

Research Article

Dissipativity-Based Fuzzy Integral Sliding Mode Control of Nonlinear Stochastic Systems

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In this paper, both dissipation and stabilization of nonlinear T-S fuzzy systems with uncertain parameters are considered. By designing a new integral sliding mode control scheme, which sufficiently utilizes the special properties of T-S fuzzy membership functions, the difficulties induced by the uncertainties and external interference in T-S fuzzy systems are overcome by this new scheme. For the time delay and uncertainty, sufficient conditions are given to ensure the strict dissipation and asymptotic stability of the corresponding sliding mode dynamics. We propose a fuzzy integral sliding mode control law to drive the system trajectory to the fuzzy switching surface in the case of uncertainty and external interference. The realization of a fuzzy sliding mode controller for an inverted pendulum system proves the feasibility and superiority of our theoretical results.

1. Introduction

Takagi–Sugeno fuzzy model is to fit the same nonlinear system by multiple linear systems and then to defuzzify it by fuzzy calculus reasoning. Its advantage is that it can express highly nonlinear complex system with less fuzzy rules and approximate to any accuracy [1]. Therefore, the fuzzy control based on Takagi–Sugeno (T-S) fuzzy model [2] has become a research hot spot because of its wide applications in intelligent control theory and various complex practical system control [3–12]. Recently, Radivel et al. [13] studied the robust H_∞ performance for discrete-time T-S fuzzy switched memristive stochastic neural networks [14] with mixed time-varying delays and switching signal design. Wu et al. [15] studied the H_∞ performance for discrete-time uncertain fuzzy-logic systems.

Sliding mode control (SMC, for short) is a kind of control strategy of variable structure control system and a kind of special nonlinear control. It can continuously change according to the current state of the system (such as the deviation and its derivatives) in the dynamic process, forcing the system to move according to the state track of the predetermined “sliding mode,” which has the advantages of

fast response and strong robustness. Therefore, it has been widely used in various complex dynamical systems, such as time-delay systems [16–19], stochastic systems [20], and Markov jump systems [21–23]. When the common sliding mode control is tracking any trajectory if there is some external disturbance, it may bring steady-state error and cannot meet the required performance index. In order to deal with this trouble, Chen et al. [24–26] proposed an integral sliding mode control (ISMC, for short), which makes the system state slide along the sliding mode surface through the switching of control variables and makes the system invariant when it is disturbed by some external disturbances. It reduces chattering and can make fast response while simplifying the structure complexity of the fuzzy control system. This kind of control has been widely used in inverted pendulum control [27–29], motor and power system control [30], and robot control [31–33].

However, these obtained results on integral sliding mode control have some limitations. More specifically, a common limitation in [34, 35] is that all input matrices of fuzzy systems are required to be the same, but many physical devices do not meet this assumption. In the fuzzy integral sliding mode control proposed in [35], the parameter matrix

is restricted to be Hurwitz, which is very special. Time delay usually reduces the performance of the system, even leads to instability, but it is common in most practical systems. In [36], the authors studied the fuzzy integral sliding mode control of continuous time fuzzy system with uncertainty and disturbance, but not considering the time delay. The dissipation and stability of T-S fuzzy stochastic system with time-varying delay are studied in [37] without stochastic disturbances. For more stability results of systems with time-varying delay, one can refer to Wu et al. [38–40] and the references therein.

This paper studies the dissipation and stability of T-S fuzzy stochastic system with different input matrices and time delay.

The main contributions of this paper are as follows:

- (1) A new T-S fuzzy system with stochastic disturbance with different input matrices and time delay is considered
- (2) A new integral fuzzy switching surface function is proposed for tackling the time delay case, and the dissipation and asymptotic stability of the sliding mode control system are obtained via this function
- (3) A novel fuzzy sliding mode controller is designed to ensure the reachability of sliding mode motion in the presence of matching uncertainties and interference signals

The remaining parts of this paper are organized as follows. Section 2 focuses on the formation and preliminary preparations. The fuzzy integral switching surface is given

in Section 3. Several sufficient conditions ensuring the strict dissipation performance and asymptotic stability of the corresponding sliding dynamic modes are also derived in this part. In Section 4, the details of the fuzzy integral sliding mode control (FISMC, for short) law is presented. Section 5 contains a FISMC simulation example for illustrating the feasibility and effectiveness of the proposed method. We make a brief conclusion in Section 6 to end this paper.

Notations: $\mathbf{R}^{m \times n}$ stands for the set of all $m \times n$ real matrices. The terms induced by symmetry are denoted by $*$. The inverse, transpose, and left inverse for an appropriated dimensioned matrix are indicated, respectively, by superscripts -1 , T , and $+$. I is adopted to denote the identity matrix. $B^\perp(x) \in \mathbf{R}^{n \times (n-m)}$ is the matrix with independent columns spanning the null space of $B(x) \in \mathbf{R}^{n \times m}$. $L_2[0, \infty)$ denotes the space about square integrable vector over $[0, \infty)$. If A is a matrix, denoted by $\|A\| = \sup\{\|Ax\| : |x| = 1\} = \sqrt{\lambda_{\max}(A^T A)}$, where $\lambda_{\max}(A)$ means the largest eigenvalue of A .

