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Research Article

Integrated Optimization of Port Rotation Direction and Fleet Deployment for Container Liner Shipping Routes

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This paper provides an integrated planning methodology for the optimization of port rotation direction and fleet deployment for container liner shipping routes with consideration of demand uncertainty. We first consider a special case that demand is deterministic. A multicommodity flow network model is developed via minimizing the total network-wide cost. Its decisions are the selection of port rotation direction and fleet deployment and container routings in the shipping network. Afterward, we address the generic case that uncertain demand is considered, which is represented by potentially realizable demand scenarios. We develop a minimax regret model to procure the least maximum regret across all the demand scenarios. The proposed models are applied to an Asia-Europe-Oceania liner shipping network with 46 ports and 12 ship routes. Results could provide the liner company with a comprehensive decision tool to simultaneously determine port rotation direction and fleet deployment when tackling uncertain demand.

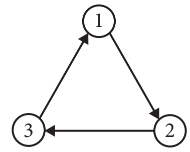
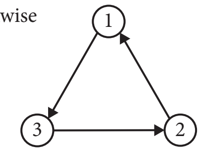
1. Introduction

Container transportation is a significant component of international transportation, which is vital to the sustainable development of international trade and global economy [1–8], among many others. In 2017, annual containerized trade volume was estimated at 1.83 billion tons [9]. Containers are transported by container liner shipping companies over their shipping networks. A shipping network operated by a particular liner company involves a number of weekly serviced ship routes. Each route is a sequence of port visits (port rotation) composing a directed loop in ways that ships visit the first port again after visiting the last one. A notable trait of shipping network is its prohibitively high operating cost. Hence, the liner company entails determining a set of efficient decisions so as to reduce the total operating cost. The decisions span the strategic, tactical, and operational stages, such as network design/alteration, fleet deployment, speed optimization, and schedule design [10].

Among these decisions, network design/alteration and fleet deployment are two important decisions at the strategic planning level. Network alteration is a special case of network design, which strives to retrofit the existing network via minor alterations (the essential difference between network alteration and network design is that network design aims at obtaining a newly designed network with minimized network-wide cost (the result may be quite different from the existing network) while network alteration attempts to enhance an existing network where the network should not deviate too much from the existing one). The alteration of port rotation direction of existing ship routes is one manner of network alteration. Fleet deployment determines which type of ship to deploy on each ship route. The liner company can benefit from economies of scale by deploying ships with large capacity. Yet, larger ships signify higher chartering and voyage costs which may be not appropriate if demand is insufficient. At the same time, future demand is not available to be predicted accurately, which is reliant on several influencing factors [11].

Uncertainty in demand complicates the analysis of the impact of port rotation direction and fleet deployment on the shipping network operation. Table 1 shows a simple

Table 1: Impact of port rotation direction and ship capacity.

O-D Demand (TEU/week) 1→3: q 2→1: q 3→2: q		Clockwise direction				Counter-clockwise direction			
Ship capacity (TEU)		2000	5000	8000	10000	2000	5000	8000	10000
Scenario I ($q = 2000$)	q_1	1000	2000	2000	2000	2000	2000	2000	2000
	q_2	1000	0	0	0	0	0	0	0
Scenario II ($q = 5000$)	q_1	1000	2500	4000	5000	2000	5000	5000	5000
	q_2	4000	2500	1000	0	3000	0	0	0
Scenario III ($q = 9000$)	q_1	1000	2500	4000	5000	2000	5000	8000	9000
	q_2	8000	6500	5000	4000	7000	4000	1000	0

Note: q_1 represents the number of containers transported by this route service for each of three O-D pairs (TEU/week); q_2 represents the number of containers transported by slot-purchasing for each of three O-D pairs (TEU/week).

network containing a sole ship route. The route visits three ports with either clockwise or counterclockwise direction. There are four types of ships whose capacities are 2000 twenty-foot equivalent units (TEU), 5000 TEU, 8000 TEU, and 10000 TEU, respectively. Under three demand scenarios, the maximum number of containers transported by the ship route itself is quite different when selecting different port rotation directions and ship types. For instance, in scenario II, if clockwise direction and 2000-TEU ship type are chosen, the maximum number of containers for each O-D pair is 1000 TEU/week. In this case, the liner company needs to purchase ship slots from other companies in order to fulfill the remaining demand. If the company selects counterclockwise direction and 5000-TEU ships, all the demand of three O-D pairs can be transported by its own ship route. In addition, the transit time and associated inventory cost of containers from origin to destination may also be various, which will cause an effect on competitiveness and market share. Therefore, the problem of determining port rotation direction and fleet deployment is not trivial especially when uncertain demand is taken into account. This study investigates the integrated optimization problem with respect to these two strategic decisions for container liner shipping routes.

1.1. Literature Review. For a liner company, the design of container liner shipping service at the strategic stage mainly subsumes network design/alteration and fleet deployment. For decades, a number of previous studies have been conducted on network design (e.g., [6, 8, 12–17]). Liner shipping network design problem is defined as follows: given a set of ports and a group of origin-destination demand pairs, network design determines the fixed cyclic itinerary of ship routes (i.e., which ports each route service should visit and in what order) such that the network-wide cost is minimized. For instance, Agarwal and Ergun [18] developed a space-time network model for liner shipping network design with cargo routing. Yet, their model did not consider transshipment costs. Meng and Wang [19] proposed a network design model incorporating hub-and-spoke structure and multiple port-calling operations. Brouer et al. [20] proved the liner shipping network design problem to be strongly NP-hard and contributed a seminal benchmark suite for global network design based on the data from the largest liner shipping company in the world (i.e., Maersk Line). In additional, some countries prohibit foreign carriers to ship cargo between two ports within the country as well as other limitations so as to protect the national trade business. Zheng et al. [21] developed a hub-and-spoke network design model considering the effect of cabotage rules. Regarding various model formulations and solution methods, we refer the readers to Brouer et al. [20]; Tran and Haasis [5] and Christiansen et al. [2] for a comprehensive review of liner shipping network design.

In reality, for an existing shipping network, the liner company can hardly reshuffle its network overnight. As a compromise, network alteration aims to improve the existing network through making minor alterations [22]. Current, studies associated with network alteration are limited, which can be divided into two patterns: (i) the disassembly and reassembly of routes with the homogeneous type of ship and (ii) the alteration of port rotation directions [23]. For example, Chen et al. [23] proposed a mixed-integer programming model to obtain the optimal port rotation directions of ship routes in a given shipping network.

