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Research Article

Optimization of Extended Warranty Cost for Multi-Component Products with Failure Interaction

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In practice, the assumption of failure independence between components is seldom valid, especially for those complex systems with complicated failure mechanism. Users can decide whether to purchase extended warranty (EW) at the end of basic warranty, and there are many factors that influence this decision, such as product reliability and EW price. In order to solve the problem of EW pricing for multi-component systems with failure interaction reasonably, considering the failure interaction characteristics between components of the multi-component systems, under the condition of type II failure interaction, this paper constructed a dependent failure rate model and developed a EW cost model of two-component systems. Thus, after optimizing the preventive maintenance (PM) strategy, this paper obtained the optimal PM interval when EW cost is the lowest, which is a win-win strategy to reduce the EW price for manufacturers and users under the premise of ensuring the manufacturer's profit demand. Finally, the validity of the model was verified by a numerical example and sensitivity analysis for important parameters was presented.

1. Introduction

Due to the fierce market competition nowadays, manufacturers seek different market competition strategies to seize market share and improve customer satisfaction, and providing attractive warranty policies is a common marketing measure. Warranty policy is a statement of the type of compensation (such as free repair/replacement, refund, and so on) and scope (period) provided by the manufacturer when the product fails [1]. In fact, warranty policies have been widely used for many purposes in commercial trade, and warranty management decisions have also received extensive attention from various disciplines [2, 3]. According to the function period of the warranty policy in the product usage phase, warranty policies can be divided into two basic types: basic warranty (BW) and extended warranty (EW). BW starts after the consumer purchases the product and ends after a certain period of time. The length of this period is called BW period or warranty period. In addition, during the product marketing process, manufacturers usually provide customers with an option to purchase EW. EW refers to the additional warranty period provided by the manufacturer for their products after the end of BW. As an optional warranty policy, EW has aroused widespread concern in the industry and academia [4]. Existing EW studies and market examples indicate that EW plays an increasingly important role for both manufacturers and users [5]. For one thing, EW has brought profit margins of 44% to 77% for some manufacturers; for another, a considerable number of users purchase EW to reduce the economic burden caused by the product failures [6]. Depending on different time span and scope of EW implementation, EW mode can be divided into two categories: partially outsourcing and fully outsourcing, as shown in Figure 1.

Partially outsourcing EW means that after BW is over, the product is handed over to the manufacturer for EW because the user cannot maintain it independently, until the user is willing to bear the product maintenance cost; meanwhile, fully outsourcing EW means that after BW is over, the product is completely handed over to the manufacturer for EW until the product is replaced. For a long

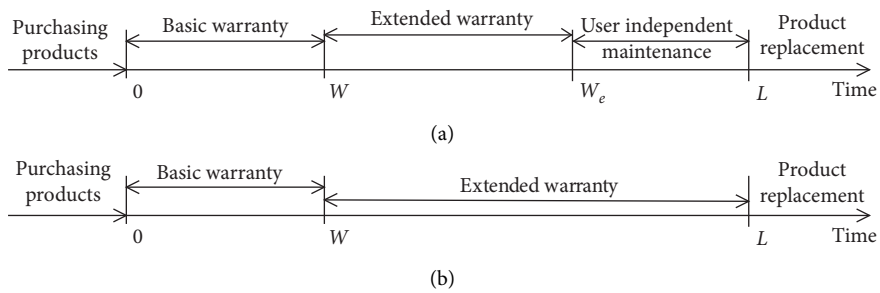

Figure 1: The partially outsourcing and the fully outsourcing modes of EW.

time, the after-sales service of many products has provided a BW [7–9], while EW is applied late. Generally speaking, EW is sold to consumers as a special product, and free maintenance services are provided for product failures that occur during EW period [10]. Although EW will bring additional costs to consumers, more and more consumers are more willing to purchase EW service because it can reduce maintenance costs caused by the product failure. At the same time, manufacturers have also strengthened their contact with users by providing EW services after the end of BW, highlighting the product quality and value, thereby enhancing market share and increasing their own profits [11]. Therefore, the research on EW is particularly important. Due to the additional cost required for manufacturers to provide consumers with EW [9], the pricing of EW has become an urgent problem to be solved. However, few studies have focused on this area [12, 13]. Through the literature research, it is found that the fundamental factor affecting the EW price is EW cost, and EW cost will vary with different maintenance strategies [14]. Generally speaking, according to the degree of recovery of the product after maintenance, maintenance strategies are mainly divided into five categories [15], namely, perfect repair, minimal repair, imperfect repair, worse repair, and worst repair. Different repair strategies have a significant impact on EW cost. Jack and Murthy [16] analyzed EW warranty cost and examined the optimal EW pricing strategy for manufacturers based on the minimal repair. Su and Shen [17] established the EW warranty cost and profit models considering the imperfect repair combined with the minimal repair. In warranty practice, the commonly used effective way to reduce the warranty costs is to implement appropriate preventive maintenance (PM) [18, 19] during the warranty process. PM can improve the product reliability and prolong the product life [20], reducing the manufacturer's warranty cost, thereby maintaining EW price at a reasonable level which is beneficial to both manufacturers and users. In the field of preventive EW studies, Wang et al. [21] discussed the PM strategy under BW and EW from the manufacturer's perspective. After each PM, the product failure rate can be reduced in proportion to the degree of maintenance. In order to reduce EW cost, Chang and Lin [22] proposed that manufacturers adopt PM strategies during the warranty period and established a profit model for manufacturers when consumers purchase EW, thereby obtaining the manufacturer's maximal profit and the corresponding optimal maintenance strategy. Soumaya et al. [23] considered the PM strategy and established an EW cost model on this basis to determine a reasonable price range for EW to achieve a win-win situation for manufacturers and consumers. Therefore, formulating an appropriate PM strategy and combining it with EW is a problem worthy of further study.

