Hindawi
Discrete Dynamics in Nature and Society
Volume 2021, Article ID 2492430, 17 pages
https://doi.org/10.1155/2021/2492430

Research Article

Availability Optimization of Multicomponent Products with Economic Dependence under Two-Dimensional Warranty

Enzhi Dong,[1] Zhonghua Cheng,[1] Rongcai Wang,[1] and Xiaona Zhang[2]

[1]*Shijiazhuang Campus of Army Engineering University of PLA, Shijiazhuang 050003, China*
[2]*Army Research Institute, Beijing, China*

Correspondence should be addressed to Zhonghua Cheng; asd3v36@163.com

Received 5 September 2021; Revised 26 October 2021; Accepted 29 October 2021; Published 23 November 2021

Academic Editor: Daqing Gong

For multicomponent products, the maintenance of every component separately will increase the downtime and reduce the availability of products during the warranty period. To solve this problem, the economic dependence between the components is considered in this paper. Firstly, a single-component two-dimensional (2D) preventive maintenance (PM) availability model is established, and the simulated annealing algorithm is adopted to calculate the optimal 2D PM interval to achieve the maximal availability of any single component. Then, to ensure that the warranty cost of each component does not exceed the budget, the PM benchmark interval is introduced, and the PM work is optimized following the method of grouping maintenance. Based on this, the 2D preventive grouping maintenance availability model of multicomponent products is established. Finally, an example is given to verify the proposed method, and the results indicate that the proposed method increases the availability of multicomponent products during the 2D warranty period.

1. Introduction

Competition is ubiquitous in the market. To gain market share, manufacturers provide product warranties besides high-quality products [1, 2]. Warranty means that the manufacturer should handle any problems encountered by the user and product failures during the warranty period of the product. According to the dimension of warranty period, warranty can be divided into one-dimensional warranty, two-dimensional (2D) warranty, and even multidimensional warranty. 2D warranty refers to the warranty period determined by time and usage, and it is usually used for automobiles, construction machinery, and other durable goods. For example, a car has a warranty of 5 years or 20,000 km [3, 4]. Figure 1 shows the 2D warranty area, where T_w and U_w refer to the warranty period determined by time and usage, respectively [5].

Most of the 2D warranty products consist of multiple components, and there is economic dependence between the components. Economic dependence means that grouping maintenance has a lower or higher cost than the sum of individual maintenance costs and it can make the product have higher or lower availability during the warranty period. For multicomponent products, if the components are maintained individually during the warranty period, frequent downtime will increase the maintenance cost and reduce the availability of the product. Therefore, it is common for manufacturers and consumers to choose an appropriate grouping maintenance method to reduce the warranty cost and improve availability. Most of the current research focuses on the reduction of warranty cost, and the existing models do not support the optimization of product availability during the warranty period.

This paper is organized as follows. In Section 2, the background of the study is introduced, and the related references are reviewed. In Section 3, the mathematical models of the 2D failure rate function and imperfect maintenance are established. In Section 4, the 2D warranty

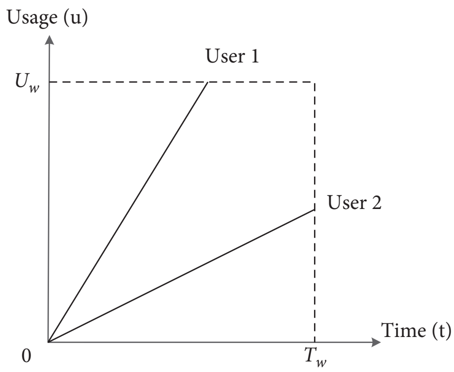


Figure 1: 2D warranty process.

product availability model is built based on the 2D imperfect preventive maintenance (PM), and the method of grouping maintenance is given. Section 5 shows the effectiveness and accuracy of the model through numerical examples. Finally, Section 6 concludes this paper.

The model notations in this paper are as follows:

(T_{0j}, U_{0j}): 2D imperfect PM interval of each component, where T_{0j} is the time interval of the jth component, and U_{0j} is the usage interval of the jth component

(T_w, U_w): 2D warranty period of the product, where T_w is the time warranty period and U_w is the usage warranty period

T_{fj}: time consumption of corrective maintenance (CM) of the jth component

T_{pj}: time consumption of imperfect PM of the jth component

T_{ft}: total time consumption of CM during the warranty period

T_{pt}: total time consumption of imperfect PM during the warranty period

T_{Dj}: PM preparation time of the jth component

$r, G(r), g(r)$: the product utilization rate, cumulative distribution function, and probability density function of utilization rate, respectively

r_σ, r_τ: the lowest and highest utilization rate in the same batch of products

r_{0j}, r_w: the ratio of PM interval U_{0j} to T_{0j} and the ratio of warranty period U_w to T_w

$\lambda(t, r)$: the failure rate function of the product

θ: the improvement factor of imperfect PM

$E[A_w(T_w, U_w)]$: the operational availability of the product within the 2D warranty period (T_w, U_w) when the PM interval is (T_{0j}, U_{0j})

C_{Dj}: PM preparation cost of the jth component

C_{fj}: cost consumption for CM of the jth component

C_{pj}: cost consumption for imperfect PM of the jth component

C_{ft}: total cost consumption of CM during the warranty period

C_{pt}: total cost consumption of imperfect PM during the warranty period

2. Related Reference Review

There have been many studies on 2D warranty. Most of the studies follow the same process, which is shown in Figure 2. First, the failure rate function of the research object is determined; then, the two-dimensional warranty area is determined and divided into several subareas; finally, the maintenance method of each subarea is determined, and the optimization model with the objectives of warranty cost [6], availability [7], cost-effectiveness ratio [8] is established. The decision variables of the optimization model include warranty period, PM interval, PM times or degree, etc.

2.1. Review of the Failure Rate Function. Determining the failure rate function of a product is often the first step in warranty modeling. The failure rate function needs to be obtained through distribution fitting and parameter estimation based on failure data. At present, there are three main methods to construct the 2D failure rate function: univariate method [3, 9], bivariate method [10, 11], and composite scale method [12]. The univariate method usually regards the usage as a random function of age, so it can effectively classify 2D problems into one-dimensional problems [13]. by using the binary data including failure time and usage, the bivariate method directly fits the bivariate distribution function $F(t, x)$ to model the first failure time. This method is more applicable when the age and usage are relatively weak and only limited warranty data can be obtained. The composite scale method combines the two scales of age and usage to define a composite scale, and it models the product failure as a counting process based on the composite scale. The comparison of the above methods indicates that the modeling process of the bivariate method and composite scale method is complicated. Also, the univariate method assumes that the user's usage rate remains unchanged, which is out of practice. Therefore, the above methods have some limitations. AFT model is a method to construct two-dimensional failure laws of products based on accelerated life testing, and it can easily incorporate segment usage into the failure rate function model. Therefore, it makes up for the ideal assumption and simple modeling process of the univariate method [14]. As for 2D warranty, Iskandar and Husniah [15], Tong et al. [16], and Li et al. [17] adopted AFT models to describe the effect of user use on the first failure time of a product. In this paper, the AFT method is used to construct the model.

2.2. Review of Maintenance Methods. Maintenance is the concrete implementation of the warranty work, and the warranty effect is reflected by the maintenance effect. Determining an appropriate maintenance method is the core of warranty service. A reasonable maintenance method plays an important role in reducing the manufacturer's warranty

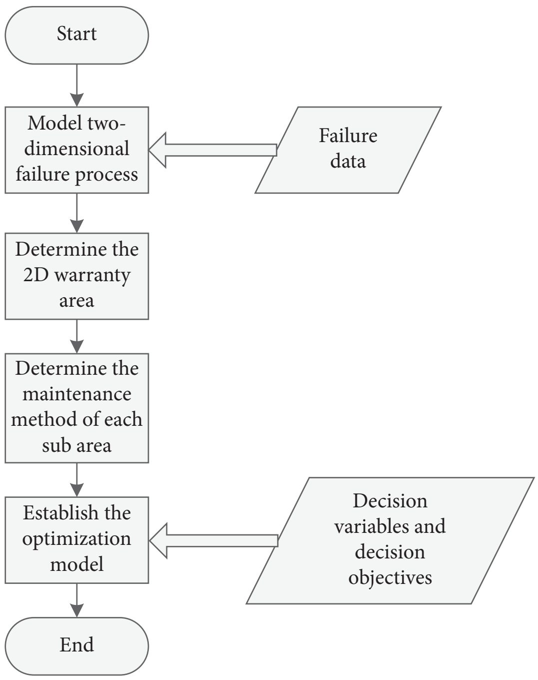


Figure 2: The 2D warranty modeling process.

cost, expanding the profit space, and improving consumer satisfaction. According to the sequence of maintenance and failure, maintenance can be divided into PM and CM. Because PM can avoid the failure with serious consequences to reduce the warranty cost and improve consumer satisfaction, it attracts much attention. References [18, 19] attempt to implement PM during the warranty period.

According to the degree of maintenance, maintenance can be divided into three types: perfect maintenance [20], imperfect maintenance [21, 22], and minimal maintenance [23]. Perfect maintenance repairs the product as new, while minimal maintenance restores the product to the failure rate level before failure. In the current engineering practice, especially after PM, the product is usually between repairing as old and repairing as new, i.e., imperfect maintenance. There are two main methods to describe the impact of imperfect PM on failure rate: failure reduction method and virtual age method. The failure reduction method expresses the imperfect PM effect as a direct decrease in the failure rate, and then the system decline rate is the same as that before maintenance, that is,

$$\lambda_{T_i^+} = \lambda_{T_i^-} - S(i, T_1, \ldots, T_i), \quad \forall i \geq 1, \tag{1}$$

where T_i is the time of imperfect PM and $S(i, T_1, \ldots, T_i)$ is the decline degree of the failure rate.