There are numerous studies that have been dedicated to the optimization of fleet deployment. Relevant studies can be separated into two categories. The first category assumes that container shipment demand is known with complete certainty. For instance, Gelareh and Meng [24] developed a mixed integer nonlinear programming model for a short-turn fleet deployment problem, in which the optimal vessel speeds for different vessel types on different routes were considered. Liu et al. [25] proposed two models associated with fleet deployment and container flow management. Results show that the joint optimization model outperforms the sequential model in terms of improving ship capacity utilization. The second category relaxes the deterministic demand assumption and addresses the fleet deployment problem with uncertain demand. For instance, Meng and Wang [26] proposed a space-time network approach to address the practical ship fleet problem under the background of week-dependent demand. In the model of Ng [27], dependencies between shipment demands on different routes were incorporated. Some other studies investigated the fleet deployment problem in conjunction with other decisions, such as network design, frequency setting, and speed optimization (e.g., [28–33]). Readers are referred to Ng [34] for the exposition of a class of fleet deployment models.

Uncertain demand can be expressed in numerous avenues. A viable way is to generate a set of potentially realizable demand scenarios based on historical demand data [35]. The liner company needs to make decisions in advance that will perform adequately under any likely to occur scenario. In other words, uncertain demand is incorporated via making it part of the decision making reasoning. Therefore, the impact of port rotation direction and fleet deployment on the shipping network operation gets more complex in the presence of uncertain demand. Numerous previous studies focus on liner shipping network design and fleet deployment, while studies of network alteration are limited. The alteration of port rotation directions is one pattern of network alteration. The mutual effect of two strategic decisions (port rotation direction and fleet deployment) gives rise to apparent influence as preliminarily described in Table 1. Nevertheless, to the authors' knowledge, formulations that thoroughly discuss the integrated optimization of port rotation direction and fleet deployment have not been presented in the literature. This study aims to remedy the above gap in which uncertain demand is taken into account as well.

1.2. Objective and Contribution. The primary objective and contribution of this study is to propose two network-level models for the integrated optimization of port rotation direction and fleet deployment. In our formulations, the first model deals with the special case that demand is assumed to be deterministic. We develop a multicommodity flow network model. The objective is to minimize the total network-wide cost, which is made up of loading and discharge cost, transshipment cost, inventory cost, slot-purchasing cost, and deployment cost. The decision variables subsume the port rotation direction, fleet deployment, and container routings of all O-D pairs. The second model addresses the generic case that deterministic demand assumption is relaxed and uncertain demand is taken into account. We develop a minimax regret model, whose objective is to procure the least maximum regret across all the potentially realizable demand scenarios. The proposed two models are applied to an Asia-Europe-Oceania container liner shipping network with 46 ports and 12 ship routes.

The remainder of this paper is organized as follows. Section 2 describes the problem and formulates two network-level models. A numerical example is presented in Section 3. Finally, conclusions are provided in Section 4.

2. Problem Statement and Model Development

Consider a liner company which operates a number of ship routes, denoted by the set R. Each ship route $r \in R$ maintains weekly service frequency. The company regularly serves a group of ports denoted by the set P. Let p_{ri} denote the port corresponding to the ith port of call on route r. The port rotation of ship route r composes a directed loop, which is expressed as

$$p_{r1} \longrightarrow p_{r2} \longrightarrow \cdots \longrightarrow p_{rN_r} \longrightarrow p_{r1}, \quad (1)$$

where N_r denotes the number of ports of call on ship route r. Define $I_r := \{1, 2, \ldots, N_r\}$ and $p_{r,N_r+1} := p_{r1}$. The voyage from port of call i to port of call $i+1$ is called leg i, and leg N_r is the voyage from port of call N_r to the first port of call. Figure 1 presents an illustrative liner service network, which contains three ship routes denoted by $R = \{1, 2, 3\}$ and seven ports denoted by $P = \{\text{HK, JK, SG, XM, CB, CN, CC}\}$. Route 1 has three legs, route 2 has five legs, and route 3 has three legs.

Define x_r^1 $(r \in R)$ as a binary decision variable which equals 1 if the port rotation direction of ship route r is reversed and 0 otherwise. Let V be the set of ship types. Ships of the same type in each route $r \in R$ are assumed to be homogeneous in their capacity, deployment cost, and other ship-specific characteristics. Let E_v (TEU) denote the capacity of ship type $v \in V$. The deployment cost of ship route $r \in R$ per week is C_{rv} if ship type $v \in V$ is selected. We define x_{rv}^2 $(r \in R, v \in V)$ as a binary decision variable which equals 1 if ships of type v are deployed on route r and 0 otherwise. Let vector $\mathbf{x} := \{x_r^1, x_{rv}^2 \mid r \in R, v \in V\}$ be the integrating decisions on port rotation direction and fleet deployment.

Represent by W the set of O-D pairs, $W = P \times P$. Based on historical demand data, the liner shipping company is available to generate a set of potentially realizable demand scenarios (denoted by Θ). Uncertain demand is represented by these generated demand scenarios. Under scenario $\omega \in \Theta$, the demand for O-D pair $(o, d) \in W$ is designated by q_ω^{od} (TEU). Decision $\mathbf{x}$ needs to be made before it is known which scenario is realized.

Let $\widehat{c}_p$ and $\widetilde{c}_p$ (USD/TEU) denote the container loading cost and discharge cost charged by port $p \in P$, respectively. If there is no direct service between some origins and destinations, containers can be transshipped, and the transshipment cost at port $p \in P$ is denoted by $\overline{c}_p$ (USD/TEU). An additional time yields due to transshipment operations at port p, which is called the connection time, denoted by t_p (h). In this study, we make the simplifying assumption that the connection time t_p at each port $p \in P$ is a fixed number. We let t_{ri} denote the transit time of containers on leg i of route r $(i \in I_r, r \in R)$. The inventory cost rate associated with the transit time of containers is denoted by α (USD/TEU/h).

Furthermore, if the liner shipping company cannot transport all the containers by its own ships, it may purchase ship slots from other shipping companies. Let g^{od} (USD/TEU) denote the cost for purchasing one slot for O-D pair $(o, d) \in W$. We do not consider empty containers for simplicity [36–38]. At the same time, we let T^{od} denote the transit time of containers of O-D pair (o, d) transported by the purchased slots for formulating the inventory cost.

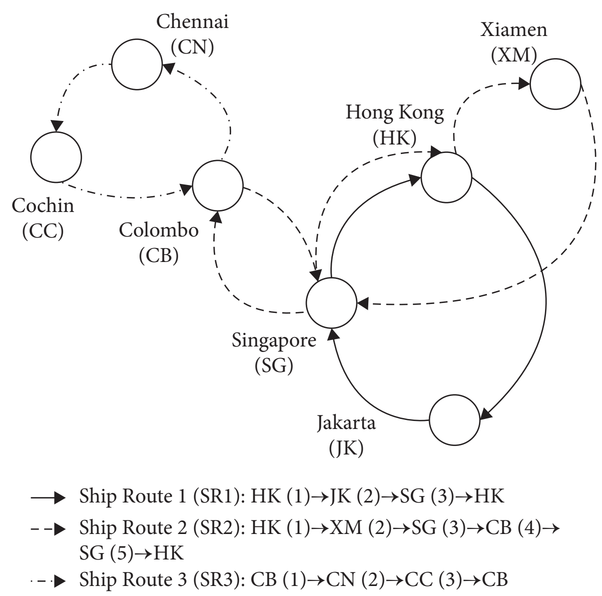


Figure 1: A liner shipping network with three ship routes [31].

