Mingxia Li[1], **Kebing Chen**[2], **Caiyun Liu**[2,3] **and Baoxiang Liu**[4]

[1]*College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*
[2]*College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*
[3]*Department of Mathematics and Computer, Tong Ling University, Tong Ling 244061, China*
[4]*Key Laboratory of Data Science and Application of Hebei Province, Tangshan 063000, China*

Correspondence should be addressed to Mingxia Li; nicemingxia@sina.cn and Kebing Chen; kbchen@nuaa.edu.cn

Received 3 July 2020; Revised 19 September 2020; Accepted 3 November 2020; Published 1 December 2020

Academic Editor: Jaime Zabalza

The interval concept lattice theory, a new method of mining objects based on interval parameters, can more accurately deal with uncertain information than the classical concept lattice theory. The optimization of interval parameters has been a problem that is not well solved. From the perspective of three-way decision space, we first combine the theories of interval concept lattice and three-way decision and then put forward interval three-way decision space theory; second, in the interval three-way decision space, the positive region, negative region, and boundary region are divided by extension of interval three-way decision concept; further, the decision loss function and three-way decision rules are extracted. Through adjusting interval parameters of the lattice structure, we could find that when parameter α is roughly 0.6, more credible decision rules will be mined and decision-making becomes more clear than that under the condition α is less than 0.6; finally, we verify the model by a "Green Products Recommendation" example.

1. Introduction

The interval concept lattice theory is the new method of mining objects based on interval parameters α and β and proposed by Liu [1] in 2012. Compared to the classical concept lattice theory [2], it can not only contribute to exploring the potential information from the uncertain system, but also deal with uncertain information more accurately. It provides the foundation for dealing with boundary samples and reducing the decision loss [3]. Interval parameters α and β can divide the object domain U into three regions according to the condition attributes U meets. This division method is similar to that in probabilistic rough set [4]. The interval parameters affect not only the concepts and lattice structure, but also the decision-making. Therefore, it is very meaningful to study interval parameter optimization problem. Although there are many general interval parameter optimization methods [5], it is of weak pertinence. In other words, the optimized interval parameters given by the general method could not be well applied to our problem. Thus, we innovate the optimization method by combining the original three-way decision theory.

To provide a reasonable semantic interpretation for probabilistic rough set [3, 4] and decision-theoretic rough set [6–10], Yao first puts forward the concept of three-way decision [11–15], which is an extension of the traditional two-way decision theory. It considers the uncertain factors in the decision-making process and takes the delayed decision as the third decision behavior in the case that the information is insufficient to decide the acceptance or rejection [16]. The recently related research based on the three-way decision theory impressing us deeply is a trisecting-and-acting model to describe the three-way decision, in which the model not only represents trisecting a whole into three parts but also devises strategies and actions to act on the three regions [12]. Liang et al. [17] involve the risk appetite of the decision-maker into three-way decisions and utilize TODIM (interactive multicriteria decision-making) as a valuable tool to handle the risk appetite character to construct risk appetite dual hesitant fuzzy three-way decisions. Jiang et al. [18] choose a strategy depending on the probability or distribution of three regions instead of the benefits or costs and propose a probabilistic movement model of three-way decision, and then give a strategy selection mechanism based on information entropy.

The last case to motivate our research is that Wei et al. [19] give the three-way concept lattices theory and indicate that they can supply much more information than classical concept lattices since they contain the positive information and negative information between objects and attributes simultaneously. Motivated by the commonality of the above two theories, we define the interval three-way decision space according to the interval concept decision loss function that depends on interval parameters α and β. Decision concepts in the interval three-way decision space will change with respect to the interval parameters, which can finally affect users' decision-making and benefit interval parameter optimization. To demonstrate the impact of interval parameters on decision rules and the optimization process of interval parameters, we use a "Green Products Recommendation" example. The reason why we choose this case mainly includes two sides. On the one side, recently the research about "green," "eco," and "sustainable" has been a hotspot issue [20, 21]. On the other hand, due to the complexity of consumers' preferences, it is definitely difficult to establish an analytical model to master the green demand of each consumer. The method of taking advantage of a priori knowledge to further conclude consumers' preferences is relatively feasible. Of course, our model can also be extended to other cases about "Recommendation."

2. Preliminaries

2.1. Three-Way Decision and Rough Set. Suppose U is a finite set of entity objects and $E(E \subseteq U \times U)$ is an equivalence relation on U set, i.e., E is reflexive, symmetric, and transitive. The equivalence class of E containing an object $x(x \in U)$ is given by $[x]_E = [x] = \{y \in U | xEy\}$. The set of all equivalence classes, $U/E = \{[x]_E | x \in U\}$, is called the quotient set and we regard it as a partition of U.

Definition 1. For a pair of thresholds $[\alpha, \beta]$ with $0 \leq \alpha < \beta \leq 1$, the $[\alpha, \beta]$-probabilistic lower and upper approximations of X are expressed as follows:

$$\begin{aligned}\underline{\operatorname{apr}}_{(\alpha,\beta)}(X) &= \bigcup\left\{[x] \in \frac{U}{E} | \Pr(X|[x]) \geq \beta\right\},\\ \underline{\operatorname{apr}}_{(\alpha,\beta)}(X) &= \bigcup\left\{[x] \in \frac{U}{E} | \Pr(X|[x]) \geq \alpha\right\}.\end{aligned} \tag{1}$$

For a subset $X(X \subseteq U)$, $\Pr(X|[x])$ denotes the conditional probability of an object in X given that the object is in equivalence class $[x]$, and in other words, $\Pr(X|[x])$ implies the confidence coefficient of such entity belonging to extension of X.

Proposition 1. *According to the lower and upper approximation, the following probabilistic positive, negative, and boundary region can be given as follows:*

$$\begin{aligned}\operatorname{POS}_{(\alpha,\beta)}(X) &= \underline{\operatorname{apr}}_{(\alpha,\beta)}(X) = \cup\left\{x \in \frac{U}{E} | \Pr(X|[x]) \geq \beta\right\},\\ \operatorname{NEG}_{(\alpha,\beta)}(X) &= \left(\overline{\operatorname{apr}}_{(\alpha,\beta)}(X)\right)^c = \left\{x \in \frac{U}{E} | \Pr(X|[x]) < \alpha\right\},\\ \operatorname{BND}_{(\alpha,\beta)}(X) &= \left(\operatorname{POS}_{(\alpha,\beta)}(X) \cup \operatorname{NEG}_{(\alpha,\beta)}(X)\right)^c = \{x \in U | \alpha \leq \Pr(X|[x]) < \beta\}.\end{aligned} \tag{2}$$

where $(\overline{apr}_{(\alpha,\beta)}(X))^c = U - \overline{apr}_{(\alpha,\beta)}(X)$. *The three probabilistic regions are pairwise disjoint and their union is the entire set* U.

Proposition 2. *The false acceptance rates in different regions are as follows:*

(i) *When* $\Pr(X|[x]) \geq \beta$, *we choose to accept. However, accepting all entities of* $[x]$ *will lead to an error. And the false acceptance rate in the positive region is as follows:*

$$\operatorname{IAE}\left(\operatorname{POS}_{(\alpha,\beta)}(X), X\right) = \frac{\left|\operatorname{POS}_{(\alpha,\beta)}(X) \cap X^c\right|}{\left|\operatorname{POS}_{(\alpha,\beta)}(X)\right|}. \tag{3}$$

(ii) *When* $\Pr(X|[x]) \leq \alpha$, *we choose to reject. Similarly, rejecting all entities of* $[x]$ *will lead to an error. And the false rejection rate in the negative region is as follows:*

$$\operatorname{IRE}\left(\operatorname{NEG}_{(\alpha,\beta)}(X), X\right) = \frac{\left|\operatorname{NEG}_{(\alpha,\beta)}(X) \cap X\right|}{|\operatorname{NEG}(X)|}. \tag{4}$$

(iii) *When the confidence coefficient is too low to warrant an acceptance, at the same time, and too high to warrant a rejection, then we choose a third option, noncommitment.*

For the boundary region, two new types of errors are introduced, namely, noncommitment for positives and noncommitment for negatives. They are defined by the following equations, respectively:

$$\begin{aligned} &\mathrm{NPE}\left(\mathrm{NEG}_{(\alpha,\beta)}(X), X\right) = \frac{\left|\mathrm{BND}_{(\alpha,\beta)}(X) \cap X\right|}{\left|\mathrm{BND}_{(\alpha,\beta)}(X)\right|}, \quad \text{for positives,} \\ &\mathrm{NNE}\left(\mathrm{POS}_{(\alpha,\beta)}(X), X\right) = \frac{\left|\mathrm{BND}_{(\alpha,\beta)}(X) \cap X^{c}\right|}{\left|\mathrm{BND}_{(\alpha,\beta)}(X)\right|}, \quad \text{for negatives.} \end{aligned} \tag{5}$$

In contrast, due to allowing certain levels of error, a probabilistic rough set model may have a smaller boundary region than a classical rough set model. The sizes of the three regions are controlled by the pair of thresholds $[\alpha, \beta]$.

