Research Article

Hamilton-Connected Mycielski Graphs*

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Jarnicki, Myrvold, Saltzman, and Wagon conjectured that if G is Hamilton-connected and not K_2 , then its Mycielski graph $\mu(G)$ is Hamilton-connected. In this paper, we confirm that the conjecture is true for three families of graphs: the graphs G with $\delta(G) > |V(G)|/2$, generalized Petersen graphs $GP(n, 2)$ and $GP(n, 3)$, and the cubes G^3 . In addition, if G is pancyclic, then $\mu(G)$ is pancyclic.

1. Introduction

All graphs considered in this paper are simple and finite. For notations and terminologies not defined here, we refer to Bondy and Murty [1]. A spanning cycle (path) of a graph is called Hamilton cycle (Hamilton path). A graph which contains a Hamiltonian path between every two vertices of G is called Hamilton-connected (HC). Mycielski [2] proved that the chromatic numbers of triangle-free graphs can be arbitrarily large by introducing a graph transformation as follows. For a graph G on vertices $V = \{v_1, v_2, \dots, v_n\}$, its Mycielski graph, denoted by $\mu(G)$, is the graph on vertices $X \cup Y \cup \{z\} = \{x_1, x_2, \dots, x_n\} \cup \{y_1, y_2, \dots, y_n\} \cup \{z\}$ with edges zy_i for all i and edges $x_i x_j$, $y_i y_j$, and $x_i y_j$ for all edges $v_i v_j$ in G . In recent years, a number of papers are devoted to various properties of Mycielski graphs, such as Hamilton-connectivity, Hamiltonicity [3–7], total chromatic number [8, 9], circular chromatic number [10–14], and connectivity [15, 16]. Fisher et al. [4] obtained the following results.

Theorem 1 (see Fisher et al. [4]). *The following results hold for a graph G :*

- (1) If G is Hamiltonian, then $\mu(G)$ is Hamiltonian
- (2) If G is not connected, then $\mu(G)$ is not Hamiltonian
- (3) If G has at least two pendant vertices, then $\mu(G)$ is not Hamiltonian

Cheng, Wang, and Liu studied Hamiltonicity and Hamilton-connectedness in Mycielski graphs of bipartite graphs.

Theorem 2 (see Cheng et al. [3]). *For a bipartite graph G , the following are true:*

- (1) If $\mu(G)$ is Hamiltonian, then G is balanced
- (2) If $\mu(G)$ is Hamiltonian, then G has a Hamilton path

In 2017, Jarnicki et al. [17] established the following results for $\mu(G)$ being Hamilton-connected or not.

Theorem 3 (see Jarnicki et al. [17]). *The following results hold for a graph G :*

- (1) If G is an odd cycle, then $\mu(G)$ is Hamilton-connected
- (2) If G is a Hamilton-connected graph with order odd, then $\mu(G)$ is Hamilton-connected
- (3) If G is an even cycle, then $\mu(G)$ is not Hamilton-connected

They posed the following conjecture.

Conjecture 1 (see Jarnicki et al. [17]). *If G is Hamilton-connected and not K_2 , then $\mu(G)$ is Hamilton-connected.*

In this paper, we confirm that the conjecture is true for three families of graphs: the graphs G with $\delta(G) > |V(G)|/2$,

generalized Petersen graphs $GP(n, 2)$ and $GP(n, 3)$, and the cubes G^3 . In addition, if G is pancyclic, then $\mu(G)$ is pancyclic.

2. Mycielski Factor

Let G be a connected graph of order n even, and $v_1 \in V(G)$. We call a connected spanning subgraph of G to be a Mycielski factor starting at v_1 if it consists of an even number of odd cycles $C_1, \dots, C_{2s}$ (possibly $s = 0$) and an even cycle C_{2s+1} with the chord (possibly empty), joined by $2s$ edges $e_1, \dots, e_{2s}$, where $e_i = v_i v_{i+1}'$ for each $i \in \{1, \dots, 2s\}$ such that $v_i v_i' \in V(C_i)$ for each $i \in \{1, \dots, 2s+1\}$, and the chord joins v_{2s+1}' and a vertex at distance even on C_{2s+1} .

Lemma 1. Assume that a graph G is Hamilton-connected. If, for any $v \in V(G)$, there exists a Mycielski factor starting at v , then $\mu(G)$ is Hamilton-connected.

