Research Article

Multiobjective Evaluation of Coevolution among Innovation Populations Based on Lotka–Volterra Equilibrium

Sheng-Yuan Wang,¹,² Wan-Ming Chen,¹ Rong Wang,² and Xiao-Lan Wu²

¹College of Economics and Management, Nanjing University of Aeronautics & Astronautics, Nanjing, Jiangsu 210016, China
²Business School, Nanjing Xiaozhuang University, Nanjing, Jiangsu 211171, China

Correspondence should be addressed to Sheng-Yuan Wang; 56439976@qq.com

Received 6 January 2021; Accepted 27 May 2021; Published 8 June 2021

Academic Editor: Maria Alessandra Ragusa

Copyright © 2021 Sheng-Yuan Wang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The collaborative evaluation of enterprise innovation populations is a hot issue. The Lotka–Volterra model is a mature method used to evaluate the interaction mechanism of populations and is widely used in innovation ecology research studies. The Lotka–Volterra model mainly focuses on the quantitative characteristics of the interactive populations. The growth mechanisms cannot explain all the synergy mechanisms of the innovative populations. The collaborative evaluation between enterprise innovation populations is a typical multiobjective evaluation problem. The multichoice goal programming model is a mature method to solve multiobjective optimization problems. This paper combines the Lotka–Volterra model and multichoice goal programming method to construct a three-stage multiobjective collaboration evaluation method based on Lotka–Volterra equilibrium. An evaluation example is used to illustrate the application process of this method. The method proposed in this paper has excellent performance in computing, parameter sensitivity analysis, and model stability analysis.

1. Introduction

Innovation activity is a systems engineering issue. It is difficult for a single organization to have all the resources necessary for innovation. It has dynamic evolution, symbiosis, and concurrence formed by the connection and transmission of material flow and talent flow among various innovation populations (IPs) under a specific innovation environment in a certain region. The innovation resources in a certain region are limited. The basic problem addressed by theoretical research and social practice in the fields of economics and management is how to allocate resources optimally [1–6]. An appropriate scale, along with the development of an innovation ecosystem, can support the effective development of the IP. In this case, the resources in the innovation ecosystem can be fully utilized, and the innovation performances can be maximized [7, 8].

The population scale in a region determines the population density. Existing studies generally use the population density index to analyze the relationship between population scale and innovation performance. Some studies found that industrial clusters have a positive impact on enterprise innovation [9, 10]. However, other studies found that industrial clusters have a negative effect on enterprise innovation [11, 12]. Given these two different conclusions, it is necessary to establish a new framework to analyze the relationships between an enterprise’s population size and its innovation ability. Therefore, the topic of population ecology should be addressed. Hannan and Freeman [13] analyzed the enterprise problem based on the logic of population ecology and founded the organizational ecology field. This present study further develops and expands the population ecology framework.

The Lotka–Volterra model is often used in population ecology to analyze the cooperative or competitive relationship of populations. Studies have shown that the introduction of Lotka–Volterra, a population competition model in biology, into market competition and diffusion has produced better analysis results [14, 15]. Lee et al. used the Lotka–Volterra model to analyze the dynamic competitive
relationship in the Korean stock market [16] and confirmed that the role of competitors will change over time. Kim, Lee, and Ahn studied the Korean mobile communications market through the Lotka–Volterra model and found the symbiotic relationship between competitors [17]. Kreng and Wang used the Lotka–Volterra model and the Lyapunov function to study the dynamic competition and balance point of PDP and LCD TV [18]. On the basis of previous studies, this paper selects interdependent innovation populations as the research object and uses the Lotka–Volterra model to study the level of synergy and balance between innovation populations in the innovation ecosystem. Different from previous studies, when using the Lotka–Volterra model, this study is not simply based on the scale of the innovation population to explore the synergy relationship, but on the number of innovation populations, input and output optimization, and other multiple objectives as the research basis to explore the relationship between innovation populations. The research organization of this article is shown in Figure 1.

As shown in Figure 1, the research organization of this article is as follows: (1) this paper uses population dynamics (Lotka–Volterra model) to analyze the influence mechanism between interactive innovation populations. (2) This paper builds multichoice goal programming (MCGP) and Lotka–Volterra-MCGP models for innovation populations scale optimization. (3) This paper uses TOPSIS method to evaluate the collaboration of innovation populations.

In this study, the interaction model of IPs is constructed based on the perspectives of ecological theory and innovation theory, which form the theoretical basis of this study. A multichoice model is used to determine the appropriate scale of the IP under resource constraints, population synergy, and maximum output targets. To resolve the multichoice optimization problem mentioned above while taking into account the collaborative development of the IP, a comprehensive method is required. Based on the perspective of resource constraints, this paper constructs a dynamic model of the growth for the IP. This paper estimates the suitability of population size by using the multichoice goal programming method. Two proposed models are constructed to obtain the appropriate population scale. This research constructs the theoretical model from two aspects, namely, input constraint and output maximization, which is theoretically innovative. Our research also has practical significance because it provides an appropriate analysis method for various innovation subjects to analyze and plan the development of IPs.

2. Literature Review

Over the past 10 years, the concept of innovation ecosystem has become popular among the rapidly growing literature [19] that typically focuses on business and strategy. Oh et al. [20] criticized the concept innovation ecosystem with regard to its usefulness and distinctiveness in relation to existing conceptualizations of innovation systems and with regard to the biologically inspired “eco” qualifier, which is arguably a flawed analogy to natural ecosystems.

Related literature uses the theory of organizational ecology to study the coevolution process of corporate populations. Baum, Korn, and Kotha empirically studied the competitive advantages and survival rate of incumbents and new entrants in the telecommunication service industry after the technical standards have been established [21]. Low and Abrahamson studied the changes in corporate population organization in three different stages of emerging industries, growing industries, and mature industries [22]. Geroski and Mazzucato used the market size model, negative feedback model, infectious disease model, and density dependence model to empirically study the evolution process of the population of the US auto industry [23]. Hannan, Carroll, Dundon, and Torres [24] and Hannan [25] used the density-dependent model to empirically study the evolution process of the corporate population of the European auto industry.

