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Research Article

The Effect of a Service Experience Cost on a Queueing System

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In this work, the authors consider the effect of a service experience cost (*SE* cost) on customer behaviour in the M/M/1 queueing system. Based on customer individual equilibrium strategy, social welfare is also analyzed in unobservable and observable cases. The *SE* cost decreases the equilibrium joining probability and social welfare in an unobservable case. However, there might exist multiple individual equilibrium thresholds in an observable case. Furthermore, numerical results show that the *SE* cost can be used as a feasible policy to make an incentive for customers and regulate the system for improved social welfare in some scenarios.

1. Introduction

Service experience is a term to describe the feeling of customers. However, the sense of each customer may be different, which depends on the service environment. For example, (1) depressed consumers may feel open in a spacious fast-food restaurant, while lonely consumers may feel at home in a little fast-food restaurant. On the contrary, lonely consumers might be more frustrated in the spacious environment, and unhappy consumers might be more agitated in the narrow environment. (2) At the library box office, a reader's experience in the 15th position of the queue among 20 readers is different from a reader's experience in the 15th position of a queue among 200 readers. The former reader thinks the library is relatively idle, and the latter feels that his sojourn time is shorter. These two cases reflect their positive mental attitude. In the same situation, the former reader might think he is at the end of the queue, and the latter thinks the library is relatively crowded. (3) A cable car has eight seats in scenic spot, and the difference between the 8th position and 9th position in the queue is not just the extra waiting time. The customer in the 9th position might feel unlucky because he misses the last cable car or feels lucky because he must be in the next cable car. Hence, service experience should be considered in the study of customer behaviour and social welfare.

Naor [1] investigated customer behaviour from the viewpoint of game-theoretic in the M/M/1 queueing system where an arriving customer can observe the queue length and obtain the individual optimal threshold, the optimal social threshold, and the maximal revenue threshold. Later, Edelson and Hilderbrand [2] considered the case when the queue length is unobservable and showed that the optimal social toll is equal to the revenue-maximizing toll. Subsequently, many scholars published their works on the customer behaviour in various queueing systems, such as unreliable queueing systems [3, 4], retrial queueing systems [5–9], queueing systems with catastrophe [10, 11], vacation queueing systems [12–17], queueing system with egalitarian processor-sharing [18], queueing systems with priority customer [19, 20], and service inventory systems [21–24]. And the following excellent monographs: Hassin and Haviv [25], Stidham [26], Hassin [27], summarized achievements in recent years.

In the above works, the benefit function of individual customers generally includes the reward (or the degree of satisfaction) received after service and the sojourn cost in the system as

$$I = R - cE[W], \tag{1}$$

where R is customer reward, c is a waiting cost for one customer per time unit, and W and $E[W]$ are sojourn time and its expectation, respectively. Furthermore, all the customers are assumed to be homogeneous, and the optimal individual strategy follows the Nash symmetric equilibrium strategy.

To reflect the effect of service experience on customer behaviour, the authors first establish a simple net benefit function as follows:

$$I = R - c_1 E[W] - c_2, \tag{2}$$

where $c_2\,(\geq 0)$ is some kind of cost corresponding to service experience, simplified as a *SE* cost. The smaller of c_2, the better of service experience and so the greater of the satisfactory degree of customers, and the greater of c_2 means an arriving customer has less motivation to enter the queue. It is worth mentioning that this cost is not connected with the sojourn time of the customer, but it reflects the actual service experience of the customer. Especially for the case of $c_2 = 0$, when the service experience reaches its optimum, the benefit function degenerates into (1).

Moreover, it is more reasonable to use the variable cost instead of the constant cost c_2. From the point of the queueing system, the traffic intensity is a measure to reflect a service environment. Therefore, the authors use the product of c_2 and P_{rob} to describe the variable *SE* cost, where P_{rob} is some kind of probability related the system state. So, the new benefit function of an individual customer is established as

$$I = R - c_1 E[W] - c_2 P_{\text{rob}}. \tag{3}$$

Although many works focused on analyzing the equilibrium behaviour of customers, to the best knowledge of the authors, only a few papers have studied service experience. This work considers an individual equilibrium strategy in unobservable and observable cases and then analyzes social welfare. The main contributions of this work are summarized as follows: (1) this is the first work that utilizes *SE* cost to investigate customer behaviour and social welfare; (2) the joining probability and social welfare are all smaller than those without *SE* cost in an unobservable case and so introducing *SE* cost is not suitable for the system manager; (3) there exist multiple individual thresholds in an observable case which is different from the unique threshold in Naor [1]; furthermore, the upper and lower bounds of thresholds are also determined; (4) numerical results illustrate that social welfare does not necessarily decrease in observable cases after *SE* cost is added; (5) compared to without *SE* cost, social welfare can be improved while maintaining the same throughput.

The rest of this work includes five sections. Section 2 presents the model description. Section 3 investigates the individual equilibrium strategy and establishes a new social welfare function in an unobservable case. Section 4 devotes to an observable case. Section 5 carries out some numerical experiments and analyzes the corresponding results. Finally, Section 6 summarizes the main conclusions.