2.2. Interval Concept Lattice

Definition 2 (see [1, 5]). Given the formal context $(U, A, R), L(U, A, R)$ is a classic concept lattice based on it. Assume the interval $[\alpha, \beta]$, $0 \le \alpha < \beta \le 1$, we have

α-upper extension M^{α}: $M^{\alpha} = \{x \mid x \in U, |f(x) \cap Y|/|Y| \ge \alpha,\ 0 \le \alpha \le 1\}$

β-lower extension M^{β}: $M^{\beta} = \{x \mid x \in U, |f(x) \cap Y|/|Y| \ge \beta,\ 0 \le \alpha < \beta \le 1\}$

Among them, Y is the intension of the concept. $|Y|$ is the number of elements contained by set Y, namely, cardinal number. M^{α} expresses the objects covered by at least $\alpha \times |Y|$ attributes from Y and M^{β} means the objects covered by at least $\beta \times |Y|$ attributes from Y.

Definition 3 (see [1, 5]). Given the formal context (U, A, R), the ternary ordered pairs $(M^{\alpha}, M^{\beta}, Y)$ are called interval concept. Among them, Y is the intension and describing the concept, M^{α} is the α-upper extension, and M^{β} is the β-lower extension.

Definition 4 (see [1, 5]). We use $L_{\alpha}^{\beta}(U, A, R)$ to express all interval concepts lattice structures in the formal context (U, A, R). If $(M_1^{\alpha}, M_1^{\beta}, Y_1) \le (M_2^{\alpha}, M_2^{\beta}, Y_2) \Longleftrightarrow Y_1 \subseteq Y_2$, "$\le$" is the partial order relation of $L_{\alpha}^{\beta}(U, A, R)$, and all concepts meeting the partial order relation constitute $L_{\alpha}^{\beta}(U, A, R)$ in formal context (U, A, R).

Definition 5. Suppose interval concept lattice $L_{\alpha}^{\beta}(U, C \cup D, R)$ is determined by formal context $U, C \cup D, R$ with the interval parameters α and β. $C = (M^{\alpha}, M^{\beta}, Y)$ is one interval concept in the lattice structure. The upper and lower extensions of interval concept divide U into three regions:

$$\begin{aligned} \mathrm{POS}_{\alpha}^{\beta}(X) &= M^{\beta} = \left\{x \mid x \in U, \frac{|f(x) \cap Y|}{|Y|} \ge \beta\right\}, \\ \mathrm{BND}_{\alpha}^{\beta}(X) &= M^{\alpha} - M^{\beta} = \left\{x \mid x \in U, \alpha \le \frac{|f(x) \cap Y|}{|Y|} \ge \beta\right\}, \\ \mathrm{NEG}_{\alpha}^{\beta}(X) &= U - M^{\beta} = \left\{x \mid x \in U, \frac{|f(x) \cap Y|}{|Y|} \ge \alpha\right\}, \end{aligned} \tag{6}$$

where a subset $X \subseteq U$. If $x \in \mathrm{POS}_{\alpha}^{\beta}(X)$, we could make the acceptance decision on x; if $x \in \mathrm{NEG}_{\alpha}^{\beta}(X)$, we could make the rejection decision on x; otherwise, we make the noncommitment decision.

2.3. Interval Three-Way Decision Space.

Combining the above theoretical basis, in order to obtain decision rules, we innovate the theory of interval three-way decision space, and the following definitions are presented.

Definition 6. When object x belongs to X $(x \in X)$, we make λ_{PP}, λ_{NP}, and λ_{BP} express the cost function of dividing an object into $\mathrm{POS}_{\alpha}^{\beta}(X)$, $\mathrm{NEG}_{\alpha}^{\beta}(X)$, and $\mathrm{BND}_{\alpha}^{\beta}(X)$, respectively. And when $x \notin X$, we make λ_{PP}, λ_{NP}, and λ_{BP} express the cost function of dividing an object into $\mathrm{POS}_{\alpha}^{\beta}(\overline{X})$, $\mathrm{NEG}_{\alpha}^{\beta}(\overline{X})$, and $\mathrm{BND}_{\alpha}^{\beta}(\overline{X})$, respectively.

We suppose $\zeta = \{a_P, a_B, a_N\}$, in which a_P, a_B, and a_N, respectively, express the possible states that the current object belongs to a particular attribute set of three regions. Under different states, the risk cost [22–24] of object x taking different division plan is shown in Table 1.

Generally, the cost function meeting the conditions of $\lambda_{\mathrm{PP}} \le \lambda_{\mathrm{BP}} < \lambda_{\mathrm{NP}}$ and $\lambda_{\mathrm{NN}} \le \lambda_{\mathrm{BN}} < \lambda_{\mathrm{PN}}$, is explained as follows: for an object x belonging to X, the risk cost of dividing it into $\mathrm{POS}_{\alpha}^{\beta}(X)$ is not more than that of dividing it into $\mathrm{BND}_{\alpha}^{\beta}(X)$, and at the same time, the risk costs of the both are less than that of dividing it into $\mathrm{NEG}_{\alpha}^{\beta}(X)$. Similarly, for an object x not belonging to X, the risk cost of dividing it into $\mathrm{NEG}_{\alpha}^{\beta}(\overline{X})$ is not more than that of dividing it into $\mathrm{BND}_{\alpha}^{\beta}(\overline{X})$, and simultaneously, the risk costs of the both are less than that of dividing it into $\mathrm{POS}_{\alpha}^{\beta}(\overline{X})$.

Definition 7. The expectation loss functions [25, 26] of taking a_P, a_B, a_N decision action are expressed by the following equations, respectively:

$$\begin{aligned} R(a_P) &= \lambda_{\mathrm{PP}}\left(\frac{\left|X \cap M^{\beta}\right|}{\left|M^{\beta}\right|}\right) + \lambda_{\mathrm{PN}}\left(\frac{\left|\overline{X} \cap M^{\beta}\right|}{\left|M^{\beta}\right|}\right), \\ R(a_B) &= \lambda_{\mathrm{BP}}\left(\frac{\left|X \cap \left(M^{\alpha} - M^{\beta}\right)\right|}{\left|M^{\alpha} - M^{\beta}\right|}\right) + \lambda_{\mathrm{BN}}\left(\frac{\left|\overline{X} \cap \left(M^{\alpha} - M^{\beta}\right)\right|}{\left|M^{\alpha} - M^{\beta}\right|}\right), \\ R(a_N) &= \lambda_{\mathrm{NP}}\left(\frac{\left|X \cap \left(U - M^{\alpha}\right)\right|}{\left|U - M^{\alpha}\right|}\right) + \lambda_{\mathrm{BN}}\left(\frac{\left|\overline{X} \cap \left(U - M^{\alpha}\right)\right|}{\left|U - M^{\alpha}\right|}\right). \end{aligned} \tag{7}$$

Table 1: Risk cost in different decision-making plans and states.

	a_P	a_B	a_N
X	λ_{PP}	λ_{BP}	λ_{NP}
X^c	λ_{PN}	λ_{BN}	λ_{NN}

Definition 8. Suppose interval concept lattice $L_\alpha^\beta(U, C\cup D, R)$ is determined by the formal context $(U, C\cup D, R)$. $C = (M^\alpha, M^\beta, Y)$ is one interval concept in the lattice structure. We call $\widetilde{C} = (M^\alpha, M^\beta, Y; R(a_P), R(a_B), R(a_N))$ interval three-way decision concept.

