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Research Article

Analysis of a High-Dimensional Mathematical Model for Plant Virus Transmission with Continuous and Impulsive Roguing Control

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Roguing and replanting are the most common strategies to control plant diseases and pests. How to build the mathematical models of plant virus transmission and consider the impact of roguing and replanting strategies on plant virus eradication is of great practical significance. In the present paper, we propose the mathematical models for plant virus transmission with continuous and impulsive roguing control. For the model with continuous control strategies, the threshold values for the existences and stabilities of multiple equilibria have been given, and the effect of roguing strategies on the threshold values is also addressed. Furthermore, the model with impulsive roguing control tactics is proposed, and the existence and stability of the plant-only and disease-free periodic solutions of the model are investigated by calculating several threshold values. Moreover, when selecting the design control strategy to minimize the threshold, we systematically analyze the existence of the optimal times of roguing infected plants within a replanting cycle, which is of great significance to the design and optimization of the prevention and control strategy of plant virus transmission. Finally, numerical investigations are given to reveal the main conclusions, and the biological implications of the main results are briefly discussed in the last section.

1. Introduction

Plant diseases are caused by plant viruses transmitted by insect vectors [1–5], which can cause serious losses in economy and increased poverty [6]. Moreover, vector activity and behavior, especially in relation to virus transmission, are key to determining severity of plant disease transmission [7–13]. Therefore, the development of corresponding prevention and control strategies of plant infectious diseases plays a very important role in protecting plants and eradicating vectors. Note that current control tactics of plant disease mainly include gene resistance, vector insect, and culture control. Gene resistance control cannot be widely used due to its high cost, and vector control through insecticide spraying has a small risk of failure due to selection for more damaging virus strains [14–16]. However, the cultural control including roguing or removal of diseased plants has been practised and could be more effective [17–20]. Therefore, a successful plant disease control program depends on a plant production system which is closely aligned with the goals of pest management [21–23].

How to evaluate the effectiveness and timeliness of these prevention and control measures needs the development of corresponding mathematical models. A large number of studies have proved that mathematical modeling approaches can help in the prediction of disease propagation in the agricultural realm [8, 13, 22, 24–27]. More recently, several mathematical models for the spread of plant diseases in populations have been analyzed mathematically and applied to control plant diseases. For example, a deterministic model with bilinear incidence was formulated to address the impact of cross protection on the spread and control of Huanglongbing (HLB) disease [22], and

the main results reveal that cross protection and removing infected trees play an important role in controlling the spread of HLD. In literature [24], the authors developed an individual-based model (IBM) of vector-borne plant pathogen spread and compared the simulation results with a classical continuous model. In [25], the authors proposed a model with time delay to study the effect of both incubation delay and latent time on the dynamics of plant disease. A more realistic model has been developed in [26] which takes within-plant cell processes, vector population dynamics, behavior, and broader ecological interactions into account. In [27], considering awareness as a controlling measure, the effect of time delay and awareness campaign on the dynamics and control of the mosaic disease has been investigated.

Recently, impulsive differential equations have been employed as the mathematical models for many real problems including pest control [16, 28–30], infectious diseases control, and drug administration [31]. Moreover, various plant disease models with impulsive control have been developed and studied [17–20, 32–34]. In [17], the authors proposed a mathematical model for plant population which aims to eradicate infected plants or maintain the number of infected plants below the economic threshold. In [18, 19], the plant disease models with impulsive cultural strategies were developed and analyzed, and the main results reveal that the plant disease could be eradicated or the number of infected plants could be maintained below the economic threshold by choosing proper control frequency and intensity. We note that those modelling methods have been developed and extended in various references [20, 32–34].

However, most of the above works only consider (a) the transmission of diseases among a single plant population, without considering the effect of the vector population on the disease transmission; (b) the continuous human intervention strategies, without considering the instantaneity of intervention strategies; and (c) the impact of pulse control on infected plants, without considering the inevitable impact of control measures on healthy plants. Considering the above shortcomings, this paper aims to develop mathematical models, fully integrate the above three factors, develop corresponding theoretical analysis and numerical techniques, and systematically and deeply analyze their impact on plant disease control. In particular, various threshold values which guarantee the existence and stability of plant-only and disease-free periodic solutions have been obtained, and the existence of the optimal times of roguing infected plants within a replanting cycle has been also investigated in more detail.

2. The Model

2.1. System Description. In this section, based on the model proposed in [21], we develop a vector-borne plant disease model with cultural control. To do this, we denote the total plant population at time t as $N(t) = X(t) + Y(t)$ and vector population as $M(t) = U(t) + V(t)$. The population flow among those compartments is shown in the schematic diagram (i.e., Figure 1), which leads to the following model:

$$\begin{cases} \dfrac{dX(t)}{dt} = r - k_1 X(t)V(t) - gX(t), \\ \dfrac{dY(t)}{dt} = k_1 X(t)V(t) - gY(t) - \alpha Y(t), \\ \dfrac{dU(t)}{dt} = b(U(t) + V(T))\left(1 - \dfrac{U(t) + V(t)}{m(X(t) + Y(t))}\right) - k_2 U(t)Y(t) - cU(t), \\ \dfrac{dV(t)}{dt} = k_2 U(t)Y(t) - cV(t), \end{cases} \tag{1}$$

where $X(t)$, $Y(t)$, $U(t)$, and $V(t)$ represent the number of susceptible plants, infectious plants, susceptible vectors, and infectious vectors, respectively. The notations and definitions of all parameters are shown in Table 1.

2.2. Basic Properties. In this section, we show the boundedness of system (1), and it follows from system (1) that we have

$$\frac{dN(t)}{dt} = r - gX(t) - (\alpha + g)(t) \le r - gN(t), \tag{2}$$

which indicates

$$N(t) \le N(0)\exp(-gt) + \frac{r}{g}(1 - \exp(-gt)) \longrightarrow \frac{r}{g}, \quad \text{for } t \longrightarrow \infty. \tag{3}$$

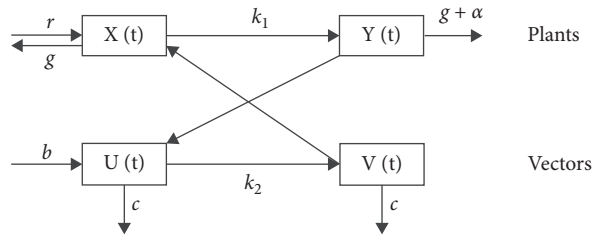

Figure 1: The transfer diagram for model (1).

Table 1: Definitions of all parameters and their baseline values.

Parameter	Interpretation	Standard value	Range	Source
r	Replanting rate	$0.005\,(\text{day}^{-1})$	0–0.025	[21]
α	Plant roguing rate	$0.003\,(\text{day}^{-1})$	0–0.033	[21]
g	Plant loss/harvesting rate	$0.003\,(\text{day}^{-1})$	0.002–0.004	[21]
m	Maximum vector abundance	$500\,(\text{plant}^{-1})$	0–2500	[21]
b	Maximum vector birth rate	$0.2\,(\text{day}^{-1})$	0.1–0.3	[21]
c	Vector mortality	$0.12\,(\text{day}^{-1})$	0.06–0.18	[21]
k_1	Infection rate	$0.008\,(\text{vector}^{-1}\text{day}^{-1})$	0.002–0.032	[21]
k_2	Acquisition rate	$0.008\,(\text{plant}^{-1}\text{day}^{-1})$	0.002–0.032	[21]

Similarly, we have

$$\frac{dM(t)}{dt} = bM(t)\left(1 - \frac{M(t)}{mN(t)}\right) - cM(t) \le (b-c)M(t) - \frac{bgM^2(t)}{mr}, \tag{4}$$

and consequently we get

$$M(t) \le \frac{M(0)\exp((b-c)t)}{1 + bgM(0)(\exp((b-c)t) - 1)/mr(b-c)} \longrightarrow \frac{mr(b-c)}{bg}, \quad \text{for } t \longrightarrow \infty. \tag{5}$$

Thus, the total plant population $N(t)$ and vector population $M(t)$ are uniformly bounded. Furthermore, the region

$$\Omega = \left\{(X(t), Y(t), U(t), V(t)) \in R_+^4 : X(t) + Y(t) \le \frac{r}{g}; U(t) + V(t) \le \frac{mr(b-c)}{bg}\right\}, \tag{6}$$

is positively invariant, which indicates that we can focus on the dynamics of system (1) on the set Ω in the following.

3. Dynamics of the Model with Continuous Control

3.1. Boundary Equilibria and Their Stabilities. System (1) has plant-only equilibrium $E_{01} = (X_0, 0, 0, 0) = (r/g, 0, 0, 0)$ and disease-free equilibrium $E_{02} = (X_0, 0, U_0, 0) = (r/g, 0, mr(b-c)/bg, 0)$ with $b > c$. By employing the next generation matrix method [35], we can obtain the basic reproduction number of model (1) as follows:

$$R_0 = R_{01}R_{02} = \sqrt{\frac{k_1 k_2 mr^2(b-c)}{bcg^2(\alpha+g)}}, \tag{7}$$

where $R_{01} = \sqrt{k_1 r/cg}$, $R_{02} = \sqrt{k_2 mr(b-c)/bg(\alpha+g)}$. R_0 represents the number of secondary cases that one infected individual will cause through the duration of the infective period, R_{01} is the expected number of plants that one vector

infects throughout its infectious life time, and R_{02} is the expected number of vectors that one plant infects throughout its infectious life time.

Theorem 1. *The plant-only equilibrium E_{01} is globally asymptotically stable if $b \le c$ and $R_{01} \le 1$.*

Proof. We introduce the following Lyapunov function:

$$V_1(t) = X - X_0 - X_0 \ln \frac{X}{X_0} + Y + M + V, \tag{8}$$

and further

$$\begin{aligned}
\dot{V}_1(t) &= \left(1 - \frac{X_0}{X}\right) X \left(r\left(\frac{1}{X} - \frac{1}{X_0}\right) - k_1 V\right) + k_1 XV - (\alpha + g)Y + (b - c)M - \frac{bM^2}{m(X+Y)} + k_2 UY - cV \\
&= r\left(2 - \frac{X}{X_0} - \frac{X_0}{X}\right) + (k_1 X_0 - c)V + (k_2 U - \alpha + g)Y + (b - c)M - \frac{bM^2}{m(X+Y)} \\
&\le r\left(2 - \frac{X_0}{X} - \frac{X}{X_0}\right) + c\left(R_{01}^2 - 1\right)V + (k_2 M - \alpha + g)Y + (b - c)M - \frac{bM^2}{m(X+Y)} \\
&\le r\left(2 - \frac{X_0}{X} - \frac{X}{X_0}\right) + c\left(R_{01}^2 - 1\right)V + (\alpha + g)\left(\frac{k_2 mr(b-c)}{(\alpha+g)bg} - 1\right)Y + (b - c)M - \frac{bM^2}{m(X+Y)}.
\end{aligned} \tag{9}$$

The arithmetical mean is great than or equal to the geometrical mean, and the function $2 - (X_0/X) - (X/X_0)$ is nonpositive for all $X > 0$, and it is equal to zero if and only if $X = X_0$. If $b \le c$ and $R_{01} \le 1$, then

$$r\left(2 - \frac{X_0}{X} - \frac{X_0}{X}\right) + c\left(R_{01}^2 - 1\right)V + (\alpha + g)\left(\frac{k_2 mr(b-c)}{(\alpha+g)bg} - 1\right)Y + (b - c)(M) - \frac{bM^2}{m(X+Y)} \le 0. \tag{10}$$

Thus, we have $\dot{V}_1 \le 0$. Furthermore, $\dot{V}_1 = 0$ only if $Y = 0$, $U = 0$, and $V = 0$, which indicates that the maximum invariant set in $\{(X_0, 0, 0, 0): \dot{V} = 0\}$ is the singleton E_{01}. By LaSalle's Invariance Principle [36], we conclude that E_{01} is globally asymptotically stable in Ω. □

Next, we show that the disease-free equilibrium is globally stable provided that $R_0 < 1$. First, the Jacobian matrix at the equilibrium E_{02} is given by

$$J_{E_{02}} = \begin{pmatrix}
-g & 0 & 0 & \dfrac{-k_1 r}{g} \\
0 & -(\alpha + g) & 0 & \dfrac{k_1 r}{g} \\
\dfrac{m(b-c)^2}{b} & \dfrac{m(b-c)^2}{b} - \dfrac{k_2 mr(b-c)}{bg} & -(b-c) & 2c - b \\
0 & \dfrac{k_2 mr(b-c)}{bg} & 0 & -c
\end{pmatrix}. \tag{11}$$

Stability of equilibrium E_{02} is determined by the eigenvalues of the Jacobian $J_{E_{02}}$ evaluated at that equilibrium. Characteristic equation at disease-free equilibrium E_{02} becomes

$$(\lambda + (b-c))(\lambda + g)\left(\lambda^2 + (\alpha + g + c)\lambda + c(\alpha + g)\left(1 - R_0^2\right)\right) = 0. \tag{12}$$

It is easy to see that two roots are $\lambda_1 = -(b-c) < 0$ and $\lambda_2 = -g < 0$, and according to Routh–Hurwitz condition, $\lambda^2 + (\alpha + g + c)\lambda + c(\alpha + g)(1 - R_0^2) = 0$ has roots with negative real parts when $R_0 < 1$. Therefore, E_{02} is stable if $R_0 < 1$. Furthermore, we have the following result for the global stability of E_{02}.