2.1. Model with Deterministic Demand. We first formulate a special model that the actual scenario realization $\omega \in \Theta$ is assumed to be already known. Then, the problem is to obtain the integrating decision $\mathbf{x}$ that minimizes the total network-wide cost when demand is deterministic.

2.1.1. Multicommodity Flow Network Representation. In this study, the shipping network is converted into a multicommodity flow (MCF) network, which is an extension of Wang and Meng [22]. Let $G = (N, A)$ represent the MCF network, where N is the set of nodes and A is the set of arcs in the network. In general, the definition of MCF network has two main steps. The first step is to define the topological structure (containing nodes and arcs) that can reflect the transport of container operations in this study. There are three categories of nodes, including source node, sink node and portcall node. There are six categories of arcs, including source arc, sink arc, original voyage arc, reversed voyage arc, transshipment arc, and slot-purchasing arc. The second step is to define the attributes of arcs (i.e., cost and capacity) as described in Table 2. Two steps are specifically described as follows.

Set N is composed of three categories of nodes. First, for each port $p \in P$, construct one source node (denoted by n_p^{src}) and one sink node (denoted by n_p^{sink}). Let N^{src} and N^{sink} denote the set of newly constructed source nodes and sink nodes, respectively. The third category of nodes is termed as calls at a port. For each ship route $r \in R$, construct one portcall node for each port of call $i \in I_r$, and all these nodes constitute the set N^{call}.

Arcs in set A are directed, which can be grouped into six categories:

(i) Source arc set A^{src}: construct an arc from n_p^{src} to each of nodes in N^{call} that represents calls at port $p \in P$, and all the newly constructed source arcs form the set A^{src}.

(ii) Sink arc set A^{sink}: construct an arc from each of nodes in N^{call} that represents calls at port $p \in P$ to n_p^{sink}, and all the newly constructed sink arcs form the set A^{sink};

(iii) Original voyage arc set A^{voy0}: construct a voyage arc for each voyage leg with original direction, and all the original voyage arcs form the set A^{voy0}.

(iv) Reversed voyage arc set A^{voy1}: construct a voyage arc for each voyage leg with reversed direction, and all the reversed voyage arcs form the set A^{voy1}.

(v) Transshipment arc set A^{tran}: if port $p \in P$ is visited more than once, construct two opposite-direction arcs between any two nodes in N^{call} that correspond to port p. All the transshipment arcs form the set A^{tran}.

(vi) Slot-purchasing arc set A^{slot}: for each O-D pair $(o, d) \in W$, construct an arc from n_o^{src} to n_d^{sink}, and all the slot-purchasing arcs form the set A^{slot}

The liner shipping network, set out in Figure 1, is taken as the example. Suppose that there is a single O-D pair, $W = \{(\text{JK}, \text{CB})\}$. Figure 2 exhibits the associated MCF network representation of this network. First, construct a source node (representing Jakarta) and a sink node (representing Colombo). The setting of the source/sink nodes enables that the containers of each O-D pair always have a unique origin and destination. Each port of call on three ship routes is indexed by a portcall node. Note that one physical port may correspond to more than one portcall node, such as Colombo which is visited twice per week.

Second, construct source arcs from the source node to all the portcall nodes that represents port Jakarta (arc 1). Construct sink arcs from portcall nodes that signify port Colombo to the sink node (arc 34 and 35). Add original voyage arcs corresponding to all the legs in the network, for example, arc 2, 3, and 4 of ship route 1. Add reversed voyage arcs to represent the voyage of ships on the legs with reversed direction, for example, arc 5, 6, and 7 of ship route 1. Construct transshipment arcs on behalf of the container transshipment operations. For instance, arc 28 means the transshipment at Singapore from the ship visiting Singapore as the third port of call on ship route 1, to the other ship visiting Singapore as the third port of call on ship route 2. Construct a slot-purchasing arc linking the source node to the sink node (arc 36).

The cost and capacity pertinent to each category of arcs in set A is summarized in Table 2. At the same time, the set of O-D pairs in the MCF network can be represented by $\widehat{W} = \{(n_o^{\text{src}}, n_d^{\text{sink}}) \mid (o, d) \in W\} \subset N^{\text{src}} \times N^{\text{sink}}$. Under scenario $\omega \in \Theta$, the demand of O-D pair $(n_o^{\text{src}}, n_d^{\text{sink}})$ in set $\widehat{W}$ is expressed as $\widehat{q}_\omega^{n_o^{\text{src}}, n_d^{\text{sink}}} = q_\omega^{od}$. For two arbitrary nodes $n, n' \in N$, define $\widehat{q}_\omega^{n,n'} = 0$ if $(n, n') \notin \widehat{W}$.

2.1.2. Model Formulation of MCF Network. To obtain the network-wide cost, we entail determining the optimal container routings of all O-D pairs in the MCF network. To this end, we define $f_{mn}^{o,\omega}$ as a continuous decision variable signifying the number of containers (TEU) that originate

Table 2: Cost and capacity of arcs in the MCF network.

Type of arc	Cost $C_{m,n}$ (USD/TEU)	Capacity $E_{m,n}$ (TEU)	Arc $(m,n) \in A$
Source arc (A^{src})	$\widehat{c}_p$	∞	$m = n_p^{\mathrm{src}}, n = i, p_{ri=p}, p \in P, i \in I_r, r \in R$
Sink arc (A^{sink})	$\overline{c}_p$	∞	$m = i, n = n_p^{\mathrm{sink}}, p_{ri} = p, p \in P, i \in I_r, r \in R$
Original voyage arc (A^{voy0})	αt_{rm}	$\sum_{v \in V} E_v (1 - x_r^1) x_{rv}^2$	$\begin{cases} m = i, n = i+1, i \in I_r \backslash \{\mathrm{N}_r\}, r \in R \\ m = i, n = 1, n = N_r, r \in R \end{cases}$
Reversed voyage arc (A^{voy1})	αt_{rn}	$\sum_{v \in V} E_v x_r^1 x_{rv}^2$	$\begin{cases} m = i, n = i-1, i \in I_r \backslash \{1\}, r \in R \\ m = i, n = N_r, i = 1, r \in R \end{cases}$
Transshipment arc (A^{tran})	$\overline{c}_p + \alpha t_p$	∞	$m = i, n = i\prime, p_{ri} = p_{r'i\prime} = p, p \in P,$ $i \in I_r, i\prime \in I_{r'}, r, r' \in R,$
Slot-purchasing arc (A^{slot})	$g^{od} + \alpha T^{od}$	∞	$m = n_o^{src}, n = n_d^{\mathrm{sink}}, (o,d) \in W$