2. Model Description

This work considers the M/M/1 queueing system where customers arrive according to a Poisson process with rate λ, and service times are independent and exponentially distributed with mean μ^{-1}. The system state at time t is characterized by $N(t)$, which denotes the number of customers in the system. The stochastic process $\{N(t), t \geq 0\}$ is a continuous-time Markov chain with state space $\Omega = \{n | n \geq 0\}$.

Assume every customer obtains a reward of R units for completing service, and there exists a waiting cost of c_1 units per time unit that the customer remains in the system. To encourage an arriving customer to enter the system, the condition

$$R - \frac{c_1}{\mu} > 0, \tag{4}$$

is necessary.

This work assumes that all customers are risk-neutral and must make decisions based on the benefit function (3). After decisions, retrials of balking customers and reneging of entering customers are not allowed. Conditioning on the number of customers in the system is observed or not, there are two cases as follows:

(i) The unobservable case where customers can not observe $N(t)$

(ii) The observable case where customers can observe $N(t)$

3. The Unobservable Case

In this case, customers can not observe the number of customers in the system and choose to join the system with a specific probability. So, all the homogeneous customers in the system follow a mixed strategy: "enter the system with probability q and balk with probability $1 - q$." The authors first analyze the individual equilibrium joining probability, which is a weakly dominant strategy. A detailed definition can be seen in Section 1.1 of [25]. Secondly, social welfare is considered after a customer makes his decision. The transition rate diagram is shown in Figure 1.

The following proposition, which has been presented in Section 2.2.1 of [28], gives the stationary distribution $p_{\text{un}}(n)$, the expected number of customers in system $E[N]$, and the expected sojourn time $E[W]$. The system is stable if and only if $\lambda q < \mu$. The authors first consider the case $\lambda < \mu$ and then present results for $\lambda \geq \mu$.

Proposition 1. *In the unobservable case of the M/M/1 queueing system, the stationary distribution $\{p_{un}(n)\colon n \geq 0\}$, the expected number of customers $E[N]$, and the expected sojourn time $E[W]$ are summarized as follows:*

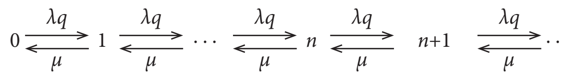


FIGURE 1: The transition rate diagram of unobservable case.

$$p_{un}(0) = 1 - \frac{\lambda q}{\mu},$$
$$p_{un}(n) = \left(1 - \frac{\lambda q}{\mu}\right)\left(\frac{\lambda q}{\mu}\right)^n,$$
$$n \geq 0, \tag{5}$$
$$E[N] = \frac{\lambda q}{\mu - \lambda q},$$
$$E[W] = \frac{1}{\mu - \lambda q}.$$

3.1. Individual Equilibrium Strategy. When a customer has no information on the system state and makes a decision based on the mean value of performance measure, the probability P_{rob} defined in equation (3) is replaced with the busy probability of the server in the steady-state, i.e., $P_{\mathrm{rob}} = (\lambda q/\mu)$, to describe the influence of *SE* cost on customer behaviour. The larger the P_{rob} probability is, the worst service experience the customer gets, and so the lower service satisfaction he obtains.

Theorem 1. *In the unobservable case of the M/M/1 queueing system, a mixed equilibrium strategy "enter the system with probability q_e" exists and q_e is uniquely given by*

$$q_e = \begin{cases} q_e^* = \dfrac{(R + c_2)\mu - \sqrt{(R - c_2)^2\mu^2 + 4c_1c_2\mu}}{2\lambda c_2}, & \text{if } R < \dfrac{c_1}{\mu - \lambda} + \dfrac{c_2\lambda}{\mu}, \\ 1, & \text{if } R \geq \dfrac{c_1}{\mu - \lambda} + \dfrac{c_2\lambda}{\mu}. \end{cases} \tag{6}$$

Proof. From equation (3), when an arriving customer enters the system, his expected net benefit is

$$I(q) = R - c_1E[W] - c_2P_{\mathrm{rob}} = R - \frac{c_1}{\mu - \lambda q} - \frac{c_2\lambda q}{\mu}, \tag{7}$$

and it is readily shown that $I(q)$ is a strictly decreasing function in $q \in [0, 1]$.

If $I(1) \geq 0$ or $R \geq (c_1/(\mu - \lambda)) + (c_2\lambda/\mu)$, the minimal of benefit function is nonnegative and so customer would like to enter the system. This proves the second part of (6).

If $I(1) < 0$, combining $I(0) > 0$ yields $(c_1/\mu) < R < (c_1/(\mu - \lambda)) + (c_2\lambda/\mu)$; a unique $q_e^* = ((R + c_2)\mu - \sqrt{(R - c_2)^2\mu^2 + 4c_1c_2\mu})/2\lambda c_2 \in (0, 1)$ can be determined by $I(q) = 0$. If the customer chooses to enter the system with probability q_e^* or balk with probability $1 - q_e^*$, the benefit always equals to zero. Hence, the customer does not feel a difference between entering and balking.