Theorem 2. *For system (1), if $R_0 \le 1$, then the disease-free equilibrium E_{02} is globally asymptotically stable.*

Proof. Define a Lyapunov function:

$$V_2(t) = \frac{k_2 mrcg(b-c)}{2k_1 r}Y^2 + \frac{bg(g+\alpha)}{2}V^2 + k_2 mr(b-c)YV, \tag{13}$$

and then the derivative of $V_2(t)$ with respect to system (1) is given by

$$\begin{aligned}
\dot{V}_2(t) &= \frac{k_2 mrcg(b-c)}{k_1 r}Y\left(k_1 XV - (g+\alpha)Y\right) + bg(g+\alpha)V\left(k_2 UY - cV\right) \\
&\quad + bg(g+\alpha)\left(k_1 XV - (g+\alpha)Y\right) + bg(g+\alpha)Y\left(k_2 UY - cV\right) \\
&= \left(k_2 mcg(b-c)X + k_2 bg(g+\alpha)U - k_2 mr(b-c)(g+\alpha) - k_2 mr(b-c)c\right)YV \\
&\quad + \left(k_2^2 mr(b-c)U - \frac{k_2 mrcg(b-c)(g+\alpha)}{k_1 r}\right)Y^2 + \left(k_1 k_2 mr(b-c)X - cbg(g+\alpha)\right) \\
&\le \frac{k_2 mrcg(b-c)(g+\alpha)}{k_1 r}\left(R_0^2 - 1\right)Y^2 + bcg(g+\alpha)\left(R_0^2 - 1\right)V^2.
\end{aligned} \tag{14}$$

Therefore, we have $\dot{V}_2(t) \le 0$ if $R_0 \le 1$, and $\dot{V}_2(t) = 0$ if $R_0 = 1$ or $Y = 0, V = 0$. Substituting $Y = 0, V = 0$ into the first and third equations of (1), one has $\dot{X}(t) = r - gX(t)$ and $\dot{U}(t) = bU(t)(1 - U(t)/mX(t)) - cU(t)$, which implies $X(t) \longrightarrow r/g$ and $U(t) \longrightarrow mr(b-c)/bg$ as $t \longrightarrow +\infty$. Again, it follows from LaSalle's Invariance Principle [36] that E_{02} is globally asymptotically stable in Ω. □

3.2. The Existence of Endemic Equilibrium and Uniform Persistence. In order to determine the existence of the endemic equilibrium, we have to look for the solution of the algebraic system of equations obtained by equating the right sides of system (1) to zero.

$$\begin{aligned}
r - k_1 X^* V^* - gX^* &= 0, \\
k_1 X^* V^* - gY^* - \alpha Y^* &= 0, \\
b\left(U^* + V^*\right)\left(1 - \frac{U^* + V^*}{m\left(X^* + Y^*\right)}\right) - k_2 U^* Y^* - cU^* &= 0, \\
k_2 U^* Y^* - cV^* &= 0.
\end{aligned} \tag{15}$$

From the first and second equations of (15), we have

$$\begin{aligned}
X^* &= \frac{r}{k_1 V^* + g}, \\
Y^* &= \frac{k_1 rV^*}{(\alpha + g)\left(k_1 V^* + g\right)}, \\
U^* &= \frac{m(b-c)\left(r(\alpha+g) + k_1 rV^*\right)}{b(\alpha+g)\left(k_1 V^* + g\right)} - V^*.
\end{aligned} \tag{16}$$

Substituting the above into the fourth equation of (15), we have

$$A\left(V^*\right)^2 + BV^* + C = 0, \tag{17}$$

where

$$\begin{aligned}
A &= -k_1^2 b(\alpha+g)\left(k_2 r + c(\alpha+g)\right) < 0, \\
B &= k_1^2 k_2 mr^2(b-c) - k_1 k_2 brg(\alpha+g) - 2k_1 bcg(\alpha+g)^2, \\
C &= bcg^2(\alpha+g)^2\left(R_0^2 - 1\right).
\end{aligned} \tag{18}$$

Equation (17) may have two roots if $\Delta > 0$, denoted by

$$\begin{aligned}
V_1^* &= \frac{-B + \sqrt{\Delta}}{2A}, \\
V_2^* &= \frac{-B - \sqrt{\Delta}}{2A},
\end{aligned} \tag{19}$$

with $\Delta = B^2 - 4AC$, and we have the following main results.

Theorem 3. *For system (1), if $R_0 > 1$, then there exists a unique endemic equilibrium $E^* = (X^*, Y^*, U^*, V^*)$; if $R_0 \le 1$, then there is no endemic equilibrium.*

Next, we will employ the persistent theory developed in literature [37] to show the uniform persistence of system (1). To do this, we let E be a closed positively invariant subset of Ω, on which a continuous flow $\mathscr{F}$ is defined. We denote the restriction $\mathscr{F}$ to ∂E by $\partial\mathscr{F}$ and note that ∂E is in general not positively invariant. Let N be the maximal invariant set of $\partial\mathscr{F}$ on ∂E. Suppose that N is a closed invariant set and there exists a cover $\{N_\alpha\}_{\alpha\in A}$ of N, where A is a nonempty index set. $N_\alpha \subset \partial E$, $N \subset \cup_{\alpha\in A} N_\alpha$, and $\{N_\alpha\}(\alpha \in A)$ are pairwise disjoint closed invariant sets. Furthermore, we propose the following hypotheses and lemmas. (H_1) All N_α are isolated invariant sets of the flow $\mathscr{F}$. $(H_2)\{N_\alpha\}_{\alpha\in A}$ is acyclic; that is, any finite subset of $\{N_\alpha\}_{\alpha\in A}$ does not form a cycle. (H_3) Any compact subset of ∂E contains, at most, finitely many sets of $\{N_\alpha\}_{\alpha\in A}$ [37].

Lemma 1 (see [37]). *Let E be a closed positively invariant subset of Ω on which a continuous flow $\mathscr{F}$ is defined. Suppose there is a constant $\varepsilon > 0$ such that $\mathscr{F}$ is point dissipative on $S[\partial E, \varepsilon] \cap E^0$ and the assumptions $(H_1 - H_3)$ hold. Then, the flow $\mathscr{F}$ is uniformly persistent, if and only if $W^+(N_\alpha) \cap S[\partial E, \alpha] \cap E^0 = \phi$ for any $\alpha \in A$, where $W^+(N_\alpha) = \{y \in \Omega, \omega(y) \subset N_\alpha\}$, and E^0 is interior of set E.*

By this lemma, we can show the uniform persistence of disease when $R_0 > 1$.

Theorem 4. *In system (1), assume that $R_0 > 1$ and the disease is initially present; then, the disease is uniformly persistent; i.e., there is a constant $\rho > 0$ such that $\liminf_{t\longrightarrow+\infty} Y(t) \ge \rho$, $\liminf_{t\longrightarrow+\infty} V(t) \ge \rho$.*

Proof. We set $E = \{(X, Y, U, V) \in \mathbf{R}^4_+ | 0 \le X + Y \le r/g, 0 \le U + V \le mr(b-c)/bg\}$, $\partial E = \{(X, Y, U, V) \in E | Y = 0, V = 0\}$, and we will show that the conditions of Lemma 1 are satisfied. Clearly, $N_\alpha = E_0 = (r/g, 0, mr(b-c)/bg, 0)$ is isolated. Hence, the covering is simply $N = E_0$, which is acyclic. Thus, the conditions $(H_1 - H_3)$ hold. We can also obtain that $\mathscr{F}$ is point dissipative by (6). Now, we show that $W^+(E_0) \cap E^0 = \phi$. Suppose this is not true; then, there exists a solution $(X(t), Y(t), U(t), V(t)) \in E^0$ such that $\lim_{t\longrightarrow+\infty} X(t) = r/g$, $\lim_{t\longrightarrow+\infty} Y(t) = 0$, $\lim_{t\longrightarrow+\infty} U(t) = mr(b-c)/bg$, $\lim_{t\longrightarrow+\infty} V(t) = 0$. For any sufficiently small constant $\varepsilon > 0$, there exists a positive constant $T = T(\varepsilon)$ such that $X(t) > r/g - \varepsilon, Y(t) < \varepsilon, U(t) > mr(b-c)/bg - \varepsilon, V(t) < \varepsilon$ for all $t \ge T$. Since $R_0 = \sqrt{k_1 k_2 mr^2(b-c)/bcg^2(\alpha+g)} > 1$, for sufficiently small ε, we have

$$R_0' = \sqrt{\frac{k_1 k_2}{c(\alpha+g)}\left(\frac{mr(b-c)}{bg} - \varepsilon\right)\left(\frac{r}{g} - \varepsilon\right)} > 1. \tag{20}$$

Note that

$$\begin{aligned} \frac{dY(t)}{dt} &\ge k_1\left(\frac{r}{g} - \varepsilon\right)V(t) - (g+\alpha)Y(t), \\ \frac{dV(t)}{dt} &\ge k_2\left(\frac{mr(b-c)}{bg} - \varepsilon\right)Y(t) - cV(t), t \ge T. \end{aligned} \tag{21}$$

Therefore, if $Y(t), V(t) \longrightarrow 0$, as $t \longrightarrow \infty$, then by a standard comparison argument and the nonnegativity, the solution $n_Y(t), n_V(t)$ of

$$\begin{aligned} \frac{dn_Y(t)}{dt} &= k_1\left(\frac{r}{g} - \varepsilon\right)n_V(t) - (\alpha+g)n_Y(t), \\ \frac{dn_V(t)}{dt} &= k_2\left(\frac{mr(b-c)}{bg} - \varepsilon\right)n_Y(t) - cn_V(t), \quad t \ge T, \end{aligned} \tag{22}$$

with initial data $n_Y(T) = Y(T), n_V(T) = V(T)$, converges to $(0, 0)$ as well. Thus, $\lim W_n(t) = 0$, where $W_n(t) > 0$ is defined by

$$W_n(t) = C_1 n_Y(t) + C_2 n_V(t), \tag{23}$$

where $C_1 = k_2, C_2 = \alpha + g/mr(b-c)/bg - \varepsilon$.

The derivative of $W_n(t)$ is given by

$$\frac{dW_n(t)}{dt} = \frac{c(\alpha+g)}{mr(b-c)/bg - \varepsilon}\left(R_0'^2 - 1\right)n_V(t) \ge 0. \tag{24}$$

Therefore, $W_n(t)$ goes to either infinity or some positive number as $t \longrightarrow \infty$, which is a contradiction of $\lim_{t\longrightarrow+\infty} W_n(t) = 0$. Thus, we have $W^+(E_0) \cap E^0 = \phi$. Then, we obtain $\lim_{t\longrightarrow+\infty} V(t) \ge \rho_1$ for some constant $\rho_1 > 0$. By the fourth equation of (1), we have $\rho_2 = k_1((r/g) - \varepsilon)\rho_1/\alpha + g$ such that $\liminf_{t\longrightarrow+\infty} Y(t) \ge \rho_2$. Denote $\rho = \min\{\rho_1, \rho_2\}$; then, $\liminf_{t\longrightarrow+\infty} Y(t) \ge \rho$ and $\liminf_{t\longrightarrow+\infty} x_2(t) \ge \rho$. The proof of Theorem 4 is completed. □

4. Dynamics of the Model with Pulse Perturbation

In this section, we extend model (1) by replacing the continuous removing of infected plants with a periodic pulse roguing control strategy, which is more realistic. Hence, we consider the following differential equation with pulse roguing strategy at fixed moments:

$$\left\{\begin{array}{l} \left.\begin{array}{l} \dfrac{\mathrm{d}X(t)}{\mathrm{d}t} = -k_1 X(t)V(t) - gX(t), \\ \dfrac{\mathrm{d}U(t)}{\mathrm{d}t} = b(U(t)+V(t))\left(1 - \dfrac{U(t)+V(t)}{m(X(t)+Y(t))}\right) - k_2 U(t)Y(t) - cU(t), \\ \dfrac{\mathrm{d}Y(t)}{\mathrm{d}t} = k_1 X(t)V(t) - gY(t), \\ \dfrac{\mathrm{d}V(t)}{\mathrm{d}t} = k_2 U(t)Y(t) - cV(t), \end{array}\right\} t \neq nT, \\ \left.\begin{array}{l} X(nT^+) = X(nT) + r, \\ U(nT^+) = (1-p)U(nT), \\ Y(nT^+) = (1-\alpha)Y(nT), \\ V(nT^+) = (1-p)V(nT), \end{array}\right\} t = nT, \end{array}\right. \tag{25}$$

where T is a fixed positive constant and denotes the period of the impulsive effect, r is the constant replanting parameter, and $n \in N$ which denotes the positive integer set. The parameter α denotes the proportion of the infected plants which is rouged at each pulse perturbation, and p represents vectors removal rate due to the infected plants being rouged; it is likely that some of the uninfected vectors and infected vectors will be removed incidentally.

4.1. The Existence and Stability of the Disease-Free Periodic Solution. If $Y(t) = 0, V(t) = 0$, then model (25) can be reduced to the following subsystem:

$$\left\{\begin{array}{l} \left.\begin{array}{l} \dfrac{\mathrm{d}X(t)}{\mathrm{d}t} = -gX(t), \\ \dfrac{\mathrm{d}U(t)}{\mathrm{d}t} = bU(t)\left(1 - \dfrac{U(t)}{mX(t)}\right) - cU(t), \end{array}\right\} t \neq nT, \\ \left.\begin{array}{l} X(nT^+) = X(nT) + r, \\ U(nT^+) = (1-p)U(nT), \end{array}\right\} t = nT. \end{array}\right. \tag{26}$$

From the first equation of (26), we can solve it in any impulsive interval $(nT, (n+1)T]$ and have

$$X(t) = X(nT^+)\exp(-g(t-nT)), t \in (nT, (n+1)T]. \tag{27}$$

Denote $X_n = X(nT^+)$; then,

$$X_{n+1} = X_n \exp(-gT) + r. \tag{28}$$

There exists a steady state $X^* = r/1 - \exp(-gT)$, which indicates that the system

$$\left\{\begin{array}{ll} \dfrac{\mathrm{d}X(t)}{\mathrm{d}t} = -gX(t), & t \neq nT, \\ X(nT^+) = X(nT) + r, & t = nT, \\ X(0^+) = X_0 & \end{array}\right. \tag{29}$$

has a positive periodic solution $X^*(t) = X^* \exp(-g(t-nT))$, which is globally asymptotically stable.