Definition 9. Given the formal context $(U, C\cup D, R)$, $\widetilde{L}_\alpha^\beta(U, C\cup D, R)$ is an interval three-way decision space composed by interval three-way decision concepts and parent-children relationships between concepts.

For a new object x, the decision rules obtained by interval three-way decision concept are composed by decision-making action a_i (i can express acceptance action P, noncommitment action B, or rejection action N) and corresponding decision loss R. We denote the decision rules as $J = (a_i, R(a_i))$.

The process of generating interval concept lattice by the formal context is essentially a process of clustering concepts. Moreover, the inclusion relation between the intension of interval concepts determines the father-children relationship in the lattice structure. In our paper, the interval concept lattice generated by the prior formal context constitutes the interval three-way decision space. The divided three decision regions by interval concept in the lattice can be regarded as decision-making rules of a new object, and the parent-children relationships in decision space can decide the next action to reduce the loss of decision-making.

3. Interval Parameter Optimization under Three-Way Decision Space

3.1. Decision Optimization Algorithm under Given Interval Parameters. In the decision-making space, an object x can make different decisions according to multiple interval three-way decision concepts. If we obtain the noncommitment decision rules J, for adventurists, they are more likely to make acceptance or rejection decision J' even though which has a relatively small loss. Here, we suppose the corresponding interval three-way decision concept of J' is $\widetilde{C}'$. When decision-makers take acceptance or rejection decision which has relatively small loss, we can reduce the loss of acceptance or rejection decision according to decision regions divided by the subconcepts of $\widetilde{C}'$. First, we give a decision optimization algorithm [27] as follows (Algorithm 1).

3.2. Three-Way Decision Space Updating with Changing Interval Parameters. The algorithm of finding decision rules of object x based on the fixed interval parameters is given in the previous section, and the updating algorithm of three-way decision space with changing parameters is given in this section. Considering that the interval parameters α and β can change from $[\alpha_0, \beta_0]$ to $[\alpha_1, \beta_1]$ and the relationship between $\alpha_0\,(\beta_0)$ and $\alpha_1\,(\beta_1)$ is indeterminate, therefore there are four kinds of cases to explain the problem of updating the interval three-way decision space. Moreover, the change of interval parameters α and β can first lead to the change of extension of interval three-way decision concepts.

Proposition 3. *When* $\alpha_1 < \alpha_0$, $\beta_1 < \beta_0$, $M^{\alpha_1} \supseteq M^{\alpha_0}$ *and* $M^{\beta_1} \supseteq M^{\beta_0}$.

Proof. Given $M^{\alpha_0} = \{x \mid x \in M, (|f(x)\cap Y|/|Y|) \geq \alpha_0 > \alpha_1\}$, $M^{\beta_0} = \{x \mid x \in M\,(|f(x)\cap Y|/|Y|) \geq \beta_0 > \beta_1\}$, $M^{\alpha_1} = M^{\alpha_0} \cup \{x_1\}$, where $\{x_1 \mid \alpha \leq (|f(x_1)\cap Y|/|Y|) \leq \alpha_1\}$. Similarly, $M^{\beta_1} = M^{\beta_0} \cup \{x_{11}\}$, where $\{x_{11} \mid \beta_1 \leq (|f(x_{11})\cap Y|/|Y|) \leq \beta_0\}$; obviously, $M^{\alpha_1} \supseteq M^{\alpha_0}$ and $M^{\beta_1} \supseteq M^{\beta_0}$. □

Proposition 4. *When* $\alpha_1 > \alpha_0$ *and* $\beta_1 > \beta_0$, $M^{\alpha_1} \subseteq M^{\alpha_0}$ *and* $M^{\beta_1} \subseteq M^{\beta_0}$.

Proof. Given $M^{\alpha_0} = \{x \mid x \in M, \alpha_1 > (|f(x)\cap Y|/|Y|) \geq \alpha_0\}$, $M^{\beta_0} = \{x \mid x \in M, \beta_1 > (|f(x)\cap Y|/|Y|) \geq \beta_0\}$, $M^{\alpha_1} = M^{\alpha_0} - \{x_1\}$, where $\{x_1 \mid \alpha \leq (|f(x_1)\cap Y|/|Y|) \leq \alpha_1\}$. Similarly, $M^{\beta_1} = M^{\beta_0} - \{x_{11}\}$, where $\{x_{11} \mid \beta_0 \leq (|f(x_{11})\cap Y|/|Y|) \leq \beta_1\}$; obviously, $M^{\alpha_1} \subseteq M^{\alpha_0}$ and $M^{\beta_1} \subseteq M^{\beta_0}$.

We assume interval parameters change into $[\alpha_1, \beta_1]$ from $[\alpha_0, \beta_0]$ and there are four kinds of cases: (i) $\alpha_1 > \alpha_0$, (ii) $\alpha_1 > \alpha_0$, (iii) $\beta_1 > \beta_0$, and (iv) $\beta_1 > \beta_0$. The first two cases imply to update the upper extension of the interval three-way decision concepts, namely, $M^{\alpha_0} \longrightarrow M^{\alpha_1}$; the other two cases mean to update the lower extension of the interval three-way decision concepts, namely, $M^{\beta_0} \longrightarrow M^{\beta_1}$. Therefore, the following four functions are given, respectively, to update the interval three-way decision concepts:

(i) Function: DCL1 $(\widetilde{C}, \alpha_0, \alpha_1)$//$\widetilde{C}$ is any node in interval three-way decision concept lattice, and$\alpha_1 > \alpha_0$

DCL1 $(\widetilde{C}, \alpha_0, \alpha_1)$
{
    Ma = $\{\phi\}$
For each x in M^{α_0} of $\widetilde{C}$:
    If $(|f(x)\cap Y|/|Y|) \geq \alpha_1$, then
        Ma = Ma $\cup\, x$
    M^{α_1} = Ma
The corresponding decision loss functions $R_{\alpha_0}^{\beta_0}(a_B)$ and $R_{\alpha_0}^{\beta_0}(a_N)$ are changed into $R_{\alpha_1}^{\beta_0}(a_B)$ and $R_{\alpha_1}^{\beta_0}(a_N)$
}

(ii) Function: DCL2 $(\widetilde{C}, \alpha_0, \alpha_1)$//$\widetilde{C}$ is any node in interval three-way decision concept lattice, and $\alpha_1 > \alpha_0$

DCL2 $(\widetilde{C}, \alpha_0, \alpha_1)$
{
    Ma = M^{α_0}
    For the upper extension Maf of any father node $\widetilde{C}F$ in $\widetilde{C}$:
{
Make maf1 = Maf − M^{α_0}
For $\forall x \in$ maf1:

Input: decision formal context $(U, C \cup D, R)$;
Interval parameters $[\alpha_0, \beta_0]$;
Object x and its condition attribute set $A\,(A \subseteq C)$;
Output: the decision rules of object x.
Step 1: according to the given formal context $(U, C \cup D, R)$, interval three-way decision space $\widetilde{L_{\alpha_0}^{\beta_0}}(U, C \cup D, R)$ will be built by interval three-way decision concepts:

$\widetilde{C} = (M^{\alpha_0}, M^{\beta_0}, Y; R(a_B), R(a_N))$ and parent-children relationship (Definitions 7–9);

Step 2: find intension Y in interval three-way decision concepts $\widetilde{C}$ if $Y - (Y \cap D) = A$, turn to Step 3; else $Y - (Y \cap D) \neq A$ and $(Y - (Y \cap D)) \cap A \neq \phi$, and turn to Step 6;
Step 3: if there are n concepts like $\widetilde{C} = (M^{\alpha_0}, M^{\beta_0}, A; R(a_P), R(a_B), R(a_N))$, n decision rules will be obtained, namely, $J_1, J_2, J_3, \ldots, J_n$, and they can constitute n-dimensional decision space, $JS = \{J_1, J_2, J_3, \ldots, J_n\}$;
Step 4: if there is only J_k meeting $R_{Pk} = \min(R_{P1}, R_{P2}, \ldots, R_{Pn})$, J_k will become the final decision rules of x, namely, making the accept or reject decision of the smallest loss as the final decision;
Step 5: if there are some J_i, J_m meeting $R_{Pi} = R_{Pm} = \min(R_{P1}, R_{P2}, \ldots, R_{Pn})$, search the subconcepts of $\widetilde{C}_i$ and $\widetilde{C}_m$. And on the basis of intension of those subconcepts, add related attribute of x, until get the only decision meeting condition;
Step 6: according to the parent-children relationship in the three-way decision space, search concepts whose intension is Y'. Y' meets the conditions of $A \subseteq Y'$ and $|Y' - A| = 1$, and turn to Step 3; else turn to Step 7;
Step 7: search the concepts whose intension is Y'', Y'' meets the conditions of $A \subseteq Y''$ and $|Y'' - A| = 2$ and turn to Step 3 to continue making decision, until the end node whose intension is $\varnothing$.
End.