In modern manufacturing systems, products are mostly composed of multiple components or subsystems. In the maintenance activities of a multi-component system, the components may be interdependent with each other in different forms [24]. Maintenance optimization of a multi-component product mainly relies on three types of dependencies between components [25] which include economic dependence, stochastic dependence, and structural dependence. Economic dependence means that the cost of group maintenance for the components in the system is higher or lower than the sum of the cost of repairing them separately [26]; structural dependence implies that components form a whole in structure, so the maintenance or replacement of a failed component will cause the maintenance actions (at least the disassembly) for other operating components in the system [27]; failure interaction is also called stochastic dependence, which mainly means that the failure of one component will affect the status of other components [28]. The "status" here includes various status metrics such as life, failure rate, and failure status. Taking the relationship between the four tires of a car as an example, if one of the four tires fails, the status of the other three tires will be affected, whose failure process will also change accordingly. However, when dealing with multi-component products, many studies often assume that the failures between components are independent of each other and use existing single-component optimization methods and models to make decisions [29, 30]. In fact, the structure of modern products has tended to integrate multiple components such as machinery, electricity, and hydraulics, and the degree of failure interaction between the product components has become closer. There are many examples of failure interaction between the components in practice. If the failures of the components are still assumed to be independent of each other, the description of the failure rules of the system will be inaccurate, causing the studies about system reliability and maintenance intervals out of reality, causing greater economic losses and even serious safety consequences. Therefore, failure interaction of this kind of components has great research value and necessity.

According to the results of failure interaction influences, they are generally divided into three categories: type I is failure interaction, that is, when one component fails, it will cause other components to fail with a certain probability μ $(0 \leq \mu \leq 1)$; type II is failure rate interaction, that is, when a component fails, it will increase the failure rate of other components to a certain extent; type III is shock damage interaction, that is, when one component in the system fails, it will cause a certain degree of random damage to other components. When it accumulates to a certain extent, it causes the component failure.

In the field of failure interaction research, Murthy and Nguyen [31] first proposed the failure interaction between the components and studied the preventive replacement strategy of two-component systems in the case of type I and type II failure interactions. Peng et al [32] studied a two-component system and proposed a failure rate interaction model based on the copula function in the case of type II failure interaction. Liu et al. [33] established a renewable and free replacement model based on the type I failure interaction, taking the warranty cost as the optimization goal. Furthermore, Lai and Chen [34] proposed a two-component system periodic replacement model that considers both type II and type III failure interactions. By introducing relative cost as the optimization objective, the expected cost rate per unit time is derived, and then the best replacement cycle with minimal cost is determined. Domestically, Wang et al. [35] proposed a new shock failure model based on the consideration of type III failure interaction, and based on this model, they studied the incomplete maintenance decision-making method under the N-type replacement strategy and obtained the minimal average cost rate of the system, verifying the feasibility of this strategy.

However, in the above studies, there is no literature to carry out EW decision-making research based on failure interaction for multi-component products. Most of the modelling studies on EW are based on the independent failure of each component of the system. Model construction and solution are less difficult, but there is a big error between the maintenance process and the actual situation. Therefore, in EW study, simply treating the components in the system as independent of each other is not in line with the actual situation. It is necessary to consider the failure interaction between the components in the system to make the repair process more in line with the actual situation. This study intends to fill this gap. With the differentiated development of market competition strategies, EW policies have become a new profit source for manufacturers. Especially for multi-component products with complex failure characteristics, optimizing maintenance strategies of EW and reducing EW cost are of great significance for enhancing market competitiveness and improving customer satisfaction. In this paper, we present a EW warranty cost model for a two-component product with type II failure interaction. Through the two-component system research, this paper optimizes the maintenance strategy during EW period under the condition of type II failure interaction and obtains the optimal PM interval and the lowest EW cost and EW price, which provides a scientific basis and managerial suggestions for manufacturers to formulate EW strategy for multi-component products with failure interaction. Compared with previous works, the following two important extensions are made in this paper. (1) Instead of assuming independent failures between components in the multi-component products, we derive the EW cost based on certain failure interaction model. (2) According to the profit demand per unit time of the manufacturer, we have come up with the scientific and specific EW price for multi-component products. Since failure interaction is a common problem in the multi-component system which will affect both the system reliability and EW cost, this paper can help decision makers better evaluate system reliability and reduce EW cost and price.

So, the rest of this paper is organized as follows. Section 2 describes model assumptions and notations. Next, to construct the EW cost model of the multi-component system, the EW price model of the multi-component system, dependent failure rate model, and imperfect repair model are presented under the logical framework in Section 3. A numerical example is then given in Section 4 to illustrate the optimization effect of EW cost of the multi-component system with failure interaction, followed by a sensitivity analysis for important parameters. Finally, the conclusions and potential future research directions are summarized in Section 5.

2. Model Assumptions and Notations

In this paper, a two-component system with failure interaction composed of a key component and a subsystem is studied, and the imperfect PM is implemented during EW period without considering the failure interaction among the components of the subsystem. During each imperfect PM interval, when the key component fails, minimal repair is implemented on it. The failure rate $\lambda_\psi(t)$ after the minimal repair remains invariant, but it will cause the subsystem failure rate $\lambda_s(t)$ to increase; otherwise, the failure of the subsystem will cause the key component to fail immediately, and the entire system needs a minimal repair at this time, and the system failure rate after the minimal repair remains invariant. By optimizing the PM interval, EW cost is minimized, thereby reducing EW price of the two-component system.

The following assumptions and notations are used in this paper.

2.1. Model Assumptions. To facilitate the research, the system modelling is based on the following assumptions:

(1) The research object is a two-component series system with failure interaction composed of a key component and a subsystem.

(2) Due to the short service time of the system in BW period, the failure rate is relatively low, so only the minimal repair after failure is implemented during BW period. During EW period, the periodic imperfect PM is implemented, and the minimal repair is implemented after the failures.

(3) The system has aging and decay characteristics, and the failure rate increases with time.

(4) The PM cost is only related to the improvement factor of imperfect repair and does not change with time and frequency.

(5) The minimal repair cost is fixed and does not change with time and frequency.

(6) The failure mode is the single failure mode without considering multiple failures.