The proof of Theorem 1 is completed.

If $\lambda \geq \mu$, the corresponding analysis also can be derived and a mixed equilibrium strategy "enter the system with probability q_e" exists and

$$q_e = q_e^*. \tag{8}$$

For the case of $c_2 = 0$, which have been shown in [26], a mixed equilibrium strategy "enter the system with probability q_e^0" exists and q_e^0 is uniquely given by

$$q_e^0 = \begin{cases} \dfrac{R\mu - c_1}{\lambda R}, & \text{if } R < \dfrac{c_1}{\mu - \lambda}, \\ 1, & \text{if } R \geq \dfrac{c_1}{\mu - \lambda}. \end{cases} \tag{9}$$

If $\lambda \geq \mu$,

$$q_e^0 = \frac{R\mu - c_1}{\lambda R}. \tag{10}$$

Compared with the unobservable M/M/1 queueing system presented in [2], the benefit function is added with an *SE* cost which causes that the joining probability q_e should not be greater than q_e^0. And through the joining probability defined in equations (6)–(9), it is readily proved that $q_e \leq q_e^0$. This fact illustrates that fewer customers are motivated to join the queue, and at this moment, the system turns less crowded. Therefore, c_2P_{rob} can be used as a factor to induce customer behaviour or regulate the whole system. □

3.2. Social Welfare. Equation (6) shows that q_e is the unique equilibrium joining probability of customers. It corresponds to the proportion of customers who will choose to enter the system for service. Based on customer behaviour, the expected social welfare per time unit is

$$S(q_e) = \lambda q_e\left(R - \frac{c_1}{\mu - \lambda q_e} - \frac{c_2\lambda q_e}{\mu}\right). \tag{11}$$

And for the case of $c_2 = 0$, the expected social welfare per time unit is

$$S^0\left(q_e^0\right) = \lambda q_e^0\left(R - \frac{c_1}{\mu - \lambda q_e^0}\right). \tag{12}$$

In reference [2], the server revenue is equal to the social welfare in the unobservable case and so it is readily shown that $S(q_e) \le S^0(q_e^0)$.

Sections 3.1 and 3.2 show that the joining probability and social welfare are smaller than the case of $c_2 = 0$. These results are reasonable and accessible, and thus the authors focus on the observable case to verify whether the same situation would happen.

4. The Observable Case

In this case, the system is same as the work of Naor [1], where a customer arriving at time t knows the exact number of customers in the system. And so Naor showed that there exists equilibrium threshold strategy, i.e., there exists threshold n such that an arriving customer enters the system if $N(t) \le n$ and balks otherwise. For this, the system has $n+1$ customers at most, and the transition rate diagram is depicted in Figure 2. And the stationary distribution $\{p_{\text{ob}}(i)\colon 0 \le i \le n+1\}$ on threshold n is

$$p_{\text{ob}}(i) = \frac{\rho^i(1-\rho)}{1-\rho^{n+2}}, \quad i = 0, 1, \ldots, n+1, \tag{13}$$

where $\rho = (\lambda/\mu)(\neq 1)$. Firstly, the individual equilibrium strategy will be investigated within the threshold strategy.

4.1. Individual Equilibrium Strategy. The exact number of customers $N(t)$ can help to determine the expected sojourn time of an arriving customer. Moreover, it can influence the service experience of customers and the service environment of the system during the service process. Since $N(t)$ is observed, the probability $P_{\text{arrival}}(i)$ that an arriving customer observes i customers upon arrival can exactly describe the real-time state of the system. Hence, the probability P_{rob} defined in equation (3) is replaced with $P_{\text{arrival}}(i)$ to derive the individual equilibrium threshold. The PASTA property shows that $P_{\text{arrival}}(i) = p_{\text{ob}}(i)$, and then the new net benefit function on threshold n of an arriving customer who observes i customers in the system can be defined as

$$\begin{aligned} I_n(i) &= R - c_1 E[W] - c_2 P_{\text{rob}} = R - c_1\frac{i+1}{\mu} - c_2 p_{\text{ob}}(i)\cdot 1_{\{i\ge 1\}},\\ &= \begin{cases} R - \dfrac{c_1}{\mu}, & \text{if } i = 0,\\[2ex] R - \dfrac{c_1(i+1)}{\mu} - \dfrac{c_2\rho^i(1-\rho)}{1-\rho^{n+2}}, & \text{if } 1 \le i \le n+1. \end{cases} \end{aligned} \tag{14}$$

$i = 0$ means the system is empty and the server only serves the arriving customer. At this time, the service experience reaches its optimum and would not generate any cost related c_2.