Lemma 2. *System (29) has a positive periodic solution $X^*(t)$; for any solution of (29), we have $X(t) \longrightarrow X^*(t)$ as $t \longrightarrow \infty$, where $X^*(t) = r \exp(-g(t-nT))/\ 1 - \exp(-gT), t \in (nT, (n+1)T]$.*

Proof. Let $X(t) = X(nT^+)\exp(-g(t-nT))$ for all $t \in (nT, (n+1)T$ be any solution of (29); then,

$$X(nT^+) = X(0^+)\exp(-ngT) + \frac{r(1-\exp(-ngT))}{1-\exp(-gT)}, \tag{30}$$

which indicates that $X(t) = (X(0^+)\exp(-ngT) + r(1- \exp(-ngT))/1 - \exp(-gT))\exp(-g(t-nT)) \longrightarrow r \exp(-g(t - nT))/1 - \exp(-gT)$ as $n \longrightarrow \infty$ and $t \in (nT, (n+1)T]$. □

Substituting the above periodic solution into the second equation of (26), one yields

$$U(t) = \frac{m(g+b-c)X^*U(nT^+)\exp((b-c)(t-nT))}{m(g+b-c)X^* + bU(nT^+)(\exp((g+b-c)(t-nT))-1)}, \quad t \in (nT, (n+1)T]. \tag{31}$$

Denote $U_n = U(nT^+)$; then,

$$U_{n+1} = \frac{(1-p)m(g+b-c)\exp((b-c)T)X^*U_n}{m(g+b-c)X^* + bU_n(\exp((g+b-c)T)-1)}. \tag{32}$$

Denote $R_1 = (1-p)\exp((b-c)T)$; then, we conclude that (32) always has a zero equilibrium provided that $R_1 < 1$ or $g+b-c=0$. When $R_1 > 1$ and $g+b-c \neq 0$, (32) has a stable positive equilibrium $U^* = mr(g+b-c)((1-p)\exp((b-c)T)-1)/b(\exp((g+b-c)T)-1)(1-\exp(-gT))$. Consequently, the system

$$\begin{cases} \dfrac{dU(t)}{dt} = bU(t)\left(1 - \dfrac{U(t)}{mX^*(t)}\right) - cU(t), & t \neq nT, \\ U(nT^+) = (1-p)U(nT), & t = nT, \\ U(0^+) = U_0 \end{cases} \tag{33}$$

has a positive periodic solution

$$U^*(t) = \frac{m(g+b-c)X^*U^*\exp((b-c)(t-nT))}{m(g+b-c)X^* + bU^*(\exp((g+b-c)(t-nT))-1)}, \quad t \in (nT, (n+1)T], \tag{34}$$

which is globally asymptotically stable. Furthermore, we have the following main result.

Lemma 3. *If $R_1 > 1$ and $g+b-c \neq 0$, then system (33) has a positive periodic solution $U^*(t)$, and for any solution of (33) we have $U(t) \longrightarrow U^*(t)$ as $t \longrightarrow \infty$.*

Proof. Denote $\overline{U}(t) = 1/U(t)$ and $\overline{U}(t) = 1/U^*(t)$; then,

$$\begin{aligned} \overline{U}(t) &= \overline{U}(nT^+)\exp(-(b-c)(t-nT)) \\ &\quad + \frac{b(1-\exp(-gT))(\exp((g+b-c)(t-nT))-1)\exp(-(b-c)(t-nT))}{mr(g+b-c)}, \\ \overline{U}^*(t) &= \overline{U}^*\exp(-(b-c)(t-nT)) \\ &\quad + \frac{b(1-\exp(-gT))(\exp((g+b-c)(t-nT))-1)\exp(-(b-c)(t-nT))}{mr(g+b-c)}. \end{aligned} \tag{35}$$

Thus, we have

$$\begin{aligned} \left|\overline{U}(t) - \overline{U}^*(t)\right| &= \left|\overline{U}(nT^+) - \overline{U}^*\right|\exp(-(b-c)(t-nT)) \\ &\leq \left|\overline{U}(nT^+) - \overline{U}^*\right|. \end{aligned} \tag{36}$$

Note that if $R_1 > 1$, then we have $|\overline{U}(nT^+) - \overline{U}^*| = |U(nT^+) - U^*/U(nT^+)U^*| \longrightarrow 0$ as $t \longrightarrow \infty$, and consequently the result of Lemma 2 follows. □

From Lemmas 1 and 2, we obtain the following results.

Lemma 4. *System (26) has boundary periodic solution $(X^*(t), 0)$, which is globally stable if $R_1 \leq 1$, and has a positive periodic solution $(X^*(t), U^*(t))$, which is globally stable if $R_1 > 1$.*

Therefore, system (25) has a plant-only periodic solution $(X^*(t), 0, 0, 0)$ and disease-free periodic solution $(X^*(t), U^*(t), 0, 0)$, and their stabilities have been addressed in the following.

Theorem 5. *The plant-only periodic solution $(X^*(t), 0, 0, 0)$ of model (25) is globally asymptotically stable in the first quadrant if $R_1 = (1-p)\exp((b-c)T) < 1$.*

Proof. Firstly, we prove the local stability of the solution $(X^*(t), 0, 0, 0)$ of model (25). Denote $X(t) = x(t) + X^*(t)$,

$Y(t) = y(t), U(t) = u(t), V(t) = v(t)$; the corresponding linear system of (25) at $(X^*(t), 0, 0, 0)$ reads as

$$\begin{cases} \left.\begin{aligned} \frac{dx(t)}{dt} &= -k_1 X^*(t) v(t) - g x(t), \\ \frac{du(t)}{dt} &= (b-c) u(t), \\ \frac{dy(t)}{dt} &= k_1 X^*(t) v(t) - g y(t), \\ \frac{dv(t)}{dt} &= -c v(t), \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} x(nT^+) &= x(nT) + r, \\ u(nT^+) &= (1-p) u(nT), \\ y(nT^+) &= (1-\alpha) y(nT), \\ v(nT^+) &= (1-p) v(nT), \end{aligned}\right\} t = nT. \end{cases} \tag{37}$$

Thus, the fundamental solution matrix $\Phi(t)$ of (37) satisfies

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} -g & 0 & 0 & -k_1 X^*(t) \\ 0 & b-c & 0 & 0 \\ 0 & 0 & -g & k_1 X^*(t) \\ 0 & 0 & 0 & -c \end{pmatrix} \Phi(t), \tag{38}$$

and $\Phi(0) = I_{4\times 4}$ is the identity matrix, which results in

$$\Phi(t) = \begin{pmatrix} e^{-gT} & 0 & 0 & \Delta_{14} \\ 0 & e^{(b-c)T} & 0 & 0 \\ 0 & 0 & e^{-gT} & \Delta_{34} \\ 0 & 0 & 0 & e^{-cT} \end{pmatrix}. \tag{39}$$

There is no need to give the exact forms of Δ_{14} and Δ_{34} as they are not required in the analysis that follows. If all eigenvalues of

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-p & 0 & 0 \\ 0 & 0 & 1-\alpha & 0 \\ 0 & 0 & 0 & 1-p \end{pmatrix} \Phi(T), \tag{40}$$

are less than one, then the plant-only periodic solution $(X^*(t), 0, 0, 0)$ is locally stable. It is easy to see that

$$\begin{aligned} \lambda_1 &= \exp(-gT) < 1, \\ \lambda_2 &= (1-p)\exp((b-c)T), \\ \lambda_3 &= (1-p)\exp(-gT) < 1, \\ \lambda_4 &= (1-p)\exp(-cT) < 1, \end{aligned} \tag{41}$$

and then $|\lambda_2| < 1$ if and only if $R_1 < 1$. According to Floquet's theory of impulsive differential equations, the plant-only periodic solution $(X^*(t), 0, 0, 0)$ is locally stable.

In the following, we prove the global attractivity of the periodic solution $(X^*(t), 0, 0, 0)$. It follows from (25) that

$$\begin{cases} \frac{dN(t)}{dt} = -gN(t), & t \neq nT, \\ N(nT^+) \leq N(nT) + r, & t = nT, \\ N(0^+) = N_0, \end{cases} \tag{42}$$

$$\begin{cases} \frac{dM(t)}{dt} = bM(t)\left[1 - \frac{M(t)}{mN(t)}\right] - cM(t), & t \neq nT, \\ M(nT^+) = (1-p)M(nT), & t = nT, \\ M(0^+) = M_0. \end{cases} \tag{43}$$

From (42), we get

$$N(nT) \le N_0 \exp(-ngT) + \frac{r(1-\exp(-ngT))}{1-\exp(-gT)} \longrightarrow \frac{r}{1-\exp(-gT)}, \quad \text{for } n \longrightarrow \infty. \tag{44}$$

Thus, $N(t)$ is uniformly bounded, and for $\varepsilon_1 > 0$ small enough, there exists a $t_1 > 0$ such that $X(t), Y(t) \le L_1$, where $L_1 = r/1 - \exp(-gT) + \varepsilon_1$. From (43), we get

$$\begin{aligned} M((n+1)T) &\le \frac{(1-p)\exp((b-c)T)M(nT)}{1+b(1-p)(\exp((b-c)T)-1)M(nT)/m(b-c)(L_1+\varepsilon_1)} \\ &\le (1-p)\exp((b-c)T)M(nT) \\ &\le ((1-p)\exp((b-c)T))^n M(0) \longrightarrow 0, \quad \text{for } n \longrightarrow \infty. \end{aligned} \tag{45}$$

Thus, if $R_1 < 1$, then $M(t)$ is uniformly bounded, and for $\varepsilon_2 > 0$ small enough, there exists a $t_2 > 0$ such that $U(t), V(t) \le \varepsilon_2$. Let $\varepsilon_2 \longrightarrow 0$; then, $U(t) \longrightarrow 0$, $V(t) \longrightarrow 0$ as $t \longrightarrow \infty$. From (25), $\mathrm{d}X(t)/\mathrm{d}t \le -gT$, $X(nT^+) = X(nT) + r$. Consider the following impulsive differential equation:

$$\begin{cases} \dfrac{\mathrm{d}Z_1(t)}{\mathrm{d}t} = -gZ_1(t), & t \ne nT, \\ Z_1(nT^+) = Z_1(nT) + r, & t = nT, \\ Z_1(0^+) = X(0^+). \end{cases} \tag{46}$$

It follows from Lemma 1 and the comparison theorem on impulsive differential equations [38] that we have $X(t) \le Z_1(t)$ and $Z_1(t) \longrightarrow X^*(t)$ as $t \longrightarrow \infty$. Hence, for $\varepsilon_3 > 0$ small enough and all large t, we have

$$X(t) \le Z_1(t) < X^*(t) + \varepsilon_3. \tag{47}$$

For simplification, we may assume that (47) holds for all $t \ge 0$. From (25), we get

$$\begin{cases} \dfrac{\mathrm{d}Y(t)}{\mathrm{d}t} \le k_1(X^*(t) + \varepsilon_3)\varepsilon_2 - gY(t), & t \ne nT, \\ Y(nT^+) = (1-\alpha)Y(nT), & t = nT, \\ Y(0^+) = Y_0. \end{cases} \tag{48}$$

Again, from the comparison theorem on impulsive differential equation, we get

$$\begin{aligned} Y((n+1)T) &\le Y(nT^+)\exp(-gT) + \exp(-gT)\int_{nT}^{(n+1)T} k_1(X^*(t)+\varepsilon_3)\varepsilon_2 \exp(gT)\mathrm{d}t \\ &= Y(nT^+)\exp(-gT) + k_1\varepsilon_2\int_{nT}^{(n+1)T}\left[\frac{r\exp(-gT)}{1-\exp(-gT)} + \varepsilon_3\right]\mathrm{d}t \\ &= Y(nT^+)\exp(-gT) + \frac{(\varepsilon_3 + r\exp(-gT))k_1\varepsilon_2 T}{1-\exp(-gT)} \\ &= (1-\alpha)Y(nT)\exp(-gT) + \frac{(\varepsilon_3 + r\exp(-gT))k_1\varepsilon_2 T}{1-\exp(-gT)}Y((n)T) \\ &\le (1-\alpha)^n\exp(-ngT) + \frac{(\varepsilon_3 + r\exp(-gT))k_1\varepsilon_2 T}{1-\exp(-gT)}\frac{1}{1-(1-\alpha)\exp(-gT)}. \end{aligned} \tag{49}$$

Let $\varepsilon_2 \longrightarrow 0$; then, $Y(nT) \longrightarrow 0$ as $n \longrightarrow \infty$. Therefore, $Y(t) \longrightarrow 0$ as $t \longrightarrow \infty$.

Next, we show $X(t) \longrightarrow X^*(t)$ as $t \longrightarrow \infty$. It follows from $V(t) < \varepsilon_2$ that for $t > t_2$ we have

$$\frac{dX(t)}{dt} \geq -k_1 X(t)\varepsilon_2 - gX(t); \tag{50}$$

then, we have

$$\begin{cases} \dfrac{dZ_2(t)}{dt} = -(k_1\varepsilon_2 + g)Z_2(t), & t \neq nT, \\ Z_2(nT^+) = Z_2(nT) + r, & t = nT, \\ Z_2(0^+) = X(0^+). \end{cases} \tag{51}$$

Note that the above system has a globally stable periodic solution $Z_2^*(t) = r\exp(-(k_1\varepsilon_2 + g)(t - nT))/1 - \exp((-(k_1\varepsilon_2 + g)T)$ with $Z_2^*(nT^+) = r/1 - \exp((-(k_1\varepsilon_2 + g)T)$. From the comparison theorem, we get $Z_2(t) \leq X(t) \leq Z_1(t), Z_2(t) \longrightarrow Z_2^*(t)$ and $Z_1(t) \longrightarrow X^*(t)$ as $t \longrightarrow \infty$. Therefore, there exists a t_3 such that $t_3 > t_2$, and for $t > t_3$ we have

$$Z_2^*(t) - \varepsilon_3 < X(t) < X^*(t) + \varepsilon_3, \tag{52}$$

with $\varepsilon_3 > 0$ small enough.