Adomavicius, Bockstedt, Gupta, and Kaufman qualitatively studied the interdependence and mechanism of the three types of technologies: components, basic common technologies, and products and applications [26]. Adner and Kapoor empirically studied the influence of the difficulty of developing upstream components and downstream complementary products on the competitive advantage of integrated innovation leaders [27].

Almost all of the above-mentioned studies are based on a certain industry, starting from the enterprise level, using the number of enterprises entering and exiting the market as the research data source, using ecological related models to verify the evolution of a single population or the coevolution relationship between two subpopulations. In reality, especially in high-tech industries, it is more common to form an interinfluenced and interdependent technological innovation ecosystem around the industrial chain. There are few relevant literatures focusing on technological populations in the innovation ecosystem, and qualitative research is the main focus.

3. The Model of Innovation Population Growth

In this section, the growth model of the innovation populations (IPs) is to be constructed. Based on the Lotka–Volterra model, an innovation population relationship model is proposed, and the equilibrium point is analyzed.

3.1. Model Construction Ideas. When there are abundant resources, populations can grow at geometric or exponential rates. As resources are depleted, population growth rate slows and eventually stops. This is known as logistic population growth. The environment limits population growth by changing birth and death rates. On average, small organisms experience increases per capita at higher rates and more variable populations, while large organisms have lower increase rates per capita and less variable populations. In view of resource constraints and the need for specialization, it is difficult for any single firm to develop and commercialize a technology-based offering from start to finish [28, 29]. Increasingly complex constellations of organizations have been emerging in the form of innovation...
ecosystems, where actors interact with each other to create, deliver, and appropriate value.

The study of enterprise IP dynamics should consider the influence of regional constraints. The ecological sense of a population is a collection of certain organisms within a given time and space. The region where the population grows is a relatively homogeneous nonlinear region that is different from the surrounding environment. There are universal temporal and spatial constraints in natural hierarchy systems.

The spatial distribution characteristics of different ecological regions such as size, shape, boundary, nature, and distance make up different ecological zones, forming the differences of ecosystem and regulating population growth. The model of enterprise IP dynamics focuses on the quantity change in the IP. Its changing rule is based on the nonlinear growth principle of biological population quantity. The growth model of most species is nonlinear in nature. The number of innovative enterprises may change rapidly with the influence of incentive-based policies and innovation resources in a given period and in a given area.

Competition and synergy within populations are also important factors, based on the intraspecific competition principle of biological populations. Competition exists within the biological population. The larger the population scale is, the more intense the competition will be. Competition among populations has the function of population size adjustment. There is also a certain competition mechanism in the IP, and this competition mechanism will suppress the excessive expansion of the IP, to some extent.

Therefore, intraspecific competition is also one of the processes for the survival of the fittest. Thus, this mechanism should be an important component of the growth model for entrepreneurial population. There is also a competition or synergy relationship between the different IPs. Based on the points raised above, this study uses the growth dynamics model of biological population theory to investigate the development characteristics of IPs.

### 3.2. Deduction of the Innovation Population Relationship Model

#### 3.2.1. Dynamics Analysis of Logistic and Lotka–Volterra Models

According to the logistic model, we construct an internal relationship model of innovation population 1 (IP1) as follows:

$$g_1(t) = \frac{dN_1(t)}{dt} = \alpha_1 N_1 \left(1 - \frac{N_1}{K_1}\right). \quad (1)$$

$g_1(t)$ indicates the population growth rate of phase $t$. $N_1(t)$ indicates the number of individuals in the population in phase $t$. Within a certain period of time (phase $t$), $K_1$ is the maximum population scale in a constant environment. Each unit occupies resources and is defined as $(1/K_1)$. $\alpha_1$ reflects the promotion of the population’s growth. $(1 - (N_1/K_1))$ reflects the retardation of growth due to the consumption of limited resources by the population.

If $g_1(t) > 0$, then $\Delta N_1(t) > 0$. The synergistic effects are dominant effects in the population. Resources within an innovation ecosystem can support an increase in the number of individuals in an IP. Thus, the growth can be sustainable.

If $g_1(t) < 0$, then $\Delta N_1(t) < 0$. The competition effect is dominant in the population. Innovation resources are less able to support the increase in the number of individuals in the IP. Thus, the growth is unsustainable.

According to the logistic model, this paper constructs an internal relationship model of innovation population 2 (IP2) as follows:

$$g_2(t) = \frac{dN_2(t)}{dt} = \alpha_2 N_2 \left(1 - \frac{N_2}{K_2}\right). \quad (2)$$

$N_2(t)$ represents the number of individuals in the innovation population 2 in period $t$. Researchers should consider the impact of the IP2 on IP1. Then, the logistic model can be modified as follows:

$$g_1(t) = \frac{dN_1(t)}{dt} = \alpha_1 N_1 \left(1 - \frac{N_1}{K_1} + \frac{\beta_{12} N_1 N_2}{K_2}\right). \quad (3)$$

$\beta_{12}$ is the influence coefficient of population 2 on population 1. If $\beta_{12} > 0$, population 2 has a synergistic effect on population 1. If $\beta_{12} < 0$, population 2 has a competitive effect on population 1. After the formation of the dependent symbiosis system, due to the promotion of population 1, the size of population 2 will also increase. The scale change of population 2 can be described as
\[
g_1(t) = \frac{dN_1(t)}{dt} = \alpha_1 N_1 \left( 1 - \frac{N_2}{K_1} + \frac{\beta_{21} N_2}{K_2} \right),
\]

where \( \beta_{21} \) is the influence coefficient of population 1 on population 2. If \( \beta_{21} > 0 \), population 1 has a synergistic effect on population 2. If \( \beta_{21} < 0 \), population 1 has a competitive effect on population 2. In the system of IP1 and IP2, the symbiosis mathematical model is

\[
\begin{align*}
g_1(t) &= \frac{dN_1(t)}{dt} = \alpha_1 N_1 \left( 1 - \frac{N_2}{K_1} + \frac{\beta_{12} N_2}{K_2} \right), \\
g_2(t) &= \frac{dN_2(t)}{dt} = \alpha_2 N_2 \left( 1 - \frac{N_1}{K_2} + \frac{\beta_{21} N_1}{K_1} \right).
\end{align*}
\]

Among them, \( 1 > \beta_{12} > 0, 1 > \beta_{21} > 0, \beta_{12} \) is the contribution of population 2 to population 1, which means that the resources that population 2 supplies to population 1 are \( \beta_{12} \) times the resources that population 2 supplies itself. According to the dependence and independence conditions, then \( 1 > \beta_{12} > 0 \). Similarly, we can get \( 1 > \beta_{21} > 0 \).