Proof. As in the assumption, let G be HC. Trivially, G has a Hamilton cycle. By Theorem 3 (2), $\mu(G)$ is HC if the order of G is odd. So, it remains to tackle the case when the order is even. Let $V(G) = \{v_1, \dots, v_{2n}\}$, where $n \geq 2$. Recall that $V(\mu(G)) = X \cup Y \cup \{z\}$. Take any two vertices

$A, B \in V(\mu(G))$. We consider five cases in terms of the location of A and B in X, Y , and $\{z\}$. $\square$

Case 1. $A \in X$ and $B \in X$.

Without loss of generality, let $A = x_1$ and $B = x_{2n}$. Since G is HC, there exists a Hamilton path P connecting v_1 and v_{2n} in G . We shall find a Hamilton path of $\mu(G)$ depending on P as follows. Zigzag up from x_1 until y_{2n} is reached. Then, jump via z to y_1 , and zigzag right until x_{2n} is reached. Formally, it is

$$x_1 - y_2 - x_3 - \dots - y_{2n} - z - y_1 - x_2 - \dots - x_{2n}, \quad (1)$$

as shown in Figure 1.

Case 2. $A \in Y$ and $B \in Y$.

Without loss of generality, let $A = y_1$ and $B = y_{2n}$. Since G is HC, there exists a Hamilton path P connecting v_1 and v_{2n} in G . Thus, there exists a neighbor, say v_2 , of v_1 . Zigzag up from y_1 to x_2 and then back to x_1 , zigzag up to x_{2n-1} and then up to x_{2n} , and zigzag left to y_3 and then up to z and y_{2n} , as shown in Figure 2. Formally,

$$y_1 - x_2 - x_1 - y_2 - \dots - x_{2n-1} - x_{2n} - y_{2n-1} - x_{2n-2} - \dots - y_3 - z - y_{2n}. \quad (2)$$

Case 3. $A \in X$ and $B \in Y$.

Without loss of generality, let $A = x_1$ and $B = y_{2n}$. Since G is HC, there exists a Hamilton path P connecting v_1 and v_{2n} in G . We are able to find a Hamilton path joining A and B : zigzag up from x_1 to x_{2n-1} and then up to x_{2n} , and zigzag left to y_1 , and then reach z and y_{2n} , as shown in Figure 3. Formally, it is

$$x_1 - y_2 - x_3 - \dots - x_{2n-1} - x_{2n} - y_{2n-1} - \dots - y_1 - z - y_{2n}. \quad (3)$$

Case 4. $A \in Y$ and $B = z$.

Without loss of generality, let $A = y_1$. Since G is HC, G has a Hamilton cycle C . Label the vertices of C as $v_1 v_2, \dots, v_{2n} v_1$. We are able to find a Hamilton path joining A and B : zigzag from y_1 to x_{2n} and then go to x_1 , and then zigzag right to y_{2n} , and finish at z , as shown in Figure 4. Formally, it is

$$y_1 - x_2 - y_3 - \dots - x_{2n} - x_1 - y_2 - \dots - y_{2n} - z. \quad (4)$$

Case 5. $A \in X$ and $B = z$.

Without loss of generality, let $A = x_1$. Let H be a Mycielski factor of G starting at v_1 , which consists of an even number of odd cycles $C_1, \dots, C_{2s}$ (possibly $s = 0$) and an even cycle C_{2s+1} with the chord (possibly empty), joined by $2s$ edges $e_1, \dots, e_{2s}$, where $e_i = v_i v_{i+1}'$ for each $i \in \{1, \dots, 2s\}$ such that $v_i v_i' \in V(C_i)$ for each $i \in \{1, \dots, 2s+1\}$, and the chord joins v_{2s+1}' and a vertex at distance even on C_{2s+1} .

If $s \geq 1$, for every integer $i \in \{1, \dots, 2s\}$, label the vertices of C_i in the clockwise order $u_1 u_2, \dots, u_{2k+1}$. One can find a Hamilton path P_i of $\mu(C_i)$ as follows:

$$x_1 y_2 x_3 y_4, \dots, x_{2k+1} y_1 x_2 y_3 x_4, \dots, y_{2k+1}, \quad (5)$$

where $u_1 = v_i$ and $u_{2k+1} = v_i'$.

Let $C_{2s+1} = w_1 w_2 \dots w_{2l}$ be an even cycle with chord $w_{2l} w_{2t}$ in H . One can find a Hamilton path P_{2s+1} of $\mu(C_i)$ as follows:

$$x_1 y_2 x_3 y_4, \dots, y_{2l}, x_{2t} y_{2t+1} x_{2t+2}, \dots, x_{2l}, y_1 x_2 y_3 x_4, \dots, y_{2t-1}, \quad (6)$$

where $w_1 = v_i$ and $w_{2l} = v_i'$.