Algorithm 1: Decision optimization algorithm under given interval parameters (GPOA).

If $(|f(x) \cap Y|/|Y|) \geq \alpha_1$, then// Y is the intension set of $\widetilde{C}$
Ma = MaUx
}
M^{α_1} = Ma
The corresponding decision loss functions $R_{\alpha_0}^{\beta_0}(\alpha_B)$ and $R_{\alpha_0}^{\beta_0}(\alpha_N)$ are changed into $R_{\alpha_0}^{\beta_0}(\alpha_B)$ and $R_{\alpha_0}^{\beta_0}(\alpha_N)$
}

(iii) Function: DCL3 $(\widetilde{C}, \beta_0, \beta_1)$//$\widetilde{C}$ is any node in interval three-way decision concept lattice, and $\beta_1 > \beta_0$

DCL3 $(\widetilde{C}, \beta_0, \beta_1)$
{
Mb = $\{\varnothing\}$
For each x in M^{β_0} of $\widetilde{C}$:
If $(|f(x) \cap Y|/|Y|) \geq \beta_1$, then
Mb = MbUx
M^{β_1} = Mb
The corresponding decision loss functions $R_{\alpha_0}^{\beta_0}(\alpha_P)$ and $R_{\alpha_0}^{\beta_0}(\alpha_B)$ are changed into $R_{\alpha_0}^{\beta_0}(\alpha_P)$ and $R_{\alpha_0}^{\beta_0}(\alpha_B)$
}

(iv) Function: DCL4 $(\widetilde{C}, \beta_0, \beta_1)$//$\widetilde{C}$ is any node in interval three-way decision concept lattice, and $\beta_0 < \beta_1$

DCL4$(\widetilde{C}, \beta_0, \beta_1)$
{
Mb = M^{β_0}
For the upper extension Mbf of any father node $\widetilde{C}F$ in $\widetilde{C}$:
{
Make mbf1 = Mbf − M^{β_0}
For $\forall x \in$ mbf1:
If $(|f(x) \cap Y|/|Y|) \geq \beta_1$, then// Y is the intension set of $\widetilde{C}$
Mb = MbUx
}
M^{β_1} = Mb
The corresponding decision loss functions $R_{\alpha_0}^{\beta_0}(\alpha_P)$ and $R_{\alpha_0}^{\beta_0}(\alpha_B)$ are changed into $R_{\alpha_0}^{\beta_0}(\alpha_P)$ and $R_{\alpha_0}^{\beta_0}(\alpha_B)$
}

Based on the four functions, when the interval parameters change, we use the method of breadth-first to visit and judge each node from the root node in the interval three-way decision space. According to the four different cases, we can update and adjust the nodes; meanwhile, delete the redundancy concepts and empty concepts from the space structure (Algorithm 2).

On the basis of the original interval three-way decision space, when interval parameters change, the extension and decision loss function of local nodes will correspondingly change in the space. The updating algorithm can help keep or update the extension and decision loss function of each node in the original space. Finally, the new interval three-way decision space is obtained. Compared with reconstruction, the updating algorithm is superior to reconstruction in the aspects of time complexity. □

3.3. Interval Parameter Optimization in Three-Way Decision Space. Through the introduction from the previous two sections, we have mastered the interval three-way decision space updating algorithm. However, the problem of interval parameters (α and β) choice has not been solved yet. It is also an important role to play on decision-making, and the optimal parameters can bring more potential information. Therefore, we will introduce the process of interval parameter optimization as follows.

Input: decision formal context $(U, C \cup D, R)\Delta$;
$\widetilde{L}_{\alpha_0}^{\beta_0}(U, C \cup D, R)$
Interval parameters (α_1, β_1);
Output: $\widetilde{L}_{\alpha_1}^{\beta_1}(U, C \cup D, R)$.
Step 1: $\widetilde{C}_1 = (M^{\alpha_0}, M^{\beta_0}, Y, R_{\alpha_0}^{\beta_0}(a_P), R_{\alpha_0}^{\beta_0}(a_B), R_{\alpha_0}^{\beta_0}(a_N))$ is the root node of $\widetilde{L}_{\alpha_0}^{\beta_0}(U, C \cup D, R)$. If $Y = \varnothing, \widetilde{C}_1$ does not change; if $Y = \varnothing$ and $\alpha_1 > \alpha_0$, call function: DCL1 $(\widetilde{C}, \alpha_0, \alpha_1)$, else call function: DCL2 $(\widetilde{C}, \alpha_0, \alpha_1)$, then update M^{α_0} to M^{α_1}, $R_{\alpha_0}^{\beta_0}(a_B)$ to $R_{\alpha_1}^{\beta_0}(a_B)$, and $R_{\alpha_0}^{\beta_0}(a_N)$ to $R_{\alpha_0}^{\beta_0}(a_N)$; as the same, if $\beta_1 > \beta_0$, call function: DCL3 $(\widetilde{C}, \beta_0, \beta_1)$, else call function: DCL4$(\widetilde{C}, \beta_0, \beta_1)$, then update M^{β_0} to M^{β_1}, $R_{\alpha_0}^{\beta_0}(a_P)$ to $R_{\alpha_0}^{\beta_1}(a_P)$, and $R_{\alpha_0}^{\beta_0}(a_B)$ to $R_{\alpha_0}^{\beta_1}(a_B)$. And $\widetilde{C_1}$ is totally updated to $(M^{\alpha_1}, M^{\beta_1}, Y, R_{\alpha_1}^{\beta_1}(a_P), R_{\alpha_1}^{\beta_1}(a_B), R_{\alpha_1}^{\beta_1}(a_N))$;
Step 2: visit each children nodes $\widetilde{C}_i$ in $\widetilde{C}_1$;
Step 3: suppose$\widetilde{C}_i = (M_i^{\alpha_0}, M_i^{\beta_0}, Y_i R_{\alpha_0}^{'\beta_0}(a_P), R_{\alpha_0}^{'\beta_0}(a_B), R_{\alpha_0}^{'\beta_0}(a_N))$. If $\alpha_1 > \alpha_2$, call function: DCL1 $(\widetilde{C}, \alpha_0, \alpha_1)$, else call function: DCL2 $(\widetilde{C}, \alpha_0, \alpha_1)$, then update $M_i^{\alpha_0}$ to $M_i^{\alpha_1}$, $R_{\alpha_0}^{'\beta_0}(a_B)$ to $R_{\alpha_1}^{'\beta_0}(a_B)$, and $R_{\alpha_1}^{'\beta_0}(a_B)$ to $R_{\alpha_1}^{'\beta_0}(a_N)$; if $M_i^{\alpha_1} = \varnothing$, delete node $\widetilde{C}_i$; otherwise, continue updating the lower extension: if $\beta_1 > \beta_0$, call function: DCL3 $(\widetilde{C}, \beta_0, \beta_1)$, else call function: DCL4 $(\widetilde{C}, \beta_0, \beta_1)$, then update $M_i^{\beta_0}$ to $M_i^{\beta_1}$, $R_{\alpha_0}^{'\beta_0}(a_P)$ to $R_{\alpha_0}^{'\beta_1}(a_P)$, and $R_{\alpha_0}^{'\beta_0}(a_B)$ to $R_{\alpha_0}^{'\beta_1}(a_B)$, and $\widetilde{C}_i$ is totally updated to $(M_i^{\alpha_1}, M_i^{\beta_1}, Y_i, R_{\alpha_1}^{'\beta_1}(a_P), R_{\alpha_1}^{'\beta_1}(a_B), R_{\alpha_1}^{'\beta_1}(a_N))$;
Step 4: for each father node $\widetilde{C}_i' = \widetilde{C}_i \longrightarrow$ Parent in $\widetilde{C}_i$, and $\widetilde{C}_i'(M_i^{\alpha_1}, M_i^{\beta_1}, Y_i, R_{\alpha_1}^{'\beta_1}(a_P), R_{\alpha_1}^{'\beta_1}(a_B), R_{\alpha_1}^{'\beta_1}(a_N))$, if $M_i^{'\alpha_1} = M_i^{\alpha_1}, M_i^{'\beta_1} = M_i^{\beta_1}$, $R_{\alpha_1}^{'\beta_1}(a_P) = R_{\alpha_1}^{'\beta_1}(a_P)$, $R_{\alpha_1}^{'\beta_1}(a_B) = R_{\alpha_1}^{'\beta_1}(a_B)$, and $R_{\alpha_1}^{'\beta_1}(a_N) = R_{\alpha_1}^{'\beta_1}(a_N)$, $\widetilde{C}_i \longrightarrow$ Parent $= \widetilde{C}_i' \longrightarrow$ parent, namely delete $\widetilde{C}_i'$;
Step 5: for each children node of $\widetilde{C}_i, \widetilde{C}_i = \widetilde{C}_i \longrightarrow$ Childrenren, turn to Step3, until visiting the final node in $\widetilde{L}_{\alpha_0}^{\beta_0}(U, C \cup D, R)$;
Step 6: output $\widetilde{L}_{\alpha_1}^{\beta_1}(U, C \cup D, R)$;
End.