Equation (5) is called the Lotka–Volterra model. The Lotka–Volterra model of dual-population or multipopulation growth is a differential dynamic system to simulate the dynamic relationship between populations in the innovation ecosystem. Based on the numerical value of \( \beta \), the type of interaction between species can be judged [32]:

1. When \( \beta_{12} = 0, \beta_{21} = 0 \), it means that the innovation populations are independent, develop independently, and do not affect each other. At this time, the Lotka–Volterra model expresses no symbiotic relationship.

2. When \( \beta_{12} < 0, \beta_{21} < 0 \), it means that the two parties compete with each other. One party grows while the other party declines. There is no symbiotic relationship between the two.

3. When \( \beta_{12} > 0, \beta_{21} < 0 \) or \( \beta_{12} < 0, \beta_{21} > 0 \), it means that one party is attached to the other party during the symbiotic evolution of the innovative population, showing a parasitic mode of constantly requesting resources from the other party to maintain its own growth.

4. When \( \beta_{12} > 0, \beta_{21} = 0 \) or \( \beta_{12} = 0, \beta_{21} > 0 \), it means that both sides of the innovative population have obtained extra high-quality resources in the evolution process, but the symbiosis coefficient of one of them is zero, indicating that it has not obtained extra resources, but no losses have been suffered, and the innovation ecosystem is now in a symbiotic mode of partial benefit.

5. When \( \beta_{12} > 0, \beta_{21} > 0 \), it means that the innovative population is in a mutually beneficial symbiosis mode. Among them, if \( \beta_{12} \neq \beta_{21} \), it means that the symbiotic relationship between the two parties is asymmetric and mutually beneficial symbiosis; when \( \beta_{12} = \beta_{21} \), it means that the innovative population has obtained equal benefits in the process of symbiotic evolution, and the resources are exchanged in equal amounts, forming a symmetric and mutually beneficial symbiosis.

In innovation activities, competition can occur between populations that use common resources. Symbiosis in the innovation ecosystem does not exclude competition. Innovative populations in completely or part of the same living space need to conduct technology, talent, and market interaction in the factor market. The competition for capital and then separating, expanding, and alliance niche occupy a more favorable living position, enhance its core competitiveness, and form new offspring to adapt to the environment of the innovation ecosystem through mass reproduction. However, when one party in the innovation ecosystem relies on another core or dominant population to obtain resources and living space, a parasitic relationship is formed. Under the parasitic relationship, the symbiotic subject has a one-way exchange of interests. Because of the one-way asymmetric exchange, this state is not extensive. Therefore, the system will gradually develop in the direction of symbiosis that is conducive to mutual dependence and mutual benefit.

3.2.2. Analysis of Equilibrium Point Stability. The equilibrium point of the evolution of the innovation populations means that the output of both parties has reached the maximum and remained stable. The following uses the stability analysis of the equilibrium point to discuss the symbiosis stability of the innovative populations 1 and 2. When the two populations reach a symbiotic stable state, the differential equations can be expressed as

\[
\begin{align*}
f_1(N_1, N_2) &= \frac{dN_1(t)}{dt} = \alpha_1 N_1 \left( 1 - \frac{N_1}{K_1} + \frac{\beta_{12} N_2}{K_2} \right) = 0, \\
f_2(N_1, N_2) &= \frac{dN_2(t)}{dt} = \alpha_2 N_2 \left( 1 - \frac{N_2}{K_2} + \frac{\beta_{21} N_1}{K_1} \right) = 0.
\end{align*}
\]

Solving the equations can get the equilibrium point of the symbiotic relationship between the two IPs:

\[
\frac{K_1 (1 + \beta_{12})}{1 - \beta_{12} \beta_{21}} \frac{K_2 (1 + \beta_{21})}{1 - \beta_{12} \beta_{21}}.
\]

The Jacobian matrix is used to solve the equilibrium point of the symbiotic evolution model of the innovation population in the innovation ecosystem. Obtain the determinant \( \det(J) \) and trace \( \text{Tr}(J) \) of the Jacobian matrix. When \( \det(J) > 0 \) and \( \text{Tr}(J) < 0 \), the local equilibrium point is in a stable state; otherwise it is not a stable equilibrium point. The
following is a comparative analysis of the determinants and traces of the Jacobian matrix (Table 1). On this basis, the stable equilibrium point and stability conditions of the symbiotic evolution model of the innovative population are obtained.

As shown in Table 1, suppose $\alpha_1 > 0$ and $\alpha_2 > 0$ and that the determinant $\text{Det}(J)$ and trace $\text{Tr}(J)$ can be determined according to the different values of $\beta_{12}$ and $\beta_{21}$, and the condition that the local equilibrium point is a stable equilibrium point is obtained. Through comparison, the symbiosis evolution model under different values corresponds to different stable equilibrium points, but the equilibrium points of partial benefit, asymmetric reciprocity, and symmetric reciprocity symbiosis are all the same $P_4$. The parasitic equilibrium point may be $P_{24}$, and $P_4$ (Table 2).

As shown in Table 2, the symbiosis evolution equilibrium point of the innovation population is related to the symbiosis model and is affected by the size of the mutual competition coefficient between the innovation populations and the maximum capacity of the respective populations.