Let $\varepsilon_2 \longrightarrow 0$, so that $Z_2(t) \longrightarrow X^*(t)$. Then, (52) becomes

$$X^*(t) - \varepsilon_3 < X(t) < X^*(t) + \varepsilon_3. \tag{53}$$

Hence, $X(t) \longrightarrow X^*(t)$ as $t \longrightarrow \infty$. This completes the proof. □

Next, to investigate the stability results of disease-free periodic solution $(X^*(t), U^*(t), 0, 0)$ of model (25), we first calculate the basic reproduction number for the impulsive system (25) by using the next infection operator for the piecewise continuous periodic system proposed in [39, 40]. Denote $X(t) = x(t) + X^*(t), Y(t) = y(t), U(t) = u(t) + U^*(t), V(t) = v(t)$. The corresponding linear system (25) at $(X^*(t), U^*(t), 0, 0)$ reads as

$$\begin{cases} W\prime(t) = G(t)W(t), & t \neq nT, \\ W(nT^+) = HW(nT), & t = nT. \end{cases} \tag{54}$$

In this system, we have $W(t) = (x(t), u(t), y(t), v(t))^T$,

$$G(t) = \begin{pmatrix} G_1(t) & G_2(t) \\ 0 & F(t) - V_r(t) \end{pmatrix}, \quad H = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix}, \tag{55}$$

with

$$F(t) = \begin{pmatrix} 0 & k_1 X^*(t) \\ k_2 U^*(t) & 0 \end{pmatrix}, \quad V_r(t) = \begin{pmatrix} g & 0 \\ 0 & c \end{pmatrix}, 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \tag{56}$$

$$G_1(t) = \begin{pmatrix} -g & 0 \\ \dfrac{bU^{*2}(t)}{mX^{*2}(t)} & b - c - \dfrac{2bU^*(t)}{mX^*(t)} \end{pmatrix}, \quad G_2(t) = \begin{pmatrix} 0 & -k_1 X^*(t) \\ \dfrac{bU^*(t)}{mX^{*2}(t)} - k_2 U^*(t) & b - \dfrac{2bU^*(t)}{mX^*(t)} \end{pmatrix} \tag{57}$$

$$H_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 - p \end{pmatrix}, H_2 = \begin{pmatrix} 1 - \alpha & 0 \\ 0 & 1 - p \end{pmatrix}. \tag{58}$$

Let $\Phi_G(t) = (\Phi_{ij}), 1 \leq i, j \leq 2$, be the fundamental solution matrix of (54). Consequently, we have $\Phi_G'(t) = G(t)\Phi_G(t)$ with appropriate initial value $\Phi_G(0) = I_4$. Further computation implies that

$$\Phi_G(t) = \begin{pmatrix} \exp(G_1(t)) & \Phi_{12}(t) \\ 0 & \Phi_{F(t)-V_r(t)}(t) \end{pmatrix}; \tag{59}$$

then, we obtain

$$H\Phi_G(T) = \begin{pmatrix} H_1 \exp(G_1(T)) & H_1\Phi_{12}(T) \\ 0 & H_2\Phi_{F(t)-V_r(t)}(T) \end{pmatrix}. \tag{60}$$

By [39, 40], the basic reproduction number for system (54) is given as follows:

$$R_2 = \rho\left(H_2\Phi_{F(t)-V_r(t)}(T)\right). \tag{61}$$

Then, based on Floquet theory, we have the following conclusion.

Theorem 6. *The disease-free periodic solution $(X^*(t), U^*(t), 0, 0)$ of model (25) is globally asymptotically stable in the first quadrant if $R_1 = (1 - p_1)\exp((b - c)T) > 1$ and $R_2 < 1$.*

Proof. To prove the global stability of $(X^*(t), U^*(t), 0, 0)$, we only need to show the global attractivity. Suppose $R_1 > 1$; then, from the boundedness of (25), there exists a $t_1 > 0$ such that $X(t), Y(t) \leq L_1$ and $U(t), V(t) \leq L_2$ with $t \geq t_1$ for every solution $(X(t), Y(t), U(t), V(t))$ of (4.1), where

$$L_1 = \frac{r}{1-\exp(-gT)} + \varepsilon_1,$$
$$L_2 = \frac{b(1-\exp(-(b-c)T))}{mL_1(b-c)(1-\exp(-(b-c)T))} + \varepsilon. \tag{62}$$

It follows from (25) that

$$\begin{cases} \dfrac{dX(t)}{dt} \le -gX(t), & t \ne nT, \\ X(nT^+) = X(nT) + r, & t = nT, \\ X(0^+) = X_0. \end{cases} \tag{63}$$

Consider the following system:

$$\begin{cases} \dfrac{dZ_3(t)}{dt} = -gZ_3(t), & t \ne nT, \\ Z_3(nT^+) = Z_3(nT) + r, & t = nT, \\ Z_3(0^+) = X(0^+). \end{cases} \tag{64}$$

From Lemma 1 and the comparison theorem on impulsive differential equations [38], we have $X(t) \le Z_3(t)$ and $Z_3(t) \longrightarrow X^*(t)$ as $t \longrightarrow \infty$. Hence, for $\varepsilon_3 > 0$ small enough and all large t, we have

$$X(t) \le Z_3(t) < X^*(t) + \varepsilon_3. \tag{65}$$

From (25), we get

$$\begin{cases} \dfrac{dY(t)}{dt} = k_1 X(t)V(t) - gY(t) \doteq f_1(Y(t), V(t)), & t \ne nT, \\ \dfrac{dV(t)}{dt} = k_2 U(t)Y(t) - cV(t) \doteq f_2(Y(t), V(t)), & t \ne nT, \\ Y(nT^+) = (1-\alpha)Y(nT), & t = nT, \\ V(nT^+) = (1-p)V(nT), & t = nT, \\ Y(0^+) = Y_0, \\ V(0^+) = V_0. \end{cases} \tag{66}$$

Consider the following system:

$$\begin{cases} \dfrac{dQ_1(t)}{dt} = k_1(X^*(t) + \varepsilon_3)Q_2(t) - gQ_1(t) \doteq g_1(Q_1(t), Q_2(t)), & t \ne nT, \\ \dfrac{dQ_2(t)}{dt} = k_2 M^*(t)Q_1(t) - cQ_2(t) \doteq g_2(Q_1(t), Q_2(t)), & t \ne nT, \\ Q_1(nT^+) = (1-\alpha)Q_1(nT), & t = nT, \\ Q_1(nT^+) = (1-p)Q_2(nT), & t = nT, \\ Q_1(0^+) = Y_0, \\ Q_2(0^+) = V_0, \end{cases} \tag{67}$$

which is equivalent to

$$\begin{cases} \dfrac{dQ(t)}{dt} = (F(t) - V_r(t))Q(t) + k_1\varepsilon_3 IQ(t), & t \ne nT, \\ Q(nT^+) = H_2 Q(nT), & t = nT, \end{cases} \tag{68}$$

where $Q(t) = (Q_1(t), Q_2(t))^T$ and I is identity matrix. By [39, 40], there exists a positive T-periodic function $\nu(t)$, such that $Q(t) = \exp(\mu_1 t)\nu(t)$ is a solution of system (68), where $\mu_1 = 1/T \ln \rho(H_2\Phi_{(F-V_r+k_1\varepsilon_3 I)}(T))$. When $R_2 < 1$, we have $\rho(H_2\Phi_{(F-V_r)}(T)) < 1$. Due to the continuity of $\rho(H_2\Phi_{(F-V_r+k_1\varepsilon_3 I)}(T))$, we can choose sufficient small ε_3 such that $\rho(H_2\Phi_{(F-V_r+k_1\varepsilon_3 I)}(T)) < 1$. Hence, we obtain $\mu_1 < 0$, and $Q(t) \longrightarrow 0$ as $t \longrightarrow \infty$. For any nonnegative initial value $(Q_1(t_2), Q_2V(t_2))^T$ of system (68), there exists an η large enough such that

$$(Q_1(t_2), Q_2(t_2))^T \le \eta(\nu_1(0), \nu_2(0))^T. \tag{69}$$

Since $\partial g_1/\partial Q_2(t) \ge 0$, $\partial g_2/\partial Q_1(t) \ge 0$, and $f_i \le g_i, i = 1, 2$, system (68) is a quasimonotone increasing system [41]; by the comparison theorem [38], we obtain

$$(Y(t), V(t)) \le \eta \exp(\mu_1(t-t_2))(\nu_1(t-t_2), \nu_2(t-t_2))^T, \quad t \ge t_2. \tag{70}$$

Hence, we have $Y(t) \longrightarrow 0, V(t) \longrightarrow 0$ as $t \longrightarrow \infty$.

Next, we shall prove that if $\lim_{t\longrightarrow\infty} Y(t)t = 0$ and $\lim_{t\longrightarrow\infty} V(t) = 0$, then $X(t) \longrightarrow X^*(t)$ and $U(t) \longrightarrow U^*(t)$ as $t \longrightarrow \infty$. First, we show $U(t) \longrightarrow U^*(t)$ as $t \longrightarrow \infty$, since $M(t) = (U(t) + V(t)) \longrightarrow M^*(t)$, when $\lim_{t\longrightarrow\infty} V(t) = 0, U(t) \longrightarrow M^*(t) = U^*(t)$. It follows from $\lim_{t\longrightarrow\infty} V(t) = 0$ that, for $\varepsilon_3 > 0$ small enough, there exists a t_3 such that $t_3 \ge t_2$ and $0 < V(t) < \varepsilon_3$ for $t \ge t_3$. Thus, for $t \ge t_3$, we have

$$(-k_1\varepsilon_3 - g)X(t) \le \frac{dX(t)}{dt} \le -gX(t), \tag{71}$$

from which the following system is obtained:

$$\begin{cases} \dfrac{dZ_4(t)}{dt} = (-k_1\varepsilon_3 - g)Z_4(t), & t \ne nT, \\ Z_4(nT^+) = Z_4(nT) + r, & t = nT, \\ Z_4(0^+) = X(0^+). \end{cases} \tag{72}$$

It follows from Lemma 1 that system (72) has a globally stable positive periodic solution $Z_4^*(t)$ for $t \in (nT, (n+1)T]$; here, $Z_4^*(t) = r\exp((-k_1\varepsilon_3 - g)(t - nT))/1 - \exp((k_1\varepsilon_3 - g)T)$ with $Z_4^*(nT^+) = r/1 - \exp((k_1\varepsilon_3 - g)T)$. By the comparison theorem, we obtain $Z_4(t) \le X(t) \le Z_3(t)$, $Z_4(t) \longrightarrow Z_4^*(t)$, and $Z_3(t) \longrightarrow X^*(t)$ as $t \longrightarrow \infty$. Therefore, there is a t_4 for $\varepsilon_4 > 0$ small enough such that $t_4 \ge t_3$; for $t \ge t_4$, we have

$$Z_4^*(t) - \varepsilon_4 \le X(t) \le X^*(t) + \varepsilon_4. \tag{73}$$

Let $\varepsilon_3 \longrightarrow 0$ in (71), so that $Z_2(t) \longrightarrow X^*(t)$. Then,

$$X^*(t) - \varepsilon_4 \le X(t) \le X^*(t) + \varepsilon_4. \tag{74}$$

Hence, $X(t) \longrightarrow X^*(t)$ as $t \longrightarrow \infty$. We have proved the global attractivity of the disease-free periodic solution $(X^*(t), U^*(t), 0, 0)$ of model (25). This completes the proof. □

4.2. Permanence. The persistence of the system indicates that plant (susceptible and infected) and vectors (susceptible and infected) can coexist. If our goal is to eliminate infected plants and vectors, then the persistence suggests that control strategies fail to achieve it. Meanwhile, the permanent condition obtained from analyzing the system can provide scientific support for us to identify the key factors that result in failure and the effectiveness of control strategies, and then guide us to establish a good control program.

Theorem 7. *System (25) is uniformly persistent if* $R_2 > 1$.

Proof. It follows from $R_2 > 1$ that $\rho(H_2\Phi_{(F-V_r)(T)}) > 1$, which indicates that there exists $\bar{\varepsilon} > 0$ such that

$$\rho\left(H_2\Phi_{(F-V_r-\bar{\varepsilon}M_1)}(T)\right) > 1, \tag{75}$$

where

$$M_1 = \begin{pmatrix} 0 & k_1 \\ k_2 & 0 \end{pmatrix}. \tag{76}$$

Let $(X(t), U(t), Y(t), V(t))$ be any solution of (25). First, we claim that we can find a positive constant η_2 such that

$$\begin{aligned} \lim_{t\longrightarrow\infty} \sup Y(t) \ge \eta_2, \\ \lim_{t\longrightarrow\infty} \sup V(t) \ge \eta_2. \end{aligned} \tag{77}$$

Otherwise, there exists t_1 such that $Y(t) < \eta_2$ and $V(t) < \eta_2$ for all $t \ge t_1$. Based on system (25), we have

$$\begin{cases} \dfrac{dX(t)}{dt} = -k_1V(t)X(t) - gX(t) \doteq f_3(X(t), U(t)), & t \ne nT, \\ \dfrac{dU(t)}{dt} = bU(t)\left(1 - \dfrac{U(t) + \eta_2}{mX(t)}\right) - k_2\eta_2U(t) - cU(t) \doteq f_4(X(t), U(t)), & t \ne nT, \\ X(nT^+) = X(nT) + r, & t = nT, \\ U(nT^+) = (1-p)U(nT), & t = nT, \end{cases} \tag{78}$$

and we take the auxiliary system

$$\begin{cases} \dfrac{d\widetilde{X}(t)}{dt} = -k_1\eta_2\widetilde{X}(t) - g\widetilde{X}(t) \doteq g_3(\widetilde{X},\widetilde{U}(t)), & t \neq nT, \\ \dfrac{d\widetilde{U}(t)}{dt} = b\widetilde{U}(t)\left(1 - \dfrac{\widetilde{U}(t)+\eta}{m\widetilde{X}(t)}\right) - k_2\eta\widetilde{U}(t) - c\widetilde{U}(t) \doteq g_4(\widetilde{X}(t),\widetilde{U}(t)), & t \neq nT, \\ \widetilde{X}(nT^+) = \widetilde{X}(nT) + r, & t = nT, \\ \widetilde{U}(nT^+) = (1-p)\widetilde{U}(nT), & t = nT. \end{cases} \tag{79}$$

Since $\partial f_3/\partial U(t) = 0$, $\partial f_4/\partial X(t) \geq 0$, and $f_i \geq g_i, i = 3, 4$, system (78) is a quasimonotone increasing system [41]. By the comparison theorem [38], we have $X(t) \geq \widetilde{X}(t)$, $U(t) \geq \widetilde{U}(t)$ and $\widetilde{X}(t) \longrightarrow X^*(t), \widetilde{U}(t) \longrightarrow U^*(t)$ as $t \longrightarrow \infty$ and $\eta_2 \longrightarrow 0$. Therefore, for the above $\overline{\varepsilon} > 0$, there exist $t_2 \geq t_1$ and $\widetilde{\eta} > 0$; for $\eta_2 < \widetilde{\eta}$, we have

$$\begin{aligned} X(t) &\geq \widetilde{X}(t) \geq X^*(t) - \overline{\varepsilon} \\ U(t) &\geq \widetilde{U}(t) \geq U^*(t) - \overline{\varepsilon}. \end{aligned} \tag{80}$$

Again, from system (25), we have

$$\begin{cases} \dfrac{dY(t)}{dt} \geq k_1(X^*(t) - \overline{\varepsilon})V(t) - gY(t), & t \neq nT, \\ \dfrac{dV(t)}{dt} \geq k_2(U^*(t) - \overline{\varepsilon})Y(t) - cV(t), & t \neq nT, \\ Y(nT^+) = (1-\alpha)Y(nT), & t = nT, \\ V(nT^+) = (1-p)V(nT), & t = nT. \end{cases} \tag{81}$$