This formula indicates that the influence of imperfect PM on failure rate is related to previous maintenance. In the virtual age method, the effect of imperfect PM is expressed as the decrease of effective age of the item, That is, after imperfect PM, the length of service of the component is reduced for a period of time.

$$\lambda_t = \lambda\left(t - \sum_{i=1}^{N_t} S(i, T_1, \ldots, T_i)\right), \quad \forall i \geq 1, \tag{2}$$

where $S(i, T_1, \ldots, T_i)$ is the effective age reduction of the components after the ith imperfect PM and N_t is the number of imperfect PM of the components during 0-t.

In [24], the impact of customers' nonpunctuality on the optimization of PM strategy and the resulting warranty costs are studied. Based on this, a non-real-time imperfect PM policy was proposed, which allows customers to advance or delay scheduled PM activities within the allowable range. Dai et al. [25] jointly optimized the number of PM and the corresponding PM level. This study suggested that the PM cost should be shared by the manufacturer and the customer in proportion so that the manufacturer's warranty cost can be minimized. Wang et al. [26] studied the PM problem of series multicomponent systems where each component has a 2D warranty period. The PM intervals of the components were combined to minimize the warranty cost of the multicomponent system during the warranty period, and the optimal PM interval was found. The study also considered the situation of PM by different mechanisms and investigated the optimal task allocation scheme. Similarly, there are many references on imperfect PM, but most of the warranty strategies are one-dimensional, and few consider the economic dependence between multicomponent.

2.3. Review of 2D Warranty. Wang and Xie [14] presented a comprehensive overview of 2D warranty policies, two-dimensional warranty cost modeling methods, and some other interesting topics. References [27–32] explored the 2D imperfect maintenance strategies aiming at the lowest warranty cost. Peng et al. [33] developed a stochastic dynamic maintenance model, and the study suggested that stochastic and dynamic utilization rate directly affects the 2D PM decision. Reference [34] developed a value-in-use risk method for dual channel (online and offline) manufacturers and optimized 2D warranty policy and pricing at the same time. By fully considering the heterogeneity of warranty service objects, references [35–38] tailored 2D warranty services for different customers. References [39–41] used PM measures in 2D warranty service to reduce warranty cost and improve availability. Using the historical claim data as the basis of modeling, the study obtained much key information. References [42–44] studied the data-driven 2D warranty decision model.

Most of the above studies take the lowest warranty cost as the decision-making goal, but they do not consider the availability of products. Besides, a few studies consider the availability of products. Su and Cheng [45] studied the optimal 2D PM strategy of equipment. Meanwhile, they considered the constraint of equipment availability during the warranty period, that is, the availability should be greater than or equal to a minimum value. A similar study is available [8]. These studies all aim at the lowest 2D warranty cost while taking into account product availability. However, in practice, the downtime of products could have serious economic, safety, or task consequences, and availability should be the primary consideration. For the 2D PM modeling of equipment such as weapons systems, the goal is to maximize the operational availability under some constraints on maintenance cost.

The extended warranty is the continuation of the basic warranty. The extended warranty service occurs after the end of the basic warranty, and consumers can decide whether to purchase this service. Su and Wang [46] considered the time when customers purchase a 2D extended warranty and studied the optimization of imperfect PM of repairable components. Reference [38] mainly studied 2D extended warranty pricing to maximize the expected profit of dealers in the product life cycle, and comprehensive factors were considered in the study, including product price fluctuation, repair learning characteristics, production scale effect, market demand fluctuation, etc. He et al. [47] established a 2D extended warranty cost model based on product failure process according to different utilization rates of consumers, purchase time of extended warranty, and PM options. Based on this model, the isoline of the win-win area and win-win extended warranty interval was obtained.

It can be seen that the current research on 2D warranty is more and more concentrated on the interests of consumers and the diversity of products and gradually highlights the importance of warranty data. Meanwhile, more and more researchers pay attention to PM because it can prevent failure or serious consequences of failure and reduce the loss caused by failure shutdown. The research on 2D extended warranty is increasing, and more and more achievements are achieved. However, most of these studies regard the research object as a single component, and the dependence between multicomponent is less studied. In engineering practice, multicomponent systems are extensively used, so it is urgent to study the 2D warranty with consideration of the dependence between multiple components.

2.4. Review of Economic Dependence Research. The important components of high-grade durable products are usually guaranteed separately. The combination of PM work of each component is conducive to reducing warranty cost and downtime and improving product availability. Maintenance optimization of a multicomponent product relies on two types of dependence. The first type of dependence is the dependence among components, such as economic dependence, stochastic dependence, and structural dependence. Many studies have been conducted on economic dependence. Zhou et al. [48] proposed a maintenance optimization method for multistate series and parallel systems, which considers the inspection intervals of economic dependence and state dependence. For a two-component system with stochastic and economic dependence, Do et al. [49] proposed a condition-based maintenance (CBM) strategy model. Dao et al. [50] explored the formulation of a selective maintenance strategy for multistate series and parallel systems with economic components. Based on this, decision-makers can select different components for maintenance to reduce maintenance cost, according to the maintenance objectives, availability of resources, maintenance time, and cost of each component. Considering the economic dependence between wind turbine components, Su and Chen [51] established a long-term average cost rate model under the CBM strategy based on renewal process theory and analyzed the optimal detection cycle of a multicomponent system. It can be seen that in the warranty research of complex multicomponent systems, the economic dependence between components is concerned by researchers, and the combination strategies of maintenance work are investigated from different aspects. However, the economic dependence of multicomponent systems in the 2D warranty strategy has not been studied. 2D warranty products are usually complex products containing many multicomponent systems. In the series multicomponent system, every PM will fail the equipment. The more the number of single components in series, the more the number of system failures and the longer the downtime. To reduce the downtime caused by the PM of multiple components in series and improve the availability of products, the method of grouping maintenance is used to optimize the PM of every single component, which mainly optimizes the PM time and usage array of every single component.

2.5. Contributions of This Work. At present, the studies on warranty cost ignore equipment availability that users are concerned about during the 2D warranty period. Meanwhile, most of these studies regard the research object as a single component, and the dependence between multicomponent, especially economic dependence, is less studied. Aiming at the deficiency of the existing studies, our study attempts to maximize the operational availability during the 2D warranty period and takes into account the constraint on maintenance cost. Also, our study investigates the 2D PM strategy of complex products including multicomponent systems. Based on this, the economic dependence between every single component is considered, and the grouping maintenance strategy of series multicomponent systems is proposed. Besides, by establishing a 2D PM availability model of a single component, the optimal PM interval is obtained. Moreover, the PM work among different components is combined, and the PM interval of every single component is reset to increase the availability of the product during the warranty period. The research results provide a scientific basis and quantitative analysis method for the formulation of complex equipment grouping maintenance strategy under 2D PM service.

3. 2D Warranty Failure Modeling

3.1. 2D Failure Rate Model. The design utilization rate needs to be considered for the product after it is purchased by a consumer. However, the real utilization rate of the product is usually not consistent with the design utilization rate, so the product presents different failure characteristics. AFT model can simulate the relationship of product failure rate function between the actual utilization rate and design utilization rate, which can better meet the modeling requirements of this paper.

In this model, $F_d(t, \varphi, \varsigma)$ represents the cumulative failure distribution function of the product under the design utilization rate r_d, where φ and ς are the scale parameters and shape parameters of the failure distribution, respectively. T_d and T_a, respectively, represent the first failure time under the design utilization rate r_d and the actual utilization rate r. The relationship between T_d and T_a can be expressed as follows:

$$\frac{T_d}{T_a} = \left(\frac{r_d}{r}\right)^{\omega}. \tag{3}$$

ω is the acceleration factor, $\omega > 0$. Under the utilization rate of r, the scale parameter of the cumulative failure distribution function changes to

$$\varphi(r) = \varphi\left(\frac{r_d}{r}\right)^{\omega}. \tag{4}$$

Meanwhile, the cumulative failure distribution function can be expressed as follows:

$$F(t, \varphi(r), \varsigma) = F_d\left(t, \varphi\left(\frac{r_d}{r}\right)^{\omega}, \varsigma\right). \tag{5}$$

Since the shape parameter does not change with the failure rate, so it is omitted in the following formula. Therefore, the failure rate function of the product is obtained as follows:

$$\lambda(t, r) = \frac{f(t, \varphi(r))}{1 - F(t, \varphi(r))}. \tag{6}$$

3.2. Imperfect PM Maintenance Modeling. The modeling of imperfect maintenance aims to describe the impact of imperfect maintenance on product failure rate. Kim et al. adopted the virtual age approach to describe the impact of imperfect maintenance on product failure rate. They found that the actual age of products can be reduced by PM, which can reduce the failure rate of products and improve the reliability of products [52]. The virtual age approach considers that after PM, the failure rate of the product at this moment is reduced to the failure rate of a certain time before the actual age, and the previous moment is the virtual age of the product. Denote the virtual age of the product after the i – th imperfect PM as $f_i\,(i \geq 1)$ and the improvement factor of imperfect PM as θ. When $\theta = 1$, the product reaches the state of "repaired as new", i.e., perfect maintenance; when $\theta = 0$, the product reaches the state of "repaired as old", i.e., minimal maintenance. For a given improvement factor θ, when the product is under imperfect PM at time t, the change of failure rate $\lambda(t)$ of the product is shown in Figure 3, where T_0 is the PM interval.

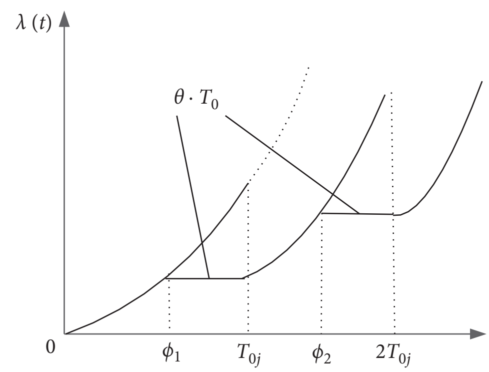


FIGURE 3: The change of product failure rate under imperfect PM.