2.2. Model Notations. The notations used in this paper are presented in Table 1.

3. Extended Warranty Mathematical Model of Multi-Component System with Failure Interaction

3.1. Extended Warranty Price Model of Multicomponent System. EW price refers to the cost that users need to pay for purchasing EW service. When formulating EW price of a multi-component system, it is usually necessary to consider two factors, that is, EW cost and the manufacturer's profit demand. In view of this, the EW price model of the multi-component system can be expressed as

$$P(t_1, t_2) = \mathrm{EC}(t_1, t_2) + \phi(t_2 - t_1), \tag{1}$$

where t_1 and t_2 are the start and end time of EW, respectively; $\mathrm{EC}(t_1, t_2)$ represents EW cost; and ϕ is the manufacturer's profit demand per unit time. It can be seen from equation (1) that under the premise of ensuring the manufacturer's profit, in order to reduce the EW price and improve the economic benefits of EW, we need to optimize the EW strategy to reduce EW cost. Reducing the EW price is not only the demands of users but also manufacturers who can also improve customer satisfaction and the competitiveness of their products by providing high-quality and low-cost EW services.

3.2. Dependent Failure Rate Model. In an actual system, when a component fails, it will lead to the changes in the state of the system, such as the increase of pressure, temperature, humidity, and so on, which will increase the failure rate of other components. In fact, due to the change of system state, the actual failure rate of the component is bigger than the failure rate at the time of the component failure. Therefore, the actual failure rate of the component at a certain time is related to the average failure times of all components in a system before that time. For a multi-component system, the actual failure rate of each component at a certain time is affected by the average number of its failures before that time and the average number of failures of other components before that time. The actual failure rate of each component at a certain time is greater than that of each component with independent failure. The failure interaction modes between the two components are shown in Figure 2.

It can be seen from Figure 2 that each failure of the component will have an impact on itself and other components in the system, and the failure between components will no longer be independent. For a system with failure interaction, the actual failure rate of each component during the operation of the system is determined by two factors: independent failure rate and dependent failure rate. Independent failure rate refers to the inherent failure rate of the component itself, which is determined by the design and manufacturing level of the component; dependent failure rate refers to the failure rate caused by the failure or failure of other components in the system. Assume that the multi-component system is composed of q components in series, and each component shows failure interaction. Then, under the condition of failure interaction, the actual failure rate of each component can be expressed in the following matrix form [36]:

$$[\lambda_a(t)] = [I][\lambda_{a0}(t)] + [\theta_{ab}(t)][\lambda_{ba}(t)], \tag{2}$$

where $[\lambda_a(t)]$ is the $q \times 1$-dimensional vector, $a = 1, 2, 3, \ldots, q$; $[\lambda_{a0}(t)]$ is the $q \times 1$-dimensional vector, which represents the independent failure rate of the component; $[\lambda_{ba}(t)]$ is the $q \times 1$-dimensional vector, $a, b = 1, 2, 3, \ldots, q$ & $a \neq b$, which denotes the dependent failure rate of the component a caused by the failure interaction of component b before the dependent failure; and $[\theta_{ab}(t)]$ is the $q \times q$-dimensional non-negative real matrix, which indicates the failure influence coefficient of the component b on the component a, $0 \leq \theta_{ab}(t) \leq 1$; when $\theta_{ab}(t)$ is 0, it means that there is no interaction between the two components; when $\theta_{ab}(t)$ is 1, it means that the failure of component b leads the component a to fail. When the rank of $\theta_{ab}(t)$ is 0, i.e., $R[\theta_{ab}(t)] = 0$, it means that all components in the system have no dependent failure influence on themselves. The failure rate curve is shown in Figure 3, which indicates the change of failure rate of component a after being affected by dependent failure, and t_0 is the starting time of dependent failure.

3.3. Imperfect Repair Model. For EW policies of multi-component systems, most of the warranty objects are the product system or subsystem with advanced technology and complex structure. Due to its advanced materials and technology, the cost of preventive replacement is usually very high. Therefore, PM usually cannot achieve the perfect update of the system [37], but the failure rate of the system after the PM actions is reduced, which is between "as good as new" and "as bad as old," that is, imperfect repair. The basis of imperfect repair modelling is to reasonably demonstrate the impact of imperfect repair on the failure rate. The virtual age method [38, 39] is the most mature and most widely applied method to update the failure rate function in accordance with the imperfect repair effect [40, 41].

In this paper, the virtual age method is used to deal with imperfect repair. Virtual age, also known as effective age, refers to the failure rate of the system dropping to the previous time $v(t)$ after the imperfect repair at time t, just as the system's age has been reduced for a period of time, and the change trend of the failure rate after that is the same as the original failure rate function. Then, $v(t)$ is the virtual age of the system, and $v(t) \leq t$. The failure rate of the system is determined by the virtual age; assuming that the kth PM

Table 1: Model notations.

W	Deadline of BW
W_e	Deadline of EW
T	Imperfect PM interval
T_p	Time of single imperfect PM
C_p	Cost of single imperfect PM for the two-component system
C_{f1}	Cost of single minimal repair for the key component
C_{f2}	Cost of single minimal repair for the subsystem
$\mathrm{EC}\ (T, W, W_e)$	EW cost of the two-component system
$\lambda_\psi(t)$	Failure rate function of the key component
$\lambda_s(t)$	Failure rate function of the subsystem
$m_{k\psi}$	Number of failures of the key component in the kth PM interval
m_{ks}	Number of failures of the subsystem in the kth PM interval
δ	Improvement factor of imperfect repair
θ	Failure influence coefficient

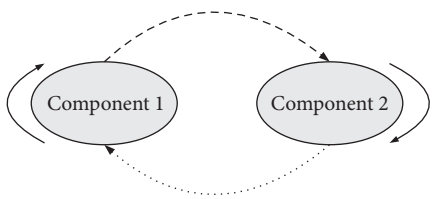

Figure 2: Failure interaction diagram.

reduces the whole virtual age before this maintenance, δ is the improvement factor, which indicates the reduction degree of virtual age. $\delta = 0$ denotes the minimal repair, and $\delta = 1$ represents the perfect repair. Thus, $0 < \delta < 1$ means the imperfect repair. On this basis, the failure rate $\lambda(t)$ of the product in the k th PM interval is as follows:

$$\lambda(t) = \lambda(t - \delta(k-1)T), \tag{3}$$

where T is the imperfect PM interval. At this time, the change curve of system failure rate under imperfect PM is shown in Figure 4.