2. Problem Formulation and Definitions

T-S fuzzy model uses fuzzy reasoning to a group of nonlinear input-output submodels. We consider a class of nonlinear stochastic system with time delay, which can be exactly depicted by a T-S fuzzy model composed of the following fuzzy rules:

Rule i : if $\theta_1(t)$ is M_{i1} , $\theta_2(t)$ is M_{i2} , \dots and $\theta_p(t)$ is M_{ip} , then

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - \tau) + B_i(u(t) + f_i(x(t), x(t - \tau))) + B_{wi}w(t), \\ y(t) = C_i x(t) + C_{di}x(t - \tau) + D_{wi}w(t), \\ x(t) = \varphi(t), \quad t \in [-\tau, 0], \quad i \in \mathcal{R} = \{1, 2, \dots, r\}, \end{cases} \quad (1)$$

where $M_{i1}, M_{i2}, \dots, M_{ip}$ are fuzzy sets, r is the number of fuzzy rules, $x(t) \in \mathbf{R}^n$ stands for the system state, $\theta_1(t), \theta_2(t), \dots, \theta_p(t)$ are the premise variables, $x(t)$ is the state vector, $u(t) \in \mathbf{R}^m$ is the input signal, $w(t) \in \mathbf{R}^q$ is the disturbance signal belonging to $L_2[0, \infty)$, $y(t) \in \mathbf{R}^p$ is the controlled output, $A_i \in \mathbf{R}^{n \times n}$, $A_{di} \in \mathbf{R}^{n \times n}$, $B_i \in \mathbf{R}^{n \times m}$, $B_{wi} \in \mathbf{R}^{n \times q}$, $C_i \in \mathbf{R}^{p \times n}$, and $D_{wi} \in \mathbf{R}^{p \times q}$ are known constant matrices, τ represents a known constant time delay, and $\varphi(t)$ is a continuous vector-valued initial function.

The nonlinear function $f_i(x(t), x(t - \tau)) \in \mathbf{R}^m$ and ΔA_i and ΔA_{di} represent the matched and unmatched uncertainties, respectively, and satisfy the conditions

$$\begin{aligned} \|f_i(x(t), x(t - \tau))\| &\leq \gamma_{1i}\|x(t)\| + \gamma_{2i}\|x(t - \tau)\|, \\ \|\Delta A_i\| &\leq \sigma_i, \\ \|\Delta A_{di}\| &\leq \sigma_{di}, \quad i \in \mathcal{R}, \end{aligned} \quad (2)$$

where $\gamma_{1i}, \gamma_{2i}, \sigma_i$ and σ_{di} ($i = 1, 2, \dots, r$) are known non-negative real scalars.

By adopting the regular fuzzy inference methodology, that is, using a singleton fuzzifier and center-average

defuzzifier a more compact presentation about the fuzzy system [1] can be transformed into the following system:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(\theta(t)) [(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - \tau) + B_i(u(t) + f_i(x(t), x(t - \tau))) + B_{wi}w(t)], \\ y(t) = \sum_{i=1}^r h_i(\theta(t)) [C_i x(t) + C_{di} x(t - \tau) + D_{wi} w(t)], \end{cases} \quad (3)$$

where $h_i(\theta(t)) = (\prod_{j=1}^n M_{ij}(\theta_j(t)) / \sum_{l=1}^r \prod_{j=1}^n M_{lj}(\theta_j(t))) \geq 0$, $h_i(\theta(t))$ are the normalized membership functions, and $\sum_{i=1}^r h_i(\theta(t)) = 1$ with $M_{ij}(\theta_j(t))$ representing the grade of membership of premise variable $\theta_j(t)$ in M_{ij} . Without loss of generality, we assume that the input matrix $\mathcal{B}(x) = \sum_{i=1}^r h_i(\theta(t)) B_i$ has full column rank.

We first present some definitions and lemmas which will be used during sliding motion and controller synthesis. For the fuzzy system (3), we first introduce the following definition about dissipativity.

Definition 1. The fuzzy system (3) is said to be strictly $(\mathcal{L}, \mathcal{Y}, \mathcal{X})$ - γ -dissipative, for any $t^* > 0$ and initial state, if the following inequality holds:

$$V(x(t)) \leq V(x(0)) + \int_0^{t^*} [r(\omega(s), y(s)) - \gamma \omega^T(s) \omega(s)] ds, \quad t \geq 0, \quad (4)$$

where $V(x(t))$ is called the storage function and $r(\omega(t), y(t))$ is called supply rate.

The supply rate $r(\omega(t), y(t))$ is usually given by $r(\omega(t), y(t)) = y^T(t) \mathcal{L} y(t) + 2y^T(t) \mathcal{Y} \omega(t) + \omega^T(t) \mathcal{X} \omega(t)$, where $\mathcal{L} \in \mathbf{R}^{p \times p}$, $\mathcal{X} \in \mathbf{R}^{q \times q}$, and $\mathcal{Y} \in \mathbf{R}^{p \times q}$ are real matrices, with \mathcal{L} and \mathcal{X} being symmetric matrices, and $\gamma > 0$ is a positive scalar.

Remark 1. Here, it is assumed that $\mathcal{L} \leq 0$ and $\mathcal{Y} \geq 0$; thus, the performance index given in Definition 1 includes H_∞ , positive realness, and passivity as special cases.

Lemma 1 (see [36]). *For any given matrix $\mathcal{Q}(x) \in \mathbf{R}^{n \times m}$ with $\text{rank}(\mathcal{Q}(x)) = l \leq m$, suppose that $\bar{\mathcal{Q}}(x) \in \mathbf{R}^{n \times l}$ is a full column rank matrix formed by any l independent columns of $\mathcal{Q}(x)$. If the distribution $\mathcal{D}(x) = \text{span} \bar{\mathcal{Q}}_p^\perp(x)$, $p = 1, \dots, n - l$, is involutive, that is,*

$$[\bar{\mathcal{Q}}_p^\perp(x), \bar{\mathcal{Q}}_q^\perp(x)] = \frac{\partial \bar{\mathcal{Q}}_q^\perp(x)}{\partial x} \bar{\mathcal{Q}}_p^\perp(x) - \frac{\partial \bar{\mathcal{Q}}_p^\perp(x)}{\partial x} \bar{\mathcal{Q}}_q^\perp(x) \in \mathcal{D}(x), \quad (5)$$

where $p, q = 1, \dots, n - l$, $\bar{\mathcal{Q}}_p^\perp$ stands for the p th column of $\bar{\mathcal{Q}}^\perp$, and $[\cdot, \cdot]$ is the Lie bracket of two vector fields. Then, the nonlinear vector $g(x) = [\bar{g}_1(x) \dots \bar{g}_l(x) 0_{1 \times (m-l)}]^T \in \mathbf{R}^{m \times 1}$

exists such that $(\partial g(x) / \partial x) = \mathcal{G}(x) = \mathcal{N}(x) \mathcal{Q}^T(x)$, where $\mathcal{N}(x) \in \mathbf{R}^{m \times m}$ is a full rank matrix.