Then, under the actual utilization rate r, the failure rate function of the product at any time can be expressed as

$$\lambda(t, r) = \begin{cases} \lambda(t - (i-1) * T_0 * \theta, r), & (i-1)T_0 \leq t < iT_0, i = 1, 2, \ldots, n, \\ \lambda(t - n * T_0 * \theta, r), & nT_0 \leq t < T_w. \end{cases} \tag{7}$$

In the study, it is assumed that there is a linear relationship between time and usage, i.e., $r = u/t$. When the product is under imperfect PM according to usage, T_0 in the failure rate function needs to be changed to U_0/r.

4. 2D Warranty Availability Model

4.1. Model Assumptions

(1) The subsystem of a product is composed of several single components in series.
(2) All failure statistics during the warranty period are independent.
(3) Every single component is repairable, and its failure rate increases with time and usage.
(4) Regular PM shall be carried out for every single component by the manufacturer, and the maintenance degree is imperfect maintenance. In case of a failure between two PM, the minimum maintenance shall be carried out by the manufacturer.
(5) The response time of the failure is ignored, that is, the product is repaired immediately after failure.
(6) PM shall be carried out when the time reaches T_{0j} or the usage reaches U_{0j}.

4.2. Model Formulation. Firstly, the availability model of single-component two-dimensional warranty is established to find the optimal PM interval of a single component, which lays the foundation for the multicomponent grouping maintenance model.

As for 2D warranty, the operational availability of the product can be expressed as

$$E\left[A_w(T_w, U_w)\right] = \frac{\text{warranty period} - \text{expected downtime}}{\text{warranty period}}. \tag{8}$$

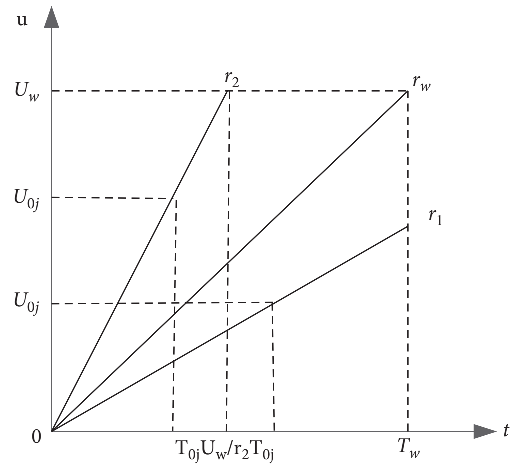


FIGURE 4: Warranty period corresponding to different utilization rates.

Within a certain period of time $[0, t]$, the minimal maintenance can be decomposed into a finite number of independent nonhomogeneous Poisson processes (NHPP). The expected minimal repair time is as follows:

$$E[N(t)] = \int_0^t \lambda(\gamma)\mathrm{d}\gamma = -\ln(1 - F(t)), \tag{9}$$

where $N(t)$ is the number of product failures in $[0, t]$; $\lambda(\gamma)$ is the failure rate function; and $F(t)$ is the cumulative failure distribution function.

Since the PM interval is not a definite value, the utilization rate r_{0j} derived from the PM interval is uncertain [53]. Two cases during modeling need to be discussed, namely, $r_{0j} \le r_W$ and $r_{0j} > r_W$, where

$$\begin{aligned} r_{0j} &= \frac{U_{0j}}{T_{0j}}, \\ r_W &= \frac{U_W}{T_W}. \end{aligned} \tag{10}$$

Because the actual utilization rate is not a fixed value, the warranty period will be different. Specifically, as shown in Figure 4, when the actual utilization rate is low, the warranty period is $[0, T_W)$; when the actual utilization rate is high, the warranty period is $[0, U_W)$.

Based on different warranty periods, there are four cases of the value of imperfect PM, as listed in Table 1.

4.2.1. $r_{0j} \le r_w$. As shown in Figure 5, in this case, $r \le r_{0j}$, $r_{0j} < r \le r_w$, and $r > r_w$ should be distinguished.

(1) $r \le r_{0j}$. When $r \le r_{0j}$, the PM interval of the components is T_{0j}. The number of PM of the components during this 2D warranty period is n_1. The expected downtime of components during the warranty period consists of two parts: total PM downtime (T_{pt}) and total postfailure maintenance downtime (T_{ft}). The expected downtime T_1 of the component during the warranty period is

$$\begin{aligned} T_1 &= T_{pt} + T_{ft} \\ &= \left(T_{pj} + T_{Dj}\right) * n_1 + T_{fj} * \sum_{i=0}^{n_1-1} \int_{i\left(T_{0j}+T_{pj}\right)}^{i\left(T_{0j}+T_{pj}\right)+T_{0j}} \lambda\left(t - i * \theta * \left(T_{0j} + T_{pj}\right), r\right)\mathrm{d}t \\ &\quad + T_{fj} * \int_{n_1\left(T_{0j}+T_{pj}\right)}^{T_w} \lambda\left(t - n_1 * \theta * \left(T_{0j} + T_{pj}\right), r\right)\mathrm{d}t. \end{aligned} \tag{11}$$

(2) $r_{0j} < r \le r_w$. When $r_{0j} < r \le r_w$, the PM interval of the components is U_{0j}/r. The number of PM of the components under the 2D warranty period is n_2. The expected downtime T_2 of the component during the warranty period is

Table 1: Values of imperfect PM in different warranty periods.

Number of imperfect PM	Warranty periods	Value
n_1	$[0, T_W)$	$\text{int}[T_w/(T_{0j}+T_p)]$
n_2	$[0, T_W)$	$\text{int}[T_w r/(U_{0j}+T_{pj}r)]$
n_3	$[0, U_W)$	$\text{int}[U_w/(U_{0j}+T_{pj}r)]$
n_4	$[0, U_W)$	$\text{int}[U_w/((T_{0j}+T_{pj})r)]$

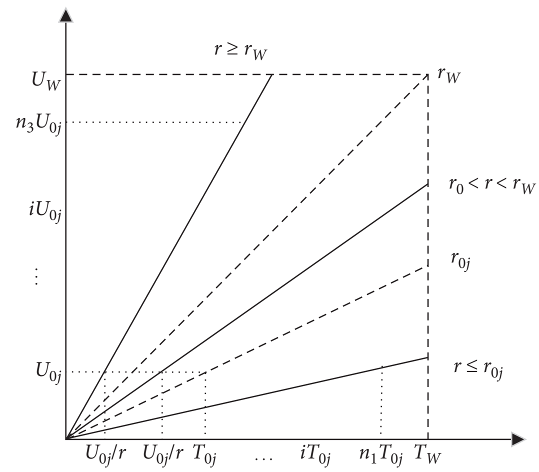


Figure 5: Two-dimensional imperfect PM cycle under $r_{0j} \leq r_W$.

$$\begin{aligned} T_2 &= T_{pt} + T_{ft} \\ &= \left(T_{pj}+T_{Dj}\right) * n_2 + T_{fj} * \sum_{i=0}^{n_2-1} \int_{i\left(\left(U_{0j}/r\right)+T_{pj}\right)}^{i\left(\left(U_{0j}/r\right)+T_{pj}\right)+\left(U_{0j}/r\right)} \lambda\left(t - i*\theta*\left(\frac{U_{0j}}{r}+T_{pj}\right), r\right)\mathrm{d}t \\ &\quad + T_{fj} * \int_{n_2\left(\left(U_{0j}/r\right)+T_{pj}\right)}^{T_w} \lambda\left(t - n_2*\theta*\left(\frac{U_{0j}}{r}+T_{pj}\right), r\right)\mathrm{d}t. \end{aligned} \tag{12}$$

(3) $r > r_w$. When $r > r_w$, the PM interval of the components is U_{0j}/r. The number of PM of the components under the 2D warranty period is n_3. The expected downtime T_3 of the component during the warranty period is

$$\begin{aligned} T_3 &= T_{pt} + T_{ft} \\ &= \left(T_{pj}+T_{Dj}\right) * n_3 + T_{fj} * \sum_{i=0}^{n_3-1} \int_{i\left(U_{0j}/r+T_{pj}\right)}^{i\left(\left(U_{0j}/r\right)+T_{pj}\right)+U_{0j}/r} \lambda\left(t - i*\theta*\left(\frac{U_{0j}}{r}+T_{pj}\right), r\right)\mathrm{d}t \\ &\quad + T_{fj} * \int_{n_3\left(U_{0j}/r+T_{pj}\right)}^{\left(U_w/r\right)} \lambda\left(t - n_3*\theta*\left(\frac{U_{0j}}{r}+T_{pj}\right), r\right)\mathrm{d}t. \end{aligned} \tag{13}$$

Then, during warranty period (T_w, U_w), when $r_{0j} \leq r_w$, the total expected availability of the component is

$$E_1\left[A_j(T_w,U_w)\right]=\int_{r_\sigma}^{r_{0j}}\frac{(T_w-T_1)}{T_w}\mathrm{d}G(r)+\int_{r_{0j}}^{r_w}\frac{(T_w-T_2)}{T_w}\mathrm{d}G(r)+\int_{r_w}^{r_\tau}\frac{((U_w/r)-T_3)}{(U_w/r)}\mathrm{d}G(r). \tag{14}$$

4.2.2. $r_{0j}>r_w$. As shown in Figure 6, in this case, $r\le r_w$, $r_w<r\le r_{0j}$, and $r>r_{0j}$ should be distinguished.