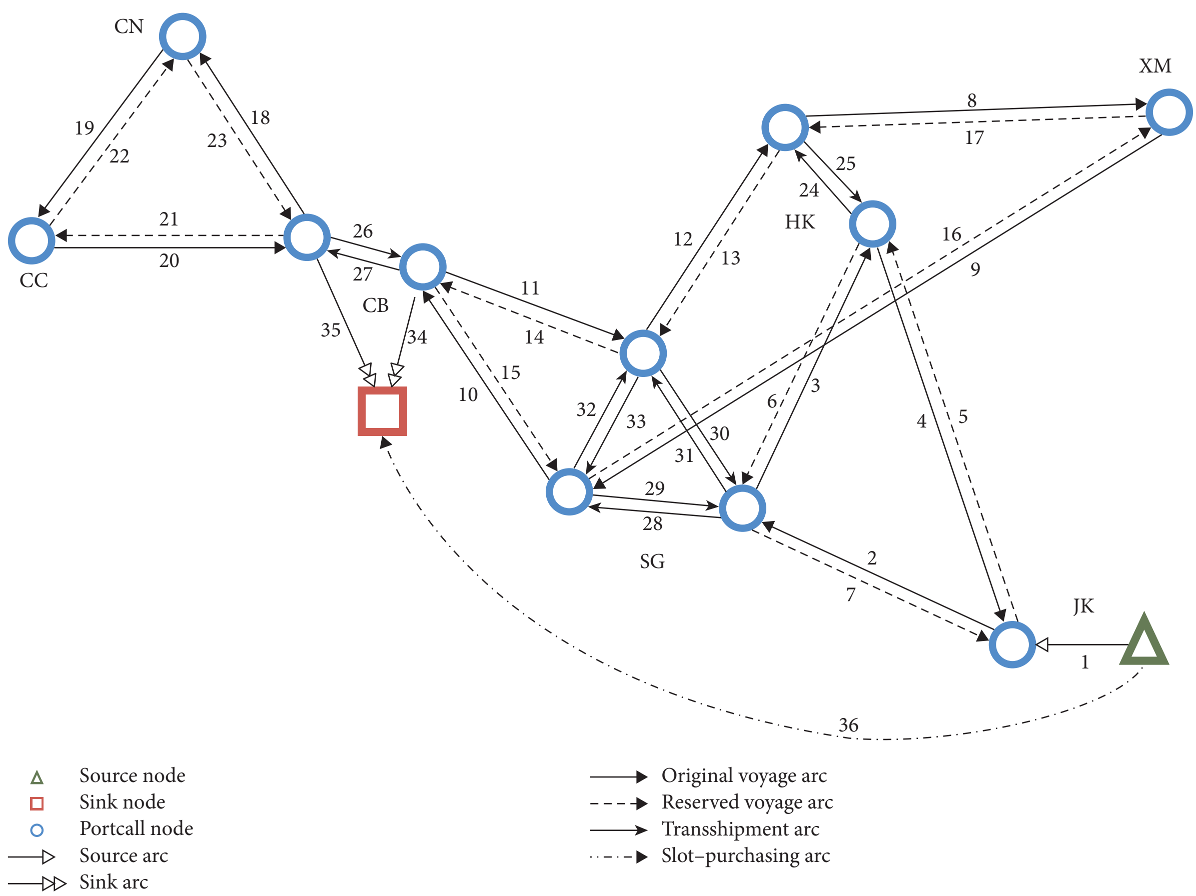


Figure 2: An illustrative O-D pair in the MCF network.

from port $o \in N^{\mathrm{src}}$ and flow on arc $(m,n) \in A$ under demand scenario $\omega \in \Theta$. The MCF network model with deterministic demand is formulated as follows:

$$[\mathrm{MCF} - \omega] Z(\omega) = \min_{x, f_{mn}^{o,\omega}} \sum_{(m,n) \in A} c_{mn} \sum_{o \in N^{src}} f_{mn}^{o,\omega} + \sum_{r \in R} \sum_{v \in V} C_{rv} x_{rv}^2, \tag{2}$$

subject to

$$\sum_{(m,n) \in A} f_{mn}^{o,\omega} - \sum_{(n,m) \in A} f_{nm}^{o,\omega} = \begin{cases} \widehat{q}_{\omega}^{on}, & n \neq o, \\ -\sum_{d \in N^{\sin k}} \widehat{q}_{\omega}^{od}, & n = o \end{cases} \quad n \in N, o \in N^{\mathrm{src}}, \tag{3}$$

$$\sum_{o \in N^{\mathrm{src}}} f_{mn}^{o,\omega} \leq \sum_{v \in V} E_v \left(1 - x_r^1\right) x_{rv}^2, \quad (m,n) \in A^{\mathrm{voy0}}, r \in R, \tag{4}$$

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le \sum_{v\in V} E_v x_r^1 x_{rv}^2, \quad (m,n) \in A^{\text{voy1}}, r \in R, \tag{5}$$

$$\sum_{v\in V} x_{rv}^2 = 1, \quad r \in R, \tag{6}$$

$$x_r^1 \in \{0,1\}, \quad r \in R, \tag{7}$$

$$x_{rv}^2 \in \{0,1\}, \quad v \in V, r \in R, \tag{8}$$

$$f_{mn}^{o,\omega} \ge 0, \quad (m,n) \in A, \quad o \in N^{\text{src}}. \tag{9}$$

The objective function, equation (2), minimizes the total network-wide cost under scenario $\omega \in \Theta$: the first term is the total cost on all the arcs in set A containing loading and discharge cost, transshipment cost, inventory cost, and slot-purchasing cost (c_{mn} denotes the cost of arc mn as depicted in Table 2), and the second term is the deployment cost of ship routes. Equation (3) is the container flow conservation equation. Equations (4) and (5) enforce the ship capacity constraint on original voyage arcs and reversed voyage arcs, respectively. Equation (6) imposes that exactly one type of ship is deployed on each ship route. Equations (7) and (8) indicate that x_r^1 and x_{rv}^2 are binary variables. Equation (9) is the nonnegativity constraint.

Equations (4) and (5) contain nonlinear terms $x_r^1 x_{rv}^2$. Based on equation (6), these two equations can be linearized as follows:

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le E_{v,\max}\left(1 - x_r^1\right), \quad (m,n) \in A^{\text{voy0}}, r \in R, \tag{10}$$

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le \sum_{v\in V} E_v x_{rv}^2, \quad (m,n) \in A^{\text{voy0}}, r \in R, \tag{11}$$

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le E_{v,\max} x_r^1, \quad (m,n) \in A^{\text{voy1}}, v \in V, r \in R, \tag{12}$$

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le \sum_{v\in V} E_v x_{rv}^2, \quad (m,n) \in A^{\text{voy1}}, r \in R, \tag{13}$$

where $E_{v,\max}$ denotes the maximum capacity of all ship types in set V, i.e. $\max_{v\in V}(E_v)$.