Moreover, through simple calculation,

$$I_n(i+1) - I_n(i) = \begin{cases} -\dfrac{c_1}{\mu} - \dfrac{c_2\rho(1-\rho)}{1-\rho^{n+2}}, & \text{if } i = 0,\\[2ex] -\dfrac{c_1}{\mu} + \dfrac{c_2\rho^i(1-\rho)^2}{1-\rho^{n+2}}, & \text{if } 1 \le i \le n. \end{cases} \tag{15}$$

For given threshold $n(\ge 0)$, $I_n(0) = R - (c_1/\mu) > 0$ is obvious from equation (4). And if $I_n(1) < 0$, this means the threshold n must be zero; that is to say, customer joins the system when the system is empty, and once the system is nonempty, no one joins in.

To get more generalized results, the authors investigate the nonzero thresholds under the condition of $I_n(1) \ge 0$ hereinafter.

For individual equilibrium threshold n, there imply two-tier meanings. Firstly, the benefit $I_n(i)(1 \le i \le n)$ of an arriving customer, who finds i customers upon arrival and decides to join, should be nonnegative. And secondly, if he finds $n+1$ customer and decides to join, his benefit $I_n(n+1)$ must be negative; if not, there exist at least $(n+2)$ customers in the system which contradicts to the system threshold n. Based on these two points, the upper bound and lower bound of thresholds can be determined. To achieve the upper bound, the following proposition is needed to illustrate the quality of benefit functions $I_n(i)$ and $I_n(n)$.

Proposition 2. *For threshold n, $I_n(n) \ge 0$ can ensure the benefits $I_n(i)(1 \le i \le n)$ are all nonnegative.*

Proof. The proof is divided into three cases:

If $\rho > 1$, equation (15) yields $I_n(i+1) - I_n(i) < 0$ and then $I_n(n) = \min\{I_n(i), 1 \le i \le n\}$.

If $\rho < 1$ and $(c_1/\mu) \ge c_2\rho(1-\rho)^2/(1-\rho^{n+2})$, equation (15) also yields $I_n(i+1) - I_n(i) \le 0$ and then $I_n(n) = \min\{I_n(i), 1 \le i \le n\}$.

If $\rho < 1$ and $(c_1/\mu) < c_2\rho(1-\rho)^2/(1-\rho^{n+2})$, then equation (15) yields $I_n(0) - I_n(1) > 0$ and $I_n(2) - I_n(1) > 0$. Hence, $i = 1$ is a local minimum point of $I_n(i)$ for $0 \le i \le 2$. Furthermore, $I_n(i+2) - 2I_n(i+1) + I_n(i) = -(c_2\rho^i(1-\rho)^3/(1-\rho^{n+2})) < 0$ shows that $I_n(i)$ is concave for $1 \le i \le n$. If $I_n(i)$ increases on $1 \le i \le n$, then $I_n(i)(1 \le i \le n)$ are all nonnegative because of $I_n(1) \ge 0$; if $I_n(i)$ increases at first and then decreases, $I_n(n) \ge 0$ can ensure $I_n(i)(1 \le i \le n)$ are all nonnegative.

The proof of Proposition 2 is completed.

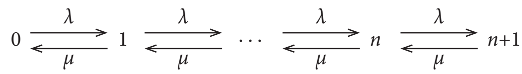


Figure 2: The transition rate diagram of observable case.

This proposition shows that all the benefits $I_n(i)\,(1 \le i \le n)$ are nonnegative so long as $I_n(n)$ is nonnegative. Hence, to achieve the upper bound of threshold n, it is just to find the maximum number of n such that $I_n(n) \ge 0$. □

Lemma 1. *In the observable M/M/1 queueing system, the individual equilibrium threshold has an upper bound n_U.*

Proof. Equation (14) yields

$$\begin{aligned} I_{n+1}(n+1) - I_n(n) &= R - \frac{c_1(n+2)}{\mu} - \frac{c_2\rho^{n+1}(1-\rho)}{1-\rho^{n+3}} - R \\ &\quad + \frac{c_1(n+1)}{\mu} + \frac{c_2\rho^{n}(1-\rho)}{1-\rho^{n+2}} \\ &= -\frac{c_1}{\mu} + \frac{c_2\rho^{n}(1-\rho)^2}{\left(1-\rho^{n+2}\right)\left(1-\rho^{n+3}\right)}, \\ \Delta^2 I_n(n) &= I_{n+2}(n+2) - 2I_{n+1}(n+1) + I_n(n) \\ &= -\frac{c_2\rho^{n}(1-\rho)^3\left(1+\rho^{n+3}\right)}{\left(1-\rho^{n+2}\right)\left(1-\rho^{n+3}\right)\left(1-\rho^{n+4}\right)} < 0. \end{aligned} \tag{16}$$

Hence, $I_n(n)$ is a discrete concave function on n. Considering $I_1(1) \ge 0$ and $I_n(n) \longrightarrow -\infty$ as $n \longrightarrow \infty$ for any ρ, there exists a unique integer n_U such that

$$\begin{cases} I_n(n) \ge 0, & \text{if } n \le n_U, \\ I_n(n) < 0, & \text{if } n > n_U. \end{cases} \tag{17}$$

So, n_U is the upper bound of threshold.