Lemma 2 (see [41]). *Let M and N be real matrices of appropriate dimensions. Then, for any matrix Δ satisfying $\Delta^T \Delta \leq I$ and scalar $\varepsilon > 0$, there holds*

$$M \Delta N + (M \Delta N)^T \leq \varepsilon^{-1} M M^T + \varepsilon N^T N. \quad (6)$$

3. Switching Surface Design and Sliding Mode Dynamics Analysis

In [38], a FISMC scheme is proposed based on the following fuzzy dynamic integral switch surface functions:

$$\begin{aligned} s(t) &= S_x x(t) - S_x x(0) + S_u u(t) - S_u u(0) \\ &+ \int_0^t \sum_{i=1}^r h_i(\theta(\alpha)) S_x [A_i x(\alpha) + B_i u(\alpha)] d\alpha \\ &- \int_0^t \sum_{i=1}^r h_i(\theta(\alpha)) S_u [F_i x(\alpha) + G_i u(\alpha)] d\alpha, \end{aligned} \quad (7)$$

where S_x, S_u, F_i , and G_i are the parameters of the surface to be designed.

Note that the requirement that $\begin{bmatrix} A_i & B_i \\ F_i & G_i \end{bmatrix}$ has to be Hurwitz in [38] is very special since each subsystem of the fuzzy system is usually inherently unstable. The developed FISMC schemes based on this type of surface lack the robustness for the case of matched uncertainty or perturbation during sliding motion. To better accommodate the characteristics of T-S fuzzy systems, the following fuzzy integral switching surface function is adopted:

$$\begin{aligned} s(t) &= \int_0^t \mathcal{G}(x) dx - \int_0^t \left[\mathcal{G}(x(\alpha)) \sum_{i=1}^r h_i(\theta(\alpha)) \sum_{j=1}^r h_j(\theta(\alpha)) (A_j x(\alpha) \right. \\ &\left. + A_{di} x(\alpha - \tau) + B_j K_j x(\alpha)) \right] d\alpha, \end{aligned} \quad (8)$$

where $\mathcal{G}(x) = \mathcal{N}(x) \mathcal{B}^T(x)$ is a projection matrix, $\mathcal{B}(x) = \sum_{i=1}^r h_i(\theta(t)) B_i$ has full column rank, and $\mathcal{N}(x) \in \mathbf{R}^{m \times m}$ is a full rank matrix. $\sum_{j=1}^r h_j(\theta(t)) K_j x(t)$ is used to stabilize the sliding motion and $K_j \in \mathbf{R}^{m \times n}$ is the design parameters.

Based on $\dot{s}(t) = 0$, the corresponding fuzzy controller is designed as follows:

$$\begin{aligned}
 u_{eq}(t) = & -(\mathcal{G}(x)\mathcal{B}(t))^{-1}\mathcal{G}(x)\sum_{i=1}^r h_i(\theta(t)) \\
 & \cdot [\Delta A_i x(t) + \Delta A_{di} x(t-\tau) + B_{wi} w(t)] \\
 & - \sum_{i=1}^r h_i(\theta(t)) f_i(x(t), x(t-\tau)) \\
 & + \sum_{j=1}^r h_j(\theta(t)) K_j x(t).
 \end{aligned} \tag{9}$$

By substituting (9) into (3), the sliding mode dynamics are given by

$$\begin{cases}
 \dot{x}(t) = \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) [(A_i + B_i K_j)x(t) + \overline{\mathcal{G}}(x)\Delta A_i x(t) + (A_{di} + \overline{\mathcal{G}}(x)\Delta A_{di})x(t-\tau) + \overline{\mathcal{G}}(x)B_{wi} w(t)], \\
 y(t) = \sum_{i=1}^r h_i(\theta(t)) [C_i x(t) + C_{di} x(t-\tau) + D_{wi} w(t)],
 \end{cases} \tag{10}$$

where $\overline{\mathcal{G}}(x) = I - \mathcal{B}(x)(\mathcal{G}(x)\mathcal{B}(x))^{-1}\mathcal{G}(x)$ is the transition matrix of unmatched uncertainties.

The following theorem is presented to ensure the asymptotic admissibility with $(\mathcal{L}, \mathcal{Y}, \mathcal{X}) - \gamma$ -dissipative performance for sliding motion (10).