As a result, model $[\text{MCF} - \omega]$ is reformulated to a mixed-integer linear programming (MILP) model, which can be solved by off-the-shelf MILP solvers.

2.2. Model with Uncertain Demand. In practice, demand is usually uncertain, which can be reflected by a set of potentially realizable demand scenarios as stated. Considering this practical circumstance, it is important to grasp the performance of decision **x** across all the demand scenarios. A minimax regret model with uncertain demand (represented via scenario set Θ) is proposed next.

Under scenario $\omega \in \Theta$, $Z(\omega)$ is the optimal objective value of model $[\text{MCF} - \omega]$ in equation (2). Let $\mathbf{x}(\omega)$ denote the corresponding optimal port rotation direction and fleet deployment solution to model $[\text{MCF} - \omega]$. Furthermore, we let $Z(\mathbf{x},\omega)$ be the generalized network-wide cost under scenario $\omega \in \Theta$ for a given decision **x**. Then, $\max_{\omega\in\Theta}[Z(\mathbf{x},\omega) - Z(\omega)]$ is called the maximum absolute regret that exhibits the worst-case deviation from optimality, for the given decision **x** over all scenarios in set Θ.

The minimax regret model is to procure the optimal decision **x** with the least maximum regret:

$$[\text{MMR} - \Theta] \min_{x, f_{mn}^{o,\omega}} \max_{\omega\in\Theta} [Z(x,\omega) - Z(\omega)]. \tag{14}$$

For each scenario $\omega \in \Theta$, the value of $Z(\omega)$ can be achieved through solving model $[\text{MCF} - \omega]$. We intend to linearize the objective function equation (14) by introducing an auxiliary continuous variable (denoted by M). The minimax regret model $[\text{MMR} - \Theta]$ is reformulated as an equivalent MILP model:

$$[\text{MMR} - \Theta 2] \min_{x, f_{mn}^{o,\omega}} M, \tag{15}$$

subject to

$$M \ge \sum_{(m,n)\in A} c_{mn} \sum_{o\in N^{src}} f_{mn}^{o,\omega} + \sum_{r\in R}\sum_{v\in V} C_{rv} x_{rv}^2 - Z(\omega), \quad \omega \in \Theta, \tag{16}$$

$$\sum_{(m,n)\in A} f_{mn}^{o,\omega} - \sum_{(n,m)\in A} f_{nm}^{o,\omega} = \begin{cases} \widehat{q}_{\omega}^{on}, & n \neq o, \\ -\sum_{d\in N^{\text{sink}}} \widehat{q}_{\omega}^{od}, & n = o \end{cases}, \quad n \in N, \ o \in N^{src}, \omega \in \Theta, \tag{17}$$

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le E_{v,\max}\left(1 - x_r^1\right), \quad (m,n) \in A^{\text{voy0}}, r \in R, \omega \in \Theta, \tag{18}$$

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le \sum_{v\in V} E_v x_{rv}^2, \quad (m,n) \in A^{\text{voy0}}, r \in R, \omega \in \Theta, \tag{19}$$

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le E_{v,\max} x_r^1, \quad (m,n) \in A^{\text{voy1}}, v \in V, r \in R, \omega \in \Theta, \tag{20}$$

$$\sum_{o\in N^{src}} f_{mn}^{o,\omega} \le \sum_{v\in V} E_v x_{rv}^2, \quad (m,n) \in A^{\text{voy1}}, r \in R, \omega \in \Theta, \tag{21}$$

$$\sum_{v\in V} x_{rv}^2 = 1, \quad r \in R, \tag{22}$$

$$x_r^1 \in \{0,1\}, \quad r \in R, \tag{23}$$

$$x_{rv}^2 \in \{0,1\}, \quad v \in V, r \in R, \tag{24}$$

$$f_{mn}^{o,\omega} \ge 0, \quad (m,n) \in A, \ o \in N^{\text{src}}, \omega \in \Theta. \tag{25}$$

Equation (16) ensures that auxiliary variable M is not less than the maximum value of $Z(\mathbf{x},\omega) - Z(\omega)$ for all scenarios in set Θ, which is equivalent to the nonlinear expression in equations (14), and (17) guarantees conservation of flow in each scenario. Equations (18)–(21) are linearized expressions of ship capacity constraint. Equation (22) indicates that each ship route only employs a single ship type. Equations (23) and (24) present that x_r^1 and x_{rv}^2 are binary variables. Equation (25) signifies that continuous variables $f_{mn}^{o,\omega}$ are nonnegative.

3. Numerical Example

The proposed two models in Section 2 are applied to an Asia-Europe-Oceania shipping network of a global liner shipping company. This network consists of 46 ports, as shown in Figure 3. Twelve ship routes are operated in the network, and the port rotation of each ship route is described in Table 3. Three types of ships are considered: 3000 TEU, 5000 TEU, and 10000 TEU. Their corresponding deployment costs are \$76,900, \$115,400, and \$173,100 per week, respectively.

The parameter values used are summarized as follows: the container loading cost $\hat{c}_p$, discharge cost $\tilde{c}_p$, and the transshipment cost $\overline{c}_p$ are \$100, \$100, and \$150 per TEU, the unit inventory cost rate is $\alpha = 0.2$ USD/TEU/h, and the connection time is $t_p = 3.5$ days (84 h) for all the ports. The slot-purchasing cost g^{od} and the transit time T^{od} are assumed to be

$$\begin{aligned} g^{od} &:= 1000 + 0.2 \times \text{distance between the two ports (n mile)}, \quad \forall (o,d) \in W, \\ T^{od} &:= 7 \times 24 + \text{distance between the two ports (n mile)/15 knots}, \quad \forall (o,d) \in W. \end{aligned} \tag{26}$$

Both model [MCF $-$ ω] and model [MMR $-$ Θ2] can be efficiently solved using CPLEX of version 12.8 with default settings, running on a desktop with Intel Core Quad CPU Q9550 @ 2.83 GHz and 8.00 G RAM.

There are 652 O-D pairs with container shipment demand in the network. Three demand scenarios are generated based on the historical container slot booking data of three months in one quarter. Specifically, the data is ordered from smallest to largest according to the weekly demand of all the O-D pairs. In this study, we partition the data into three quantiles of (nearly) equal sizes, which are denoted by low-demand quantile, medium-demand quantile, and high-demand quantile, respectively. For each of three quantiles, the mean of demand of the O-D pairs is viewed as the corresponding demand scenario.

First, model [MCF $-$ ω] is solved for each demand scenario. Table 4 describes the optimal computational results of three demand scenarios. Results show that both the optimized port rotation direction and fleet deployment of twelve routes are significantly different under different demand scenarios. For example, in scenario I, the number of ship routes with reversed port rotation direction is 6 and ten routes are suggested to employ 3000-TEU ship type. Yet, in scenario II, there are only three ship routes that are recommended to reverse their port rotation direction (i.e., route 4, 10, and 11). Meantime, ship route 9 needs to utilize 10000-TEU ship type so as to satisfy all the O-D pairs in scenario II. Note that the slot purchasing cost equals 0 for three demand scenarios. It indicates that the liner company prefers to transporting all the demand by its own ship routes (even if deploying ships with larger capacity) rather than purchasing slots from other companies due to relatively high slot-purchasing cost per TEU.