Algorithm 2: Interval three-way decision space updating algorithm based on changing parameters (SPDA).

3.3.1. The Basic Idea. According to the given decision formal context, decision rules of various parameters will be obtained through this algorithm. First, we determine the number n of attributes in the formal context and divide α and β, respectively, by the equal step length. Here, we suppose the step length $\lambda = 1/n$, and then α_i is i/n, $(i = 1, 2, 3, \ldots, n)$. Due to having preliminarily researched [3] the values of α, we found that when α is in the median (roughly 0.5), the stability of the lattice structure can be ensured. So first we initialize $\alpha_0 = 1/2$ and $\beta_0 = 1$, build the interval three-way decision space, and then further mine decision rules. When interval parameters change by the equal step, we update the original space and obtain the new concepts, lattice structure, and decision rules. Finally, the optimal decision rules of object x and the best interval parameters are found under those decision rules.

3.3.2. Algorithm Design. The original three-way decision space obtains constantly updating with the respect of interval parameters. Both concept and lattice structure will change, and further, the decision rules of object x will be influenced. Remarkably, the changing of these decision rules is not sudden, but gradually guides to make clear decisions of acceptance or rejection. Therefore, adventurist can make a clear decision, but not noncommitment, which not only saves the cost of time, but also rationally improves the decision-making efficiency. When users make relatively accurate or pleasant decisions, at this time, the values of interval parameters can be considered to be the best parameters in this formal context.

4. Example Analysis

For the ease of understanding and exposition, we set parameter β value to 1 and explore the effect of α on decision rules. We choose ten objects to demonstrate the above algorithms and receive the meaningful decision rules. The following example is about "Green Products Recommendation." Some attributes describing the feature of green products, such as condition attribute set $\{a, b, c, d\}$, where "a" expresses "green packing," "b" expresses "green technology," "c" expresses "green raw materials," and "d" expresses "environmental certification of manufacturer"; decision attributes are "e" and "f," which express that some green products can be recommended to "consumer e" and "consumer f." When a green product has a green package, but without green technology during the production, made of green raw materials and by environmental certification holder, its corresponding condition attribute set is $\{1, 0, 1, 1\}$. Whereafter, the decision formal context is shown in Table 2.

Through simply preprocessing this formal context, we unify the representation of decision attributes and condition attributes. For example, if a green product is of the condition attributes of $\{1, 0, 1, 1\}$, consumer e will not consider purchasing this green product, while consumer f would like to purchase it as object 1 showing. The converted form context is shown in Table 3.

4.1. Model Verification. According to the formal context (U, A, R) as shown in Table 3, there are 6 attributes (A), including 4 condition attributes and 2 decision attributes, and 10 objects (U). We assume the new object x is a "green packing" product, and we aim at recommending the new

Table 2: Form context with decision.

Object	a	b	c	d	Decision
1	1	0	1	1	F
2	0	1	0	0	Ef
3	0	0	1	0	NA
4	0	1	0	1	Ef
5	1	1	1	0	Ef
6	1	0	0	1	ef
7	1	0	1	1	ef
8	0	0	1	1	f
9	1	1	1	0	ef
10	0	1	0	1	f

Table 3: Converted form context.

Object	a	b	c	d	e	f
1	1	0	1	1	0	1
2	0	1	0	0	1	1
3	0	0	1	0	0	0
4	0	1	0	1	1	1
5	1	1	1	0	1	1
6	1	0	0	1	1	1
7	1	0	1	1	1	1
8	0	0	1	1	0	1
9	1	1	1	0	1	1
10	0	1	0	1	0	1

Input: decision formal context $(U, C \cup D, R)$;
Object x and condition attribute set $A\,(A \subseteq C)$;
Output: the decision rules of x;
The interval parameters under optimal decision.
Step 1: determine the number n of attributes in the formal context, and set the step length $\lambda = 1/n$;
Step 2: initialize $\alpha(\alpha_1, \beta)(\alpha_1, \beta)$, $\beta = 1$, and build the interval three-way decision space $\widetilde{L_\alpha^\beta}(U, C \cup D, R)$;
Step 3: put $\widetilde{L_\alpha^\beta}(U, C \cup D, R)$ into Algorithm 1 (GPOA);
Step 4: in the output of Algorithm 1 (GPOA), if the accept loss is 0 and the reject loss is 0, turn to Step 5;
Step 5: make $\alpha = \alpha + \lambda\beta = \beta - \lambda$, and update three-way decision space according to Algorithm 2 (SPDA);
Step 6: turn to Steps 3 and 4;
Step 7: compare these decision rules of x, and output the optimal decision and interval parameters.
End.

Algorithm 3: Interval parameter optimization algorithm in three-way decision space (IPOA).

Table 4: Three-way decision concept by $\alpha = 3/6$ and $\beta = 1$

Concept $\widetilde{C}$	Upper extension M^α	Lower extension M^β	Intension Y	Accept loss $R(a_P)$	Noncommitment loss $R(a_B)$	Reject loss $R(a_N)$
$\widetilde{C}_1$	{12345679}	{5679}	Ae	0	5.5	0
$\widetilde{C}_2$	U	{15679}	Af	0	7.6	0
$\widetilde{C}_3$	{245679}	{59}	abe	0	9	0
$\widetilde{C}_4$	{135679}	{579}	ace	0	4.33	7.5
$\widetilde{C}_5$	{1356789}	{1579}	acf	0	6.67	15
$\widetilde{C}_6$	{1345679}	{67}	ade	0	6.2	5
$\widetilde{C}_7$	{1345678910}	{167}	adf	0	7.67	15
$\widetilde{C}_8$	{123456789}	{59}	$abce$	0	6	0
$\widetilde{C}_9$	U	{59}	$abcf$	0	8.125	0
$\widetilde{C}_{10}$	{1234567910}	ϕ	$abde$	0	6.67	0
$\widetilde{C}_{11}$	{1345678910}	{17}	$acdf$	0	8	15
$\widetilde{C}_{12}$	{13456789}	ϕ	$abcde$	0	6.375	7.5
$\widetilde{C}_{13}$	{1345678910}	ϕ	$abcdf$	0	8.22	15

product to the potential consumers (e and f). On the one hand, our model avoids the waste of information resources by pushing the product information to partial consumers instead of all consumers, and on the other hand, it causes less customer churn than the classical model which is only for precise consumers. Set $\lambda_{PP} = 0, \lambda_{BP} = 9, \lambda_{NP} = 15, \lambda_{PN} = 17, \lambda_{BN} = 2$, and $\lambda_{NN} = 0$. According to the previous three steps of Algorithm 3, we can obtain the initial interval three-way decision concepts as shown in Table 4.