Theorem 1. For given scalars $\mu > 0$ and $\gamma > 0$, matrices $0 \geq \mathcal{L} \in \mathbf{R}^{p \times p}$, $\mathcal{Y} \in \mathbf{R}^{p \times q}$, and $\mathcal{X} \in \mathbf{R}^{q \times q}$ with \mathcal{L} and \mathcal{X} are symmetric, the sliding mode dynamics (10) is robust asymptotically stable and strictly $(\mathcal{L}, \mathcal{Y}, \mathcal{X}) - \gamma$ -dissipative if

there exist symmetric positive-definite matrices $P \in \mathbf{R}^{n \times n}$ and $Q \in \mathbf{R}^{n \times n}$, matrices $\overline{K}_j \in \mathbf{R}^{m \times n}$, $j = 1, 2, \dots, r$, $M \in \mathbf{R}^{n \times n}$, and scalars $\varepsilon_{ki} > 0$, $k = 1, 2, 3, 4, 5, 6$, $i = 1, 2, \dots, r$ such that the inequalities

$$\Lambda_{ii} < 0, \tag{11}$$

$$\Lambda_{ij} + \Lambda_{ji} < 0, \quad i < j, \tag{12}$$

hold, where $i = 1, 2, \dots, r$, $\Lambda_{ij} = \begin{bmatrix} \overline{\Xi}^{ij} & \sigma_{idi} \\ * & \varepsilon_{ji} \end{bmatrix}$,

$$\begin{aligned}
 \Xi^{ij} &= \begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} & A_{di}M^T & -MC_i^T \mathcal{Y} & MC_i^T \mathcal{X} \\ * & \Xi_{22}^i & \mu A_{di}M^T & 0 & 0 \\ * & * & \Xi_{33}^i & -MC_{di}^T \mathcal{Y} & MC_{di}^T \mathcal{X} \\ * & * & * & \Xi_{44}^i & D_{wi}^T \mathcal{X} \\ * & * & * & * & \mathcal{X} \end{bmatrix}, \\
 \sigma_{idi} &= \begin{bmatrix} \sigma_i M & 0 & 0 & \sigma_i M & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{di} M & 0 & 0 & \sigma_{di} M & 0 \\ 0 & 0 & B_{wi}^T & 0 & 0 & B_{wi}^T \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
 \varepsilon_{ji} &= \begin{bmatrix} -\varepsilon_{1i} & 0 & 0 & 0 & 0 & 0 \\ * & -\varepsilon_{2i} & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_{3i} & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{4i} & 0 & 0 \\ * & * & * & * & -\varepsilon_{5i} & 0 \\ * & * & * & * & * & -\varepsilon_{6i} \end{bmatrix}, \\
 \Xi_{11}^{ij} &= A_i M^T + M A_i^T + B_i \bar{K}_j + \bar{K}_j^T B_i^T + \varepsilon_{1i} I + \varepsilon_{2i} I + \varepsilon_{3i} I + Q, \\
 \Xi_{22}^i &= -\mu M^T - \mu M + \mu^2 \varepsilon_{4i} I + \mu^2 \varepsilon_{5i} I + \mu^2 \varepsilon_{6i} I, \\
 \Xi_{33}^i &= -Q, \\
 \Xi_{44}^i &= -D_{wi}^T \mathcal{Y} - \mathcal{Y}^T D_{wi} - \mathcal{X} + \gamma I, \\
 \Xi_{12}^{ij} &= P - M^T + \mu M A_i^T + \mu \bar{K}_j^T B_i^T.
 \end{aligned} \tag{13}$$

Moreover, the parameter matrices K_j in (8) are given by

$$K_j = \bar{K}_j M^{-T}. \tag{14}$$

Proof. For the fuzzy system (10), we consider the following Lyapunov function:

$$V(x(t), t) = x^T(t) \bar{P} x(t) + \int_{t-\tau}^t x^T(z) \bar{Q} x(z) dz, \tag{15}$$

where $\bar{P} = \bar{M} P \bar{M}^T$ and $\bar{Q} = \bar{M} Q \bar{M}^T$.

The derivative of $V(x(t), t)$ along fuzzy system (9) is calculated as

$$\begin{aligned}
 \dot{V}(x(t), t) &= x^T(t) \bar{P} \dot{x}(t) + \dot{x}^T(t) \bar{P} x(t) + x^T(t) \bar{Q} x(t) - x^T(t-\tau) \bar{Q} x(t-\tau) \\
 &= \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) [x^T(t) \bar{P} \dot{x}(t) + \dot{x}^T(t) \bar{P} x(t)] + x^T(t) \bar{Q} x(t) - x^T(t-\tau) \bar{Q} x(t-\tau).
 \end{aligned} \tag{16}$$

From the Leibniz–Newton formula, the following equation is true for any scalar $\mu > 0$ and matrix $\overline{M} = M^{-1}$ with appropriate dimension

$$2 \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) \left[x^T(t) \overline{M} + \mu \dot{x}^T(t) \overline{M} \right] \cdot \begin{bmatrix} -\dot{x}(t) + (A_i + B_i K_j + \overline{\mathcal{F}}(x) \Delta A_i) x(t) \\ +(A_{di} + \overline{\mathcal{F}}(x) \Delta A_{di}) x(t - \tau) + \overline{\mathcal{F}}(x) B_{wi} w(t) \end{bmatrix} = 0. \quad (17)$$

On the contrary, by substituting the value of $\mathcal{G}(x)$ into $\overline{\mathcal{G}}(x)$, we have

$$\begin{aligned} \overline{\mathcal{G}}(x) &= I - \mathcal{B}(x) [\mathcal{G}(x) \mathcal{B}(x)]^{-1} \mathcal{G}(x) = I - \mathcal{B}(x) \mathcal{B}^+(x), \\ \overline{\mathcal{G}}^T(x) &= I - \mathcal{G}^T(x) [\mathcal{G}(x) \mathcal{B}(x)]^{-T} \mathcal{B}^T(x) = I - \mathcal{B}(x) \mathcal{B}^+(x). \end{aligned} \quad (18)$$

Then, $\mathcal{G}(x) = \mathcal{G}^T(x)$ and

$$\overline{\mathcal{G}}^T(x) \overline{\mathcal{G}}(x) = [I - \mathcal{B}(x) \mathcal{B}^+(x)] [I - \mathcal{B}(x) \mathcal{B}^+(x)] = \overline{\mathcal{G}}(x). \quad (19)$$