Lemma 1 shows that threshold n has an upper bound n_U so that $I_n(n) \ge 0$ for all $n \le n_U$. And combining with Proposition 2, all the benefits $I_n(i)\,(1 \le i \le n \le n_U)$ are nonnegative.

Next, it is turn to seek for the lower bound of threshold n through the benefit $I_n(n+1)$ of an arriving customer, who finds $n+1$ customers upon arrival and decides to join. If $I_n(n+1) > 0$, an arriving customer enters the system and therefore the system contains $n+2$ customers which contradicts the policy of threshold n. So, to achieve the lower bound of threshold or all the equilibrium thresholds, it is necessary to find the minimal $n(\le n_U)$ satisfying $I_n(n+1) \le 0$.

The following theorem shows how to find the lower bound and all the possible thresholds. □

Theorem 2. . *In the observable M/M/1 queueing system, there exist at least one individual equilibrium threshold n_e.*

Proof. Equations (14) and (17) show that $I_{n_U}(n_U+1) \le I_{n_U+1}(n_U+1) < 0$; therefore, individual equilibrium threshold can be derived by backward extrapolation: starting from the subscript n_U to 1, let $n_1 - 1$ be the subscript of the first positive term of $I_n(n+1)$, then we have

$$I_{n_U}(n_U+1), \ldots, I_{n_1+1}(n_1+2), \quad I_{n_1}(n_1+1) < 0, I_{n_1-1}(n_1) \ge 0. \tag{18}$$

Specially, if $I_n(n+1) < 0$ for all $1 \le n \le n_U$, i.e.,

$$I_{n_U}(n_U+1), \ldots, I_2(3), \quad I_1(2) < 0, \tag{19}$$

define $n_1 = 1$. Denote $n_L = n_1$, and then $n_e \in \{n_L, n_L+1, \ldots, n_U\}$ are all individual equilibrium thresholds.

The above theorem shows that any positive integer $n_e \in \{n_L, n_L+1, \ldots, n_U\}$ can be regarded as individual threshold of customer, which is different from the results presented in Naor [1].

Now, there may exist multiple thresholds after *SE* cost is introduced. So what is the effect of them on the system? The numerical experiment illustrates that more social welfare can be obtained based on the threshold n_U or n_L. The following section gives the social welfare function.

For the case of $c_2 = 0$, Naor [1] shows that the unique individual equilibrium threshold is

$$n_e^0 = \left\lfloor \frac{R\mu}{c_1} - 1 \right\rfloor. \tag{20}$$

From Theorem 2, it is difficult to obtain the explicit expression of n_e, which leads n_e and n_e^0 cannot be compared although the concrete expression of n_e^0 is solved in equation (20). For this reason, the comparison is performed numerically in Section 5. However, after introducing *SE* cost, there exist multiple thresholds rather than the unique threshold in the observable case. And this modification prompts not only customers to have more choices for joining threshold but also the system manager achieves more social welfare. □

4.2. Social Welfare. Given the individually equilibrium threshold n_e defined from Theorem 2, similar to Section 3.2, the social welfare per time unit is

$$S(n_e) = \lambda \sum_{i=0}^{n_e} p_{\text{ob}}(i)\left\{R - \frac{c_1(i+1)}{\mu} - c_2 p_{\text{ob}}(i)\cdot 1_{\{i\ge 1\}}\right\}. \tag{21}$$

For the case of $c_2 = 0$, the social welfare per time unit

$$S^0\left(n_e^0\right) = \lambda \sum_{i=0}^{n_e^0} p_{\text{ob}}(i)\left[R - \frac{c_1(i+1)}{\mu}\right], \tag{22}$$

has been defined in [1]. By replacing n with n_e or n_e^0 in equation (13), the corresponding stationary distribution $p_{\text{ob}}(i)$ can be derived.

Since the expression of n_e is implicit, these two social welfare functions can not be compared directly. So, some numerical experiments are shown in Section 5. Remarkably, the tendency of social welfare is different from the result in the unobservable case, and the detailed results can be seen in Section 5.

5. Numerical Experiments

Section 3.1 shows that the joining probabilities q_e and q_e^0 satisfy $q_e \le q_e^0$. This fact leads to the social welfare $S(q_e)$ and $S^0(q_e^0)$ have the same relationship between them. Therefore, this section gives priority to the observable case and investigates the tendency of individual thresholds n_e and n_e^0 and the social welfare $S(n_e)$ and $S^0(n_e^0)$ by considering the following system parameters:

$$\begin{aligned} R &= 2.5, \\ c_1 &= 1.5, \\ \mu &= 3. \end{aligned} \tag{23}$$

Firstly, the authors consider the tendency of thresholds n_e^0, n_U, and n_L. For the special case of $c_2 = 0$, n_e^0 is independent of λ and remains at a constant value (see Figure 3). If $c_2 \ne 0$, after the *SE* cost $c_2 P_{\text{rob}}$ is introduced, the cost that customer has to be afforded is added and so the upper and lower bounds n_U and n_L of individual threshold n_e are smaller than n_e^0. Hence, $n_e^0 \ge n_U \ge n_L$.