Further, we consider an auxiliary system

$$\begin{cases} \dfrac{d\widetilde{Y}(t)}{dt} = k_1X^*(t)\widetilde{V}(t) - g\widetilde{Y}(t) - \overline{\varepsilon}\widetilde{V}(t), & t \neq nT, \\ \dfrac{d\widetilde{V}(t)}{dt} = k_2U^*(t)\widetilde{Y}(t) - c\widetilde{V}(t)\overline{\varepsilon}\widetilde{Y}(t), & t \neq nT, \\ \widetilde{Y}(nT^+) = (1-\alpha)\widetilde{Y}(nT), & t = nT, \\ \widetilde{V}(nT^+) = (1-p)\widetilde{V}(nT), & t = nT, \end{cases} \tag{82}$$

which can be rewritten as

$$\begin{cases} \begin{pmatrix} \dfrac{d\widetilde{Y}(t)}{dt} \\ \dfrac{d\widetilde{V}(t)}{dt} \end{pmatrix} = \left((F(t) - V_r(t)) - \overline{\varepsilon}M_1\right)\begin{pmatrix} \widetilde{Y}(t) \\ \widetilde{V}(t) \end{pmatrix}, & t \neq nT, \\ \begin{pmatrix} \widetilde{Y}(nT^+) \\ \widetilde{V}(nT^+) \end{pmatrix} = H_2\begin{pmatrix} \widetilde{Y}(nT) \\ \widetilde{V}(nT) \end{pmatrix}, & t = nT, \end{cases} \tag{83}$$

where $F(t)$, $V_r(t)$, and H_2 can be seen in (56) and (58). It follows from the results shown in [39, 40] that there exists a positive T-periodic function $W(t) = (W_1(t), W_2(t))$ such that $v_2(t) = \exp(\mu_2 t)W(t)$ is a solution of (83), where $\mu_2 = 1/T \ln \rho(H_2\Phi_{(F-V_r-\overline{\varepsilon}M_1)}(T))$ and $v_2(t) = (\widetilde{Y}(t), \widetilde{V}(t))$. From (75), $\mu_2 > 0$. Choose $t_3 \geq t_2$ and $\eta_2 > 0$ such that $W(t_2) \geq \eta_2 v_2(0)$; we obtain

$$W(t) \geq \eta_2 \exp(\mu_2(t - t_2))v_2(t - t_2), \quad t \geq t_3. \tag{84}$$

By the comparison theorem [38], we obtain that, for all $t \geq t_3$,

$$\begin{aligned} Y(t) &\geq \widetilde{Y}(t) \geq \eta_2 \exp(\mu_2(t - t_2))v_2(t - t_2), \\ V(t) &\geq \widetilde{V}(t) \geq \eta_2 \exp(\mu_2(t - t_2))v_2(t - t_2). \end{aligned} \tag{85}$$

As a result, we obtain $W(t) \longrightarrow \infty$ as $t \longrightarrow \infty$. Consequently, $(\widetilde{Y}(t), \widetilde{V}(t)) \longrightarrow +\infty$ as $t \longrightarrow \infty$, which conflicts with $\widetilde{Y}(t) < \eta_2$ and $\widetilde{V}(t)) < \eta_2$. Thus,

$$\begin{aligned} \lim_{t\longrightarrow\infty} \sup Y(t) &\geq \eta_2, \\ \lim_{t\longrightarrow\infty} \sup V(t) &\geq \eta_2. \end{aligned} \tag{86}$$

Furthermore, we have the following two possibilities.

(i) $Y(t) \geq \eta_2$ and $V(t) \geq \eta_2$ for all large t.

(ii) $Y(t)$ and $V(t)$ oscillations about η_2 for all large t.

If case (i) holds, then the result is obtained. Next, we consider case (ii). Because $\lim_{t\longrightarrow\infty}\sup Y(t) \geq \eta_2$ and $\lim_{t\longrightarrow\infty}\sup V(t) \geq \eta_2$, we can choose a $t_1 \in (n_1T, (n_1+1)T]$ such that $Y(t_1) \geq \eta_2$ and $V(t_1) \geq \eta_2$. The above discussion implies that there exists $t_2 \in (n_2T, (n_2+1)T]$, such that $Y(t_2) \geq \eta_2$ and $V(t_2) \geq \eta_2$, where $n_2 - n_1 \geq 0$ is finite. Then, we will consider the solution of system (25) in the interval $[t_1, t_2]$:

$$\frac{dY(t)}{dt} = k_1X(t)V(t) - gY(t) \geq -gY(t), \tag{87}$$

which results in the following relations:

$$\begin{aligned} Y(t) &\geq Y(t_1)\exp(-g(t-t_1)) \geq \eta_2\exp(-g(t_2-t_1)) \\ &\geq \eta_2\exp(-g(n_2-n_1)T). \end{aligned} \tag{88}$$

Similarly, it follows from

$$\frac{dV(t)}{dt} = k_2U(t)Y(t) - cV(t) \geq -cV(t) \tag{89}$$

that we have

$$\begin{aligned} V(t) &\geq V(t_1)\exp(-c(t-t_1)) \geq \eta_2\exp(-c(t_2-t_1)) \\ &\geq \eta_2\exp(-c(n_2-n_1)T). \end{aligned} \tag{90}$$

Let $\sigma_1 = \min\{\eta_2\exp(-g(n_2-n_1)T), \eta_2\exp(-c(n_2-n_1)T)\}$; then, $\sigma_1 > 0$ cannot be infinitely small and $n_2 - n_1 \geq 0$ is finite. We get $Y(t) \geq \sigma_1 > 0$ and $V(t) \geq \sigma_1 > 0$.

For $t > t_2$, we take the same steps and get another positive σ_2. As a consequence of this, we obtain the sequence σ_i, where $\sigma_i = \min\{\eta_2\exp(-g(n_{i+1}-n_i)T), \eta_2\exp(-c(n_2-n_1)T)\}$, for $i = 1, 2, \ldots,$ is noninfinitesimal when $n_{i+1} - n_i \geq 0$ is finite. In this case, the solution of system (25) satisfies $Y(t) \geq \sigma_i > 0$ and $V(t) \geq \sigma_i > 0$ for all $[t_i, t_{i+1}]$, where $t_i \in (n_iT, n_{i+1}T]$. Let $\sigma = \min\{\sigma_i, i = 1, 2, \ldots,\}$; we obtain $Y(t) \geq \sigma > 0$ and $V(t) \geq \sigma > 0$. This completes the proof. □

5. Optimum Times of Roguing Control

Note that, in reality, replanting the susceptible and removing the infected plants are usually not at the same time, and the infected plants could be removed several times within each replanting period. The key question is whether there exist optimal times of removing the infected plants within each replanting period such that the threshold value reaches its minimum. To address this question, we extend model (25) by taking the multiple pulse controls as follows:

$$\begin{cases} \left.\begin{aligned} \frac{dX(t)}{dt} &= -k_1X(t)V(t) - gX(t), \\ \frac{dU(t)}{dt} &= b(U(t)+V(t))\left(1 - \frac{U(t)+V(t)}{m(X(t)+Y(t))}\right) - k_2U(t)Y(t) - cU(t), \\ \frac{dY(t)}{dt} &= k_1X(t)V(t) - gY(t), \\ \frac{dV(t)}{dt} &= k_2U(t)Y(t) - cV(t), \end{aligned}\right\}, \quad t \neq \{\tau_n, t_n^i\}, \\ \left.\begin{aligned} X(t^+) &= X(t) + r, \\ U(t^+) &= U(t), \\ Y(t^+) &= Y(t), \\ V(t^+) &= V(t), \end{aligned}\right\}, \quad t = \tau_n, \\ \left.\begin{aligned} X(t^+) &= X(t), \\ U(t^+) &= (1-p)U(t), \\ Y(t^+) &= (1-\alpha)Y(t), \\ V(t^+) &= (1-p)V(t), \end{aligned}\right\}, \quad t = t_n^i, \end{cases} \tag{91}$$

where $\tau_n (n = 0, 1, 2, \ldots,)$ and $t_n^i (i = 1, 2, \ldots, k)$ are impulsive point series at which the replanting and removing actions have been taken, respectively. Further, we assume $\tau = \tau_{n+1} - \tau_n$ for all $n = 0, 1, 2, \ldots,$, where τ is the period of replanting. For this case, model (91) is called a period system with period τ if there exists a positive integer k such that $t_{n+1}^k - t_n^k = \tau$, where $t_n^i \in [\tau_n, \tau_{n+1}]$ for all $i = 1, 2, \ldots, k$. This implies that, in each period τ, the infected plants have been removed k times. Denote $\Delta_i = t_n^{i+1} - t_n^i, i = 0, 1, 2, \ldots, k$, where $\Delta_0 = \tau_n, \Delta_{k+1} = \tau_{n+1} - t_n^k$. Firstly, the basic properties of subsystem

$$\begin{cases} \left.\begin{aligned} &\frac{\mathrm{d}X(t)}{\mathrm{d}t} = -gX(t), \\ &\frac{U(t)}{\mathrm{d}t} bU(t)\left(1 - \frac{U(t)}{mX(t)}\right) - cU(t), \end{aligned}\right\}, \quad t \neq \left\{\tau_n, t_n^i\right\}, \\ \left.\begin{aligned} &X\left(t^+\right) = X(t) + r, \\ &U\left(t^+\right) = U(t), \end{aligned}\right\}, \quad t = \tau_n, \\ \left.\begin{aligned} &X\left(t^+\right) = X(t), \\ &U\left(t^+\right) = (1 - p)U(t), \end{aligned}\right\}, \quad t = t_n^i \end{cases} \tag{92}$$

play a key role in analyzing the disease control.

In any given time interval $(\tau_n, \tau_{n+1}]$, we integrate the first equation of system (92) and have $X(t) = X(\tau_n^+)\exp(-g(t - \tau_n))$. At time τ_{n+1}, one replanting strategy is applied, which results in

$$X\left(\tau_{n+1}^+\right) = X\left(\tau_n^+\right)\exp(-g\tau) + r. \tag{93}$$

Denote $X_{\tau_n} = X(\tau_n^+)$; then, we have the following difference equation: $X_{\tau_{n+1}} = X_{\tau_n}\exp(-g\tau) + r$, which has a unique steady state $\overline{X}^* = r/1 - \exp(-g\tau)$. Therefore, from Lemma 1, the subsystem

$$\begin{cases} \frac{\mathrm{d}X(t)}{\mathrm{d}t} = -gX(t), \quad t \neq \tau_n, \\ X\left(t^+\right) = X(t) + r, \quad t = \tau_n, \\ X\left(0^+\right) = X_0 \end{cases} \tag{94}$$

has a positive periodic solution $\overline{X}^*(t) = \overline{X}^*\exp(-g(t - \tau_n))$ which is globally asymptotically stable.

Further, substituting the above solution into the second equation of (92) and integrating the second equation of system (92) from τ_n to t_n^1 yields

$$U(t) = \frac{U\left(\tau_n^+\right)\exp\left((b - c)(t - \tau_n)\right)}{1 + bU\left(\tau_n^+\right)/m(g + b - c)\overline{X}^*\left(\exp\left((g + b - c)(t - \tau_n)\right) - 1\right)}, \quad t \in \left(\tau_n, t_n^1\right]. \tag{95}$$

At time t_n^1, one removing strategy is applied, which results in

$$U\left(t_n^{1+}\right) = \frac{(1 - p)U\left(\tau_n^+\right)\exp\left((b - c)\Delta_1\right)}{1 + bU\left(\tau_n^+\right)/m(g + b - c)\overline{X}^*\left(\exp\left((g + b - c)\Delta_1\right) - 1\right)}. \tag{96}$$

Again, integrating the second equation of system (92) from t_n^1 to t_n^2 yields

$$U(t) = \frac{U\left(t_n^{1+}\right)\exp\left(\left(b - c\right)\left(t - t_n^1\right)\right)}{1 + bU\left(t_n^{1+}\right)/m(g + b - c)\overline{X}^*\left(\exp\left(\left(g + b - c\right)\left(t - t_n^1\right)\right) - 1\right)}, \quad t \in \left(t_n^1, t_n^2\right]. \tag{97}$$

Further, at time t_n^2, one removing tactics is applied and

$$U\left(t_n^{2+}\right) = \frac{(1-p)^2 U\left(\tau_n^+\right)\exp\left((b-c)\left(\Delta_1+\Delta_2\right)\right)}{1+bU\left(\tau_n^+\right)/m(g+b-c)\overline{X}^*\left(\exp\left((g+b-c)\Delta_1\right)-1+(1-p)\exp\left((b-c)\Delta_1\right)\left(\exp\left((g+b-c)\Delta_2\right)\right)-1\right)}. \tag{98}$$

By induction, we can see that

$$\begin{aligned} U(t) = {} & \frac{(1-p)^k U\left(\tau_n^+\right)\exp\left(\sum_{i=1}^{k}\Delta_i\right)\exp\left((b-c)\left(t-t_n^k\right)\right)}{1+bU\left(\tau_n^+\right)/m(g+b-c)\overline{X}^*\left(\sum_{i=1}^{k}(1-p)^{(i-1)}\exp\left((b-c)\right.\right.} \\ & \left.\left.\sum_{j=1}^{i-1}\Delta_j\right)\left(\exp\left((g+b-c)\Delta_i\right)-1\right)+(1-p)^k\left(\exp\left((g+b-c)\left(t-t_n^k\right)\right)-1\right)\right), \end{aligned} \tag{99}$$

for all $t \in (t_n^k, \tau_{n+1}]$. At time τ_{n+1}, we only need to replant healthy plants once without removing infected plants.

$$U\left(\tau_{n+1}^+\right) = \frac{(1-p)^k U\left(\tau_n^+\right)\exp\left((b-c)\tau\right)}{1+bU\left(\tau_n^+\right)/m(g+b-c)\overline{X}^*\left(\sum_{i=1}^{k+1}(1-p)^{i-1}\exp\left((b-c)\sum_{j=1}^{i-1}\Delta_j\right)\left(\exp\left((g+b-c)\Delta_i\right)-1\right)\right)}. \tag{100}$$