Because of the interdependence between the two populations, the population size cannot be zero, so the points $P_1$, $P_2$, and $P_3$ are discarded. $P_4$ point corresponds to the scale of populations 1 and 2, respectively ($(K_1(1 + \beta_{12})/1 - \beta_{12}\beta_{21}), (K_2(1 + \beta_{21})/1 - \beta_{12}\beta_{21})$). At the same time, the conditions for $P_4$ to be meaningful are

$$
\begin{align*}
K_1(1 + \beta_{12}) & > 0, \\
1 - \beta_{12}\beta_{21} & > 0,
\end{align*}
$$

(8)

The only nonnegative solution can be obtained by solving the equations above. That is, the equilibrium point is ($(K_1(1 + \beta_{12})/1 - \beta_{12}\beta_{21}), (K_2(1 + \beta_{21})/1 - \beta_{12}\beta_{21})$). This equilibrium point represents the equilibrium state of innovation resources occupied by the IP of the enterprise and the IP of the scientific research institution.

4. Output-Oriented Population Size Optimization Model

4.1. Objective Planning Method. Objective programming is an effective method for solving the multiobjective programming problem. Its basic idea is to determine a desired value (objective value or ideal value) for each objective function of the multiobjective programming problem. However, due to the limitations of various conditions, these objective values are often impossible to achieve. Therefore, positive or negative deviation variables are introduced into each objective function to represent the situation where the objective value is either exceeded or not reached. To distinguish the importance of each objective, the priority and weighting coefficient of the objective are introduced. Then, constraint equations are established for all objective functions. From this new set of constraints, the scheme to minimize the combination deviation is obtained. The foundations of the objective programming model are simple and easy to understand, and the model and its hypothesis are in line with reality. Compared with other methods, the objective programming method has more flexibility, effectiveness, and convenience in use and implementation when dealing with multiobjective problems.

4.2. Multichoice Goal Programming. In recent years, multichoice goal programming (MCGP) has been widely used to resolve many practical decision-making problems. Chang et al. [33] integrated MCGP and fuzzy mathematics methods, according to different strategic directions of LCD and acrylic plate manufacturers. They considered the multiobjective expectation level and fuzzy relationship, which helped decision makers select the best supplier. Lee et al. [34] solved the problem of engineering technology selection in product design by combining MCGP, AHP, and QFD. Chen et al. [35] proposed a three-layer MCGP method to help forest managers obtain appropriate solutions for forest resource allocation. The multiple-choice goal programming (MCGP) method proposed by Chang [36, 37] is described as follows:

Objective function: Min $\sum_{i=1}^{n} (d_i^+ + d_i^-) + \sum_{i=1}^{n} (e_i^+ + e_i^-)$,

Constraints:

$$
\begin{align*}
f_i(x) - d_i^+ + d_i^- & = g_i, \quad i = 1, 2, \ldots, n, \\
x & \in X = \{x_1, x_2, \ldots, x_m\}, \\
g_i - e_i^+ + e_i^- & = g_i, \quad i = 1, 2, \ldots, n, \\
g_{i_{\text{min}}} \leq g_i \leq g_{i_{\text{max}}}, \quad i = 1, 2, \ldots, n, \\
d_i^+, d_i^-, e_i^+, e_i^- & \geq 0, \quad i = 1, 2, \ldots, n, \\
X \in F \quad (F \text{ is the set of feasible solutions}).
\end{align*}
$$

(9)
MCGP is a linear form of objective programming, which can be solved by some common linear programming software. This is because, in minimizing the objective function, the objective function can be infinitely close to the value of the objective. In the same way, in minimizing the objective function, the objective value can also approach the upper bound of the objective infinitely.

4.3. Multi-choice Goal Programming Embedding with Lotka–Volterra Equilibrium. Embed the Lotka–Volterra model and the symbiotic population equilibrium point as constraints into the MCGP model to obtain the Lotka–Volterra-MCGP model:

Here, \(d_i^+, d_i^-\) indicate the value of the \(i\)-th goal exceeding and not reaching the expected value of the goal. \(f_i(x)\) is the function of the \(i\)-th object. \(X\) is the decision variable, representing \(m\) alternatives \((x_1, x_2, \ldots, x_m)\). \(g_i\) is the expected level for the \(i\)-th goal.

\(e_i^+\) and \(e_i^-\) are close to positive and negative deviation variables of \(|g_i - g_{i,\text{max}}|, g_{i,\text{min}}\) and \(g_{i,\text{max}}\) are the lower and upper limits of the target for \(g_i\).

MCGP is a linear form of objective programming, which can be solved by some common linear programming software. This is because, in minimizing the objective function, the objective function can be infinitely close to the value of the objective. In the same way, in minimizing the objective function, the objective value can also approach the upper bound of the objective infinitely.

Table 1: Equilibrium point and stability conditions of innovation populations.

<table>
<thead>
<tr>
<th>Equilibrium point</th>
<th>Det (J)</th>
<th>Tr (J)</th>
<th>Stability conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1(0,0))</td>
<td>(\alpha_i \alpha_j)</td>
<td>(\alpha_i + \alpha_j)</td>
<td>Unstable</td>
</tr>
<tr>
<td>(P_2(N_1,0))</td>
<td>(-\alpha_i \alpha_j (\beta_{21}))</td>
<td>(-\alpha_i + \alpha_j (1 + \beta_{21}))</td>
<td>(\beta_{21} &lt; -1)</td>
</tr>
<tr>
<td>(P_3(n,0))</td>
<td>(-\alpha_i \alpha_j (\beta_{12}))</td>
<td>(-\alpha_i + \alpha_j (1 + \beta_{12}))</td>
<td>(\beta_{12} &lt; 0)</td>
</tr>
<tr>
<td>(P_4(K_1(1 + \beta_{12})/\beta_{12}, K_2(1 + \beta_{21})/\beta_{21}))</td>
<td>(\alpha_i \alpha_j (1 - \beta_{12})(1 - \beta_{21})/\beta_{12} \beta_{21})</td>
<td>(\alpha_i \alpha_j (1 - \beta_{12})(1 - \beta_{21})/\beta_{12} \beta_{21})</td>
<td>(\beta_{12} &lt; 1, \beta_{12} &lt; 1)</td>
</tr>
</tbody>
</table>

Table 2: Evolution mode among innovation populations.