It implies that $\overline{\mathcal{G}}(x)$ is an idempotent matrix. Thus, its eigenvalues can only be ones or zeros. Since $\text{rank}(\mathcal{B}(x) \mathcal{B}^+(x)) < n$, the rank of $I - \mathcal{B}(x) \mathcal{B}^+(x)$ cannot be zero. Then, there is at least one eigenvalue of $\overline{\mathcal{G}}(x)$ is one, which implies that

$$\|\overline{\mathcal{G}}(x)\| = 1. \quad (20)$$

Thus, by Lemma 2, there exist positive scalars $\varepsilon_{ki}, k = 1, \dots, 6$ such that

$$2x^T(t) \overline{M} \overline{\mathcal{G}}(x) \Delta A_i x(t) \leq \varepsilon_{1i}^{-1} \sigma_i^2 x^T(t) x(t) + \varepsilon_{1i} x^T(t) \overline{M} \overline{M}^T x(t), \quad (21)$$

$$2x^T(t) \overline{M} \overline{\mathcal{G}}(x) \Delta A_{di} x(t - \tau) \leq \varepsilon_{2i}^{-1} \sigma_{di}^2 x^T(t - \tau) x(t - \tau) + \varepsilon_{2i} x^T(t) \overline{M} \overline{M}^T x(t), \quad (22)$$

$$2x^T(t) \overline{M} \overline{\mathcal{G}}(x) B_{wi} w(t) \leq \varepsilon_{3i}^{-1} w^T(t) B_{wi}^T B_{wi} w(t) + \varepsilon_{3i} x^T(t) \overline{M} \overline{M}^T x(t), \quad (23)$$

$$2\mu \dot{x}^T(t) \overline{M} \overline{\mathcal{G}}(x) \Delta A_i x(t) \leq \varepsilon_{4i}^{-1} \sigma_i^2 \dot{x}^T(t) x(t) + \varepsilon_{4i} \mu^2 \dot{x}^T(t) \overline{M} \overline{M}^T \dot{x}(t), \quad (24)$$

$$2\mu \dot{x}^T(t) \overline{M} \overline{\mathcal{G}}(x) \Delta A_{di} x(t - \tau) \leq \varepsilon_{5i}^{-1} \sigma_{di}^2 \dot{x}^T(t - \tau) x(t - \tau) + \varepsilon_{5i} \mu^2 \dot{x}^T(t) \overline{M} \overline{M}^T \dot{x}(t), \quad (25)$$

$$2\mu \dot{x}^T(t) \overline{M} \overline{\mathcal{G}}(x) B_{wi} w(t) \leq \varepsilon_{6i}^{-1} w^T(t) B_{wi}^T B_{wi} w(t) + \varepsilon_{6i} \mu^2 \dot{x}^T(t) \overline{M} \overline{M}^T \dot{x}(t). \quad (26)$$

Substituting (17) and (21)–(26) into (16), we have

$$\begin{aligned} & \dot{V}(x(t), t) - y^T(t) \mathcal{L} y(t) - 2y^T(t) \mathcal{Y} w(t) - w^T(t) (\mathcal{X} - \gamma I) w(t) \\ & \leq \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) \left\{ x^T(t) \overline{P} \dot{x}(t) + \dot{x}^T(t) \overline{P} x(t) + x^T(t) \overline{Q} x(t) - x^T(t - \tau) \overline{Q} x(t - \tau) + 2 \left[x^T(t) \overline{M} + \mu \dot{x}^T(t) \overline{M} \right] \right. \\ & \quad \cdot \left[-\dot{x}(t) + (A_i + B_i \times K_j + \overline{\mathcal{F}}(x) \Delta A_i) x(t) + (A_{di} + \overline{\mathcal{F}}(x) \Delta A_{di}) x(t - \tau) + \overline{\mathcal{F}}(x) B_{wi} w(t) \right] \\ & \quad \left. - y^T(t) \mathcal{L} y(t) - 2y^T(t) \mathcal{Y} w(t) - w^T(t) (\mathcal{X} - \gamma I) w(t) \right\} \\ & \leq \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) \xi^T(t) \overline{\Lambda}_{ij} \xi(t) = \sum_{i=1}^r h_i(\theta(t)) \xi^T(t) \overline{\Lambda}_{ii} \xi(t) \\ & \quad + \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) \xi^T(t) (\overline{\Lambda}_{ij} + \overline{\Lambda}_{ji}) \xi(t), \end{aligned} \quad (27)$$

where

$$\bar{\Lambda}_{ij} = \begin{bmatrix} \bar{\Xi}_{11}^{ij} & \bar{\Xi}_{12}^{ij} & \bar{M}A_{di} & -C_i^T \mathcal{Y} \\ * & \bar{\Xi}_{22}^i & \mu \bar{M}A_{di} & 0 \\ * & * & \bar{\Xi}_{33}^i & -C_{di}^T \mathcal{Y} \\ * & * & * & \bar{\Xi}_{44}^i \end{bmatrix} - \begin{bmatrix} C_i^T \\ 0 \\ -C_{di}^T \\ D_{wi}^T \end{bmatrix} \mathcal{X} \begin{bmatrix} C_i^T \\ 0 \\ -C_{di}^T \\ D_{wi}^T \end{bmatrix}^T,$$