Denote $U_{\tau_n} = U\left(\tau_n^+\right)$; then, we have the following difference equation:

$$U_{\tau_{n+1}} = \frac{(1-p)^k U_{\tau_n}\exp\left((b-c)\tau\right)}{1+bU\tau_n/m(g+b-c)\overline{X}^*\sum_{i=1}^{k+1}(1-p)^{i-1}\exp\left((b-c)\sum_{j=1}^{i-1}\Delta_j\right)\left(\exp\left((g+b-c)\Delta_i\right)-1\right)}. \tag{101}$$

Define $R_3 = (1-p)^k\exp\left((b-c)\tau\right)$, and we have the following: if $R_3 \le 1$, then the above difference equation always has a zeros equilibrium; if $R_3 > 1$, then it has a stable positive equilibrium:

$$\overline{U}^* = \frac{(1-p)^k\exp\left((b-c)\tau\right)-1}{b/m(g+b-c)\overline{X}^*\sum_{i=1}^{k+1}(1-p)^{i-1}\exp\left((b-c)\sum_{j=1}^{i-1}\Delta_j\right)\left(\exp\left((g+b-c)\Delta_i\right)-1\right)}. \tag{102}$$

Thus, the system

$$\begin{cases} \dfrac{\mathrm{d}U(t)}{\mathrm{d}t} = bU(t)\left(1-\dfrac{U(t)}{mX^*(t)}\right)-cU(t), & t \ne \left\{\tau_n, t_n^i\right\}, \\ U\left(t^+\right) = (1-p)U(t), & t = t_n^i, \\ U\left(0^+\right) = U_0 \end{cases} \tag{103}$$

has a positive periodic solution

$$\overline{U}^*(t)=\begin{cases}\dfrac{\overline{U}^*\exp\left((b-c)(t-\tau_n)\right)}{1+b\overline{U}^*/m(g+b-c)\overline{X}^*\left(\exp\left((g+b-c)(t-\tau_n)\right)-1\right)}, & t\in\left(\tau_n,t_n^1\right],\\[2ex] \dfrac{\left(1-p\right)\overline{U}^*t\,expn((b-c)\Delta_1)qexph\left(\left(b-c\right)\left(t-t_n^1\right)\right)}{1+b\overline{U}^*/m(g+b-c)\overline{X}^*\left(\exp\left((g+b-c)\Delta_1\right)-1+(1-p)\exp\left((b-c)\Delta_1\right)\left(\exp\left(\left(g+b-c\right)\left(t-t_n^1\right)\right)-1\right)\right)}, & t\in\left(t_n^1,t_n^2\right],\\[2ex] \vdots\\[2ex] \dfrac{(1-p)^k\overline{U}^*\exp\left((b-c)\sum_{i=1}^{k}\Delta_i\right)\exp\left((b-c)\left(t-t_n^k\right)\right)}{1+b\overline{U}^*/m(g+b-c)\overline{X}^*\left(\sum_{i=1}^{k}(1-p)^{(i-1)}\exp\left((b-c)\sum_{j=1}^{i-1}\Delta_j\right)\left(\exp\left((g+b-c)\Delta_i\right)-1\right)+(1-p)^k\exp\left((b-c)\sum_{i=1}^{k}\Delta_i\right)\left(\exp\left(\left(g+b-c\right)\left(t-t_n^k\right)\right)-1\right)\right)}, & t\in\left(t_n^k,\tau_{n+1}\right],\end{cases} \tag{104}$$

which indicates that we obtain the complete expression of periodic solution for system (92) over the time interval $\tau_n < t \le \tau_{n+1}$.

Lemma 5. *System (91) has two nonnegative periodic solutions* $(\overline{X}^*(t),0)$ *and* $(\overline{X}^*(t),\overline{U}^*(t))$. *Moreover, if* $R_3 \le 1$, *then for any solution* $(X(t),U(t))$ *of (91), we have* $X(t)\longrightarrow\overline{X}^*(t), U(t)\longrightarrow 0$ *as* $t\longrightarrow\infty$; *if* $R_3>1$, *then we have* $X(t)\longrightarrow\overline{X}^*(t), U(t)\longrightarrow\overline{U}^*(t)$ *as* $t\longrightarrow\infty$.

Therefore, system (91) has a plant-only periodic solution $(\overline{X}^*(t),0,0,0)$ and disease-free positive periodic solution $(\overline{X}^*(t),\overline{U}^*(t),0,0)$. Due to the complexity of the model, it is possible but very complex to obtain the threshold conditions for the global stability of the above two periodic solutions. Therefore, we will use the numerical analysis method to discuss the influence of the control strategy on the stability of the above two boundary periodic solutions in the following.

6. Numerical Investigations

Due to the high dimension of the system, it is challenging to study the global dynamics of the system after integrating the pulse effect. Therefore, in this section, we employ the parameters given in Table 1 as the baseline parameters for more systematic numerical analysis. This can not only help to further determine the existing main conclusions, but also help to find new dynamic behaviors and reveal the key factors of plant disease control.

According to the main results related to model (1), if $b\le c$ and $R_{01}\le 1$, then the plant-only equilibrium E_{01} is globally asymptotically stable; if $b>c$ and $R_0\le 1$, then the disease-free equilibrium E_{02} is globally asymptotically stable; and if $R_0>1$, then the disease is uniformly persistent. To confirm those results, we show the stability of the plant-only equilibrium of system (1) for $\alpha=0, b=0.1, c=0.18$ in Figure 2(a), and stability of the disease-free equilibrium of system (1) for $\alpha=0.02, b=0.2, c=0.12, R_0=0.7906<1$ in Figure 2(b). If we choose $\alpha=0, b=0.2, c=0.12$ with $R_0=1.9365>1$, then the plant disease is persistent, as shown in Figure 2(c). The effects of roguing rate α on the basic reproduction number R_0 are shown in Figure 3, from which we can see that R_0 is a strictly monotonic decreasing function of α. This reveals that increasing the roguing rate can effectively reduce the threshold R_0, and then the disease could be successfully controlled.

In Figure 4(a), we show the stability of the plant-only periodic solution of model (25) with parameter set $\alpha=0.95, T=15, p=0.75$, and $R_1<1$; in Figure 4(b), we show the stability of the disease-free periodic solution of model (25) with parameter set $\alpha=0.95, T=30, p=0.85, R_1<1$, and $R_2<1$. If we fix the roguing rate $\alpha=0.8$, $T=30$, and $p=0.6$, then $R_1>1$ and $R_2>1$, and then the numerical simulations reveal that the disease is persistent (Figure 4(c)). Figure 5 shows how different period T and vectors removal rate p affect the threshold value R_1. However, due to the complexity of model (25), we cannot get the analytical expression of the threshold R_2; thus, we use the methods proposed in [39, 40] to calculate the threshold value R_2. We fix the parameter set $T=30$, $p=0.85$, and $r=0.4$, and let the roguing rate α vary. It follows from Figure 6(a) that the threshold value R_2 is a monotonically decreasing function with respect to roguing rate α. Similarly, for the parameter sets shown in Figure 6, we see that R_2 is a monotonically decreasing function with respect to vectors removal rate p, as shown in Figure 6(b). On the other hand, R_2 is a monotonically increasing function with respect to impulsive period T or replanting rate r, as shown in Figures 6(c) and 6(d), respectively. All those results indicate that increasing the roguing rate α and vectors removal rate p and reducing the impulsive period T and the replanting rate r can effectively reduce the threshold R_2. Thus, it is conducive to the control of plant disease. Figure 7 further shows the effects of different parameter sets on the threshold condition R_2.

According to the theoretical analyses for model (91), we obtain two threshold values R_3 and R_τ^k, and if $R_3\le 1$, then the plant-only periodic solution $(\overline{X}^*(t),0,0,0)$ is globally asymptotically stable. Indeed, in Figure 8(a), we capture the stability of the plant-only periodic solution $(\overline{X}^*(t),0,0,0)$. In Figure 9, we fix the replanting period $\tau=60$ and let vectors removal rate $p=(0.95,0.9,0.75,0.65)$ vary. The simulation

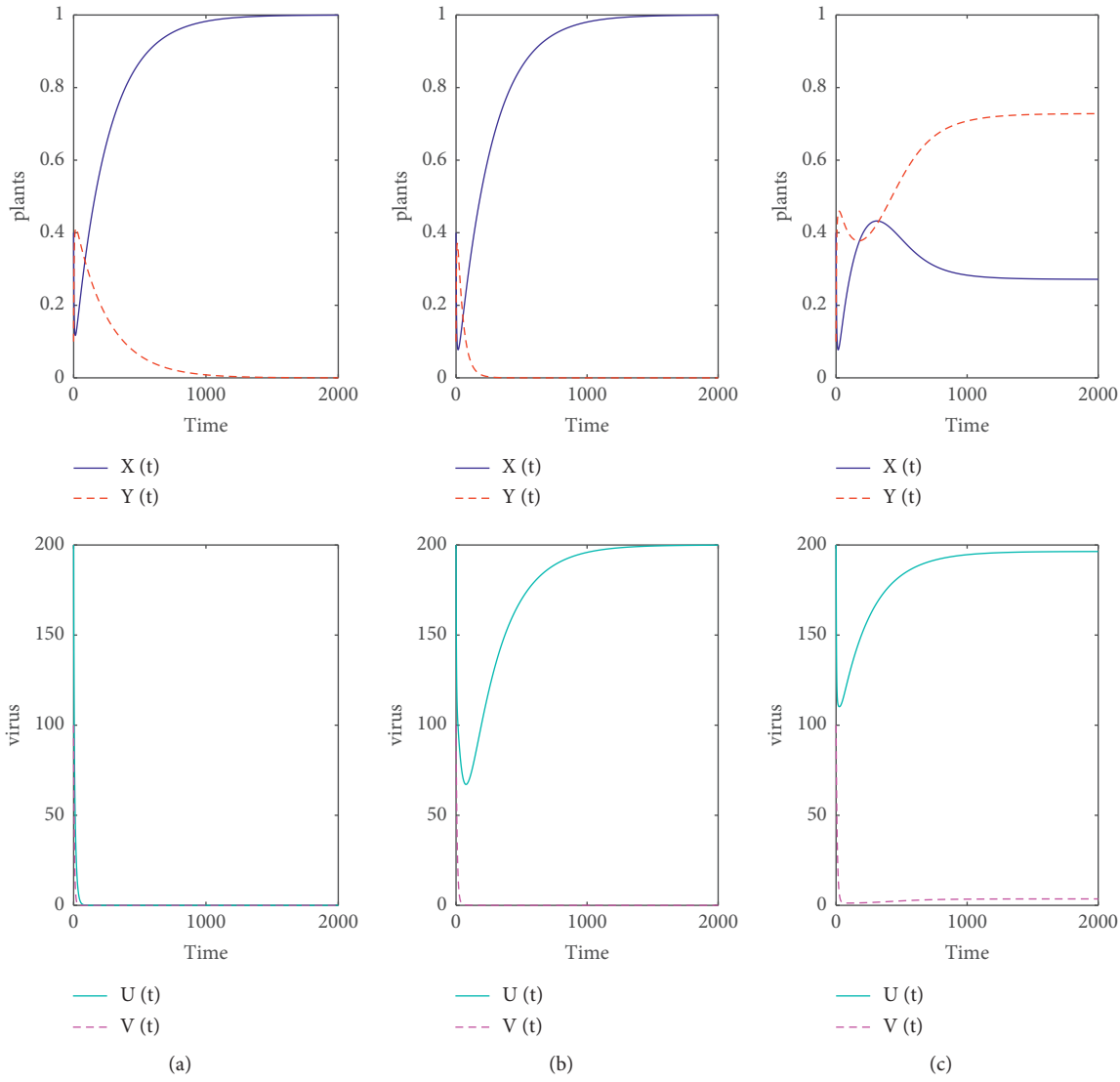

Figure 2: Stabilities of the equilibria $E_{0i}\,(i = 1, 2)$ and E^* for model (1). The set of parameter values is as follows: $r = 0.004, k_1 = 0.003, g = 0.004, m = 500, k_2 = 0.003$. (a) $\alpha = 0, b = 0.1, c = 0.18$; (b) $\alpha = 0.02, b = 0.2, c = 0.12, R_0 = 0.7906 < 1$; (c) $\alpha = 0$, $b = 0.2$, $c = 0.12, R_0 = 1.9365 > 1$.

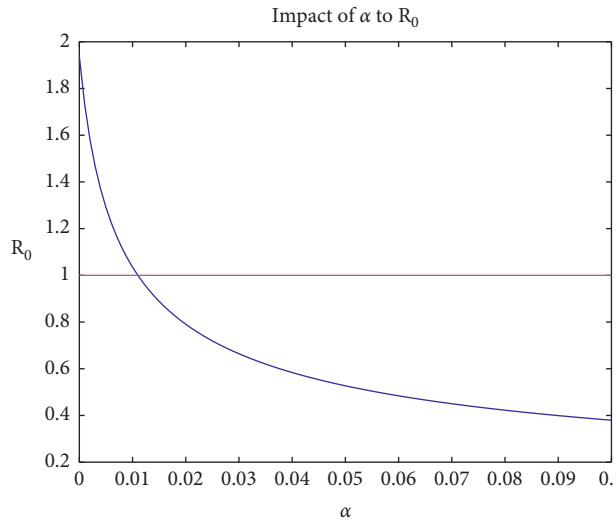

Figure 3: The effects of parameter α on the R_0.