(1) $r\le r_w$. When $r\le r_w$, the PM interval of the components is T_{0j}. The number of PM of the components under the 2D warranty period is n_1. The expected downtime T_4 of the component during the warranty period is

$$\begin{aligned}T_4&=T_{pt}+T_{ft}\\&=\left(T_{pj}+T_{Dj}\right)*n_1+T_{fj}*\sum_{i=0}^{n_1-1}\int_{i\left(T_{0j}+T_{pj}\right)}^{i\left(T_{0j}+T_{pj}\right)+T_{0j}}\lambda\left(t-i*\theta*\left(T_{0j}+T_{pj}\right),r\right)\mathrm{d}t\\&\quad+T_{fj}*\int_{n_1\left(T_{0j}+T_{pj}\right)}^{T_w}\lambda\left(t-n_1*\theta*\left(T_{0j}+T_{pj}\right),r\right)\mathrm{d}t.\end{aligned} \tag{15}$$

(2) $r_w<r\le r_{0j}$. When $r_w<r\le r_{0j}$, the PM interval of components is T_{0j}. The number of PM of the components under the 2D warranty period is n_4. The expected downtime T_5 of the component during the warranty period is

\

$$\begin{aligned}T_5&=T_{pt}+T_{ft}\\&=\left(T_{pj}+T_{Dj}\right)*N_5+T_{fj}*\sum_{i=0}^{N_5-1}\int_{i\left(T_{0j}+T_{pj}\right)}^{i\left(T_{0j}+T_{pj}\right)+T_{0j}}\lambda\left(t-i*\theta*\left(T_{0j}+T_{pj}\right),r\right)\mathrm{d}t\\&\quad+T_{fj}*\int_{N_5*T_{0j}}^{(U_w/r)}\lambda\left(t-N_5*\theta*\left(T_{0j}+T_{pj}\right),r\right)\mathrm{d}t.\end{aligned} \tag{16}$$

(3) $r>r_{0j}$. When $r>r_{0j}$, the PM interval of the components is U_{0j}/r. The number of PM of the components under the 2D warranty period is n_3. The expected downtime T_6 of the component during the warranty period is

$$\begin{aligned}T_6&=T_{pt}+T_{ft}\\&=\left(T_{pj}+T_{Dj}\right)*N_6+T_{fj}*\sum_{i=0}^{N_6-1}\int_{i\left(\left(U_{0j}/r\right)+T_{pj}\right)}^{i\left(\left(U_{0j}/r\right)+T_{pj}\right)+\left(U_{0j}/r\right)}\lambda\left(t-i*\theta*\left(\frac{U_{0j}}{r}+T_{pj}\right),r\right)\mathrm{d}t\\&\quad+T_{fj}*\int_{N_6\left(\left(U_{0j}/r\right)+T_{pj}\right)}^{(U_w/r)}\lambda\left(t-N_6*\theta*\left(\frac{U_{0j}}{r}+T_{pj}\right),r\right)\mathrm{d}t.\end{aligned} \tag{17}$$

Then, during warranty period (T_w,U_w), when $r_{0j}>r_w$, the total expected availability of the component is

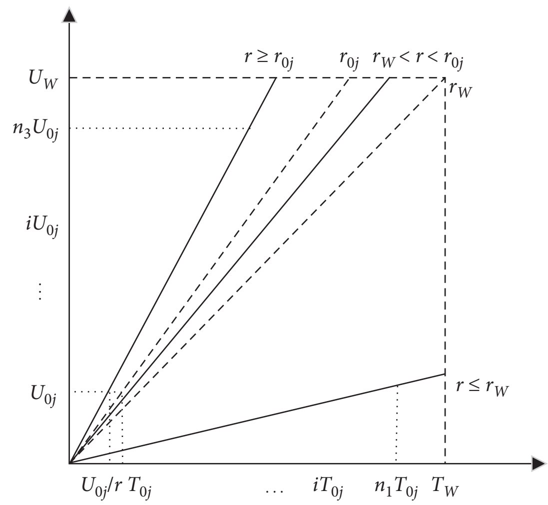


FIGURE 6: Two-dimensional imperfect PM cycle under $r_{0j} > r_w$.

$$E_2\left[A_j\left(T_w, U_w\right)\right] = \int_{r_\sigma}^{r_w} \frac{\left(T_w - T_4\right)}{T_w} \mathrm{d}G(r) + \int_{r_w}^{r_{0j}} \frac{\left(\left(U_w/r\right) - T_5\right)}{\left(U_w/r\right)} \mathrm{d}G(r) + \int_{r_{0j}}^{r_\tau} \frac{\left(\left(U_w/r\right) - T_6\right)}{\left(U_w/r\right)} \mathrm{d}G(r). \tag{18}$$

Thus, in the 2D warranty period (T_w, U_w), when the product utilization rate r follows a certain distribution $G(r)$ under the periodic 2D imperfect PM, the average product availability can be expressed as

$$E_\Sigma\left[A_j\left(T_w, U_w\right)\right] = \begin{cases} \int_{r_\sigma}^{r_{0j}} \frac{\left(T_w - T_1\right)}{T_w} \mathrm{d}G(r) + \int_{r_{0j}}^{r_w} \frac{\left(T_w - T_2\right)}{T_w} \mathrm{d}G(r) + \int_{r_w}^{r_\tau} \frac{\left(\left(U_w/r\right) - T_3\right)}{\left(U_w/r\right)} \mathrm{d}G(r), & r_{0j} \leq r_w, \\ \int_{r_\sigma}^{r_w} \frac{\left(T_w - T_4\right)}{T_w} \mathrm{d}G(r) + \int_{r_w}^{r_{0j}} \frac{\left(\left(U_w/r\right) - T_5\right)}{\left(U_w/r\right)} \mathrm{d}G(r) + \int_{r_{0j}}^{r_\tau} \frac{\left(\left(U_w/r\right) - T_6\right)}{\left(U_w/r\right)} \mathrm{d}G(r), & r_{0j} > r_w. \end{cases} \tag{19}$$

The optimal PM interval of a single component (T_{0j}^*, U_{0j}^*) that maximizes the total expected availability of the product during the warranty period is solved, respectively. Based on this, the optimal PM time and usage array of each component can be obtained:

$$\begin{cases} T_j^* = \left[T_{j1}^*, \ldots, T_{jc}^*, \ldots, T_{jN}^*\right], & \left(T_{jc}^* = cT_{0j}^*\right), \\ U_j^* = \left[U_{j1}^*, \ldots, U_{jc}^*, \ldots, U_{jN}^*\right], & \left(U_{jc}^* = cU_{0j}^*\right). \end{cases} \tag{20}$$

For the series multicomponent system, when a single component fails, the whole system fails. Therefore, before the PM work is combined, the availability of a multicomponent system is as follows:

$$A\left(T_W, U_W\right) = 1 - \sum_{j=1}^{S} \left\{1 - E_\Sigma\left[A_j^*\left(T_W, U_W\right)\right]\right\}, \tag{21}$$

where S is the number of components and $E_\Sigma\left[A_j^*\left(T_W, U_W\right)\right]$ is the availability of a single component in PM planning based on the optimal PM interval.

Each PM needs a certain PM preparation time. If the PM work of each component is combined, the total downtime of the system in the 2D warranty period can be reduced, and the availability of the system can be improved.

Based on this, the PM benchmark interval (T_m, U_m) is introduced, where T_m and U_m are the minimal values of PM time intervals and usage intervals for all single components.

$$\begin{cases} T_m = \min\left(T_{01}^*, T_{02}^*, \ldots, T_{0j}^*\right), \\ U_m = \min\left(U_{01}^*, U_{02}^*, \ldots, U_{0j}^*\right). \end{cases} \tag{22}$$

In this case, the PM work of every single component is slightly advanced or delayed, and the integral multiple of T_m and U_m are taken as the optimal time and usage of PM. Then, a new array of optimized PM time and usage $(T_{j\,\mathrm{new}}^*, U_{j\,\mathrm{new}}^*)$ can be obtained as

$$\begin{cases} T_{j\,\text{new}}^{*} = \left[T_{j1}^{\text{new}^*}, T_{j2}^{\text{new}^*}, \ldots, T_{jN}^{\text{new}^*}\right], & T_{jN}^{\text{new}^*} = \left[\dfrac{T_{jN}^{*}}{T_m}\right] \cdot T_m, \\ U_{j\,\text{new}}^{*} = \left[U_{j1}^{\text{new}^*}, U_{j2}^{\text{new}^*}, \ldots, U_{jN}^{\text{new}^*}\right], & U_{jN}^{\text{new}^*} = \left[\dfrac{U_{jN}^{*}}{U_m}\right] \cdot U_m, \end{cases} \tag{23}$$

where "[*]" is to round"*".

Figure 7 shows the optimal combination of PM work in the time dimension.

It should be ensured that the total number of PM of components during the warranty period is unchanged. Thus, after adjustment, if the last PM time (or usage) of component j exceeds the warranty period T_W (or U_W), the PM work is advanced to the maximal integral multiple of the benchmark interval within the warranty period. Then, T_j^* and U_j^* in $A(T_W, U_W)$ are replaced with $T_{j\,\text{new}}^*$ and $U_{j\,\text{new}}^*$ to obtain the optimized availability $A^*(T_W, U_W)$.

5. 2D Warranty Cost Model

The establishment process of the 2D warranty cost model is similar to that of the availability model. The 2D warranty cost consists of two parts, i.e., PM cost (C_{ft}) and CM cost (C_{pt}). According to formulas (11) to (13) and formulas (15) to (18), T_{fj}, T_{pj}, and T_{Dj} are replaced with C_{fj}, C_{pj}, and C_{Dj} respectively. In this way, the warranty cost expectation under different utilization rates is obtained, i.e., C_1 to C_6. The expected cost during the 2D warranty period is

$$EC = \begin{cases} \displaystyle\int_{r_\sigma}^{r_{0j}} C_1 dG(r) + \int_{r_{0j}}^{r_w} C_2 dG(r) + \int_{r_w}^{r_\tau} C_3 dG(r), & r_{0j} \le r_w, \\ \displaystyle\int_{r_\sigma}^{r_w} C_4 dG(r) + \int_{r_w}^{r_{0j}} C_5 dG(r) + \int_{r_{0j}}^{r_\tau} C_6 dG(r), & r_{0j} > r_w. \end{cases} \tag{24}$$

This paper attempts to maximize the availability of components during the 2D warranty period. Since the total maintenance cost should be lower than a specified budget, the nonlinear programming model with the constrained condition is

$$\begin{cases} \max & E_\Sigma\left[A_j(T_w, U_w)\right] \\ \text{s.t.} & EC \le C_0, \end{cases} \tag{25}$$

where C_0 is the maximum cost that the manufacturer can bear.