$$\begin{aligned} \bar{\Xi}_{11}^{ij} &= \bar{M}A_i + A_i^T \bar{M}^T + \bar{M}B_i K_j + K_j^T B_i^T \bar{M}^T + \varepsilon_{1i} \bar{M} \bar{M}^T + \varepsilon_{2i} \bar{M} \bar{M}^T + \varepsilon_{3i} \bar{M} \bar{M}^T + \varepsilon_{1i}^{-1} \sigma_{di}^2 I + \varepsilon_{4i}^{-1} \sigma_{di}^2 I + Q, \\ \bar{\Xi}_{12}^{ij} &= -\bar{M} + \mu A_i^T \bar{M}^T + \mu K_j^T B_i^T \bar{M}^T + \bar{P}, \\ \bar{\Xi}_{22}^i &= -\mu \bar{M} - \mu \bar{M}^T + \varepsilon_{4i} \mu^2 \bar{M} \bar{M}^T + \varepsilon_{5i} \mu^2 \bar{M} \bar{M}^T + \varepsilon_{6i} \mu^2 \bar{M} \bar{M}^T, \\ \bar{\Xi}_{33}^i &= \varepsilon_{2i}^{-1} \sigma_{di}^2 I + \varepsilon_{5i}^{-1} \sigma_{di}^2 I - Q, \\ \bar{\Xi}_{44}^i &= \varepsilon_{3i}^{-1} B_{wi}^T B_{wi} + \varepsilon_{6i}^{-1} B_{wi}^T B_{wi} - D_{wi}^T \mathcal{Y} - \mathcal{Y}^T D_{wi} - \mathcal{X} + \gamma I, \\ \xi(t) &= \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t-\tau) & w^T(t) \end{bmatrix}^T. \end{aligned} \tag{28}$$

Pre- and postmultiplying both sides of (11) and (12) with $\text{diag}\{\bar{M}, \bar{M}, \bar{M}, I, I, I, I, I, I, I\}$ and its transpose, respectively, and applying the Schur complement theorem, we have $\bar{\Lambda}_{ii} < 0$ and $\bar{\Lambda}_{ii} + \bar{\Lambda}_{ji} < 0$, that is,

$$\dot{V}(x(t), t) < \Psi(t), \tag{29}$$

where $\Psi(t) = y^T(t) \mathcal{X} y(t) + 2y^T(t) \mathcal{Y} w(t) + w^T(t) (\mathcal{X} - \gamma I) w(t)$.

Under zero initial conditions, we have $t = 0, V(x(t), t) = 0$ and $t \rightarrow \infty, V(x(t), t) \geq 0$. Integrating both sides of (29) from 0 to t results in $\int_0^t \Psi(t) dt \geq V(x(t), t) - V(x(0), 0) \geq 0$, which ensures (4), that is, sliding mode dynamics (10) is strictly $(\mathcal{X}, \mathcal{Y}, \mathcal{X}) - \gamma$ -dissipative. When $w(t) = 0, \dot{V}(x(t), t) \leq y^T(t) \mathcal{X} y(t) + \xi^T(t) \bar{\Lambda}_{ij} \xi(t) \leq \xi^T(t) \bar{\Lambda}_{ij} \xi(t) \leq -\lambda \|x(t)\|^2$ with $\lambda = \lambda_{\min}(-\bar{\Lambda}_{ij})$. Then, we obtain that sliding mode dynamics (10) is asymptotic stability. This completes the proof.

Remark 2. In order to sufficiently utilize the characteristics of T-S fuzzy model, a fuzzy integral switching surface (8) is designed by introducing an integral term of state-dependent projection matrices. The precondition for the establishment of the new surface lies in the existence of the nonlinear vector function $g(x) \in \mathbf{R}^{m \times 1}$ satisfying $(\partial g(x)/\partial x) = \mathcal{G}(x)$. Then, according to Lemma 1, the requirement can be satisfied if the distribution span $B_i^+, i = 1, 2, \dots, n-m$ is involutive, with B_i^+ being the i th column about B_i^+ . Compared with the system matrix with very strict conditions

adopted in [34, 36, 42], the significant advantage of this constraint is that the input matrices B_i can be diverse in different subsystems. In addition, it can be found from (10) and (19) that, in the corresponding sliding motion, the matched uncertainty is completely rejected and the unmatched one and external disturbance are not amplified in the sliding motion, while the switching surface function proposed in [43] cannot achieve the role of (10) and (19).

4. FISMC Law

In this section, we synthesize a FISMC law. According to the FISMC law, the trajectories of fuzzy system (3) can be driven onto the prespecified fuzzy integral switching surface (8) with some uncertainties and external disturbances.

For the fuzzy system (3), the FISMC controller for rule j is designed as follows:

Model Rule j : IF $\theta_1(t)$ is M_{i1}, \dots , and $\theta_p(t)$ is M_{ip} , THEN

$$u(t) = \sum_{j=1}^r h_j(\theta(t)) \left[K_j x(t) - \rho(t) (\mathcal{G}(x) \mathcal{B}(x))^{-1} \frac{s(t)}{\|s(t)\|} \right]. \tag{30}$$

where $\|s(t)\| \neq 0$ and

$$\begin{aligned} \rho(t) &= \nu + \sum_{i=1}^r h_i(\theta(t)) (\sigma_i \|\mathcal{G}(x)\| \|x(t)\| + \sigma_{di} \|\mathcal{G}(x)\| \|x(t-\tau)\| + \gamma_{1i} \|\mathcal{G}(x) \mathcal{B}(x)\| \times \|x(t)\| \\ &\quad + \gamma_{2i} \|\mathcal{G}(x) \mathcal{B}(x)\| \|x(t-\tau)\| + \|\mathcal{G}(x) B_{wi}\| \|w(t)\|), \end{aligned} \tag{31}$$

with ν being a positive constant.

Theorem 2. Consider the fuzzy stochastic time-delay system (3). Suppose the fuzzy integral switching surface is adopted in (8) with K_j being solved by (13), and the FISMC is constructed as (30). Then, the trajectories of fuzzy system (3) can be driven onto the predefined fuzzy switching surface $s(t) = 0$ with matched or unmatched uncertainties and external disturbance.