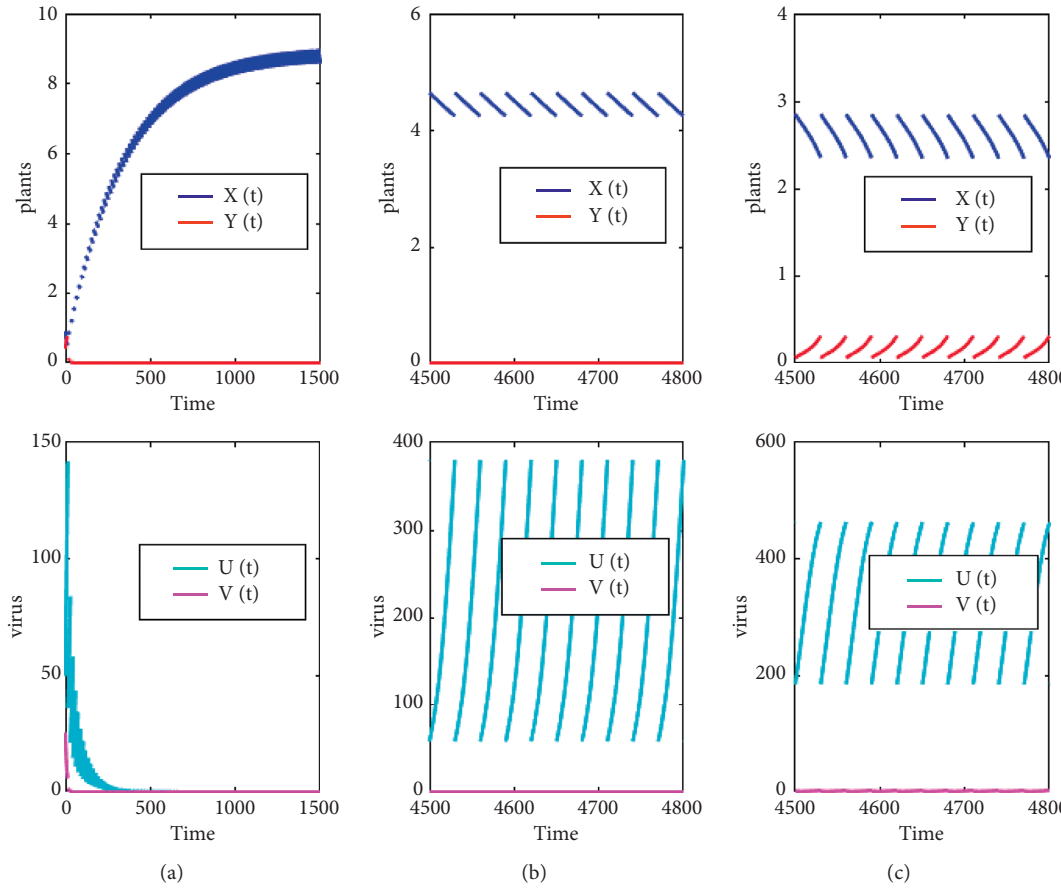

Figure 4: Existence and stability of periodic solutions for model (25). The set of parameter values is as follows: $r = 0.4, k_1 = 0.003, g = 0.003, b = 0.2, m = 500, k_2 = 0.003, c = 0.12$. (a) $T = 15, p = 0.75, \alpha = 0.9, R_1 = 0.83 < 1$; (b) $T = 30, p = 0.85, \alpha = 0.95, R_1 = 1.6535 > 1, R_2 = 0.9331 < 1$; (c) $T = 30, p = 0.6, \alpha = 0.8, R_1 = 4.4093 > 1, R_2 > 1$.

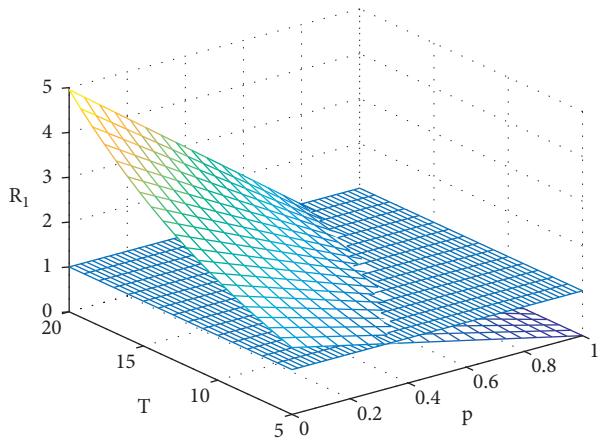

Figure 5: The effects of the parameter sets on the threshold values R_1.

results indicate that the times of roguing control k is inversely proportional to vectors removal rate p. In order to maintain the threshold value R_3 less than one, we find that from Figure 9 the times of roguing control k should be increased significantly as vectors removal rate p is decreasing. Figure 10 further shows the effects of different parameter sets on the threshold condition R_3. Note that the value of R_3 appears to be quite sensitive to small changes in removal rate p, times of roguing control k, and replanting rate r.

In order to discuss the influence of the control strategies on the stability of the disease-free positive periodic solution $(\overline{X}^*(t), \overline{U}^*(t), 0, 0)$, similar to the calculation of R_2, we can obtain a threshold $R_\tau^k = \rho(H_2^k \Phi_{\overline{F}(t)-V_r(t)}(\tau))$; here, $V_r(t)$ and H_2 can be seen in (56) and (58).

$$\overline{F}(t) = \begin{pmatrix} 0 & k_1 \overline{X}^*(t) \\ k_2 \overline{U}^*(t) & 0 \end{pmatrix}. \tag{105}$$

Due to the complexity of model (90) and the inability to calculate the exact expression of R_τ^k, we let the times of roguing control k vary and fix the other parameters. The numerical results show that if $k = 3$ and $R_3 > 1$, then the threshold value $R_\tau^k < 1$, which indicates that the disease-free periodic solution is globally asymptotically stable (Figure 8(b)). If $k = 1, 2$ and $R_3 > 1$, then the threshold value $R_\tau^k > 1$, which indicates that the endemic periodic solution is persistent (Figure 8(c)).

Therefore, the optimal control of plant diseases is in fact to choose suitable times of roguing control k such that $R_\tau^k < 1$ and $R_\tau^{k-1} > 1$. In Figure 11, we fix the replanting period τ and

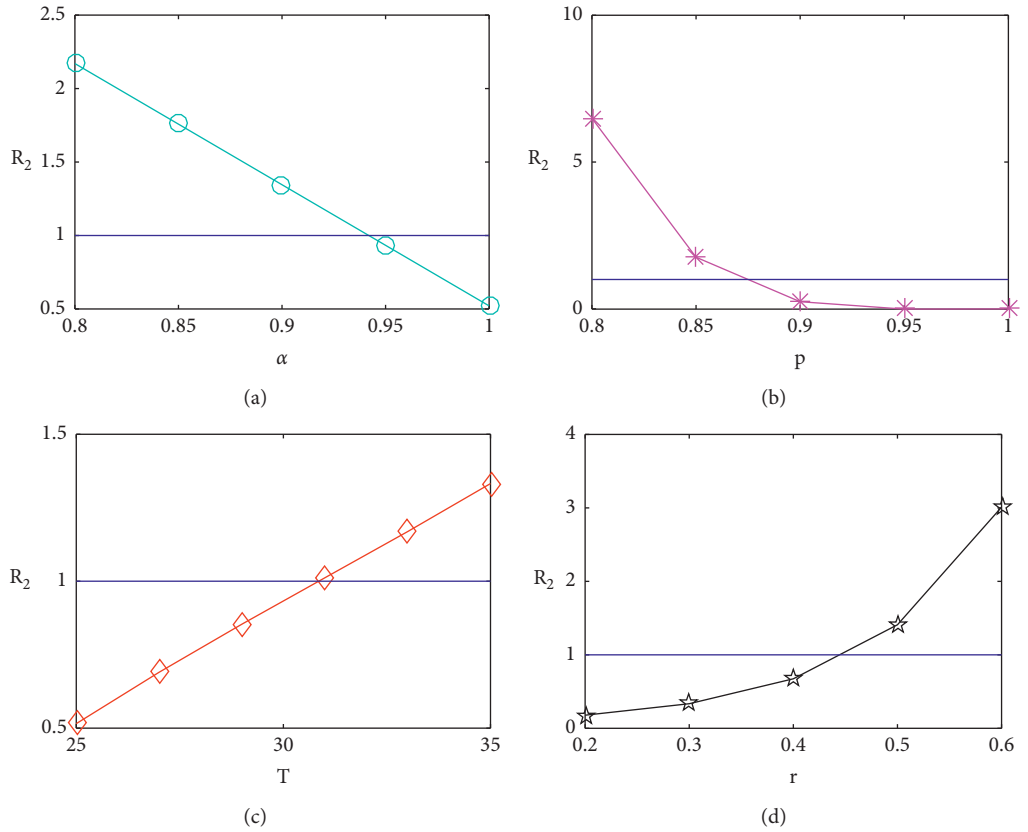

FIGURE 6: The effects of the parameter sets on the threshold level R_2. The set of parameter values is as follows: $b = 0.2, c = 0.12, m = 500, k_1 = 0.003, k_2 = 0.003, g = 0.003$. (a) $T = 30, p = 0.85, r = 0.4$; (b) $T = 30, \alpha = 0.85, r = 0.4$; (c) $p = 0.82, \alpha = 0.82, r = 0.4$; (d) $T = 30, \alpha = 0.9, p = 0.87$.

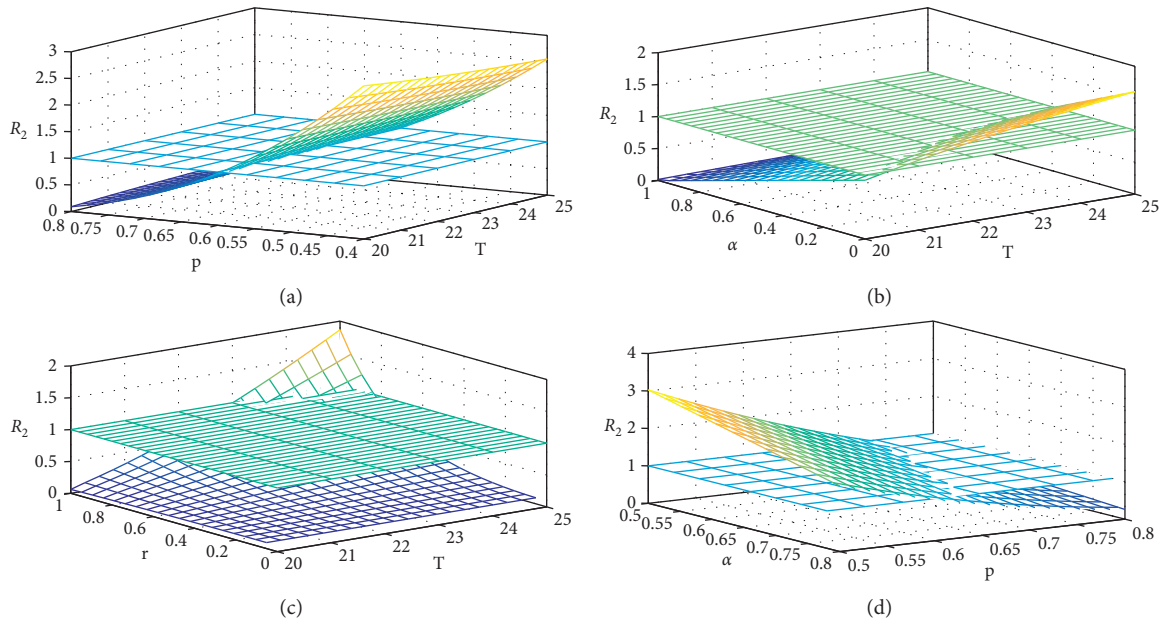

FIGURE 7: The effects of the parameter sets on the threshold level R_2. The set of parameter values is $b = 0.2, c = 0.12, m = 500, k_1 = 0.003, k_2 = 0.003, g = 0.003$. (a) $r = 0.4, \alpha = 0.9$; (b) $r = 0.4, p = 0.8$; (c) $p = 0.8, \alpha = 0.9$; (d) $r = 0.3, T = 22$.

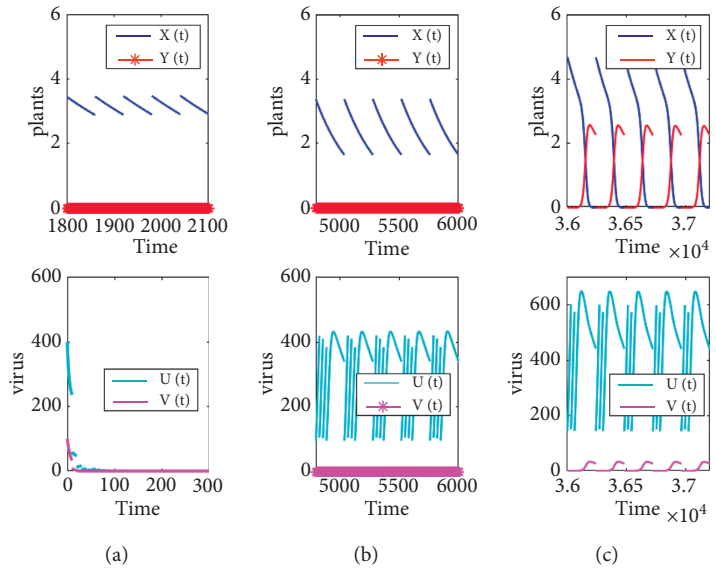

FIGURE 8: The existence and stability of the periodic solution $(\overline{X}^*(t), 0, 0, 0)$ of model (90). The set of parameter values is as follows: $k_1 = 0.003, g = 0.003, b = 0.2, m = 500, k_2 = 0.003, c = 0.12$. (a) $\tau = 60, \Delta_i = \Delta = 10, p = 0.75, r = 0.4, \alpha = 0.9, k = 4, R_3 \leq 1$; (b) $\tau = 240$, $\Delta_i = \Delta = 30, p = 0.75, r = 0.8, \alpha = 0.95, k = 3, R_3 > 1, R_\tau^k < 1$; (c) $\tau = 240, \Delta_i = \Delta = 30, p = 0.75, r = 0.8, \alpha = 0.95, b = 0.2, k = 2, R_3 > 1 R_\tau^k > 1$.