Thus, $(\cup_{i=1}^{2s+1} P_i) \cup \{v_i v_i' : 1 \leq i \leq 2s\} \cup \{y_{2t-1} z\}$ is a Hamilton path of $\mu(G)$ from joining x_1 and z .

3. Hamiltonian Connectedness

Theorem 4. Assume that G is a Hamilton-connected graph of order $n \geq 3$. If $\delta(G) \geq (n/2) + 1$, then $\mu(G)$ is Hamilton-connected.

Proof. Let v be a vertex of G . We consider a Hamilton cycle C of G . Let u be a neighbor of v on C . Since $d(u) \geq (n/2) + 1$, it has a neighbor at distance even on C . By Lemma 1, $\mu(G)$ is HC.

The k th power of a graph G , denoted by G^k , is a graph with the same vertex set as G in which two vertices are adjacent if and only if their distance in G is at most k . Thus,

FIGURE 1: A Hamilton path from A to B.

FIGURE 2: A Hamilton path from A to B.

$G^1 = G$. We need the following result due to Karaganis [18]. $\square$

Theorem 5 (see Karaganis [18]). *The cube G^3 of every connected graph G of order $n \geq 3$ is Hamilton-connected.*

Theorem 6. *For any connected graph G of order $n \geq 3$, $\mu(G^3)$ is Hamilton-connected.*

Proof. By Theorem 5, G^3 is HC for G . Since $\mu(H^3)$ is a spanning subgraph of $\mu(G^3)$ for any spanning graph H of G , to show $\mu(G^3)$ is HC, it suffices to show that $\mu(T^3)$ is HC for any tree T of order $n \geq 3$. Since T^3 is HC, by Theorem 3 (1), we may assume that n is even. By Lemma 1, it remains to show that T^3 has a Mycielski factor starting from each vertex $v \in V(T)$.

Let w be a neighbor of v in T , and let T_v and T_w be the components of $T - vw$ containing v and w , respectively. Let n_v and n_w be the order of T_v and T_w , respectively. Let v' be a neighbor of v in T_v , and let w' be a neighbor of w in T_w .

Case 1: both n_v and n_w are at least 3.

Subcase 1.1: both n_v and n_w are odd.

By Theorem 5, both T_v^3 and T_w^3 are HC. Let C_v and C_w be Hamilton cycles of T_v and T_w , respectively. One can see that $C_v \cup C_w + v'w$ is a Mycielski factor of T^3 starting at v .

Subcase 1.2: both n_v and n_w are even.

By the induction hypothesis, T_v^3 has a Hamilton path $P_{vv'}$ joining v and v' , and T_w^3 has a Hamilton path $P_{ww'}$ joining w and w' . One can see that $P_{vv'} \cup P_{ww'} + vw + v'w' + v'w$ is a Mycielski factor of T^3 starting at v .

Case 2: $\min\{n_v, n_w\} \leq 2$.

Subcase 2.1: $\min\{n_v, n_w\} = n_v \leq 2$.

Subcase 2.1.1: $\min\{n_v, n_w\} = n_v = 1$.

Since $n_v + n_w = n$ is an even number at least 3, n_w is an odd number at least 3. By Theorem 5, let $C_{ww'}$ be a Hamilton cycle of T_w^3 containing ww' . It is easy to see that $C_{ww'} + vw + v'w'$ is a Mycielski factor of T^3 starting at v .

Subcase 2.1.2: $\min\{n_v, n_w\} = n_v = 2$.

If $n_w = 2$, then $n = 4$. Trivially, $T_w^3 = K_4$ has a Mycielski factor starting at v .

If $n_w \neq 2$, then n_w is an even number at least 4. By Theorem 5, let $P_{ww'}$ be a Hamilton path of T_w^3 . It can be seen that $P_{ww'} + vw + w'v' + v'w$ is a Mycielski factor starting at v .

Subcase 2.2: $\min\{n_v, n_w\} = n_w \leq 2$.

If $T \cong K_{1,n-1}$, then $T^3 \cong K_n$ has a Mycielski factor starting at v . Next, we assume that $T \not\cong K_{1,n-1}$. We can choose a neighbor w of v such that $n_w \geq 2$. Combining with our assumption that $\min\{n_v, n_w\} = n_w \leq 2$, we have $n_w = 2$. By Theorem 5, let $P_{vv'}$ be a Hamilton path of T_v^3 . It can be checked that $P_{vv'} + vw + ww' + v'w' + v'w$ is a Mycielski factor starting at v .