<table>
<thead>
<tr>
<th>Influencing factor value</th>
<th>Symbiotic relationship</th>
<th>Stable equilibrium point</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_{12} \beta_{21} &lt; 0)</td>
<td>Parasitic relationship</td>
<td>(P_3, P_4)</td>
</tr>
<tr>
<td>(\beta_{12} = 0, \beta_{21} &lt; 0) or (\beta_{21} = 0, \beta_{12} &lt; 0)</td>
<td>Favor symbiosis</td>
<td>(P_4)</td>
</tr>
<tr>
<td>(\beta_{12} &gt; 0, \beta_{21} &gt; 0, \beta_{12} \neq \beta_{21})</td>
<td>Asymmetric symbiosis</td>
<td>(P_4)</td>
</tr>
<tr>
<td>(\beta_{12} &gt; 0, \beta_{21} &gt; 0, \beta_{12} = \beta_{21})</td>
<td>Symmetry symbiosis</td>
<td>(P_4)</td>
</tr>
</tbody>
</table>

The objective function is:

\[
\text{Objective function: Min} \sum_{i=1}^{n} (d_i^+ + d_i^-) + \sum_{i=1}^{n} (e_i^++ e_i^-),
\]

\[
f_i(x) - d_i^+ + d_i^- = g_i, \quad i = 1, 2, \ldots, n,
\]

\[
x \in X = \{x_1, x_2, \ldots, x_m\},
\]

\[
g_i - e_i^+ + e_i^- = g_{i,\text{max}}, \quad i = 1, 2, \ldots, n,
\]

\[
g_{i,\text{min}} \leq g_i \leq g_{i,\text{max}}, \quad i = 1, 2, \ldots, n,
\]

\[
d_i^+, d_i^-, e_i^+, e_i^- \geq 0, \quad i = 1, 2, \ldots, n,
\]

\[
X \in F \quad (F\text{ is the set of feasible solutions}),
\]

\[
x_1 = \frac{M_1(1 + \beta_{12})}{1 - \beta_{12} \beta_{21}}, \quad x_2 = \frac{M_2(1 + \beta_{21})}{1 - \beta_{12} \beta_{21}} \quad \frac{x_1}{x_2} = \frac{M_1(1 + \beta_{12})}{M_2(1 + \beta_{21})},
\]

\[
0 < \beta_{12} < 1, \quad 0 < \beta_{21} < 1.
\]
5. An Example

Based on the classic Cobb Douglas production function, this paper chooses the accumulated assets of R and D investment as the main investment indicator and adds R and D human capital to the model robustness test part to measure enterprise innovation investment. For the research on enterprise innovation output, many studies use the number of patents to measure the output of the innovation population. However, the technical characteristics of different industries are different, and the tendency to apply for patent protection is also different. The method of using patent numbers to measure the innovation performance of enterprises is mostly used in the research of a single industry [40]. At this time, the company’s new product sales can be used to measure innovation output [41, 42]. As an auxiliary verification, this article adds new product sales to measure the output of enterprise innovation populations in the model robustness test part.

5.1. The Relationship between Sample Data and Variables.

The two related innovation populations (IP₁ and IP₂) have a synergistic effect between them. Variable interpretation and data selection are as follows (shown in Table 3):

- (1) $P₁$: population scale of IP₁ (number of enterprises in IP₁, unit: number)
- (2) $P₂$: population scale of IP₂ (number of enterprises in IP₂, unit: number)
- (3) $E₁$: R&D expenditure for $P₁$ (unit: 100 million yuan)
- (4) $E₂$: R&D expenditure for $P₂$ (unit: 100 million yuan)
- (5) $O₁$: innovation output of $P₁$ (expressed by the number of granted patents)
- (6) $O₂$: innovation output of $P₂$ (expressed by the number of granted patents)

5.2. Optimization Model Interpretation.

In this case, the number of authorized patents is taken as a measure of innovation output. We use the objective solution of MCGP and the solution of equilibrium value to construct the suitability of population.

The related functions and parameters are listed below:

- $F₁(X) = 10.38x₁ + 11.59x₂$ (innovation output goal, the more the better)
- $F₂(X) = 0.09x₁ + 0.30x₂$ (R&D expenditure goal, the less the better)

In basis of MCGP-achievement, this problem can be formulated as follows:

\[
\begin{align*}
\text{Min}, & \quad d₁^* + d₂^* + d₃^* + d₄^* + e₁^* + e₂^* + e₃^* + e₄^*, \\
\text{s.t.,} & \quad y₁ = 10.38x₁ + 11.59x₂ - d₁^* - d₄^*; \text{for output goal, the more the better}, \\
& \quad y₂ = 0.09x₁ + 0.30x₂ - d₄^* - d₃^*; \text{for R&D expenditure goal, the less the better}, \\
& \quad y₁ - e₁^* - e₃^* = O₇, y₁ ≤ O₇, \\
& \quad y₂ - e₂^* - e₄^* = E₇, y₂ ≤ E₇, \\
& \quad d₁^* > 0, d₄^* > 0, d₃^* > 0, d₂^* > 0, e₁^* > 0, e₃^* > 0, e₂^* > 0, e₄^* > 0, x₁ > 0, x₂ > 0.
\end{align*}
\]
Table 3: Relevant data for the example.