3.4. Extended Warranty Cost Model of Two-Component System. Most of the multi-component systems have long service life and high maintenance cost. After a long-term EW, users will develop a certain degree of independent maintenance ability. Through market research, it is found that most users will choose to purchase the partially outsourcing EW service, considering the benefits of the system in the later stage of usage. In the partially outsourcing EW mode, EW starts from the end of BW and ends when the user is willing to bear the maintenance cost of the system. Therefore, the model mainly solves the problem of system EW pricing. On the premise of ensuring the profit demand of manufacturers, it is in the interests of both manufacturers and users to reduce EW cost by optimizing the PM strategy. The optimization model of EW cost is as follows:

$$\begin{cases} \min \mathrm{EC}(T, W, W_e), \\ \text{s.t.}, \\ \phi = \phi_0, \end{cases} \tag{4}$$

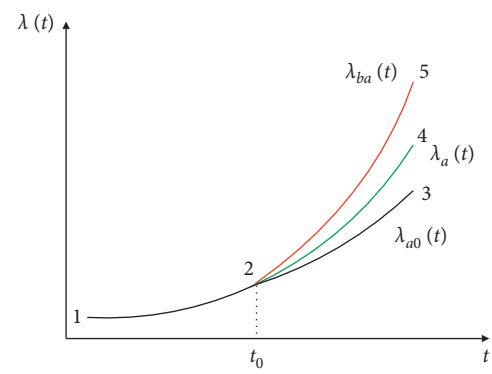

Figure 3: Failure rate curve.

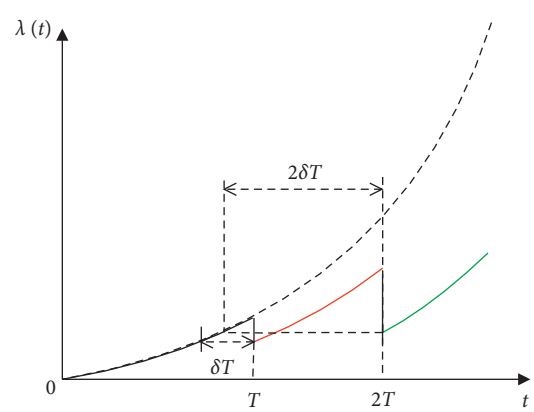

Figure 4: Virtual age method diagram.

where T is the interval of imperfect PM, W is the BW period, $[W, W_e]$is the EW period, and ϕ denotes the manufacturer's profit demand. In fact, the optimization model is to minimize EW cost by optimizing the PM interval under the premise of ensuring the manufacturer's profit demand.

Assume that the system failure is found immediately and the minimal repair does not affect the system failure rate. Only minimal repair after the failures is implemented during

BW period and PM intervals of EW. The characteristic of minimal repair is a nonhomogeneous Poisson process (NHPP). The expected minimal repair number of the system in $[0, t]$ is about

$$E[N(t)] = \int_0^t \lambda(s)\,\mathrm{d}s, \tag{5}$$

where $N(t)$ is the number of system failures in $[0, t]$ and $\lambda(s)$ is the system failure rate.

During EW period, the imperfect PM is implemented on the system with a cycle T. The cost in each PM interval includes the sum of the minimal repair cost for the key component failures, the sum of the minimal repair cost for the system failures, and the PM cost of the system. Therefore, EW cost of the multi-component system can be expressed as

$$\mathrm{EC}(T, W, W_e) = nC_p + \sum_{k=1}^{n} \mathrm{EC}_f^k(T) + \mathrm{EC}_f\left(W + n\left(T + T_p\right), W_e\right), \tag{6}$$

where n is the number of PM in EW period; $\mathrm{EC}_f^k(T)$ is the expected minimal repair cost in the kth PM interval; and $\mathrm{EC}_f(W + n(T + T_p), W_e)$ is the expected minimal repair cost in $[W + n(T + T_p), W_e]$, that is, the expected minimal repair cost in the last PM interval.

Due to the implementation of PM in EW period, we can get the following results:

$$n = \mathrm{int}\left[\frac{W_e - W}{T + T_p}\right], \tag{7}$$

where "int" denotes rounding down.

During the PM intervals, the system is in a state of shutdown, and the system age can be considered unchanged during this period, so the impact of PM time on the system failure rate can be ignored. In the kth PM interval, the failure rate $\lambda_{k\psi}(t)$ of the key component is as follows:

$$\lambda_{k\psi}(t) = \begin{cases} \lambda_\psi(t), & k = 1, \\ \lambda_\psi[t - \delta(W + (k-1)T)], & k = 2, 3, 4, \ldots, n. \end{cases} \tag{8}$$

When the key component fails, the failure rate $\lambda_s(t)$ of the subsystem will increase. Through the dependent failure rate model, the actual failure rate $\lambda_{ks}(t)$ of the subsystem in the kth PM interval can be derived as follows:

$$\lambda_{ks}(t) = \begin{cases} \lambda_s(t) + \dfrac{\theta}{2} m_{k\psi}\lambda_{k\psi}(t), & k = 1, \\ \lambda_s[t - \delta(W + (k-1)T)] + \theta\left\{\displaystyle\sum_{i=1}^{k}\left[m_{i\psi}\lambda_\psi(t - \delta(W + (i-1)T))\right] - \dfrac{1}{2} m_{k\psi}\lambda_{k\psi}(t)\right\}, & k = 2, 3, 4, \ldots, n. \end{cases} \tag{9}$$