For fixed λ, the benefit of customers is decreasing with the increase in c_2. Therefore, the upper bound n_U and lower bound n_L of thresholds are all decreasing. On the contrary, for fixed c_2, the system turns to more crowded, the probability of the idle state of server $p_{\text{ob}}(0)$ is decreasing with the increase in λ, and correspondingly, the probabilities of other states $p_{\text{ob}}(i)$ may be greater than before. Hence, the customer's benefit, the upper bound n_U, and lower bound n_L are all decreasing. This tendency can be seen in Figure 3.

Secondly, the authors consider the tendency of social welfare functions. For fixed c_2, the thresholds n_U and n_L are kept as a constant, respectively, when $\lambda < 2.5$. With the increase in λ, more customers join the system and then lead to more social welfare. For greater $\lambda(\ge 2.5)$, the thresholds are no longer constants. With the increase in λ, the threshold turns to decrease and hence the number of customers in the system also decreases which would lead to a decrease in social welfare. So, the graph of the social welfare function is very similar to the graph of the unimodal function.

For fixed λ, the steady-state probabilities $p_{\text{ob}}(i)$ are constants, and however the social welfare $S(n_U)$ and $S(n_L)$ do not necessarily increase when c_2 decreases. For smaller $\lambda(< 2.5)$, Figure 3 illustrates that the thresholds n_U and n_L are independent of c_2 and kept as constants, and so the benefit of customer $I_n(i)$ is decreasing with the decrease in c_2. Hence, the smaller c_2 is beneficial to reach more social welfare. This corresponds to the tendency of $S(n_U)$ and $S(n_L)$ in Figure 4. For greater $\lambda(\ge 2.5)$, the thresholds n_U and n_L and the number of customers in system would decrease with the increase in c_2, and then the sojourn cost of customer would decrease, too. And the *SE* cost $c_2 p_{\text{ob}}(i)$ would increase. Hence, the relationship between those two coefficients c_1 and c_2 play a critical role in the sum of sojourn cost and *SE* cost, which causes that the social welfare does not occupy the decreasing monotonicity on c_2. For example, for $\lambda = 5.5$, the social welfare $S(n_U)$ in the case of $c_2 = 2$ is greater than that of $c_2 = 0$ in Figure 4(a).

Besides, for the cases of $c_2 = 0, 0.5, 1$, the thresholds n_U and n_L do not vary for arbitrary λ and the corresponding social welfare functions $S(n_U)$ and $S(n_L)$ are all unimodal functions. However, for the case of $c_2 = 2$, $S(n_U)$ is not truly unimodal in the interval of $[0.5, 5.5]$. For $0.5 \le \lambda \le 4.1$, $n_U = 3$ and $S(n_U)$, as a unimodal function, reaches its maximum at $\lambda = 2.9$. However, for $\lambda \ge 4.3$, $n_U = 2$ and $S(n_U)$ begins to increase not decrease. A similar result also can be seen for $c_2 = 2.5$. For greater c_2, this tendency illustrates that the unimodality of social welfare is transformed after adding *SE* cost. As the authors mentioned above, the number of customers in the system would decrease with the increase in c_2 so that the sojourn cost would decrease. Although the *SE* cost is added, the social welfare $S(n_U)$ keeps increasing. Hence, among the two factors of social welfare, the sojourn cost plays more weight than *SE* cost for greater λ.

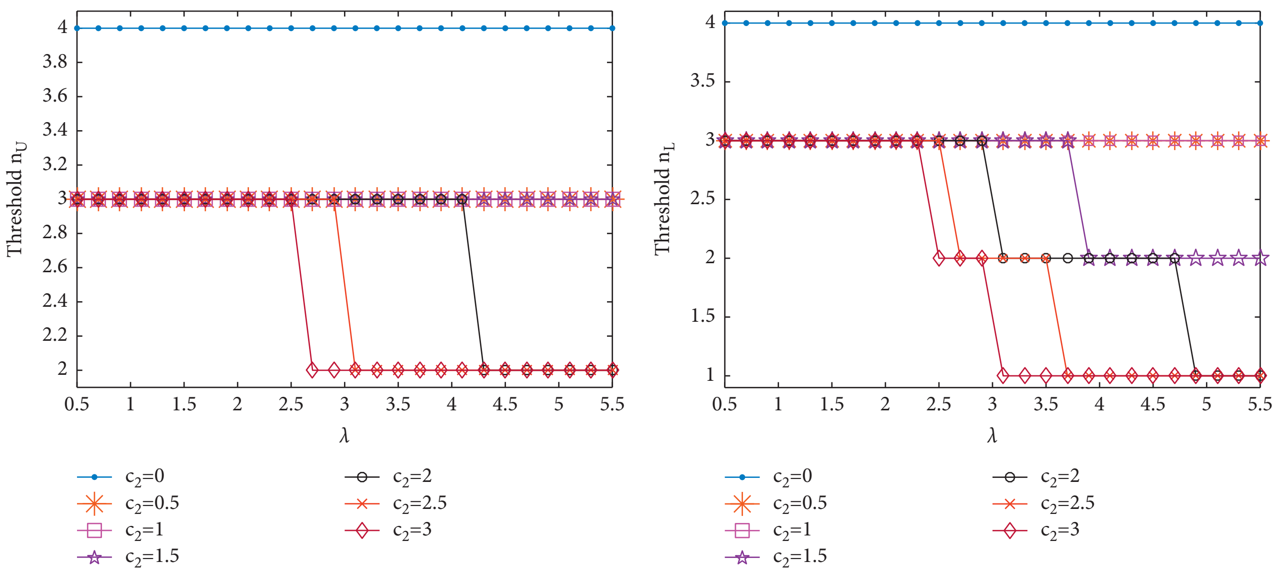
Is the social welfare increased at the expense of reduced throughput? No. Figures 4 and 5 illustrate that social welfare can be increased while maintaining the same throughput.