<table>
<thead>
<tr>
<th>Area</th>
<th>$P_1$</th>
<th>$E_1$</th>
<th>$O_1$</th>
<th>$K_1$</th>
<th>$P_2$</th>
<th>$E_2$</th>
<th>$O_2$</th>
<th>$K_2$</th>
<th>$O_T$</th>
<th>$E_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>19186</td>
<td>1610</td>
<td>157887</td>
<td>16448</td>
<td>1190</td>
<td>417</td>
<td>22214</td>
<td>1262</td>
<td>180101</td>
<td>2027</td>
</tr>
<tr>
<td>$A_2$</td>
<td>18872</td>
<td>1456</td>
<td>166445</td>
<td>14874</td>
<td>1113</td>
<td>345</td>
<td>21586</td>
<td>1044</td>
<td>188031</td>
<td>1801</td>
</tr>
<tr>
<td>$A_3$</td>
<td>14150</td>
<td>1328</td>
<td>131966</td>
<td>13567</td>
<td>998</td>
<td>632</td>
<td>14843</td>
<td>658</td>
<td>146809</td>
<td>1113</td>
</tr>
<tr>
<td>$A_4$</td>
<td>12283</td>
<td>1239</td>
<td>172787</td>
<td>12658</td>
<td>944</td>
<td>248</td>
<td>13938</td>
<td>750</td>
<td>186725</td>
<td>1487</td>
</tr>
<tr>
<td>$A_5$</td>
<td>11133</td>
<td>1080</td>
<td>186220</td>
<td>11033</td>
<td>909</td>
<td>208</td>
<td>12362</td>
<td>629</td>
<td>198582</td>
<td>1288</td>
</tr>
<tr>
<td>$A_6$</td>
<td>7712</td>
<td>964</td>
<td>118919</td>
<td>9848</td>
<td>795</td>
<td>113</td>
<td>9406</td>
<td>327</td>
<td>128325</td>
<td>1072</td>
</tr>
<tr>
<td>$A_7$</td>
<td>2257</td>
<td>525</td>
<td>71781</td>
<td>5363</td>
<td>714</td>
<td>333</td>
<td>6726</td>
<td>1008</td>
<td>78507</td>
<td>858</td>
</tr>
<tr>
<td>$A_8$</td>
<td>2236</td>
<td>571</td>
<td>14616</td>
<td>5833</td>
<td>632</td>
<td>115</td>
<td>1823</td>
<td>348</td>
<td>16439</td>
<td>686</td>
</tr>
<tr>
<td>$A_9$</td>
<td>2159</td>
<td>417</td>
<td>46976</td>
<td>4260</td>
<td>713</td>
<td>300</td>
<td>3618</td>
<td>908</td>
<td>50594</td>
<td>717</td>
</tr>
<tr>
<td>$A_{10}$</td>
<td>2508</td>
<td>684</td>
<td>23925</td>
<td>6988</td>
<td>693</td>
<td>113</td>
<td>2429</td>
<td>342</td>
<td>26354</td>
<td>797</td>
</tr>
<tr>
<td>$A_{11}$</td>
<td>2257</td>
<td>571</td>
<td>14616</td>
<td>5833</td>
<td>632</td>
<td>115</td>
<td>1823</td>
<td>348</td>
<td>16439</td>
<td>686</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>1831</td>
<td>401</td>
<td>7835</td>
<td>4097</td>
<td>658</td>
<td>290</td>
<td>1600</td>
<td>878</td>
<td>9435</td>
<td>691</td>
</tr>
</tbody>
</table>

Figure 2: Interpopulation model of IP$_1$ and IP$_2$.

Considering the symbiotic relationship [43], new constraints need to be added to the above model:

\[
\begin{align*}
\text{s.t.,} \\
-x_1 &= \frac{k_1(1 + \beta_{12})}{1 - \beta_{12} \cdot \beta_{21}}, \text{equilibrium value for IP}_1, \\
-x_2 &= \frac{k_2(1 + \beta_{21})}{1 - \beta_{21} \cdot \beta_{12}}, \text{equilibrium value for IP}_2, \\
0.1 &< \beta_{12} < 1, \text{for bound of } \beta_{12}, \\
0.1 &< \beta_{21} < 1, \text{for bound of } \beta_{21}, \\
x_1 &= \frac{k_1(1 + \beta_{12})}{k_2(1 + \beta_{21})}, \text{ratio constraint between IP}_1 \text{ and IP}_2. 
\end{align*}
\]

(12)

The problem is solved using the LINGO [44] software and is shown in Figure 2.

As seen in Table 4, there is a significant difference between the observed value and the optimized value. The population scale of IP$_1$ and IP$_2$ does not belong to the appropriate number of population. Innovation ecosystem can carry out self-organized evolution. Cooperation among populations and among enterprises within populations can promote a win-win situation among multiple populations. However, the negative cases in reality challenge the self-organization evolution of the innovation ecosystem. The reason lies in the lack of detailed and in-depth analysis of the mechanisms at play in innovation ecosystem. The innovation ecosystem is a complex system, and its operation is inevitably affected by the interaction between the subsystems.

5.3. Evaluation. This paper evaluates the level of synergy among innovative populations in different regions based on the similarity between sample values and optimized values. The higher the similarity, the better the synergy. The evaluation matrix is $A$.

\[
A = [a_{ij}]_{12 \times 2},
\]

\[
a_{ij} = |LMP_{ij} - IP_i|.
\]

(13)

This study uses the entropy method to determine the weight of the two populations in the collaborative evaluation. Use the technique for order preference by similarity to an ideal solution (TOPSIS) method to evaluate the similarity, and sort the regions in the sample at the level of population coordination. The ideal scale of the two populations can be regarded as two criteria for evaluating the synergy of the innovation system.

5.3.1. The Calculation of Evaluation Weight. Entropy weight is an objective weight method. The advantage of entropy weight method reduces the subjective impact of decision makers and increases objectivity [45]. Entropy was originally a concept in thermodynamics to calculate the degree of confusion. Shannon applied it to information theory, making it one of the ways to deal with uncertainty [46]. The less the entropy value is, the more the information that can be provided will be. Therefore, the criterion can be assigned a bigger weight [47]. The concept of entropy weight has been widely used in several fields.