Then, the number of failures of the key component in the kth PM interval is as follows:

$$m_{k\psi} = \int_{W + (k-1)\left(T + T_p\right)}^{W + kT + (k-1)T_p} \lambda_{k\psi}(t)\mathrm{d}t. \tag{10}$$

The average failure times of the subsystem in the kth PM interval are as follows:

$$m_{ks} = \int_{W + (k-1)\left(T + T_p\right)}^{W + kT + (k-1)T_p} \lambda_{ks}(t)\mathrm{d}t. \tag{11}$$

Therefore, it can be concluded that in the kth PM interval, the total minimal repair cost of the two-component system is

$$\mathrm{EC}_f^k(T) = m_{k\psi}C_{f1} + m_{ks}C_{f2}. \tag{12}$$

Similarly, the average failure number of the key component in $[W + n(T + T_p), W_e]$ can be expressed as

$$m_{(n+1)\psi} = \int_{W+n\left(T+T_p\right)}^{W_e} \lambda_{(n+1)\psi}(t)\mathrm{d}t. \tag{13}$$

The average failure time of the subsystem in $[W + n(T + T_p), W_e]$ can be expressed as follows:

$$m_{(n+1)s} = \int_{W+n\left(T+T_p\right)}^{W_e} \lambda_{(n+1)s}(t)\mathrm{d}t. \tag{14}$$

Therefore, the minimal repair cost of the two-component system in $[W + n(T + T_p), W_e]$ is

$$\mathrm{EC}_f\left(W + n\left(T + T_p\right), W_e\right) = m_{(n+1)\psi}C_{f1} + m_{(n+1)s}C_{f2}. \tag{15}$$

In conclusion, EW cost of the two-component system under imperfect PM is as follows:

$$\begin{aligned} \mathrm{EC}(T, W, W_e) = {} & nC_p + \\ & C_{f1}\left[\sum_{k=1}^{n}\int_{W+(k-1)\left(T+T_p\right)}^{W+kT+(k-1)T_p} \lambda_{k\psi}(t)\mathrm{d}t \int_{W+n\left(T+T_p\right)}^{W_e} \lambda_{(n+1)\psi}(t)\right] \\ & + C_{f2}\left[\sum_{k=1}^{n}\int_{W+(k-1)\left(T+T_p\right)}^{W+kT+(k-1)T_p} \lambda_{ks}(t)\mathrm{d}t \int_{W+n\left(T+T_p\right)}^{W_e} \lambda_{(n+1)s}(t)\mathrm{d}t\right]. \end{aligned} \tag{16}$$

4. Numerical Example

4.1. Problem Description. Diesel engine is the core component of a certain type of complex construction machinery. Its performance affects whether the construction machinery can complete the function of energy output and power traction. Diesel engine mainly includes turbocharger, oil pump, moving parts, valve train, and other important functional units. The failure of the turbocharger is usually caused by the failure of other components; the failure of the moving parts is mainly fatigue failure, and the dependent failure accounts for a small proportion; the failure of the valve train is usually also a fatigue failure, and at the same time, the occurrence of its failure will cause the turbocharger failure; oil pump failure is mainly of fatigue type, and there is almost no dependent failure, and its failure will also cause turbocharger failure. According to research and analysis, the diesel engine can be regarded as a two-component system with failure interaction composed of a turbocharger and subsystem (the remaining components of the diesel engine). It is known that the turbocharger failure law obeys the following two-parameter Weibull distribution:

$$\lambda_{\psi}(t) = \frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1}, \tag{17}$$

where the shape parameter $\alpha = 2$ and the scale parameter $\beta = 1000$. To facilitate the description and simplify the calculation, the time unit below is uniformly set to days, which is calculated as 360 days per year. During each PM interval, when the turbocharger fails, the minimal repair is implemented on it, and the failure rate $\lambda_{\psi}(t)$ remains unchanged after the maintenance. But it will cause the failure rate $\lambda_s(t)$ of the subsystem to increase; the failure of the subsystem will lead the turbocharger to fail immediately, and the entire diesel engine system needs a minimal repair.

Other parameters of the proposed model are given in Table 2.

4.2. Model Solution. In the above model, EW cost will change with the change of PM interval T and the EW deadline W_e. Suppose the value range of the PM interval T is $[0.1, 3\,\text{years}]$, and the value range of EW deadline W_e is $[3, 10\,\text{years}]$, whose change steps are all 0.1 years. On the basis of the above data assumption, we can get the change law of EW cost of diesel engine with different PM interval T and EW deadline W_e, as shown in Figure 5. The jumping points on the surface are caused by different PM intervals.

Figure 5 demonstrates that system EW cost is influenced by two variables of T and W_e. Moreover, there is a minimal point for each curve that system EW cost changes with T. That means we can obtain the optimal PM intervals. At the same time, if T is fixed, system EW cost will always increase as W_e grows. This is in line with the law that the failure rate of each component of the system will continue to increase over time and the system maintenance costs increase accordingly.

In order to facilitate the analysis and research on the change law of EW cost, the three-dimensional graph of EW cost is fixed at $T = 360$ days and $W_e = 1080$ days for dimensionality reduction analysis, respectively, as shown in Figure 6.

Table 2: The values of parameters in the presented numerical example.

Parameters	Values
W	2 years
T_p	2 days
C_p	500 CNY
C_{f1}	800 CNY
C_{f2}	1200 CNY
$\lambda_s(t)$	5×10^{-4} per day
δ	0.8
θ	0.5

It can be seen from Figure 6 that when the PM interval is fixed, the system EW cost increases with the increase of EW period; when EW period is determined, there is an optimal PM interval to minimize the system EW cost.

In order to verify the effectiveness of PM strategy, the system EW cost is calculated when only the minimal repair after the failure is implemented during EW period, as shown in Figure 7.

It can be seen from Figure 7 that EW cost increases with the increase of EW period when only the minimal repair after the failure is implemented. Obviously, when EW period is the same, EW cost only with minimal repair after the failure is higher than that with PM during EW period.