Proof. Consider the following Lyapunov function:

$$U(t) = \frac{1}{2} s^T(t) s(t). \quad (32)$$

According to the fuzzy switching surface (8), we have

$$\begin{aligned} \dot{s}(t) = & \mathcal{G}(x) \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) \\ & \cdot \left[(\Delta A_i - B_i K_j) x(t) + \Delta A_{di} x(t - \tau) + B_i u(t) \right. \\ & \left. + B_{wi} w(t) + B_i f_i(x(t), x(t - \tau)) \right]. \end{aligned} \quad (33)$$

By (33), the derivative of $U(t)$ along above equation can be calculated as

$$\begin{aligned} \dot{U}(t) = & s^T(t) \dot{s}(t) = s^T(t) \mathcal{G}(x) \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) \left[(\Delta A_i - B_i K_j) x(t) + \Delta A_{di} x(t - \tau) + B_i u(t) \right. \\ & \left. + B_{wi} w(t) + B_i f_i(x(t), x(t - \tau)) \right] \\ = & s^T(t) \mathcal{G}(x) \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) \left\{ \begin{aligned} & (\Delta A_i - B_i K_j) x(t) + \Delta A_{di} x(t - \tau) + B_i \\ & \left[K_j x(t) - \rho(t) (\mathcal{G}(x) \mathcal{B}(x))^{-1} \frac{s(t)}{\|s(t)\|} \right] + B_{wi} w(t) + B_i f_i(x(t), x(t - \tau)) \end{aligned} \right\} \\ = & s^T(t) \mathcal{G}(x) \sum_{i=1}^r h_i(\theta(t)) \sum_{j=1}^r h_j(\theta(t)) \left(\begin{aligned} & \Delta A_i x(t) + \Delta A_{di} x(t - \tau) - \\ & \left(\begin{aligned} & \nu + \sigma_i \|\mathcal{G}(x)\| \|x(t)\| + \sigma_{di} \|\mathcal{G}(x)\| \|x(t - \tau)\| \\ & + \gamma_{1i} \|\mathcal{G}(x) \mathcal{B}(x)\| \|x(t)\| + \gamma_{2i} \|\mathcal{G}(x) \times \mathcal{B}(x)\| \|x(t - \tau)\| \\ & + \|\mathcal{G}(x) B_{wi}\| \|w(t)\| \end{aligned} \right) \\ & B_i (\mathcal{G}(x) \mathcal{B}(x))^{-1} \frac{s(t)}{\|s(t)\|} + B_{wi} w(t) + B_i f_i(x(t), x(t - \tau)) \end{aligned} \right) \\ \leq & \sum_{i=1}^r h_i(\theta(t)) \left\{ \begin{aligned} & \|s(t)\| \|\mathcal{G}(x)\| (\sigma_i \|x(t)\| + \sigma_{di} \|x(t - \tau)\| + \|B_{wi}\| \|w(t)\|) + \|s(t)\| \|\mathcal{G}(x) \mathcal{B}(x)\| \\ & (\gamma_{1i} \|x(t)\| + \gamma_{2i} \|x(t - \tau)\|) - s^T(t) \mathcal{G}(x) \left[\begin{aligned} & \nu + \sigma_i \|\mathcal{G}(x)\| \|x(t)\| + \sigma_{di} \|\mathcal{G}(x)\| \|x(t - \tau)\| \\ & + \gamma_{1i} \|\mathcal{G}(x) \mathcal{B}(x)\| \|x(t)\| + \gamma_{2i} \\ & \times \|\mathcal{G}(x) \mathcal{B}(x)\| \|x(t - \tau)\| + \|\mathcal{G}(x) B_{wi}\| \|w(t)\| \end{aligned} \right] \\ & B_i (\mathcal{G}(x) \mathcal{B}(x))^{-1} \frac{s(t)}{\|s(t)\|} \end{aligned} \right\}. \end{aligned} \quad (34)$$

So, we have

$$\dot{U}(t) \leq -\nu \|s(t)\| < 0, \quad \text{for } \|s(t)\| \neq 0. \quad (35)$$

It is shown from (35) that the state variables for the fuzzy system can be driven onto the prespecified fuzzy surface $s(t) = 0$ in a finite lapse of time with matched/unmatched

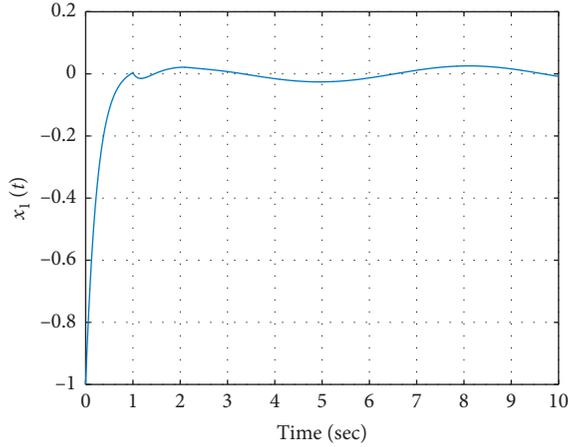


FIGURE 1: Trajectories of state $x(t)$.