For example, Mohsen [48] combined entropy with fuzzy VIKOR for the risk assessment of equipment failure. Sengül et al. [49] combined entropy with fuzzy TOPSIS approach for ranking RE supply systems. Shad et al. [50] combined entropy with AHP and GIS in green building assessment.
Hafezalkotob [51] integrated entropy and subjective weight for the engineering design. They compared the ranking results with other methods and found that the entropy method is suitable for application in similar problems. The calculation of Shannon’s entropy weight is presented as follows [52].

Assume \( m \) alternatives (\( A_1, A_2, A_m \)) and \( n \) criteria (\( C_1, C_2, C_n \)) for a decision problem. Then initial decision matrix is

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\]

(14)

Step 1: normalize the evaluation matrix:

\[
 r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}
\]

(15)

Step 2: compute entropy:

\[
e_j = -\frac{1}{\ln m} \sum_{i=1}^{m} \ln r_{ij}, \quad j = 1, 2, \ldots, n.
\]

(16)

Step 3: the weights of each criterion are calculated:

\[
w_j = \frac{1 - e_j}{\sum_{i=1}^{n} (1 - e_j)}, \quad j = 1, 2, \ldots, n.
\]

(17)

5.3.2. Order Preference by Similarity to an Ideal Solution.

Technique for order preference by similarity to an ideal solution (TOPSIS) is a popular method proposed by Hwang and Yoon [53] to determine the best alternative. The main rule of TOPSIS is that the best alternative should have shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution [54]. This method has been widely adopted to solve problems in many different fields. The algorithm of the TOPSIS method is presented as follows.

Step 1: construct the normalized decision matrix \( R \):

\[
r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}.
\]

(18)

Step 2: construct weighted normalized decision matrix \( V \):

\[
 v_{ij} = w_j r_{ij},
\]

\[
 \sum_{j=1}^{n} w_j = 1.
\]

(19)

\( w_j \) is the weight of the \( j \)-th criterion.

Step 3: determine the positive-ideal solution (PIS) and negative-ideal solution (NIS), denoted, respectively, as \( A^+ \) and \( A^- \), which are defined in the following way:

\[
 A^+ = \left\{ \left( \max_{j \in J} v_{ij} \right) \text{ or } \left( \min_{j \in J'} v_{ij} \right) \right\}, \quad i = 1, 2, \ldots, m
\]

\[
 = \left\{ v_1^+, v_2^+, \ldots, v_n^+ \right\},
\]

(20)

\[
 A^- = \left\{ \left( \min_{j \in J} v_{ij} \right) \text{ or } \left( \max_{j \in J'} v_{ij} \right) \right\}, \quad i = 1, 2, \ldots, m
\]

\[
 = \left\{ v_1^-, v_2^-, \ldots, v_n^- \right\},
\]

(21)

Calculated based on the data in the example: \( w_1 = 0.262, \ w_2 = 0.738 \).
where \(J\) and \(J'\) are sets of benefit and cost criteria, respectively.

**Step 4:** calculate the distances of each alternative from positive-ideal solution (PIS) and negative-ideal solution (NIS):

\[
S^+_i = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_{ij}^+)^2}, \quad i = 1, 2, \ldots, m, \quad (22)
\]

\[
S^-_i = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_{ij}^-)^2}, \quad i = 1, 2, \ldots, m. \quad (23)
\]

**Step 5:** calculate the closeness coefficient and rank the order of alternatives:

\[
C^+_i = \frac{S^{-}_i}{S^+_i + S^-_i}, \quad 0 < C^+_i < 1, \quad i = 1, 2, \ldots, m, \quad (24)
\]

where \(C^+_i \in [0, 1]\) with \(i = 1, 2, \ldots, m\). The best alternative can therefore be found according to the preference order of \(C^+_i\). The value is the more the better. If \(C^+_i\) is close to 1, it indicates the alternative \(A_i\) is closer to the PIS.

As shown in Table 5, this paper can successfully get the collaborative evaluation ranking of innovation populations in different regions.

### Table 5: Result of TOPSIS.

<table>
<thead>
<tr>
<th>Area</th>
<th>(V_{11})</th>
<th>(V_{12})</th>
<th>(S^+_i)</th>
<th>(S^-_i)</th>
<th>(C^+_i)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.011</td>
<td>0.035</td>
<td>0.165</td>
<td>0.083</td>
<td>0.334</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>0.024</td>
<td>0.008</td>
<td>0.014</td>
<td>0.088</td>
<td>0.866</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>0.011</td>
<td>0.016</td>
<td>0.087</td>
<td>0.093</td>
<td>0.517</td>
<td>2</td>
</tr>
<tr>
<td>A4</td>
<td>0.056</td>
<td>0.016</td>
<td>0.098</td>
<td>0.063</td>
<td>0.391</td>
<td>3</td>
</tr>
<tr>
<td>A5</td>
<td>0.085</td>
<td>0.029</td>
<td>0.163</td>
<td>0.042</td>
<td>0.205</td>
<td>8</td>
</tr>
<tr>
<td>A6</td>
<td>0.050</td>
<td>0.071</td>
<td>0.255</td>
<td>0.035</td>
<td>0.120</td>
<td>12</td>
</tr>
<tr>
<td>A7</td>
<td>0.048</td>
<td>0.069</td>
<td>0.249</td>
<td>0.037</td>
<td>0.130</td>
<td>10</td>
</tr>
<tr>
<td>A8</td>
<td>0.031</td>
<td>0.049</td>
<td>0.203</td>
<td>0.059</td>
<td>0.225</td>
<td>6</td>
</tr>
<tr>
<td>A9</td>
<td>0.062</td>
<td>0.052</td>
<td>0.216</td>
<td>0.030</td>
<td>0.122</td>
<td>11</td>
</tr>
<tr>
<td>A10</td>
<td>0.050</td>
<td>0.041</td>
<td>0.185</td>
<td>0.046</td>
<td>0.200</td>
<td>9</td>
</tr>
<tr>
<td>A11</td>
<td>0.032</td>
<td>0.053</td>
<td>0.212</td>
<td>0.056</td>
<td>0.209</td>
<td>7</td>
</tr>
<tr>
<td>A12</td>
<td>0.024</td>
<td>0.034</td>
<td>0.161</td>
<td>0.072</td>
<td>0.309</td>
<td>5</td>
</tr>
</tbody>
</table>

5.4. **Computational Experience.** The superiority of the model can be tested by the model operation time. Sample data of each year is formulated as MCGP and Lotka–Volterra-MCGP and then solved by LINGO on a PC with CPU time of 3.2 GHz. The average relative performance of MCGP and Lotka–Volterra-MCGP is measured by CPU time. The CPU times of MCGP and Lotka–Volterra-MCGP are both the same of 00:00:00 (hh:mm:ss). The two models have no significant difference in computing time, and both can be solved quickly.