6. Numerical Application

6.1. Problem Description. In the social survey of a certain type of freight car, it is found that the use of this type of equipment can be measured in dimensions: time and usage (mileage). The power plant system of this kind of car is a series three-component system, and each component has a 2D warranty period. To improve the availability within the warranty period and reduce the loss of system downtime, the warranty contract signed between the manufacturer and the users indicates that the manufacturer should provide 2D imperfect PM service regularly for users and take the minimum maintenance for the failure that occurs in the PM interval period. The warranty cost shall be borne by the manufacturer. The power plant system is the "heart" of the freight car, so the user has high requirements for its availability. According to the contract, the manufacturer must ensure that the availability of the system is as high as possible. Meanwhile, since the manufacturer's maintenance cost budget is limited, the warranty cost cannot exceed the maximum value that the manufacturer can bear.

Assuming that the basic 2D warranty period of the power plant system is 3 years and 3×10^4 km, and the cumulative failure distribution function of the jth component of the power plant system under design utilization rate r_d follows Weibull distribution:

$$F_{jd}(t, \varphi) = 1 - \exp\left(-\left(\frac{t}{\varphi}\right)^{\varsigma}\right). \tag{26}$$

As for utilization rate r, the cumulative failure distribution function of the jth component is as follows:

$$F_j(t, \varphi(r)) = 1 - \exp\left(-\left(\frac{r}{r_d}\right)^{k\varsigma}\left(\frac{t}{\varphi}\right)^{\varsigma}\right). \tag{27}$$

Therefore, the failure rate function of the jth component can be expressed as

$$\lambda_j(t, r) = \frac{f(t; \alpha(r))}{1 - F(t; \alpha(r))} = \frac{\varsigma}{\varphi^{\varsigma}}\left(\frac{r}{r_d}\right)^{\omega\varsigma} t^{\varsigma - 1}. \tag{28}$$

Through statistical analysis of the same type of power plant system, the approximate distribution of the utilization rate can be obtained. To facilitate analysis and calculation, it is assumed that the utilization rate follows a Weibull distribution. The maximal utilization rate is 10×10^4 km and the lower limit of utilization is 0.1×10^4 km.

$$g(r) = \frac{\delta}{\eta}\left(\frac{r}{\eta}\right)^{\delta - 1} e^{-(r/\eta)^{\delta}}, \quad (0.1 < r < 10), \tag{29}$$

where δ is the shape parameter and η is the scale parameter.

Through the survey, other parameters of the components in the power plant system are listed in Table 2.

6.2. Model Solving. A numerical method is adopted to calculate the 2D PM availability of a single component. The flowchart is shown in Figure 8.

As shown in Figure 8, let T_{0j} take the value in [0.1 years, 3 years] with a step of 0.1 year, and let U_{0j} take the value in $[0.1 \times 10^4$ km, 3×10^4 km] with a step of 0.1×10^4 km. Then, calculate the corresponding availability during the warranty period of each group (T_{0j}, U_{0j}) and store the calculation result. Finally, 900 groups (T_{0j}, U_{0j}) are generated.

Similarly, the trend of the cost during the warranty period of each component can be obtained, as shown in Figure 9.

Figures 10(a)–10(c) correspond to the trend of availability of component 1, component 2, and component 3, respectively. The availability of a single component corresponding to different (T_{0j}, U_{0j}) has the highest value. That

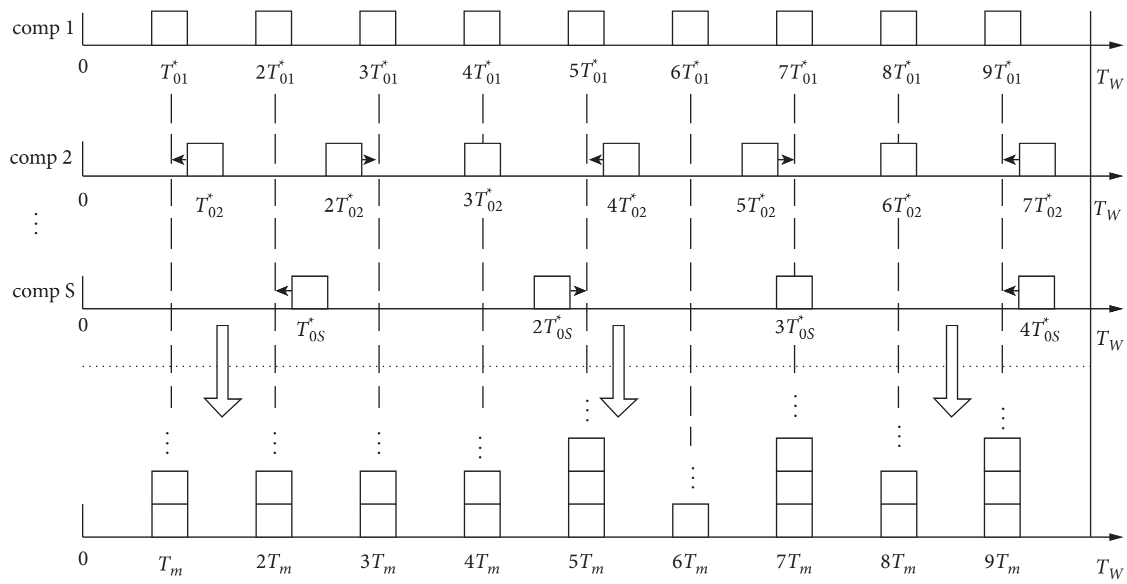


Figure 7: Optimization of PM work for components in the time dimension.

Table 2: Parameter settings.

Component	φ	ς	r_d	ω	θ	T_{fj} (days)	T_{pj} (days)	T_{Di} (days)	δ	η	C_{fj} (CNY)	C_{pj} (CNY)	C_{Di} (CNY)	C_0 (CNY)
1	1.8	2.2	1.0	1.7	0.9	1	3	1	2	5	600	1500	500	
2	1.4	1.6	1.0	1.4	0.9	1	2	1	2	5	800	2000	500	50000
3	1.1	1.2	1.0	1.6	0.9	2	3	1	2	5	650	1800	500	

is, there is a group (T^*_{0j}, U^*_{0j}) that can be regarded as the 2D PM interval to maximize the single component availability during the warranty period. A simulated annealing algorithm is used to solve the maximal value of single-component availability and its corresponding (T^*_{0j}, U^*_{0j}). The simulated annealing algorithm is an intelligent algorithm with global search function, which can obtain more accurate values than numerical algorithms.

Figures 11(a)–11(c) correspond to the iterative process of the simulated annealing algorithm for solving the maximum availability of component 1, component 2, and component 3 within the warranty period, respectively. Since the simulated annealing algorithm is usually used to solve the minimal value of the objective function, the absolute value of the best availability should be taken. The results show that when the PM interval of component 1 is $(2.786\,\text{years}, 1.004 \times 10^4\,\text{km})$, the maximal availability is 0.903 during the warranty period, and the cost is 46134 CNY, which is less than the maximum cost acceptable to the manufacturer; when the PM interval of component 2 is $(1.669\,\text{years}, 1.507 \times 10^4\,\text{km})$, the maximal availability is 0.958 during the warranty period, and the cost is 17966 CNY, which is less than the maximum cost acceptable to the manufacturer; when the PM interval of component 3 is $(2.99\,\text{years}, 2.98 \times 10^4\,\text{km})$, the maximal availability is 0.904 during the warranty period, and the cost is 19290 CNY, which is less than the maximum cost acceptable to the manufacturer.

Dimension reduction analysis is conducted on the change trend of availability of each component. Meanwhile, when the PM interval of one dimension is determined, the variation trend of availability with the PM interval of another dimension is studied. Figures 12(a)–12(f) illustrate the availability change of components 1–3, respectively. Figures 12(a)–12(c) show the availability change of every single component under different $U0_j$, and Figures 12(d)–12(f) illustrate the availability change of every single component under different $T0_j$.

It can be seen from Figure 12 that the availability of a single component changes with the PM interval. If the PM interval of any dimension is fixed, there is an optimal PM interval of another dimension to maximize the availability of components.

According to the model introduced in Section 4.2, the PM interval of the system is adjusted to the minimum value of the PM interval of every single component, following the method of grouping maintenance. Based on this, the PM time or usage of every single component is optimized and combined. The availably with and without PM measures is listed in Table 3.

6.3. Results Analysis. During the warranty period, if PM measures are not taken for each component, and only minimal repair is conducted after failure, then the availability of each component during the 2D warranty period can be calculated separately. The comparison of the availability with the highest availability with PM measures is listed in Table 4.