Thirdly, the authors consider the tendency of throughput. For fixed λ, the probability $p_{\text{ob}}(i)$ is kept as a constant. When c_2 increases, the benefit of customer $I_n(i)$ decreases and then he has less motivation to join the system. Hence, the system threshold n and the throughput $T(n) = \mu[1 - p_{\text{ob}}(0)] = \mu(1 - (1-\rho)/(1-\rho^{n+2}))$ are all decreasing. And Figure 5 illustrates the tendency of $T(n_U)$ and $T(n_L)$.

For fixed c_2, especially for $c_2 = 0, 0.5, 1, 1.5$, n_U is independent of λ and so throughput $T(n_U)$ is increasing on λ. This tendency can be seen in Figure 5(a), and the similar tendency also can be seen in Figure 5(b) for $c_2 = 0, 0.5, 1$.

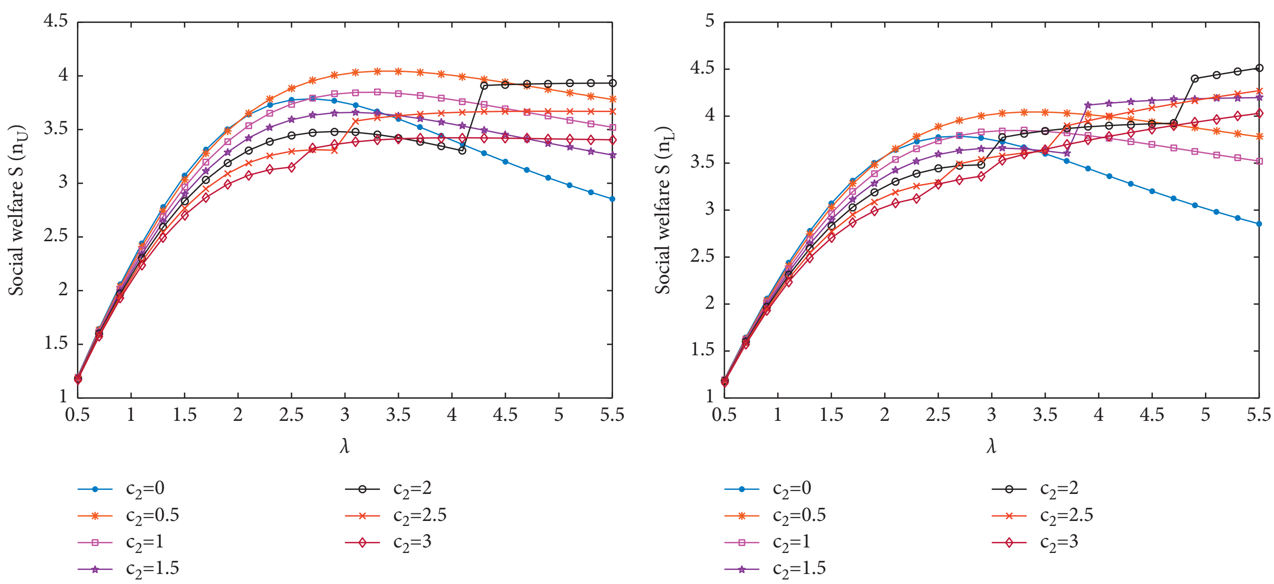
For $c_2 = 2, 2.5, 3$, the throughput $T(n_U)$ is increasing on λ when threshold n_U is kept unchanged. And with the increase in λ, the threshold n_U must be declined to a lower value. At this point, the throughput $T(n_U)$ decreases firstly and then increases again until the threshold n_U declines to another lower value. For example, for the case of $c_2 = 2.5$, n_U decreases one unit at $\lambda = 3.1$ and then the throughput $T(n_U)$ firstly decreases at $\lambda = 3.1$ and then increases again. More complicated tendency on $T(n_L)$ can be seen in Figure 5(b) for the case of $c_2 = 2.5$.