5.5. **Sensitivity Analysis of \(\beta_{12}\) and \(\beta_{21}\).** \(\beta\) (\(\beta_{12}\) and \(\beta_{21}\)) has significant sensitivity in the model. When \(\beta_{12}\) and \(\beta_{21}\) change in the same direction and the same amount, the model optimization result remains unchanged. The asynchronous changing of \(\beta_{12}\) and \(\beta_{21}\) significantly affects the model optimization results. The optimization result changed even more because of the changing of \(\beta_{21}\). Therefore, \(\beta_{21}\) is more sensitive than \(\beta_{12}\). The changing rate of product population 2 (\(\Delta LMP_2\)) is significantly higher than that of product population 1 (\(\Delta LMP_1\)). Product population 2 is more sensitive to \(\beta\).

5.6. **Model Robustness Analysis.** In order to test the robustness of the Lotka–Volterra-MCGP model, this paper changes the relevant constraints and model parameters to obtain the following new model:
Min, \( d_i^t + d_i^{-} + d_j^t + d_j^{-} + d_3^t + d_3^{-} + e_1^t + e_1^{-} + e_2^t + e_2^{-} + e_3^t + e_3^{-}; \)

\[
y_1 = 3.02x_1 + 4.03x_2 - d_1^t + d_1^{-}, \text{for output goal, the more the better,}
\]

\[
y_2 = 0.09x_1 + 0.30x_2 - d_2^t + d_2^{-}, \text{for R&D expenditure goal, the less the better,}
\]

\[
y_3 = 26x_1 + 73x_2 - d_3^t + d_3^{-}, \text{for human capital constraints,}
\]

\[
y_1 - e_1^t + e_1^{-} = O_T, y_1 \leq O_T,
\]

\[
y_2 - e_2^t + e_2^{-} = E_T, y_2 \leq E_T,
\]

\[
y_2 - e_3^t + e_3^{-} = H_T, y_3 \leq H_T,
\]

\[
d_1^t > 0, d_2^t > 0, d_2^{-} > 0, d_3^t > 0, e_1^t > 0, e_1^{-} > 0, e_2^t > 0, e_2^{-} > 0, e_3^t > 0, e_3^{-} > 0,
\]

s.t., \( d_3^t > 0, d_3^{-} > 0, e_3^t > 0, e_3^{-} > 0, \)

\[
x_1 > 0, x_2 > 0,
\]

\[
x_1 = \frac{k_1 (1 + \beta_{12})}{1 - \beta_{12} \cdot \beta_{21}}, \text{equilibrium value for IP}_1,
\]

\[
x_2 = \frac{k_2 (1 + \beta_{21})}{1 - \beta_{12} \cdot \beta_{21}}, \text{equilibrium value for IP}_2,
\]

0.1 < \( \beta_{12} < 1, \text{for bound of } \beta_{12}, \)

0.1 < \( \beta_{21} < 1, \text{for bound of } \beta_{21}, \)

\[
\frac{x_1}{x_2} = \frac{k_1 (1 + \beta_{12})}{k_2 (1 + \beta_{21})}, \text{ratio constraint between IP}_1 \text{ and IP}_2.
\]  

Model optimization results are shown in Table 6. As shown in Table 6, this paper can successfully get the optimization results of innovation populations with robustness test model.

6. Discussion

Enterprise population interaction is a common topic [55]. This kind of relationship is ubiquitous in the innovation system [56]. Scholars have studied similar problems from resource constraints [57] to organizational ecology [58]. And, it is popular to use the Lotka–Volterra model to analyze the competition of innovation resource between two enterprises [59]. In this paper, Lotka–Volterra and MCGP are combined to construct a multiagent and multiobjective optimization model. Based on the optimization results, the population collaborative evaluation is carried out. This method takes into account the mathematical characteristics
of the two models, integrates the functions of the two models, and expands the application fields of the two models. It is more suitable for analyzing practical problems.

7. Conclusions

This paper combines the Lotka-Volterra model and the multiobjective decision model to build an analysis path that takes both input and output perspectives into account. At the same time, the paper used optimization values to evaluate the population size suitability, which makes the evaluation more operable. The results show that the combination of Lotka-Volterra model and multichoice goal programming model can better evaluate the scale suitability of IPs within a region.

In the enterprise innovation ecosystem, multiple related populations affect each other. There are also interactive behaviors within the IP of enterprises. This kind of mutually influential behavior can be expressed as coordination or competition. Due to the constraints of the total resources in the innovation environment, the collaboration between populations or within populations may not necessarily promote innovation. The total resource variable is set in the model, which fully reflects the resource constraint mechanism. The innovation ecosystem can carry out self-organized evolution, and cooperation among populations and among enterprises within populations can promote a win-win situation among multiple populations. However, the fact that there are negative cases in reality challenges the self-organized evolution of the innovation ecosystem. The reason lies in the lack of detailed and in-depth analysis of the mechanisms at play in the innovation ecosystem. The innovation ecosystem is a complex system, and its operation is inevitably affected by the interaction between the subsystems.

In this study, the different life cycles of IP development are not considered, and the demand for resources of IPs in different life cycle are different. Future studies could consider the characteristics of population life cycle development and add life cycle factors into the special analysis model.

Data Availability

The experimental data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Social Science Foundation of China (no. 20BGL203). The authors thank Professor Ching-Ter Chang of Chang Gung University for his suggestions on this paper.

References