Then, model [MMR $-$ Θ2] is solved to obtain the optimal network when three demand scenarios are simultaneously considered (see Table 5). The optimal decisions on port rotation direction and fleet deployment of twelve routes are not identical to either of three demand scenarios. Five routes are chosen to reverse their port rotation direction (i.e., route 1, 3, 4, 10, and 11). The number of ship routes that select 3000-TEU ship type, 5000-TEU ship type and 10000-TEU ship type are 7, 4 and 1, respectively.

Table 6 further compares the results of absolute regret pertinent to four various decisions **x**. The first three columns denotes the optimal **x** of model [MCF $-$ ω] for each demand scenario in Table 4. The last column denotes the optimal decision **x** of model [MMR $-$ Θ2] in Table 5. The last row presents the maximum regret across all three demand scenarios. Though $\mathbf{x}(\text{I})$, $\mathbf{x}(\text{II})$, and $\mathbf{x}(\text{III})$ is the optimal port rotation direction and fleet deployment decisions for the corresponding single demand scenario, their performance gets worse when any of the other two various scenarios occurs. For example, when $\mathbf{x}(\text{I})$ is applied to demand scenario III, the absolute regret attains \$ 10.53 million per week since about 6.4% demand needs to be transported through purchasing slots from other liner companies. Therefore, the issue of demand uncertainty cannot be neglected. Model [MMR $-$ Θ2] procures that the least maximum regret across all three scenarios is \$1.13 million per week. Furthermore, results in Table 6 show that the maximum regret of $\mathbf{x}(\text{I})$ (regarding the optimal decision of low-demand scenario) is apparently higher than the results of $\mathbf{x}(\text{II})$ and $\mathbf{x}(\text{III})$ (regarding the optimal decisions of medium-demand scenario and high-demand scenario, respectively). It indicates that deploying ships with relatively larger capacity seems to be a better choice for the liner company. Otherwise, purchasing slots from other liner companies will results in a significant increase of operating cost when confronting abruptly high demand.

In addition, we analyze the leg-based ship capacity utilization, which is termed as the number of containers on one leg divided by the associated route capacity. Figure 4 depicts the fluctuation of ship capacity utilization of 12 routes under three demand scenarios. Results exhibit that route 6 is the busiest ship route that the ship capacity utilization of the majority of its legs exceeds 50%. On the

Figure 3: Ports of the shipping network (source: [22]).

Table 3: Port rotation of twelve ship routes.

ID	Ports of call
1	Tokyo⟶Nagoya⟶Kobe⟶Kwangyang⟶Xingang⟶Shanghai⟶Yantian⟶Ho Chi Minh
2	Kaohsiung⟶ Hong Kong⟶Shanghai⟶Qingdao⟶Xingang⟶Dalian⟶Busan⟶Tokyo
3	Sydney⟶Xiamen⟶Shanghai⟶Dalian⟶Busan⟶Kobe⟶Yokohama
4	Kaohsiung⟶Hong Kong⟶Ningbo⟶Qingdao⟶Brisbane⟶Sydney⟶Melbourne
5	Fremantle⟶Sydney⟶Melbourne⟶Adelaide⟶Fremantle⟶Singapore
6	Jakarta⟶Ho Chi Minh⟶Laem Chabang⟶Singapore⟶Chittagong⟶Chennai⟶Colombo
7	Colombo⟶Cochin⟶Nhava Sheva⟶Karachi⟶Jebel Ali⟶Salalah
8	Singapore⟶Port Klang⟶Manila⟶Busan⟶Shanghai⟶Xiamen⟶Chiwan⟶Hong Kong
9	Salalah⟶Sokhna⟶Aqabah⟶Jeddah⟶Singapore
10	Southampton⟶Thamesport⟶Hamburg⟶Bremerhaven⟶Rotterdam⟶Antwerp⟶Zeebrugge⟶Le Havre
11	Antwerp⟶Rotterdam⟶Hamburg⟶Thamesport⟶Port Klang⟶Yantian⟶Ningbo⟶Shanghai⟶Dalian
12	Singapore⟶Rotterdam⟶Bremerhaven⟶Hamburg⟶Antwerp⟶Jebel Ali⟶Singapore⟶Hong Kong⟶ Ningbo⟶Busan⟶Manila

Table 4: Optimal results of model $[MCF - \omega]$ for each demand scenario.

Optimal Results	Demand scenario		
	Scenario I	Scenario II	Scenario III
Total demand (TEU)	22054	27040	33028
Total cost ($ millions/week)	15.05	18.08	21.61
Load/discharge cost	4.41	5.41	6.61
Inventory cost	2.59	3.16	3.92
Transshipment cost	3.59	4.38	5.37
Deployment cost	4.46	5.13	5.71
Slot-purchasing cost	0	0	0
	Port rotation direction		
Original direction Count route ID	**6** 2, 6, 7, 8, 9, 12	**9** 1, 2, 3, 4, 5,6, 7, 8, 9, 12	**7** 1, 2, 5, 6,7, 8, 12
Reversed direction Count route ID	**6** 1, 3, 4, 5, 10, 11	**3** 4, 10, 11	**5** 3, 4, 9, 10, 11
	Fleet deployment		
3000-TEU ship Count route ID	**10** 1, 2, 3, 4, 5, 6,7, 10, 11, 12	**9** 1, 2, 3, 4, 5,6, 7, 10, 12	**6** 1, 2, 3, 4, 5,7
5000-TEU ship Count route ID	**2** 8, 9	**2** 8, 11	**5** 6, 8, 10, 11,12
10000-TEU ship Count route ID	**0**	**1** 9	**1** 9

Table 5: The optimal network of model [MMR – Θ2].