In 1969, Watkins [19] introduced the notion of the generalized Petersen graph $GP(n, k)$, $1 \leq k \leq n$, as follows. The vertex set is $\{u_i, v_i; 1 \leq i \leq n\}$, and the edge set is $\{u_i u_{i+1}, v_i v_{i+k}, u_i v_i\}$, where the subscript arithmetic performs modulo n using the residues $1, \dots, n$. In 1971, Frucht et al. [20] showed that $GP(n, k)$ is vertex-transitive if and only if $k^2 \equiv \pm 1 \pmod{n}$ or $(n, k) = (10, 2)$. Next, we consider the Hamilton-connectedness of the Mycielski graph of the generalized Petersen graphs $GP(n, 2)$ and $GP(n, 3)$. $\square$

Theorem 7 (see Alspach and Liu [21]). *The generalized Petersen graph $GP(n, 2)$ with $n \geq 6$ is Hamilton-connected if and only if $n \equiv 1, 2, 3 \pmod{6}$.*

Theorem 8. *If $GP(n, 2)$ is Hamilton-connected for $n \geq 6$, then $\mu((GP(n, 2)))$ is Hamilton-connected.*

Proof. In view of Lemma 1, it suffices to show that $GP(n, 2)$ has a Mycielski factor starting at any $v \in V(GP(n, 2))$. We consider two cases:

Case 1: $n \equiv 1$ or $3 \pmod{6}$.

Since n is odd, $GP(n, 2)$ is vertex-transitive, and we may assume that $v = u_1$, without loss of generality. Let C_1 and C_2 be the outer cycle and inner cycle of $GP(n, 2)$. Let v be a vertex of $GP(n, 2)$. It is clear that $C_1 \cup C_2 + u_2 v_2$ is a Mycielski factor of $GP(n, 2)$ starting at v .

Case 2: $n \equiv 2 \pmod{6}$.

By the symmetry, it suffices to tackle two possibilities according to the location of v in $GP(n, 2)$: v lies on the outer cycle or inner cycle of $GP(n, 2)$. Without loss of generality, let $v = u_1$ or $v = v_1$.

First, for the case when $n = 8$, we can find a Mycielski factor F_n of $GP(n, 2)$ as follows:

FIGURE 3: A Hamilton path from A to B.

FIGURE 4: A Hamilton path from Y to z.

$$F_8 = \begin{cases} C_8 + u_2u_3 & \text{if } v = u_1, \\ C_8 + u_1u_2 & \text{if } v = v_1, \end{cases} \quad (7)$$

where

$$C_8 = \begin{cases} u_1v_1v_3u_3u_4v_4v_6u_6u_5v_5v_7u_7u_8v_8v_2u_2 & \text{if } v = u_1, \\ v_1v_3u_3u_2v_2v_4u_4u_5v_5v_7u_7u_6v_6v_8u_8u_1 & \text{if } v = v_1. \end{cases} \quad (8)$$

For $n = 14$, by inserting 12 new vertices to C_8 of F_8 , we get C_{14} as illustrated in Figures 5–7 for the case that v lies in the outer cycle and for the case that v lies in the inner cycle as illustrated in Figures 6, 8, and 9. For the case when $n \geq 20$, by inserting 12 new vertices to F_{n-6} with type A insertion, we obtain a Mycielski factor F_n of $GP(n, 2)$ starting at v . $\square$

Theorem 9 (see Alspach and Liu [21]). *The generalized Petersen graph $GP(n, 3)$ with $n \geq 6$ is Hamilton-connected if and only if n is odd.*

Theorem 10. *If $GP(n, 3)$ is Hamilton-connected, then $\mu(GP(n, 3))$ is Hamilton-connected.*

Proof. Since $GP(n, 3)$ is Hamilton-connected, by Theorem 9, n is an odd number at least 7. Let v be a vertex of $GP(n, 3)$. In view of Lemma 1, it suffices to show that $GP(n, 3)$ has a Mycielski factor starting at v . We consider two cases:

Case 1: $n \equiv 1$ or $5 \pmod{6}$.

Since n is odd, $GP(n, 3)$ is vertex-transitive; by the symmetry, we may assume that $v = u_1$, without loss of generality. Let C_1 and C_2 be the outer cycle and inner cycle of $GP(n, 3)$. It is clear that $C_1 \cup C_2 + u_2v_2$ is a Mycielski factor of $GP(n, 3)$ starting at v .

Case 2: $n \equiv 3 \pmod{6}$.

By the symmetry, it suffices to tackle two possibilities according to the location of v in $GP(n, 3)$: v lies on the outer cycle or inner cycle of $GP(n, 3)$. Without loss of generality, let $v = u_1$ or $v = v_1$.