Due to the new object x with "a" condition attribute, we only need observing the concepts that contain condition attribute "a" and the rest of interval three-way decision concepts will be omitted on account of length limits.

First, we build the lattice structure as Figure 1 shows based on three-way decision concepts in Table 4, where $\widetilde{C}_{\{a,b,c,d,e,f\}}$ refers to the concept with $\{a,b,c,d,e,f\}$ intension and $\varnothing$ extension, and $\widetilde{C}_{\varnothing}$ represents the concept with $\varnothing$ intension and {12345678910} extension. Through the lattice structure, we can obviously find the parent-child relationships between concepts. For example, $\widetilde{C}_{12} \longrightarrow \widetilde{C}_{8} \longrightarrow \widetilde{C}_{3}$; their intensions have the relationship of inclusion. The smaller the intension is (e.g., $\widetilde{C}_{3}$), the higher the possibility of loss of noncommitment is. Not until we pointed out $\widetilde{C}_{12}$ did the acceptance decision obtained.

From the perspective of decision-making, when $\alpha = 3/6$ and $\beta = 1$, according to the decision loss of $\widetilde{C}_1$ and $\widetilde{C}_2$, we can see that noncommitment will bring about a certain loss. However, it will be a loss to make the decision of acceptance or rejection, corresponding to step 4 of Algorithm 3. Actually, these decision rules in this case are not meaningful because they still do not send a clear message of acceptance or rejection. For this new "green packing" product, we could not recommend it to consumer e or f yet. Next, we run step 5 of Algorithm 3 and will obtain the result as Table 5 shows.

In addition, to obtain the helpful decision-making in the case $\alpha = 3/6$ and $\beta = 1$, we could add condition attributes to the object x which may be given over a period of observing. For example, when adding condition attribute "c" to the object x, according to the decision loss of $\widetilde{C}_4$ and $\widetilde{C}_5$ we can draw a conclusion: the loss value of rejecting to recommend the object x for "consumer e" is 7.5; the loss value of rejecting to recommend the object x for "consumer f" is 15. As a consequence, the object x who includes condition attributes $\{ac\}$ should be recommended to "consumer f." Similarly, after adding condition attribute "d," the object x should be still recommended to "consumer f." And thus when we face with a new object x which is "green packing" product, it is hard to immediately make a clear decision on "product recommendation." Instead, we need to spend some time on discovering more product information, which is beneficial to make a clear decision. It is definitely the implication of noncommitment decision.

When $\alpha = 4/6$ and $\beta = 1$, it is easy to see the number of concepts is more than that under $\alpha = 3/6$ and $\beta = 1$. The parent-child relationships between concepts are obtained in Figure 2. Similarly, we find $\widetilde{C}_{14} \longrightarrow \widetilde{C}_{9} \longrightarrow \widetilde{C}_{3}$ and make the acceptance decision from $\widetilde{C}_9$. Compared to the case $\alpha = 3/6$ and $\beta = 1$, the efficiency of decision-making is obviously improved.

According to the decision loss of $\widetilde{C}_1$, we can find that "consumer e" accepts or rejects the new object x ("green packing" product) will not bring about any loss. Nevertheless, we make decision of noncommitment on "green packing" product will bring about a certain loss. Therefore, from $\widetilde{C}_1$, this decision rule is of no guiding significance. From $\widetilde{C}_2$, it is obviously willing for "consumer f" to accept the "green packing" product. In conclusion, when $\alpha = 4/6$ and $\beta = 1$, there is no need to add condition attribute to promote decision-making, and some practical significance decisions can be directly obtained by the three-way decision concepts. Interestingly, compared to the results under the condition $\alpha = 3/6$ and $\beta = 1$, we indicate that in the case $\alpha = 3/6$ and $\beta = 1$, the new object will be more efficiently recommended to "consumer f." Therefore, our algorithm will end. To highlight the validity of our model, we will give the case $\alpha = 5/6$ and $\beta = 1$ and $\alpha = 6/6$ and $\beta = 1$ as Tables 6 and 7 show.

When $\alpha = 5/6$ and $\beta = 1$, although the three-way decision concept is more clear than before, some necessary decision concepts like whose intension is $\{abdf\}$ are missing. It is likely to cause the customer churn because the recommendation is too accurate. And the parent-child relationships between concepts are shown in Figure 3. There are four concepts mattering the condition attribute "a," namely, $\widetilde{C}_1, \widetilde{C}_2, \widetilde{C}_3$, and $\widetilde{C}_4$. And the number of concepts will start to decrease with respect to parameter α. From the perspective of optimizing parameters, we will consider $\alpha = 4/6$ as the optimal parameter of interval concept lattice. According to $\widetilde{C}_1$, we easily draw the conclusion that the "green packing" product should be recommended to "consumer f." Furthermore, when the green product is with attribute "c" or "d," the result is the same (being recommended to "consumer f"). Even although recommending the green product which is with attributes "ac" to "consumer e" will not result in loss, rejecting to recommend to "consumer f" could bring about more loss ($12.5 > 6.43$).

Similarly, when $\alpha = 6/6$ and $\beta = 1$, we can obtain the lattice structure (three-way decision space) as Figure 4 shows. There are obvious parent-child relationships, $\widetilde{C}_4 \longrightarrow \widetilde{C}_2 \longrightarrow \widetilde{C}_1$ and $\widetilde{C}_4 \longrightarrow \widetilde{C}_3 \longrightarrow \widetilde{C}_1$; meanwhile, these decision concepts all imply that the "green packing" product is definitely recommended to "consumer f." The case is the same as the classical model where the objects should completely meet the condition attributes from Y. Therefore, some potential consumers may be ignored.

4.2. Model Comparison and Instruction. To further illustrate the model, the loss value of each decision under variable parameters is given in below trend charts. Here, we assume that the new object x only is of condition attribute "a." When parameter α changes by the equal step, the trend chart of making "e-decision" (acceptance, noncommitment, or rejection of "consumer e") on the new object x is shown in Figure 5(a), where the horizontal axis shows the loss value and the vertical axis expresses the value of parameter α.

From Figure 5(a), we find that the loss value of accepting or rejecting the "green product" for "consumer e" is 0 and that of noncommitment decision is 5.5, based on the three-way decision concept whose intension set is $\{ae\}$ in the condition from $\alpha = 1/6$ to $\alpha = 4/6$. These decision rules are obviously of no practical significance. From the perspective

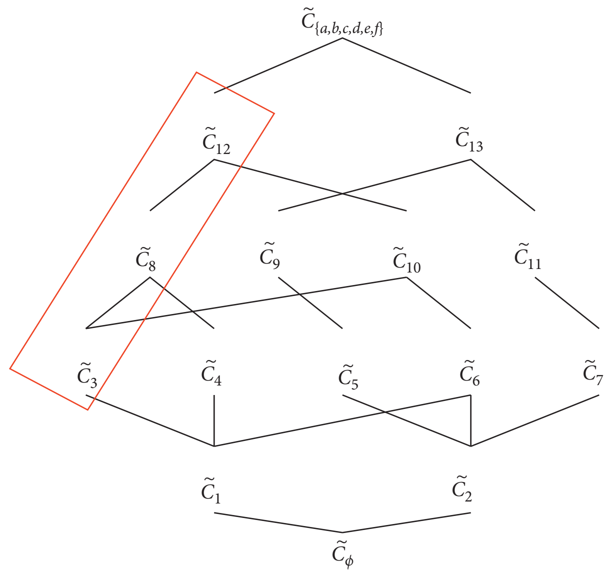

FIGURE 1: Lattice structure of $\alpha = 3/6$ and $\beta = 1$.