It can be seen from Table 4 that after PM measures are taken during the warranty period, the availability of the three components is significantly improved. Specifically, the

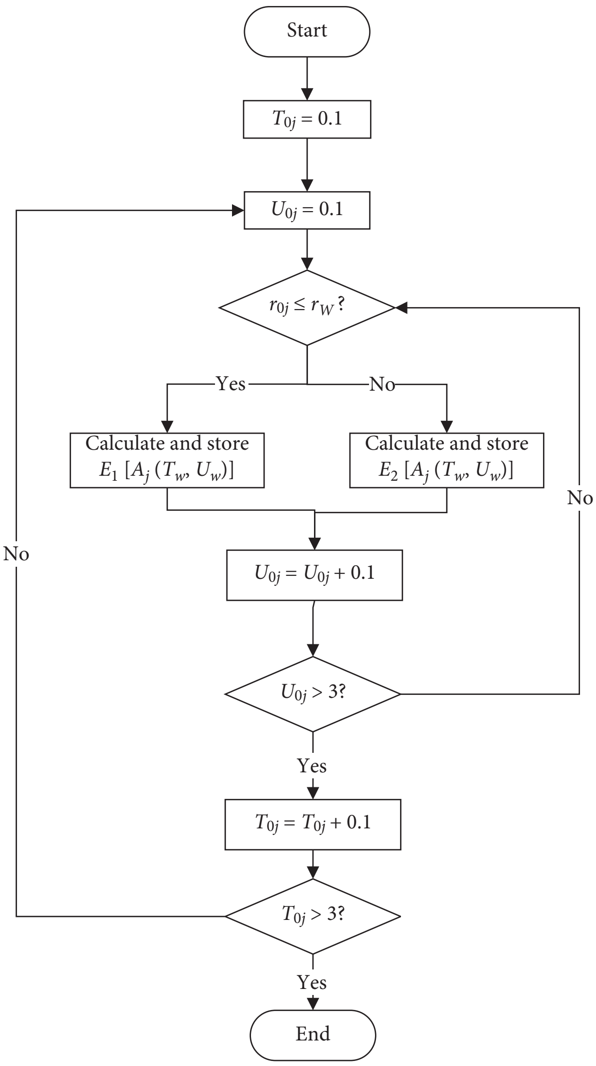
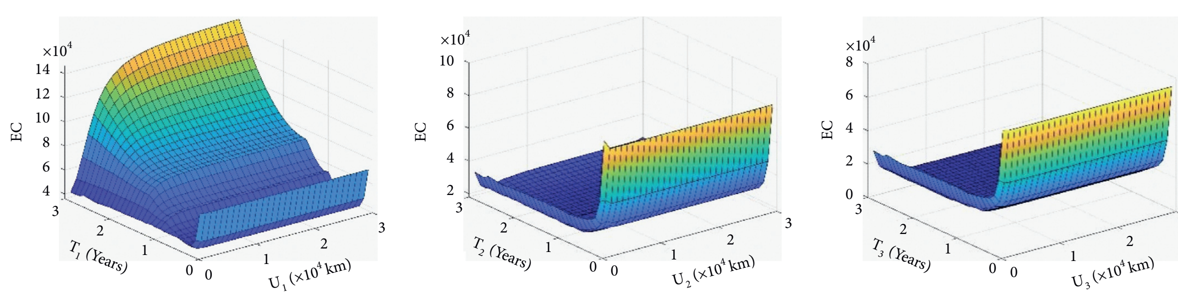


Figure 8: Flowchart of the numerical method for solving the 2D PM availability of a single component.



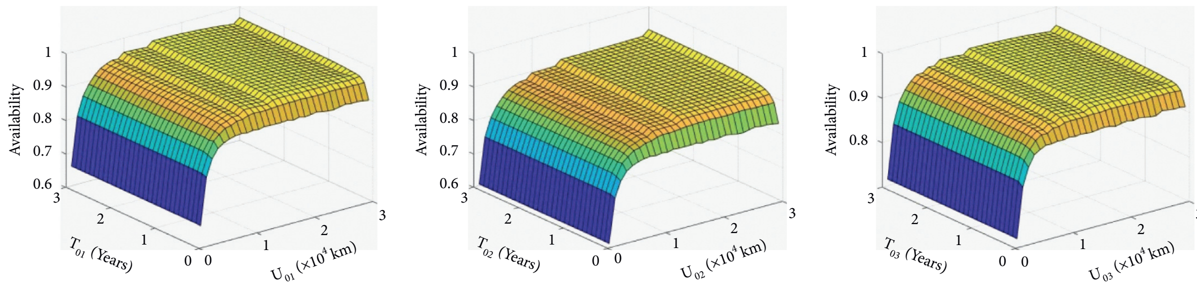
(a) (b) (c)

Figure 9: Variation trend of cost during the warranty period of each component.

availability of components 1–3 is increased by 0.3%, 4%, and 3.2%, respectively. The availability of the power plant system is increased by 10% during the warranty period, indicating that PM can reduce the probability of failure during the warranty period and greatly improve the availability of the system.

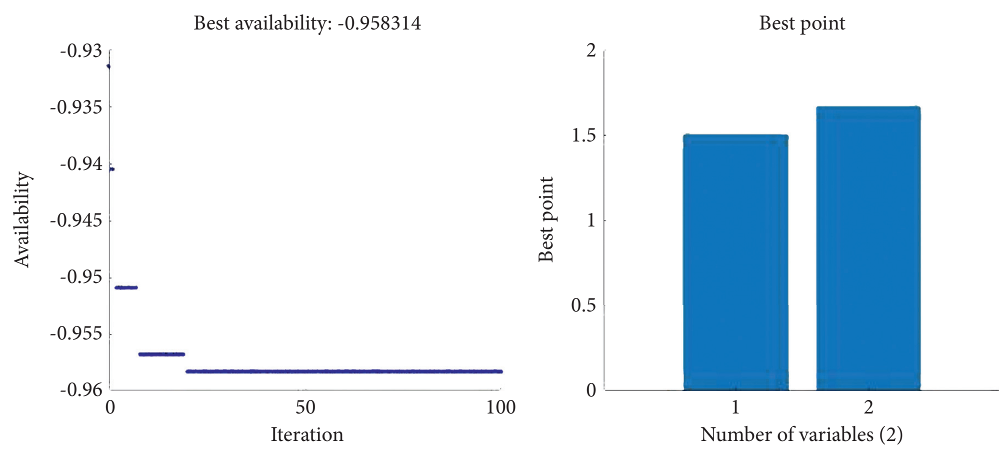
According to the model described in Section 4.2, the PM work of the components is combined, and the availability after the combination is calculated. The availability before and after the combination is listed in Table 5.

It can be seen from Table 5 that if the PM plan of each component is strictly implemented, the availability of the series multicomponent system is 0.765 during the warranty period. Following the proposed combinatorial optimization

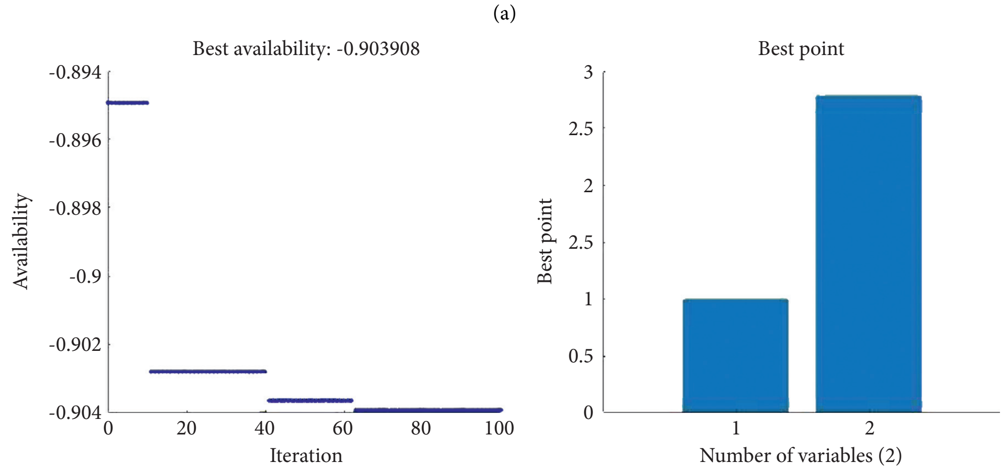


(a) (b) (c)

Figure 10: Variation trend of availability during the warranty period of each component.

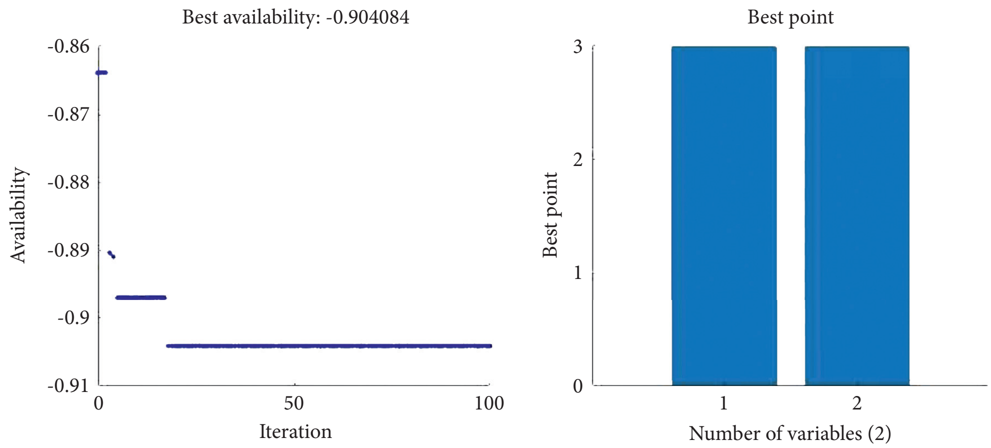


(a)



(b)

Figure 11: Continued.



(c)

FIGURE 11: The iterative process of the simulated annealing algorithm.