First, for the case when $n = 9$, we can find a Mycielski factor F_n of $GP(n, 3)$ starting at v as follows:

$$F_9 = \begin{cases} C_9 + v_1v_7 & \text{if } v = u_1, \\ C_9 + u_1u_9 & \text{if } v = v_1, \end{cases} \quad (9)$$

where $C_9 = u_1u_2v_2v_5v_8u_8u_9v_9v_6v_3u_3u_4u_6u_7v_7v_4v_1$.

For $n = 15$, by inserting 12 new vertices to C_9 of F_9 , we get C_{15} as illustrated in Figures 10–12. For the case when $n \geq 21$, by inserting 12 new vertices to F_{n-6} with type B insertion, we obtain a Mycielski factor F_n of $GP(n, 3)$ starting at v . $\square$

4. Pancyclicity

In this section, we show that if a graph G is pancyclic, then $\mu(G)$ is also pancyclic.

Theorem 11. *If G is pancyclic, then $\mu(G)$ is pancyclic.*

Proof. Let G be a pancyclic graph of order n . Since $\mu(G)$ contains G as its subgraph, $\mu(G)$ contains a cycle of length l for each $l \in \{3, \dots, n\}$.

Now, we find a cycle of length $n + 1$ in $\mu(G)$. Take a cycle C of length $n - 2$ in G . Without loss of generality, let $P = v_1v_2, \dots, v_{n-2}$ be a path resulting from C deleting an edge. It can be seen that $x_1x_2, \dots, x_{n-2}y_1zy_{n-2}x_1$ is a cycle of length $n + 1$, as illustrated in Figure 13. In a similar way, one can find a cycle of length $n + 2$ in $\mu(G)$ in terms of a cycle of length $n - 1$ in G .

Next, we will find a cycle of length $n + k$ in $\mu(G)$ for each $k \in \{3, \dots, n + 1\}$. Take a Hamilton cycle C . Without loss of

FIGURE 5: F_8 from u_1 to u_2 in $GP(8, 2)$.

FIGURE 6: Type A insertion.

FIGURE 7: F_{14} obtained from F_8 by type A insertion in $GP(8, 2)$. FIGURE 8: F_8 from v_1 to u_1 in $GP(8, 2)$. FIGURE 9: F_{14} obtained from F_8 by type B insertion in $GP(8, 2)$.

generality, let $C = v_1 v_2, \dots, v_n v_1$ in G . We consider two cases according to the parity of k :

Case 1: k is odd.

One can find a cycle of length $n+k$ in $\mu(G)$, as shown in Figure 14. Formally, it is

$$x_1 - y_2 - x_3 - \dots - y_{k-1} - z - y_1 - \dots - x_{k-1} - x_k - \dots - x_n - x_1. \quad (10)$$

Case 2: k is even.

Zigzag up from x_1 to x_{k-1} and left to x_{k-2} , then zigzag left to y_1, z, y_{k-1} , and x_k , and go right to x_n and back to x_1 , as shown in Figure 15. Formally, it is

$$x_1 - \dots - x_{k-2} - x_{k-1} - y_{k-2} - \dots - y_1 - z - y_{k-1} - x_k - \dots - x_n - x_1. \quad (11)$$

□

FIGURE 10: F_9 from u_1 to v_1 in $GP(9,3)$. FIGURE 11: Type B insertion. FIGURE 12: F_{15} obtained from F_9 by type B insertion in $GP(9,3)$. FIGURE 13: Finding a cycle C_{n+1} in $\mu(G)$ from a cycle C_{n-2} in G . FIGURE 14: A cycle of length $n+k$ in $\mu(G)$ if k is odd. FIGURE 15: A cycle of length $n+k$ in $\mu(G)$ if k is even. $\square$

5. Conclusion

In this paper, we introduce the notion of the Mycielski factor of a graph. If a graph G has a Mycielski factor starting at v for any $v \in V(G)$, then $\mu(G)$ is Hamilton-connected. Applying this result, we are able to show that if a graph G belongs to three (well-defined) families of graphs, then $\mu(G)$ is Hamilton-connected. However, the full conjecture of Jarnicki, Myrvold, Saltzman, and Wagon is not yet solved. We also prove that if G is pancyclic, then $\mu(G)$ is pancyclic.

One of the reviewers proposed the following two interesting problems.

Zhong et al. [7] showed that the line graph of the generalized Petersen graph $GP(n, k)$ is always Hamilton-connected. Is it easy to show that the Mycielski graph of $L(GP(n, k))$ is Hamilton-connected?

It is known that the line graph of a Hamilton-connected graph G is also Hamilton-connected. Is $\mu(L(G))$ Hamilton-connected if $L(G)$ is Hamilton-connected? [22].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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