ID	Ship type	Ports of call
1	3000TEU	Tokyo⟶Ho Chi Minh⟶Yantian⟶Shanghai⟶Xingang⟶Kwangyang⟶Kobe⟶Nagoya
2	3000TEU	Kaohsiung⟶Hong Kong⟶Shanghai⟶Qingdao⟶Xingang⟶Dalian⟶Busan⟶Tokyo
3	3000TEU	Sydney⟶Yokohama⟶Kobe⟶Busan⟶Dalian⟶Shanghai⟶Xiamen
4	3000TEU	Kaohsiung⟶Hong Kong⟶Ningbo⟶Qingdao⟶Brisbane⟶Sydney⟶Melbourne
5	3000TEU	Fremantle⟶Sydney⟶Melbourne⟶Adelaide⟶Fremantle⟶Singapore
6	3000TEU	Jakarta⟶Ho Chi Minh⟶Laem Chabang⟶Singapore⟶Chittagong⟶Chennai⟶Colombo
7	3000TEU	Colombo⟶Cochin⟶Nhava Sheva⟶Karachi⟶Jebel Ali⟶Salalah
8	5000TEU	Singapore⟶Port Klang⟶Manila⟶Busan⟶Shanghai⟶Xiamen⟶Chiwan⟶Hong Kong
9	10000TEU	Salalah⟶Sokhna⟶Aqabah⟶Jeddah⟶Singapore
10	5000TEU	Southampton⟶Le Havre⟶Zeebrugge⟶Antwerp⟶Rotterdam⟶Bremerhaven⟶Hamburg⟶Thamesport
11	5000TEU	Antwerp⟶Dalian⟶Shanghai⟶Ningbo⟶Yantian⟶Port Klang⟶Thamesport⟶Hamburg⟶Rotterdam
12	5000TEU	Singapore⟶Rotterdam⟶Bremerhaven⟶Hamburg⟶Antwerp⟶Jebel Ali⟶Singapore⟶Hong Kong⟶Ningbo⟶Busan⟶Manila

Table 6: Results of absolute regret (unit: $ million/week).

	$\mathbf{x}(\mathrm{I})$	$\mathbf{x}(\mathrm{II})$	$\mathbf{x}(\mathrm{III})$	$\mathbf{x}(\Theta)$
Demand scenario I	—	0.70	1.25	1.13
Demand scenario II	2.10	—	0.47	0.37
Demand scenario III	10.53	2.92	—	0.09
Maximum regret	10.53	2.92	1.25	1.13

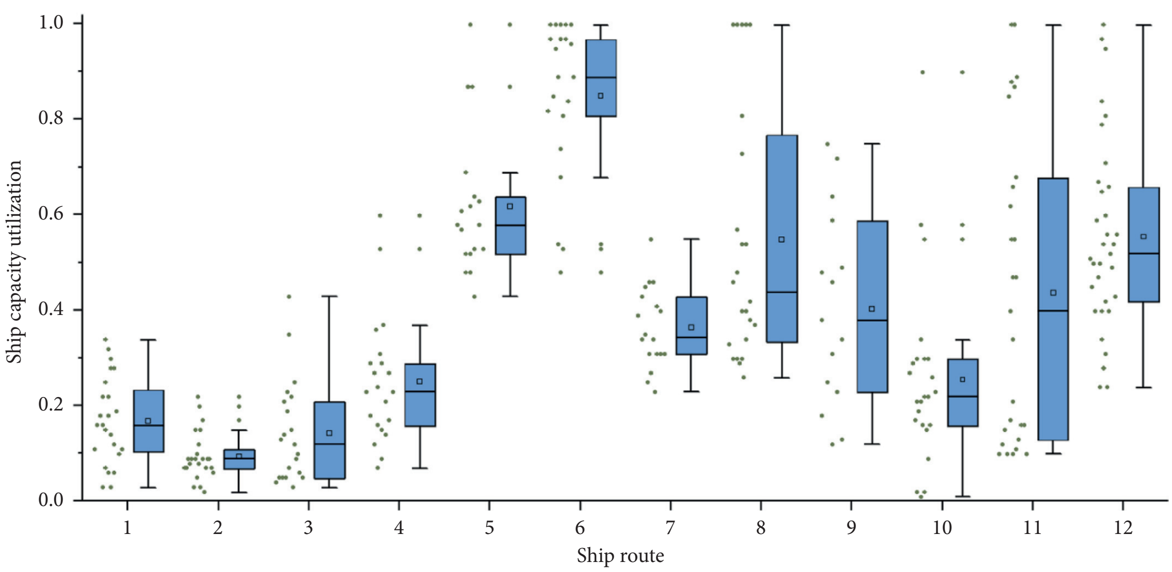


Figure 4: Ship capacity utilization of twelve ship routes.

contrary, route 2 has the lowest leg-based ship capacity utilization, whose average value is only around 9.5%. Regarding the particularly low-utilized route (like route 2), ship capacity is less likely to be fully used even in the event of confronting abruptly high demand. In such case, the associated remaining ship slots can be open to other companies in advance in a relatively low slot-purchasing cost. If ship capacity utilization of some routes is always in a low condition, these routes can be considered canceled or access to some adjacent ports in order to attract more demand.

4. Conclusions

This paper focused on the optimization of port rotation direction and fleet deployment for container liner shipping routes. When demand is assumed to be deterministic, a multicommodity flow network model was formulated. The objective aimed to minimize the total network-wide cost, which consists of loading and discharge cost, transshipment cost, inventory cost, slot-purchasing cost, and deployment cost. Later, we relaxed deterministic demand assumption and accounted for uncertain demand which is represented by a set of potentially realizable demand scenarios. A minimax regret model was developed, with the objective of least maximum regret across all the demand scenarios. Finally, a case study of an Asia-Europe-Oceania container liner shipping network with 46 ports and 12 ship routes was conducted based on CPLEX solver. The computational results indicate that the practical implication of the proposed models is beneficial to reduce the operating cost especially when uncertain demand is considered.

Admittedly, our proposed models come with some limitations. The further improvements and future research directions are as follows: (i) in this study, the minimax regret model is used to handle uncertain demand. Other methods such as stochastic programming model and robust optimization model can be systematically compared to explore the effect of demand uncertainty; (ii) more practical circumstances of the liner shipping business can be incorporated into the model formulation, such as berth resource allocation in container terminal, which influences the arrival and departure time of ships. The authors recommend that future studies could focus on these issues.

Data Availability

The data presented in this study are available on request from the first author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] B. D. Brouer, C. V. Karsten, and D. Pisinger, “Optimization in liner shipping,” *4OR*, vol. 15, no. 1, pp. 1–35, 2017.

[2] M. Christiansen, E. Hellsten, D. Pisinger, D. Sacramento, and C. Vilhelmsen, “Liner shipping network design,” *European Journal of Operational Research*, vol. 286, no. 1, pp. 1–20, 2020.

[3] J.-X. Dong and D.-P. Song, “Quantifying the impact of inland transport times on container fleet sizing in liner shipping services with uncertainties,” *Spectrum*, vol. 34, no. 1, pp. 155–180, 2012.

[4] Q. Meng, T. Wang, and S. Wang, “Multi-period liner ship fleet planning with dependent uncertain container shipment demand,” *Maritime Policy & Management*, vol. 42, no. 1, pp. 43–67, 2015.

[5] N. K. Tran and H. D. Haasis, “Literature survey of network optimization in container liner shipping,” *Flexible Services and Manufacturing Journal*, vol. 27, no. 2, pp. 139–179, 2015.

[6] T. Wang, Q. Meng, S. Wang, and X. Qu, “A two-stage stochastic nonlinear integer-programming model for slot allocation of a liner container shipping service,” *Transportation Research Part B: Methodological*, vol. 150, pp. 143–160, 2021.