4.3. Result Analysis. In the partially outsourcing EW mode, EW period is a definite value, and on this basis, when the system EW cost is optimized, then a reasonable EW price is determined. According to the above analysis and calculation, some optimization results are shown in Table 3.

In Table 3, T^* represents the optimal PM interval under different EW periods, EC represents the optimal EW cost, C_0 represents the corresponding EW cost under different EW periods when PM is not implemented, and RC means the optimization degree of the diesel engine EW cost before and after PM. By comparing the data in the table, we can draw some conclusions and regular patterns of the model proposed. For the multi-component system with failure interaction, the imperfect PM strategy can reduce the EW cost of the system. After implementing the optimized PM strategy, EW cost of the diesel engine system has been greatly reduced, which fully proves the accuracy and feasibility of the model proposed. By comparing the data in column EC in the table, it can be seen that with the increase of EW period, the optimization effect of PM on EW cost remains stable. This is because the failure rate function of diesel engine system is the first power function, and the increase rate of failure rate does not change with the increase of time, so the effect of PM on the reduction of failure rate remains stable. According to the plans in the table, EW cost and optimal PM interval of diesel engine system can be obtained, which can provide information support and basis for manufacturers and users to make EW decisions and then determine EW price scientifically in EW contract.

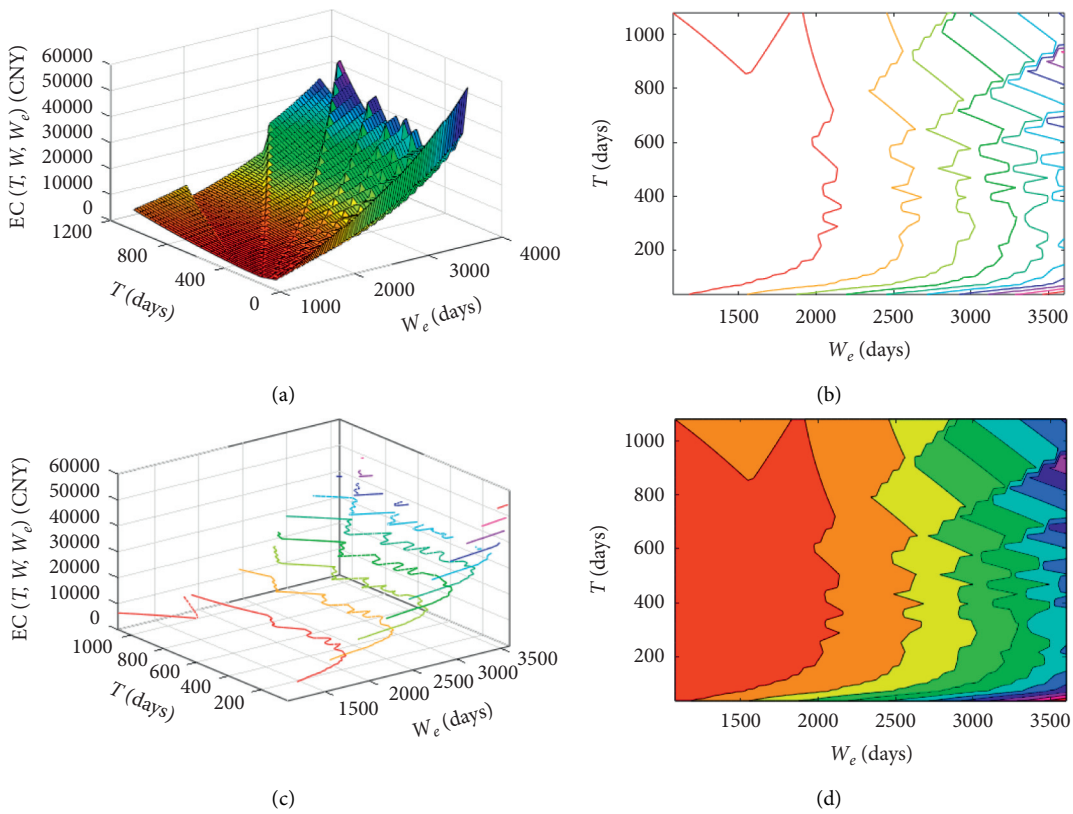

Figure 5: Change trend of system EW cost. (a) 3D change trend of system extended warranty cost. (b) 2D contour curves. (c) 3D contour curves. (d) 2D contour curves filled with color.

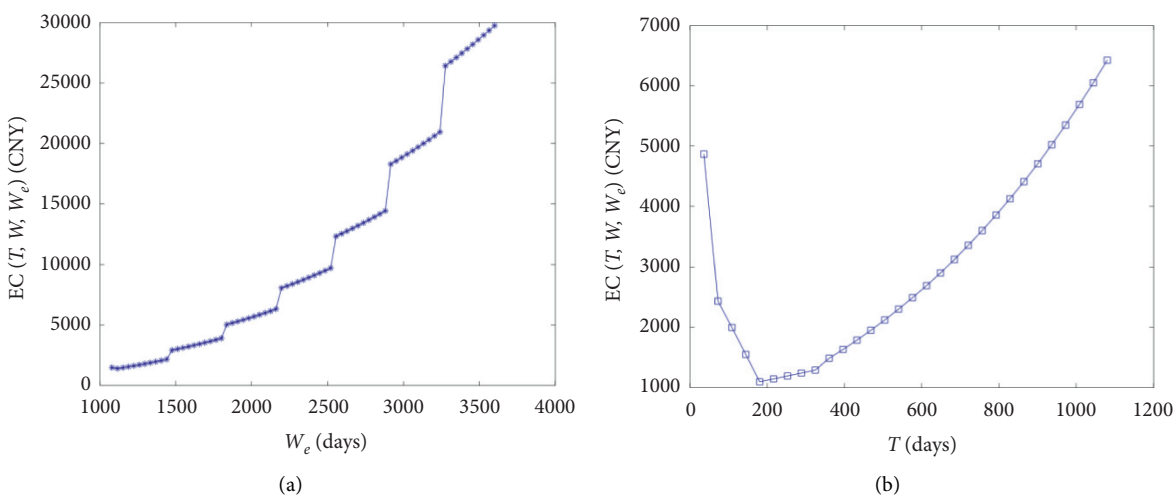

Figure 6: EW cost change when $T = 360$ days and $W_e = 1080$ days.