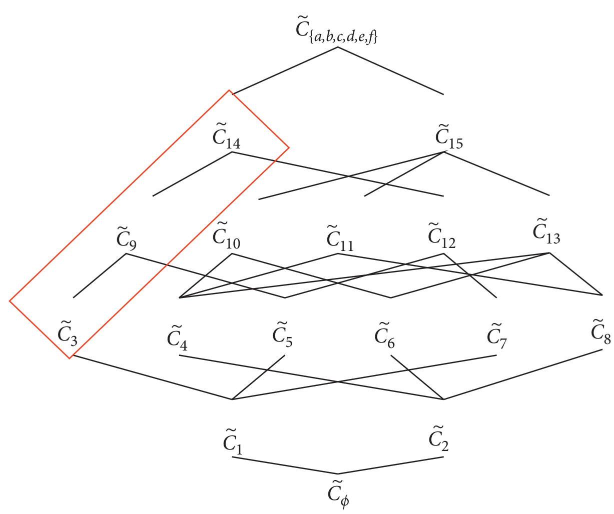

FIGURE 2: Lattice structure of $\alpha = 4/6$ and $\beta = 1$.

TABLE 5: Three-way decision concept by $\alpha = 4/6$ and $\beta = 1$.

Concept $\tilde{C}$	Upper extension M^α	Lower extension M^β	Intension Y	Accept loss $R(a_P)$	Noncommitment loss $R(a_B)$	Reject loss $R(a_N)$
$\tilde{C}_1$	{12345679}	{5679}	*ae*	0	5.5	0
$\tilde{C}_2$	{15679}	{15679}	*af*	0	0	12
$\tilde{C}_3$	{245679}	{59}	*abe*	0	9	0
$\tilde{C}_4$	{124567910}	{58}	*abf*	0	9	7.5
$\tilde{C}_5$	{135679}	{579}	*ace*	0	4.33	7.5
$\tilde{C}_6$	{1356789}	{1579}	*acf*	0	6.67	15
$\tilde{C}_7$	{1345679}	{67}	*ade*	0	6.2	5
$\tilde{C}_8$	{1345678910}	{167}	*adf*	0	7.83	15
$\tilde{C}_9$	{579}	{5}	*abce*	0	5	6.43
$\tilde{C}_{10}$	{1579}	{59}	*abcf*	0	7.83	12.5
$\tilde{C}_{11}$	{14567910}	ϕ	*abdf*	0	9	10
$\tilde{C}_{12}$	{135679}	{7}	*acde*	0	6.2	7.5
$\tilde{C}_{13}$	{1356789}	{17}	*acdf*	0	7.6	15
$\tilde{C}_{14}$	{579}	ϕ	*abcde*	0	5	6.43
$\tilde{C}_{15}$	{1579}	ϕ	*abcdf*	0	7.83	12.5

Table 6: Three-way decision concept by $\alpha = 5/6$ and $\beta = 1$.

Concept $\widetilde{C}$	Upper extension M^{α}	Lower extension M^{β}	Intension Y	Accept loss $R(a_P)$	Noncommitment loss $R(a_B)$	Reject loss $R(a_N)$
$\widetilde{C}_1$	{15679}	{15679}	Af	0	0	12
$\widetilde{C}_2$	{579}	{579}	Ace	0	0	6.43
$\widetilde{C}_3$	{1579}	{1579}	acf	0	0	12.5
$\widetilde{C}_4$	{167}	{167}	adf	0	0	12.86

Table 7: Three-way decision concept by $\alpha = 6/6$ and $\beta = 1$.

Concept $\widetilde{C}$	Upper extension M^{α}	Lower extension M^{β}	Intension Y	Accept loss $R(a_P)$	Noncommitment loss $R(a_B)$	Reject loss $R(a_N)$
$\widetilde{C}_1$	{15679}	{15679}	af	0	0	12
$\widetilde{C}_2$	{1579}	{1579}	acf	0	0	12.5
$\widetilde{C}_3$	{167}	{167}	adf	0	0	12.86
$\widetilde{C}_4$	{17}	{17}	$acdf$	0	0	13.125

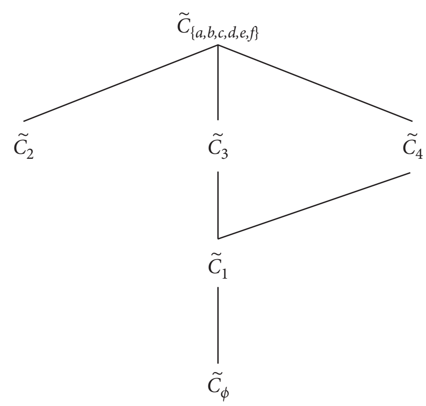

Figure 3: Lattice structure of $\alpha = 5/6$ and $\beta = 1$.

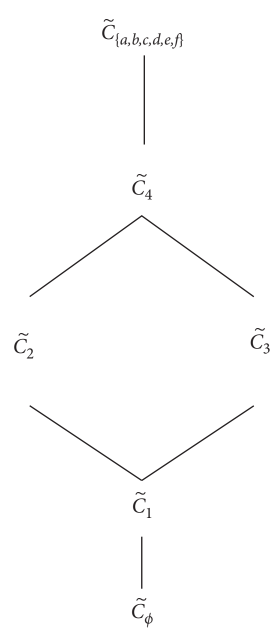

Figure 4: Lattice structure of $\alpha = 6/6$ and $\beta = 1$.

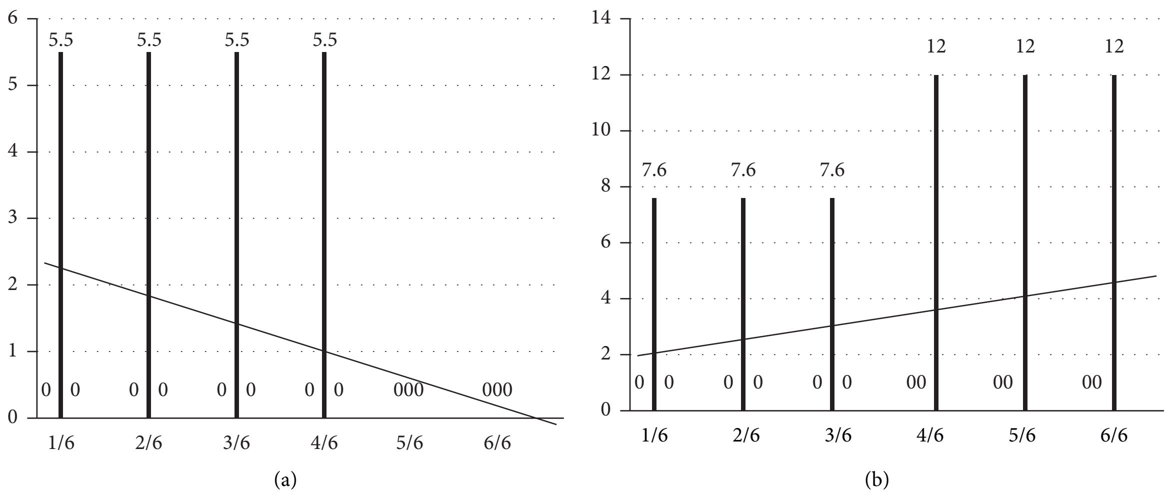

Figure 5: Trend chart of decision with changing α: (a) e-decision and (b) f-decision.

of the parameter α, when α is less than the value (roughly 0.6), it results in the slight effect of uncertain information on the decision-making. In other words, the information obtained from the condition of small α is too vague to make a clear decision of acceptance or rejection. On the intension set $\{ae\}$ side, it is not advisable for decision-maker to seek the three-way decision concept whose intension set is $\{ae\}$. According to the original form context, recommending the object x with condition attribute "a" to "consumer e" itself is ambiguous. Therefore, from Figure 5(a) we cannot obtain the meaningful implication of decision-making and the trend of line is descending.