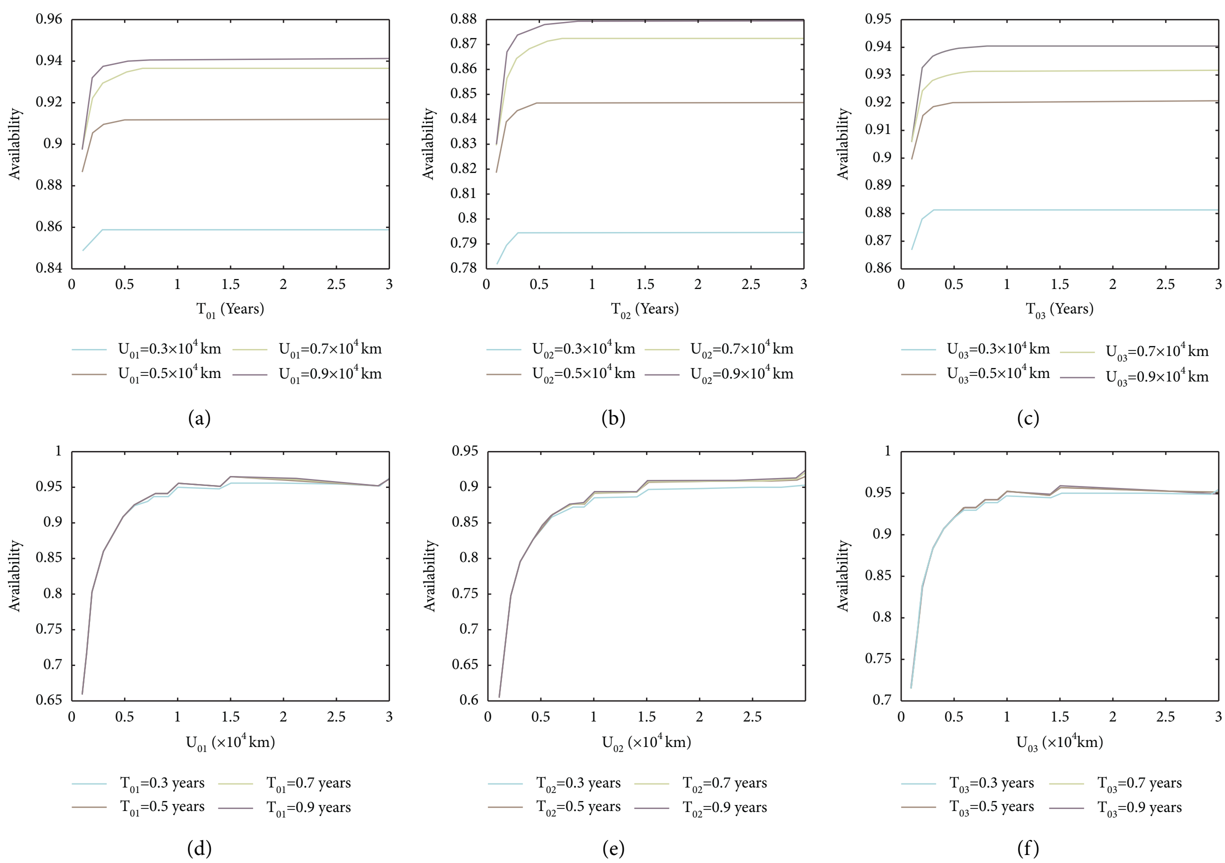


FIGURE 12: Dimension reduction analysis of availability change of components.

method, the PM work of every single component is combined, and the availability of the series multicomponent system is 0.824, which is 7.7% higher than that before optimization. This result indicates that the combinatorial optimization method can improve the availability of the power plant system.

Compared with that before optimization, the PM of every single component is no longer cyclical, but it is implemented by an integral multiple of the benchmark PM interval. The system performs a PM in the time dimension and two PM in the usage dimension. Under different utilization rates, the time and usage of PM for a single component are listed in Tables 6 and 7, where "+" indicates PM, and "–" indicates no PM.

It can be seen from Table 6 that after the PM work of each component is combined, when the actual utilization rate of

Table 3: Comparison of the availability with PM measures and without PM measures.

Component number	Before grouping maintenance		Cost (CNY)	Acceptable or not	After grouping maintenance	
	Time interval (years)	Usage interval ($\times 10^4$ km)			Time interval (years)	Usage interval ($\times 10^4$ km)
1	2.768	1.004	46134	Yes		
2	1.669	1.507	17966	Yes	1.669	1.004
3	2.99	2.98	19290	Yes		

Table 4: Comparison of the availability with PM measures and without PM measures.

Component number	No PM measures	With PM measures			Rate of change (%)
	$E_{\Sigma}[A_j(T_w,U_w)]$	T_{0j}^*(years)	U_{0j}^*($\times 10^4$ km)	$E_{\Sigma}[A_j^*(T_W,U_W)]$	
1	0.9	2.768	1.004	0.903	0.3
2	0.921	1.669	1.507	0.958	4
3	0.876	2.99	2.98	0.904	3.2
$A(T_w,U_w)$	0.697			0.765	10

Table 5: Maximal availability before and after grouping maintenance.

Component number	T_{0j}^* (years)	U_{0j}^* ($\times 10^4$ km)	$E_{\Sigma}[A_j^*(T_W,U_W)]$	$A(T_w,U_w)$	T_m (years)	U_m ($\times 10^4$ km)	$A^*(T_W,U_W)$	Rate of change (%)
1	2.768	1.004	0.903					
2	1.669	1.507	0.958	0.765	1.669	1.004	0.824	7.7
3	2.99	2.98	0.904					

Table 6: PM time of components.

Component number	r($\times 10^4$ km/year)	T_m
1	$r \leq 0.36$	+
2	$r \leq 0.9$	+
3	$r \leq 1$	+

Table 7: PM usage of components.

Component number	r($\times 10^4$ km/year)	U_m	$2U_m$
1	$r > 0.36$	+	−
2	$r > 0.9$	+	−
3	$r > 1$	−	+

each component is less than or equal to r_{0j}, the deadline of the warranty period first arrives in the time dimension. When the utilization rates of components 1, 2, and 3 are less than 0.36×10^4 km/year, 0.9×10^4 km/year, and 1×10^4 km/year respectively, the PM of the three components will be performed once in 1.669 years. At this time, the PM time of components 1 and 3 will be advanced.

It can be seen from Table 7 that after the PM work of each component is combined, when the actual utilization rate of each component is greater than r_{0j}, the deadline of the warranty period first reaches in the usage dimension. Specifically, when the utilization rates of components 1, 2, and 3 are greater than 0.36×10^4 km/year, 0.9×10^4 km/year, and 1×10^4 km/year respectively, imperfect PM needs to be conducted in the usage dimension; When the usage reaches 1.004×10^4 km, imperfect PM needs to be conducted on components 1 and 2 jointly. When the usage reaches 2.008×10^4 km, imperfect PM needs to be conducted on component 3.

It can be seen from Tables 5 and 6 that according to different utilization rates, the PM work of each component is carried out at certain intervals in the time dimension or usage dimension, indicating that the optimization method can provide a reference for formulating grouping maintenance schemes for the power plant system.

7. Conclusions

This paper studies the grouping maintenance strategy for 2D series multicomponent systems. First, the PM benchmark interval is introduced based on the 2D PM availability model of a single component. Then, by adjusting the PM time of every single component, the PM work of every single component is combined, and the 2D PM availability model of series multicomponent systems is established. Finally, taking a power plant system as an example, the results show that after adopting the grouping maintenance strategy, the 2D warranty availability of the power plant is significantly improved, which fully verifies the effectiveness of the grouping maintenance strategy proposed in this paper. The PM grouping maintenance optimization model established in this paper can provide theoretical and technical support for the formulation of 2D warranty schemes.

There are still some issues worth studying in the future. Firstly, the application of advanced technologies makes the product life longer and longer. Also, manufacturers tend to provide extended warranty service, which has become an important way for manufacturers to make profits. During the extended warranty period, it is an interesting research topic to consider the economic dependence between multicomponent to reduce the manufacturer's warranty cost and improve the product availability. Secondly, the dependence among multicomponent also includes stochastic dependence and structural dependence. The formulation of preventive maintenance strategies based on stochastic dependence and structural dependence needs to be investigated.

Data Availability

All data, models, and code generated or used during the study appear in the submitted article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research work was financially supported by the National Natural Science Foundation of China (no. 71871219).

References

[1] D. K. Manna, S. Pal, and S. Sinha, "A use-rate based failure model for two-dimensional warranty," *Computers & Industrial Engineering*, vol. 52, no. 2, pp. 229–240, 2007.

[2] B. Sarkar and S. Saren, "Product inspection policy for an imperfect production system with inspection errors and warranty cost," *European Journal of Operational Research*, vol. 248, no. 1, pp. 263–271, 2016.

[3] H.-G. Kim and B. M. Rao, "Expected warranty cost of two-attribute free-replacement warranties based on a bivariate exponential distribution," *Computers & Industrial Engineering*, vol. 38, no. 4, pp. 425–434, 2000.

[4] M. Jung and D. S. Bai, "Analysis of field data under two-dimensional warranty," *Reliability Engineering & System Safety*, vol. 92, no. 2, pp. 135–143, 2007.

[5] D. N. P. Murthy, "Product warranty and reliability," *Annals of Operations Research*, vol. 143, no. 1, pp. 133–146, 2006.

[6] B. P. Iskandar, D. N. P. Murthy, and N. Jack, "A new repair-replace strategy for items sold with a two-dimensional warranty," *Computers & Operations Research*, vol. 32, no. 22, pp. 669–682, 2005.

[7] Z. H. Cheng, Q. Wang, Y. S. Bai, R. D. Zhao, and S. K. Guo, "Optimization model of two-dimensional warranty equipment availability," *Journal of Academy of Armored Force Engineering*, vol. 33, no. 3, pp. 34–39, 2019.

[8] Z. H. Cheng, Z. Y. Yang, and H. B. Yang, "Optimization model of equipment two-dimensional warranty strategy under preventive maintenance," *Industrial Engineering Journal*, vol. 20, no. 4, pp. 79–86, 2017.

[9] Y.-S. Huang, W.-Y. Gau, and J.-W. Ho, "Cost analysis of two-dimensional warranty for products with periodic preventive maintenance," *Reliability Engineering & System Safety*, vol. 134, pp. 51–58, 2015.

[10] K. D. Majeske, "A non-homogeneous Poisson process predictive model for automobile warranty claims," *Reliability Engineering & System Safety*, vol. 92, no. 3, pp. 243–251, 2007.

[11] Y.-S. Huang, E. Chen, and J.-W. Ho, "Two-dimensional warranty with reliability-based preventive maintenance," *IEEE Transactions on Reliability*, vol. 62, no. 4, pp. 898–907, 2013.