In summary, the following managerial recommendations are available for manufacturers and users to refer to. Providing EW services for multi-component products with failure interaction can bring a steady income for manufacturers. Manufacturers can satisfy users by providing good services and then appropriately increase the profit demand per unit time, thereby increasing the revenue. Moreover, appropriate imperfect PM activities can effectively reduce the failure rate and the number of failures of the product and improve product reliability. Manufacturers need to improve production technology and improve maintenance levels to further improve product reliability and reduce the number of failures during EW period, thereby reducing EW cost and increasing the EW benefits. In addition, from the perspective of users, purchasing EW services can greatly reduce the maintenance cost caused by product failures. Users can reasonably determine the length of EW period according to their actual needs and product performance requirements.

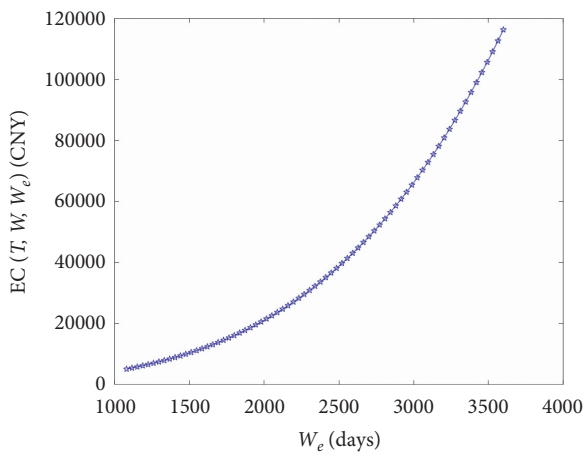

Figure 7: EW cost change only with minimal repair after failure.

Table 3: Optimization plans corresponding to different EW periods.

Plan	W_e (days)	T^* (days)	EC (CNY)	C_0 (CNY)	RC (%)
1	1080	180	1099.41	5100.10	78.44
2	1260	288	1589.67	7021.47	77.36
3	1440	360	2161.59	9436.49	77.09
4	1620	468	2988.67	12432.51	75.96
5	1800	360	3902.37	16104.43	75.77
6	1980	432	5056.27	20554.69	75.40
7	2160	360	6311.37	25893.33	75.63
8	2340	324	7928.22	32237.90	75.41
9	2520	360	9697.93	39713.54	75.58
10	2700	396	11858.78	48452.95	75.53
11	2880	360	14433.14	58596.36	75.37
12	3060	468	17593.58	70291.59	74.97
13	3240	360	20949.85	83694.00	74.97
14	3420	540	25654.16	98966.50	74.08
15	3600	360	29742.65	116279.60	74.42

Assuming that the manufacturer's profit demand for providing EW service for diesel engines is $\phi_0 = 900$ CNY per year, then based on the data in Table 3 combined with the EW price model, EW price under different EW plans can be obtained. The decision results of EW price from plan 1 to plan 15 are shown in Table 4.

In Table 4, $P(T^*, W_e)$ represents the optimal EW price, P_r means the EW price without implementing PM strategy, and RP indicates the proportion of decline in EW price after implementing PM strategy.

As can be seen from Table 4, optimizing the PM strategy during EW period can effectively reduce the system EW price and improve the economy and effectiveness for users to purchase EW. The data in Table 4 under different EW periods can provide a scientific basis for scientifically formulating the EW price of diesel engines. On the basis of the above analysis and plans in Table 4, some specific action plans for manufacturers and users can be obtained. When manufacturers are doing marketing management, they can reduce the EW price as much as possible while meeting their own profit demand, increase users' enthusiasm for buying EW services, and expand market share. In addition, users can selectively purchase EW services and combine their own needs to reduce the product utilization cost and improve utilization efficiency.

Table 4: Price decision plans corresponding to different EW periods.

Plan	W_e (days)	$P(T^*, W_e)$ (CNY)	P_r (CNY)	RP% (%)
1	1080	1999.41	6000.10	66.68
2	1260	2939.67	8371.47	64.88
3	1440	3961.59	11236.49	64.74
4	1620	5238.67	14682.51	64.32
5	1800	6602.37	18804.43	64.89
6	1980	8206.27	23704.69	65.38
7	2160	9911.37	29493.33	66.39
8	2340	11978.22	36287.90	66.99
9	2520	14197.93	44213.54	67.89
10	2700	16808.78	53402.95	68.52
11	2880	19833.14	63996.36	69.01
12	3060	23443.58	76141.59	69.21
13	3240	27249.85	89994.00	69.72
14	3420	32404.16	105716.53	69.35
15	3600	36942.65	123479.60	70.08

4.4. Sensitivity Analysis. In the above model, EW cost of the multi-component system with failure interaction is closely related to the system parameters and maintenance parameters. Their different values have different degrees of influence on the system maintenance strategy, which are analyzed below.

4.4.1. Influence Analysis of System Parameter θ. The basis of the multi-component system with failure interaction modelling is to construct the dependent failure rate model. Parameter θ represents the influence degree of dependent failure among various components in the system. In order to further verify the influence of failure influence coefficient θ on the system EW cost, $T = 360$ days and $W_e = 1080$ days are fixed, respectively, and the change curves of EW cost corresponding to different values are calculated, as shown in Figure 8.

It can be seen from Figure 8 that the system EW cost increases with the increase of the failure influence coefficient θ. When EW period is short, the influence of the failure influence coefficient θ on the system EW cost is not obvious because the number of system failures is a bit less. Failure influence coefficient θ reflects the performance of the equipment, which can only be improved from the perspective of design.