Meanwhile, the trend chart of making "f-decision" (acceptance, noncommitment, or rejection of "consumer f") on the new object x is shown in Figure 5(b). According to Figure 5(b), when α is from 1/6 to $\alpha = 3/6$, "consumer f" still cannot make a clear decision of accepting or rejecting the object x. Nevertheless, when $\alpha = 4/6$, the decision starts to be clear and we would like to recommend the object x to "consumer f" because the loss value of rejection is 12 and acceptance is 0. When α continues increasing, we find the loss value of rejection is still 12. Obviously, we should choose "acceptance" (recommending the object x to "consumer f"). The trend of line is ascending and it means that the greater the parameter α is, the clearer the decision-making is.

From both figures, it is easy to see that the loss function of three-way decision concept has a shift at the condition of roughly $\alpha = 4/6$. In other words from this condition, the decision-making starts to be explicit, and from the perspective of parameter optimization, we could consider the parameter α at its most optimal. By the way, when we further consider adding the condition attribute of the new object x, it also contributes to the efficiency of decision-making.

5. Conclusion

The interval parameters $[\alpha, \beta]$ in the concept lattice affect the concepts and decision space generated by decision formal context. The article is mainly taking the decision loss function values as the rules of decision-making based on three-way decision space. With the change of interval parameters, different three-way decision spaces are obtained. In addition, the decision rules will be explored from the three-way decision concept in this space. It is obvious that the decision rules of the same object x are different under different interval parameters. However, there definitely exist the optimal interval parameters to make the decision rules more sufficient and clear. Until making the decision of acceptance or rejection, according to the example, we can consider that the best interval parameters (roughly more than 0.6) are obtained. The conclusion of the optimal parameters provided by our paper is the same as the best parameters previously obtained by the parameter optimization model [5] of interval concept lattice. Although there are different ideas for dealing with the problem of interval parameter optimization, finally roughly similar conclusions were drawn, which has provided a reliable basis for selecting parameter problem of interval concept lattice application. Subsequently, we will continue the study of optimization problem of interval parameters under the setting of green supply chain investment.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (nos. 71971113 and 71571100), Anhui Provincial Colleges and Universities Outstanding Talents

Cultivation Project (no. gxgnfx2019042), and Natural Science Research Project of Colleges and Universities in Anhui Province (no. KJ2019A0701).

References

[1] B. X. Liu and C. Y. Zhang, "A new concept lattice structure: interval concept lattice," *Computer Science*, vol. 39, no. 8, pp. 273–277, 2012.

[2] R. Wille, "Restructuring lattice theory: an approach based on hierarchies of concepts,," in *Ordered Sets*, I. Rival, Ed., pp. 445–470, Reidel, Dordrecht-Boston, UK, 1982.

[3] J. Yao, Y. Yao, and W. Ziarko, "Probabilistic rough sets: approximations, decision-makings, and applications," *International Journal of Approximate Reasoning*, vol. 49, no. 2, pp. 253-254, 2008.

[4] Y. Yao, "Probabilistic rough set approximations," *International Journal of Approximate Reasoning*, vol. 49, no. 2, pp. 255–271, 2008.

[5] M. X. Li, C. Y. Zhang, L. Y. Wang, and B. X. Liu, "Parameters optimization and interval concept lattice update with change of parameters," *ICIC Express Letters*, vol. 10, no. 2, pp. 339–434, 2016.

[6] Y. Y. Yao and S. K. M. Wong, "A decision theoretic framework for approximating concepts," *International Journal of Man-Machine Studies*, vol. 37, no. 6, pp. 793–809, 1992.

[7] Y. Y. Yao, "Decision-theoretical Rough set models," in *Proceedings of Rough Sets Knowledge Technology (RSKT'07)*, pp. 1–12, Toronto, Canada, May 2007.

[8] D. Liu, T. Li, and D. Ruan, "Probabilistic model criteria with decision-theoretic rough sets," *Information Sciences*, vol. 181, no. 17, pp. 3709–3722, 2011.

[9] G. L. Tang, F. Chiclana, and P. Liu, "A decision-theoretic rough set model with q-rung orthopair fuzzy information and its application in stock investment evaluation," *Applied Soft Computing Journal*, vol. 91, pp. 2–15, 2020.

[10] C. Zhang, D. Li, and J. Liang, "Multi-granularity three-way decisions with adjustable hesitant fuzzy linguistic multigranulation decision-theoretic rough sets over two universes," *Information Sciences*, vol. 507, pp. 665–683, 2020.

[11] Y. Y. Yao, "Three-way decision: an interpretation of rules in rough set theory," in *Proceeding of 4th International Conference on Rough Sets and Knowledge Technology*, pp. 642–649, Gold Coast, Australia, July 2009.

[12] Y. Y. Yao, "An outline of a theory of three-way decisions," in *Proceedings of the 8th International RSCTC Conference, LNCS (LNAI)*, pp. 1–17, Chengdu, China, August 2012.

[13] Y. Yao, "Three-way decisions with probabilistic rough sets," *Information Sciences*, vol. 180, no. 3, pp. 341–353, 2010.

[14] Y. Yao, "The superiority of three-way decisions in probabilistic rough set models," *Information Sciences*, vol. 181, no. 6, pp. 1080–1096, 2011.

[15] Y. Yao, "Three-way decisions and cognitive computing," *Cognitive Computation*, vol. 8, no. 4, pp. 543–554, 2016.

[16] T. X. Wang, H. X. Li, X. Z. Zhou, B. Huang, and H. B. Zhu, "A prospect theory-based three-way decision model," *Knowledge-Based Systems*, vol. 203, pp. 106–129, 2020.

[17] D. Liang, M. Wang, Z. Xu, and D. Liu, "Risk appetite dual hesitant fuzzy three-way decisions with TODIM," *Information Sciences*, vol. 507, pp. 585–605, 2020.

[18] C. Jiang, D. Guo, Y. Duan, and Y. Liu, "Strategy selection under entropy measures in movement-based three-way decision," *International Journal of Approximate Reasoning*, vol. 119, pp. 280–291, 2020.

[19] L. Wei, L. Liu, J. Qi, and T. Qian, "Rules acquisition of formal decision contexts based on three-way concept lattices," *Information Sciences*, vol. 516, pp. 529–544, 2020.

[20] Z. Y. Zhang, D. X. Fu, and Q. Zhou, "Optimal decisions of a green supply chain under the joint action of fairness preference and subsidy to the manufacturer," *Discrete Dynamics in Nature and Society*, vol. 2020, Article ID 9610503, 18 pages, 2020.

[21] V. Albino, A. Balice, and R. M. Dangelico, "Environmental strategies and green product development: an overview on sustainability-driven companies," *Business Strategy and the Environment*, vol. 18, no. 2, pp. 83–96, 2009.

[22] H. Li, L. Zhang, X. Zhou, and B. Huang, "Cost-sensitive sequential three-way decision modeling using a deep neural network," *International Journal of Approximate Reasoning*, vol. 85, pp. 68–78, 2017.

[23] H. Li and X. Zhou, "Risk decision making based on decision-theoretic rough set: a three-way view decision model," *International Journal of Computational Intelligence Systems*, vol. 4, no. 1, pp. 1–11, 2011.

[24] H. Li, L. Zhang, B. Huang, and X. Zhou, "Sequential three-way decision and granulation for cost-sensitive face recognition," *Knowledge-Based Systems*, vol. 91, pp. 241–251, 2016.

[25] X. Y. Jia, W. Li, L. Shang et al., "An adaptive learning parameters algorithm in three-way decision-theoretic rough set model," *Acta Electronica Sinica*, vol. 39, no. 11, pp. 2520–2525, 2011.

[26] X. Y. Jia, W. Li, and L. Shang, "A simulated annealing algorithm for learning thresholds in three-way decision-theoretic rough set model," *Journal of Chinese Computer System*, vol. 34, no. 11, pp. 2604–2606, 2013.

[27] L. Y. Wang, C. Y. Zhang, and B. X. Liu, "Dynamic strategy regulation model of three-way decisions based on interval concept lattice and its application," *Computer Engineering and Applications*, vol. 52, no. 24, pp. 80–84, 2016.