Under the condition of same throughput $T(n_u)$, for example, $c_2 = 0.5, 1, 1.5$, Figure 4(a) shows that the greater c_2 means the less social benefit $S(n_u)$. And this tendency also holds true for smaller $\lambda(< 1.3)$ and $c_2 = 0$. However, for



(a) (b)

Figure 3: The individual thresholds n_U and n_L vs. λ.



(a) (b)

Figure 4: The social welfare $S(n_U)$ and $S(n_L)$ vs. λ.

greater $\lambda(>3.5)$, the social benefit $S(n_u)$ for the case of $c_2 = 0$ is the absolutely lowest among the four scenarios even if the throughput $T(n_e^0)$ is slightly greater than $T(n_u)$ of the other three scenarios. Besides this, the similar tendency about c_2 and $S(n_L)$ can also been found in Figures 4 and 5.

Generally, after introducing cost coefficient c_2, the thresholds n_e are smaller than before. However, the social welfare increases instead of decreasing in many situations keeping the acceptable throughput. These findings illustrate that rather than simply considering the impact

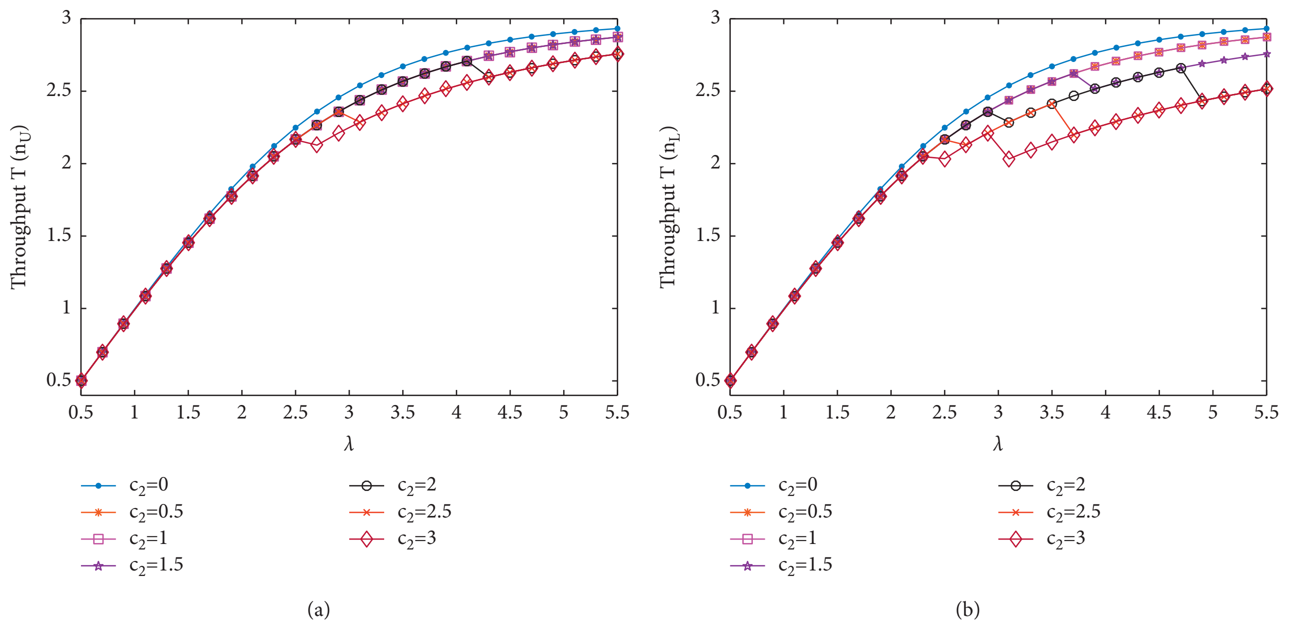


Figure 5: The throughputs $T(n_U)$ and $T(n_L)$ vs. λ.

of waiting time on customers, introducing service experience cost into the benefit function plays an unexpected role. Furthermore, when the customer flow is relatively large, how to enhance the service quality and improve the service environment is a concern that the managers need to consider in the future.

6. Conclusions

To complete the service, the customer needs to afford an inevitable sojourn cost that depends on the number of customers. In this work, the authors introduce an extra *SE* cost to establish the benefit function of customers. In the unobservable case, some intuitional results are reached. For example, joining probability and social welfare are all smaller than before. And the individual threshold is no exception in the observable case. However, in the situation of greater customer flow, numerical results show that the unimodality of social welfare is transformed, different from the result of Naor [1]. More importantly, after *SE* cost is added, social welfare can be improved while maintaining the same throughput. The reason for this interesting result owes to the multiple possible thresholds. Hence, *SE* cost can be a potential future focus to investigate the service system.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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