[7] Q. Yao, L. Xu, and Q. Zhang, “Container slot allocation for time-sensitive cargo in maritime transportation: a one-phase model with consideration of port congestion,” *Discrete Dynamics in Nature and Society*, vol. 2021, Article ID 6622291, 11 pages, 2021.

[8] L. Zhen, Y. Wu, S. Wang, and G. Laporte, “Green technology adoption for fleet deployment in a shipping network,” *Transportation Research Part B: Methodological*, vol. 139, pp. 388–410, 2020.

[9] UNCTAD, “Review of maritime transportation 2018,” in *Proceedings of the United Nations Conference on Trade and Development*, New York , NY, USA, 2018, https://unctad.org/en/PublicationsLibrary/rmt2018_en.pdf>.

[10] Q. Meng, S. Wang, H. Andersson, and K. Thun, “Containership routing and scheduling in liner shipping: overview and future research directions,” *Transportation Science*, vol. 48, no. 2, pp. 265–280, 2014.

[11] G. R. Patil and P. K. Sahu, “Estimation of freight demand at Mumbai Port using regression and time series models,” *KSCE Journal of Civil Engineering*, vol. 20, no. 5, pp. 2022–2032, 2016.

[12] X. Chen, Z. Liu, K. Zhang, and Z. Wang, “A parallel computing approach to solve traffic assignment using path-based gradient projection algorithm,” *Transportation Research Part C: Emerging Technologies*, vol. 120, Article ID 102809, 2020.

[13] Y. Gu, X. Fu, Z. Liu, X. Xu, and A. Chen, “Performance of transportation network under perturbations: reliability, vulnerability, and resilience,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 133, p. 101809, 2020.

[14] A. Krogsgaard, D. Pisinger, and J. Thorsen, “A flow-first route-next heuristic for liner shipping network design,” *Networks*, vol. 72, no. 3, pp. 358–381, 2018.

[15] Z. Liu, Q. Meng, S. Wang, and Z. Sun, “Global intermodal liner shipping network design,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 61, pp. 28–39, 2014.

[16] J. Mulder and R. Dekker, “Methods for strategic liner shipping network design,” *European Journal of Operational Research*, vol. 235, no. 2, pp. 367–377, 2014.

[17] C. E. M. Plum, D. Pisinger, and M. M. Sigurd, “A service flow model for the liner shipping network design problem,” *European Journal of Operational Research*, vol. 235, no. 2, pp. 378–386, 2014.

[18] R. Agarwal and Ö. Ergun, “Ship scheduling and network design for cargo routing in liner shipping,” *Transportation Science*, vol. 42, no. 2, pp. 175–196, 2008.

[19] Q. Meng and S. Wang, “Liner shipping service network design with empty container repositioning,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 47, no. 5, pp. 695–708, 2011.

[20] B. D. Brouer, J. F. Alvarez, C. E. M. Plum, D. Pisinger, and M. M. Sigurd, “A base integer programming model and benchmark suite for liner-shipping network design,” *Transportation Science*, vol. 48, no. 2, pp. 281–312, 2014.

[21] J. Zheng, Q. Meng, and Z. Sun, “Impact analysis of maritime cabotage legislations on liner hub-and-spoke shipping network design,” *European Journal of Operational Research*, vol. 234, no. 3, pp. 874–884, 2014.

[22] S. Wang and Q. Meng, “Reversing port rotation directions in a container liner shipping network,” *Transportation Research Part B: Methodological*, vol. 50, pp. 61–73, 2013.

[23] J. Chen, S. Jia, S. Wang, and Z. Liu, “Subloop-based reversal of port rotation directions for container liner shipping network alteration,” *Transportation Research Part B: Methodological*, vol. 118, pp. 336–361, 2018.

[24] S. Gelareh and Q. Meng, "A novel modeling approach for the fleet deployment problem within a short-term planning horizon," *Transportation Research Part E: Logistics and Transportation Review*, vol. 46, no. 1, pp. 76–89, 2010.
[25] X. Liu, H. Q. Ye, and X. M. Yuan, "Tactical planning models for managing container flow and ship deployment," *Maritime Policy & Management*, vol. 38, no. 5, pp. 487–508, 2011.
[26] Q. Meng and S. Wang, "Liner ship fleet deployment with week-dependent container shipment demand," *European Journal of Operational Research*, vol. 222, no. 2, pp. 241–252, 2012.
[27] M. Ng, "Container vessel fleet deployment for liner shipping with stochastic dependencies in shipping demand," *Transportation Research Part B: Methodological*, vol. 74, pp. 79–87, 2015.
[28] H. Andersson, K. Fagerholt, and K. Hobbesland, "Integrated maritime fleet deployment and speed optimization: case study from RoRo shipping," *Computers & Operations Research*, vol. 55, pp. 233–240, 2015.
[29] Y.-F. Huang, J.-K. Hu, and B. Yang, "Liner services network design and fleet deployment with empty container repositioning," *Computers & Industrial Engineering*, vol. 89, pp. 116–124, 2015.
[30] R. Neamatian Monemi and S. Gelareh, "Network design, fleet deployment and empty repositioning in liner shipping," *Transportation Research Part E: Logistics and Transportation Review*, vol. 108, pp. 60–79, 2017.
[31] S. Wang, Z. Liu, and X. Qu, "Minimax regret model for liner shipping fleet deployment with uncertain demand," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2549, no. 1, pp. 45–53, 2016.
[32] T. Wang, Q. Meng, S. Wang, and Z. Tan, "Risk management in liner ship fleet deployment: a joint chance constrained programming model," *Transportation Research Part E: Logistics and Transportation Review*, vol. 60, pp. 1–12, 2013.
[33] L. Zhen, Y. Hu, S. Wang, G. Laporte, and Y. Wu, "Fleet deployment and demand fulfillment for container shipping liners," *Transportation Research Part B: Methodological*, vol. 120, pp. 15–32, 2019.
[34] M. Ng, "Revisiting a class of liner fleet deployment models," *European Journal of Operational Research*, vol. 257, no. 3, pp. 773–776, 2017.
[35] P. Kouvelis and G. Yu, *Robust Discrete Optimization and its Applications*, Kluwer Academic Publishers, Boston, MA, USA, 1997.
[36] M. H. Akyüz and C. Y. Lee, "Service type assignment and container routing with transit time constraints and empty container repositioning for liner shipping service networks," *Transportation Research Part B: Methodological*, vol. 88, pp. 46–71, 2016.
[37] D.-P. Song and J.-X. Dong, "Cargo routing and empty container repositioning in multiple shipping service routes," *Transportation Research Part B: Methodological*, vol. 46, no. 10, pp. 1556–1575, 2012.
[38] D.-P. Song and J.-X. Dong, "Long-haul liner service route design with ship deployment and empty container repositioning," *Transportation Research Part B: Methodological*, vol. 55, pp. 188–211, 2013.