[12] T. Duchesne and J. Lawless, "Alternative time scales and failure time models," *Lifetime Data Analysis*, vol. 6, no. 2, pp. 157–179, 2000.

[13] J. Lawless, J. Hu, and J. Cao, "Methods for the estimation of failure distributions and rates from automobile warranty data," *Lifetime Data Analysis*, vol. 1, no. 3, pp. 227–240, 1995.

[14] X. Wang and W. Xie, "Two-dimensional warranty: a literature review," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 232, no. 3, pp. 284–307, 2018.

[15] B. P. Iskandar and H. Husniah, "Optimal preventive maintenance for a two dimensional lease contract," *Computers & Industrial Engineering*, vol. 113, pp. 693–703, 2017.

[16] P. Tong, X. Song, and L. Zixian, "A maintenance strategy for two-dimensional extended warranty based on dynamic usage rate," *International Journal of Production Research*, vol. 55, no. 19, pp. 5743–5759, 2017.

[17] X. Li, Z. Liu, Y. Wang, and M. Li, "Optimal burn-in strategy for two-dimensional warranted products considering preventive maintenance," *International Journal of Production Research*, vol. 57, no. 17, pp. 5414–5431, 2019.

[18] Y. H. Chun, "Optimal number of periodic preventive maintenance operations under warranty," *Reliability Engineering & System Safety*, vol. 37, no. 3, pp. 223–225, 1992.

[19] C. Richard, I. Cassady, and I. Mohammed, "A generic model of equipment availability under imperfect maintenance," *IEEE Transactions on Reliability*, vol. 54, no. 4, pp. 564–571, 2005.

[20] Z. H. Cheng, Z. Y. Yang, Z. Zhao, Y. B. Wang, and Z. W. Li, "Preventive maintenance strategy optimizing model under two-dimensional warranty policy," *Eksploatacja i Niezawodnosc-Maintenance and Reliability*, vol. 17, no. 3, pp. 365–372, 2015.

[21] M. Park and H. Pham, "Cost models for age replacement policies and block replacement policies under warranty," *Applied Mathematical Modelling*, vol. 40, no. 9-10, pp. 5689–5702, 2016.

[22] D. T. Nguyen, Y. Dijoux, and M. Fouladirad, "Analytical properties of an imperfect repair model and application in preventive maintenance scheduling," *European Journal of Operational Research*, vol. 256, no. 2, pp. 439–453, 2017.

[23] S. Chukova, R. Arnold, and D. Q. Wang, "Warranty analysis: an approach to modeling imperfect repairs," *International Journal of Production Economics*, vol. 89, no. 1, pp. 57–68, 2004.

[24] X. Wang, L. Li, and M. Xie, "An unpunctual preventive maintenance policy under two-dimensional warranty," *European Journal of Operational Research*, vol. 282, no. 1, pp. 304–318, 2020.

[25] A. Dai, G. Wei, Z. Zhang, and S. He, "Design of a flexible preventive maintenance strategy for two-dimensional warranted products," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 234, no. 1, pp. 74–87, 2020.

[26] Q. Wang, Z. H. Cheng, and Y. S. Bai, "Study on two-dimensional warranty cost optimization model of complex parts," *Journal of Gun Launch and control*, vol. 41, no. 2, pp. 98–103, 2020.

[27] M. Park, K. M. Jung, and H. P. Dong, "Optimal post-warranty maintenance policy with repair time threshold for minimal repair," *Reliability Engineering & System Safety*, vol. 111, pp. 147–153, 2013.

[28] C.-Y. Cheng, X. Zhao, M. Chen, and T.-H. Sun, "A Failure-rate-reduction periodic preventive maintenance model with delayed initial time in a finite time period," *Quality Technology & Quantitative Management*, vol. 11, no. 3, pp. 245–254, 2014.

[29] K. Shahanaghi, R. Noorossana, S. G. Jalali-Naini, and M. Heydari, "Failure modeling and optimizing preventive maintenance strategy during two-dimensional extended warranty contracts," *Engineering Failure Analysis*, vol. 28, pp. 90–102, 2013.

[30] G.-J. Wang, Y.-L. Zhang, and R. C. M. Yam, "Preventive maintenance models based on the generalized geometric process," *IEEE Transactions on Reliability*, vol. 66, no. 4, pp. 1380–1388, 2017.

[31] Q. W. Hu, Y. S. Bai, J. M. Zhao, and W. B. Cao, "Modeling spare parts demands forecast under two-dimensional preventive maintenance policy," *Mathematical Problems in Engineering*, vol. 2015, Article ID 728241, 9 pages, 2015.

[32] X. Wang and C. Su, "A two-dimensional preventive maintenance strategy for items sold with warranty," *International Journal of Production Research*, vol. 54, no. 19, pp. 5901–5915, 2016.

[33] S. Z. Peng, W. Jiang, and W. H. Zhao, "A preventive maintenance policy with usage-dependent failure rate thresholds under two-dimensional warranties," *Iise Transactions*, vol. 13, no. 1, pp. 136–145, 2020.

[34] A. A. Taleizadeh and M. Mokhtarzadeh, "Pricing and two-dimensional warranty policy of multi-products with online and offline channels using a value-at-risk approach," *Computers & Industrial Engineering*, vol. 148, no. 3, pp. 15–30, 2020.

[35] J. Baik, D. N. P. Murthy, and N. Jack, "Two-dimensional failure modeling with minimal repair," *Naval Research Logistics*, vol. 51, no. 3, pp. 345–362, 2010.

[36] X. Wang and Z. S. Ye, "Design of customized two-dimensional extended warranties considering use rate and heterogeneity," *Iise Transactions*, vol. 11, no. 2, pp. 170–190, 2020.

[37] R. Zheng and C. Su, "A flexible two-dimensional basic warranty policy with two continuous warranty regions," *Quality and Reliability Engineering International*, vol. 36, no. 6, pp. 2003–2018, 2020.

[38] Z. He, D. F. Wang, S. G. He, Y. W. Zhang, and A. S. Dai, "Two-dimensional extended warranty strategy including maintenance level and purchase time: a win-win perspective," *Computers & Industrial Engineering*, vol. 41, no. 2, pp. 15–30, 2020.

[39] Q. Wang, Z. H. Cheng, Q. T. Gan, Y. S. Bai, J. Q. Zhang, and M. J. Peng, "Cost optimization of two-dimensional warranty products under preventive maintenance," *Mathematical Problems in Engineering*, vol. 16, no. 2, pp. 30–50, 2021.

[40] R. C. Wang, Z. H. Cheng, and Q. Wang, "Research on availability model of two-dimensional warranty products based on imperfect maintenance," *IOP Conference Series: Materials Science and Engineering*, vol. 1043, no. 3, pp. 741–766, 2021.

[41] H. H. Zhu, *Two-dimensional Warranty Modeling and Optimization Based on Preventive Maintenance*, Shijiazhuang Railway University, Zhuji, China, 2020.

[42] A. Dai, Z. Zhang, P. Hou, J. Yue, S. He, and Z. He, "Warranty claims forecasting for new products sold with a two-dimensional warranty," *Journal of Systems Science and Systems Engineering*, vol. 28, no. 6, pp. 715–730, 2019.

[43] P. Tong and C. L. Liu, "Designing differential service strategy for two-dimensional warranty based on warranty claim data under consumer-side modularisation," *Proceedings of the Institution of Mechanical Engineers Part O-Journal of Risk and Reliability*, vol. 234, no. 3, pp. 550–561, 2019.

[44] K. S. Lin and Y. X. Chen, "Analysis of two-dimensional warranty data considering global and local dependence of heterogeneous marginals," *Reliability Engineering & System Safety*, vol. 207, no. 2, pp. 53–70, 2021.

[45] C. Su and L. Cheng, "Two-dimensional preventive maintenance optimum for equipment sold with availability-based warranty," *Proceedings of the Institution of Mechanical Engineers Part O Journal of Risk and Reliability*, vol. 233, no. 4, pp. 648–657, 2018.

[46] C. Su and X. Wang, "A two-stage preventive maintenance optimization model incorporating two-dimensional extended warranty," *Reliability Engineering & System Safety*, vol. 155, pp. 169–178, 2016.

[47] D. F. Wang, Z. He, S. G. He, Z. Zhang, and Y. Zhang, "Dynamic pricing of two-dimensional extended warranty considering the impacts of product price fluctuations and repair learning," *Reliability Engineering and System Safety*, vol. 210, no. 3, pp. 78–89, 2021.

[48] Y. F. Zhou, Z. S. Zhang, T. R. Lin, and L. Ma, "Maintenance optimisation of a multi-state series–parallel system considering economic dependence and state-dependent inspection intervals," *Reliability Engineering and System Safety*, vol. 111, no. 2, pp. 42–60, 2013.

[49] P. Do, A. Roy, P. Scarf, and I. Benoit, "Modelling and application of condition-based maintenance for a two-component system with stochastic and economic dependencies," *Reliability Engineering and System Safety*, vol. 182, no. 3, pp. 42–60, 2019.

[50] C. D. Dao, M. J. Zuo, and M. Pandey, "Selective maintenance for multi-state series-parallel systems under economic dependence," *Reliability Engineering & System Safety*, vol. 121, pp. 240–249, 2014.

[51] C. Su and W. Chen, "Condition based maintenance optimization of wind turbine system considering economic correlation of components," *Journal of Southeast University (Medical Science Edition)*, vol. 46, no. 5, pp. 1007–1012, 2016.

[52] C. S. Kim, I. Djamaludin, and D. N. P. Murthy, "Warranty and discrete preventive maintenance," *Reliability Engineering & System Safety*, vol. 84, no. 3, pp. 301–309, 2004.

[53] B. P. Iskandar and D. Murthy, "Repair-replace strategies for two-dimensional warranty policies," *Mathematical & Computer Modelling*, vol. 38, no. 11-13, pp. 1233–1241, 2003.