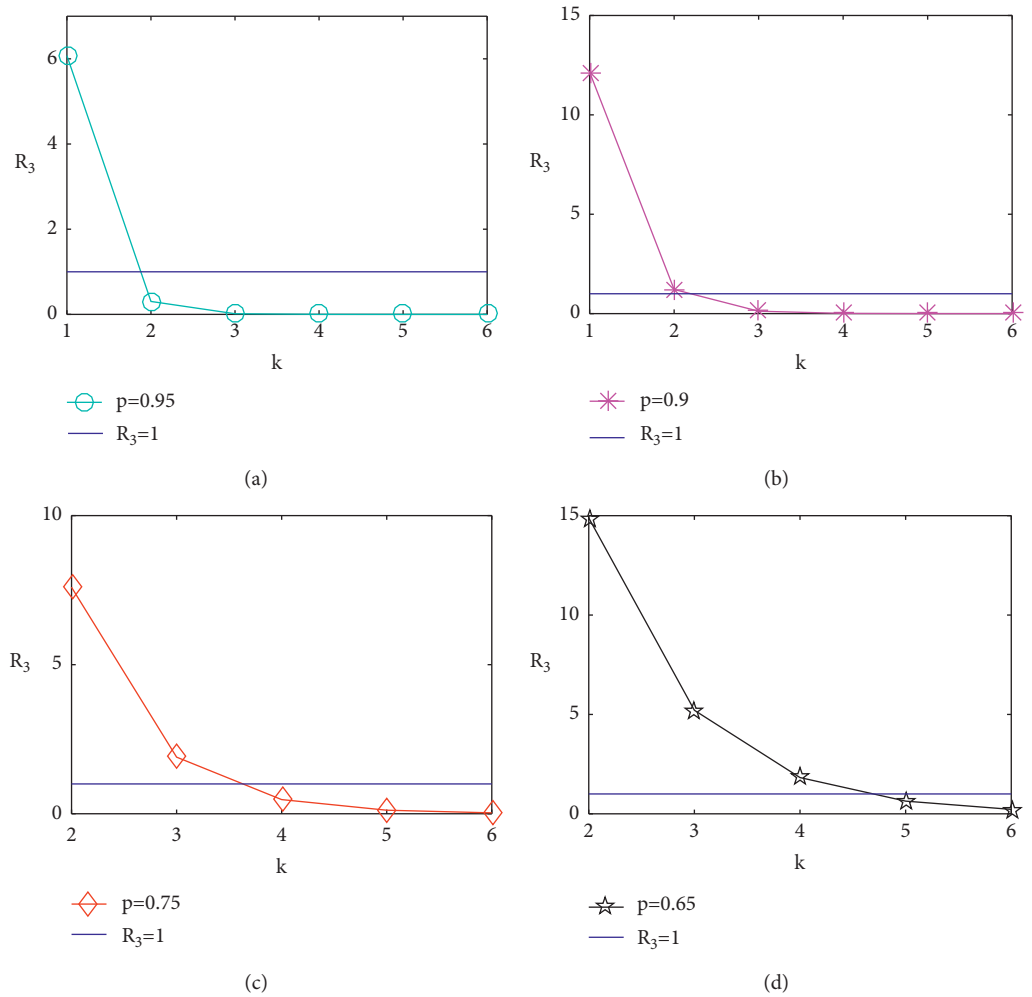

FIGURE 9: The effects of the times of roguing control k and the vectors removal rate p on the threshold level R_3. The set of parameter values is as follows: $\tau = 60, b = 0.2, c = 0.12$, and $k = 1, 2, 3, \ldots, 6$. (a) $p = 0.95$; (b) $p = 0.9$; (c) $p = 0.75$; (d) $p = 0.65$.

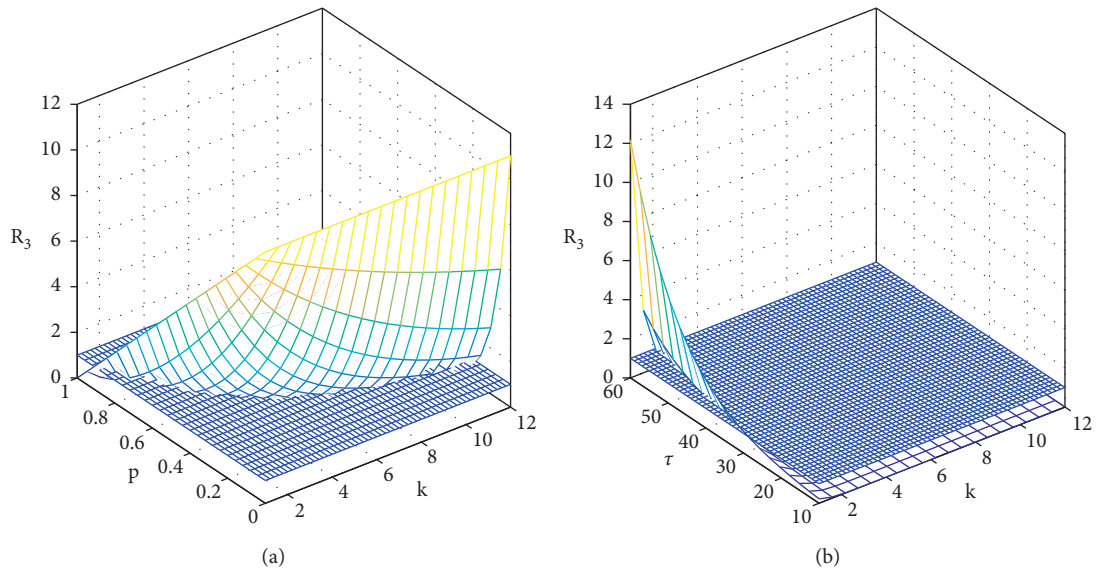

FIGURE 10: The effects of the times of roguing control k and parameter sets on the threshold level R_3. The set of parameter values is as follows: $b = 0.2, c = 0.12$, and $k = 1, 2, 3, \ldots, 6$. (a) $\tau = 30$; (b) $p = 0.9$.

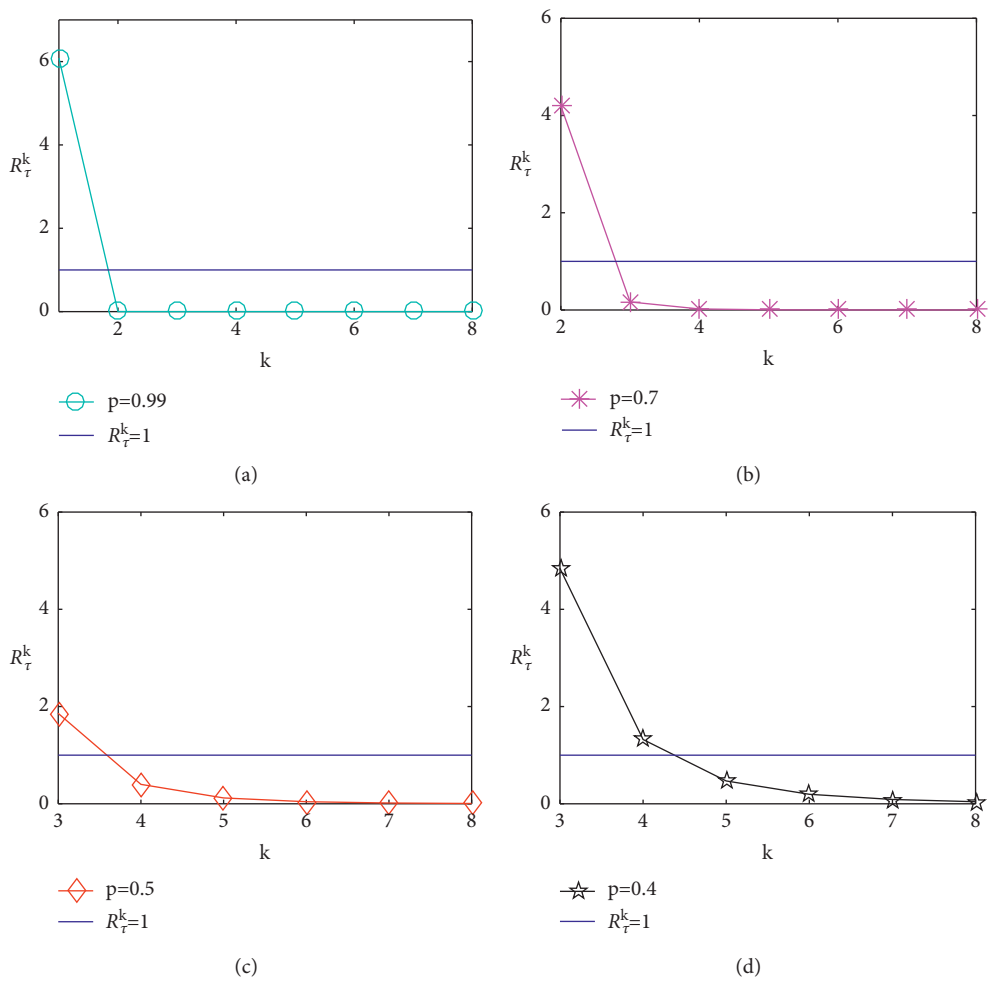

FIGURE 11: The effects of the times of roguing control k and the vectors removal rate p on the threshold level R_τ^k. The set of parameter values is as follows: $b = 0.2, c = 0.12, m = 500, k_1 = 0.003, k_2 = 0.003, g = 0.003, r = 0.8, \tau = 240, \Delta_i = \Delta = 30$, and $k = 1, 2, 3, \ldots, 8$. (a) $\alpha = 0.99$, $p = 0.99$; (b) $\alpha = 0.95, p = 0.7$; (c) $\alpha = 0.95, p = 0.5$; (d) $\alpha = 0.95, p = 0.4$.

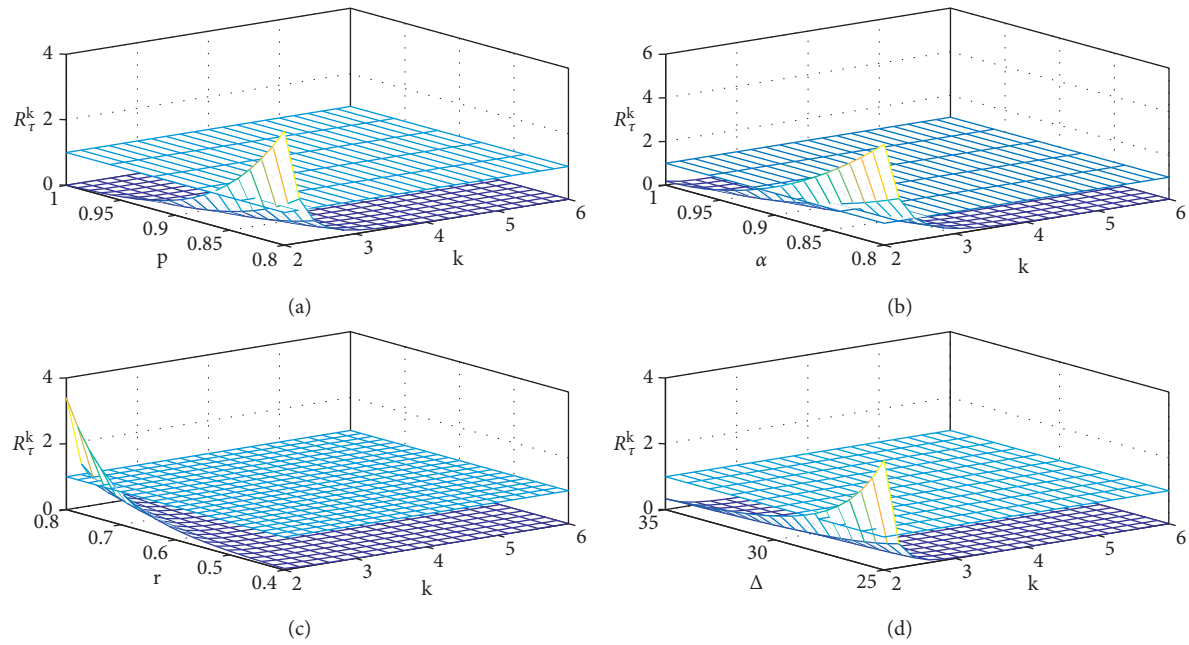

FIGURE 12: The effects of the times of roguing control k and parameter sets on the threshold level R_τ^k. The set of parameter values is as follows: $b = 0.2, c = 0.12, k_1 = 0.003, k_2 = 0.003, g = 0.003, \tau = 240,$ and $k = 1, 2, 3, \ldots, 6$. (a) $\alpha = 0.95, \Delta_i = \Delta = 30, r = 0.8$; (b) $p = 0.9$, $\Delta_i = \Delta = 30, r = 0.8$; (c) $\alpha = 0.87, p = 0.87, \Delta_i = \Delta = 30$; (d) $\alpha = 0.9, p = 0.9, r = 0.8, \Delta_i = \Delta$.

the removal period $\Delta_i = \Delta$ and let roguing rate α and the vectors removal rate p vary. The simulation results indicate that the vectors removal rate can affect the times of controls when the infected plants are removed. This shows that the vectors removal is important when removing infected plants. Figure 12 shows the effects of different parameter sets on the threshold condition R_τ^k. All simulation results shown in Figure 12 indicate that R_τ^k appears to be quite sensitive to small changes in roguing rate α and vectors removal rate p, replanting rate r, and removal period Δ.

7. Discussion

Based on the characteristics of plant diseases transmitted by insect vectors [7, 8], we proposed the mathematical models with continuous control strategy and pulse control strategy with single-pulse sequence of multipulse sequences, developed the corresponding threshold theoretical analysis and numerical techniques to address the threshold dynamics of the model, and obtained the corresponding threshold conditions [10].

The main results for the model with continuous removing of infected plants reveal that we can either control the death rate of vector population (i.e., $b < c$ here) such that the plant-only equilibrium is globally asymptotically stable, or control the plant roguing rate such that the basic reproductive number R_0 is less than one, and consequently the plant disease-free equilibrium is globally asymptotically stable. All those indicate that the infected plants and infected vectors could be completely eradicated if we implement the removal control strategies properly.

According to the fact that infected plants are removed and some vectors are removed at the same time, we develop a pulse control model with fixed moments, which allows us to choose the period of impulsive control strategies such that the plant-only periodic solution and plant disease-free periodic solution are globally asymptotically stable. Based on the above purpose, we employ the relevant theory of impulsive differential equations to address the threshold dynamics of the proposed model, and the threshold values which guarantee the global stability of plant-only and plant disease-free periodic solutions have been given. The threshold conditions show that the infected plants and infected vectors can be completely eradicated if we implement the cultural control strategies relatively frequently.

When considering the strategy of removing infected plants multiple times in a replanting cycle, a more realistic model has been proposed, which allows us to analyze and design the optimal control strategy such that the threshold value reaches its minimum. The optimal times of roguing infected plants within a replanting cycle can be numerically calculated, which can help us to design and optimize the prevention and control strategy of plant virus transmission. Finally, various numerical investigations have been given to confirm the main conclusions and reveal the optimal control strategies, and the biological implications of the main results have been briefly discussed in this section.

As we all know, insect vectors of plant diseases live directly on the corresponding infected plants. Therefore, when the corresponding infected plants or even healthy plants are removed, a large number of insect vectors will inevitably be removed. The above factors are undoubtedly beneficial to the prevention and control of plant virus disease transmission, but how to formulate and describe the above factors, develop corresponding mathematical models, and

carry out systematic analysis is challenging, which is undoubtedly one of our important work in a later stage.

Data Availability

There were no data used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgments

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