4.4.2. Influence Analysis of Maintenance Parameter δ. The basis of imperfect repair modelling is to demonstrate the impact of imperfect repair on the product failure rate. In this paper, the improvement factor δ of imperfect repair is used to express the reduction degree of virtual age of the system, so as to construct the failure rate model of the system. In order to further study the influence of the improvement factor δ on the system EW cost and make the contrast effect more obvious, fix $T = 36$ days and $W_e = 3600$ days, respectively. And we obtain the change curves of EW cost corresponding to different values of δ, as shown in Figure 9.

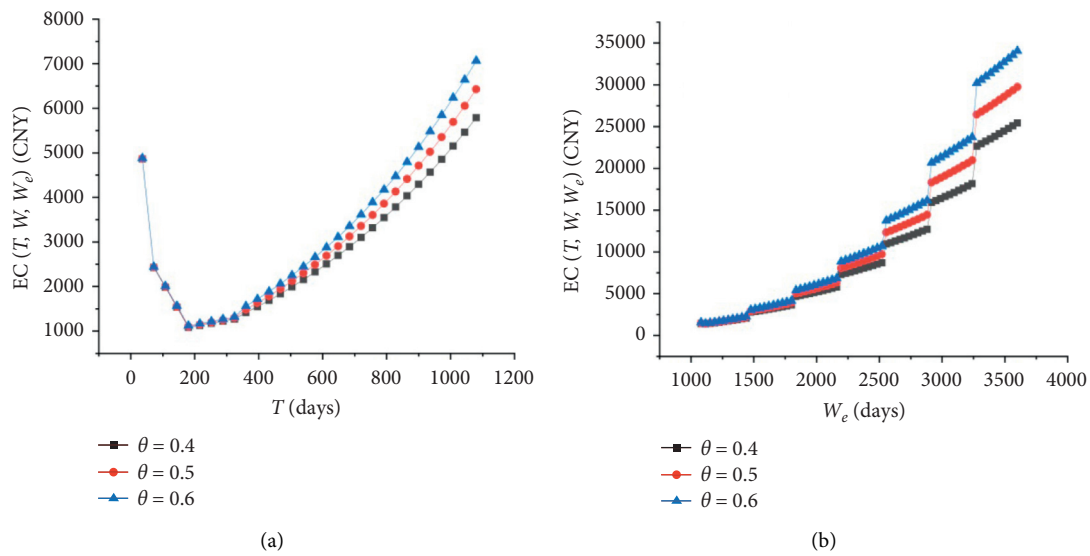

Figure 8: EW cost change with failure influence coefficient θ when W_e = 1080 days and $T = 360$ days.

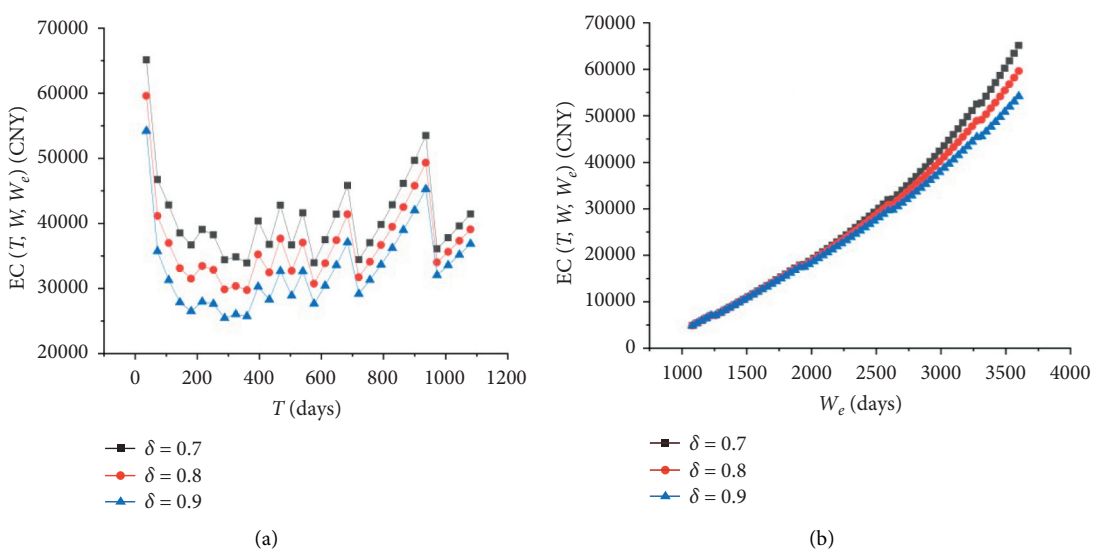

Figure 9: EW cost change with improvement factor δ when $W_e = 3600$ days and $T = 36$ days.

As can be seen from Figure 9, the system EW cost decreases with the increase of the improvement factor δ. Due to the increase of the improvement factor δ, each PM reduces the virtual age of the system to a greater extent, thereby increasing the improvement degree of the system failure rate, reducing the number of minimal repair after the failures, and reducing the system EW cost. The improvement factor is usually related to the technical ability of maintenance, which can be optimized by improving the maintenance level of workers, maintenance facilities, and equipment conditions.

5. Conclusions and Future Research Directions

In EW policy making and pricing analysis of the multi-component system with failure interaction, the dependent failure between components and the individual differences of failure interaction in the failure process should be considered. Based on this, this paper establishes the EW cost model of a two-component system with failure interaction and proposes the optimal PM decision scheme under different EW periods. Through the example analysis, the necessity of considering the failure interaction between components in EW cost analysis of the two-component system is illustrated. The comparison with only implementing minimal repair after the failures during EW period also verifies the effectiveness of the PM strategy. At the same time, the influence of different system parameters and maintenance parameters on the final EW warranty decision is also illustrated in the parameter sensitivity analysis, which can provide scientific and effective information support and basis for manufacturers and users to make EW decision. In

addition, based on Weibull distribution, this paper describes the rule of system failure and constructs the EW cost model. In fact, similar ideas and methods can be used to establish the EW cost model of the multi-component system with failure interaction when the components obey other probability distributions, such as exponential distribution, uniform distribution, and so on, and the system availability constraints can be added to optimize the maintenance strategy.

Data Availability

All data, models, and codes generated or used during the study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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