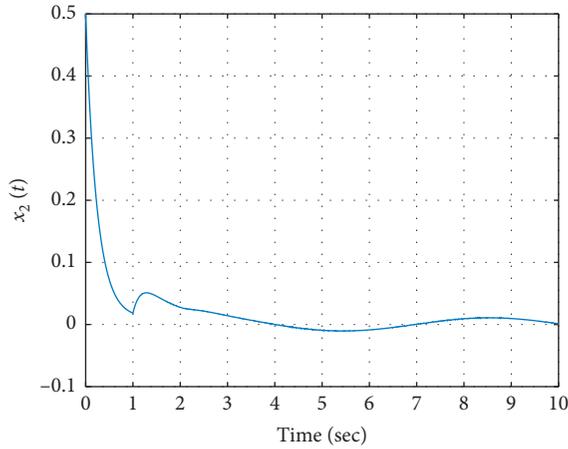


FIGURE 2: Trajectories of state $x(t)$.

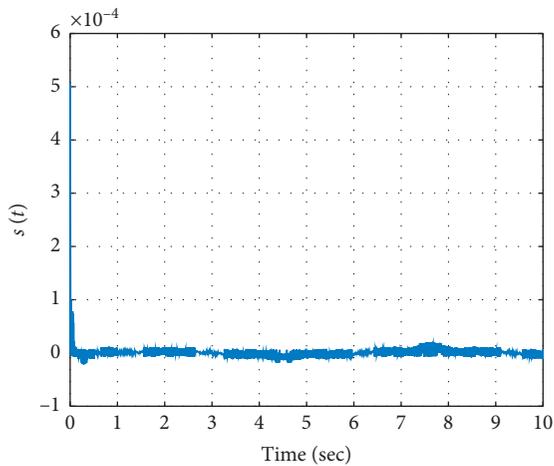


FIGURE 3: Switching surface function $s(t)$.

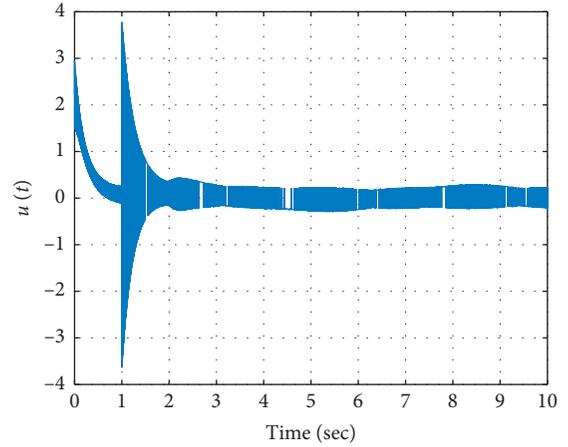


FIGURE 4: Control input $u(t)$.

uncertainties and external disturbance. This completes the proof.

Remark 3. By analyzing the proposed fuzzy integral switching surface (8), the fuzzy sliding mode controller (30) is designed. Compared with [34, 42], the main advantage of this novel fuzzy integral sliding mode controller is that different input matrices are allowed to be used, and the strict requirements for local input matrices are eliminated. Consequently, it enlarges the range of its applications. It is worth noting that the uncertainties proposed in this paper are usually unknown. For this case, a new sliding mode controller may need to be designed. One may draw on the experience of the observer-based descriptor system considered in [44] for tackling this issue.

5. Simulation Example

In order to illustrate the feasibility and effectiveness of our results, we provide a numerical example.

Consider the following T-S fuzzy stochastic system:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 h_i(\theta(t)) [(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - \tau) \\ \quad + B_i(u(t) + t f_i n(x(t), x(t - \tau))) + B_{wi}w(t)], \\ y(t) = \sum_{i=1}^2 h_i(\theta(t)) [C_i x(t) + C_{di} x(t - \tau) + D_{wi} w(t)], \end{cases} \quad (36)$$

where the model parameters are given as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} -3 & 1 \\ 1 & 5 \end{bmatrix}, & A_2 &= \begin{bmatrix} -4 & 0.5 \\ 0.5 & 2 \end{bmatrix}, & A_{d1} &= \begin{bmatrix} -0.1 & 0.5 \\ 0.2 & -0.3 \end{bmatrix}, \\ A_{d2} &= \begin{bmatrix} 0.1 & -0.5 \\ -0.2 & 0.3 \end{bmatrix}, & B_1 &= \begin{bmatrix} 0 \\ -3 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0 \\ -1 \end{bmatrix}, & B_{w1} &= \\ B_{w2} &= \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, & C_1 &= [0.1 \quad -0.1], & C_2 &= [0 \quad -0.1], & C_{d1} &= \\ C_{d2} &= [0.1 \quad 0.2], & D_{w1} &= D_{w2} = 0.01, & |x_1(t)| &\leq 1, & h_1 &= (x_1^2(t)) \end{aligned}$$

$/4$, $h_2 = (4 - x_1^2(t))/4$, and $f_1(x(t), x(t - \tau)) = 2x_1(t)$, $f_2(x(t), x(t - \tau)) = x_2(t)$, $\gamma_{11} = 2$, $\gamma_{12} = \gamma_{21} = 0$, $\gamma_{22} = 3$.

Choose $\mathcal{X} = 1.5$, $\mathcal{Y} = 1.5$, $\mathcal{Z} = -1.5$, $\mu = 1.0$, and $\gamma = 0.5$, and let $\sigma_1 = \sigma_2 = 0.14$ and $\sigma_{d1} = \sigma_{d2} = 0.15$. Solving the LMI conditions in Theorem 1, by (13), we have $K_1 = [-0.2474 \ 2.6148]$, $K_2 = [-0.4959 \ 3.9088]$.

Let $\mathcal{N}(x) = (-20/2x_1^2 + 4)$; we obtain $\mathcal{G}(x) = \mathcal{G}(x) = \mathcal{N}(x)\mathcal{B}^T(x) = [0 \ 5]$, and we can get the corresponding fuzzy integral switch surface offered in (8) as follows:

$$s(t) = 5x_2(t) - 5x_2(0) - \int_0^t \left[[0 \ 5] \sum_{i=1}^2 h_i(\theta(\alpha)) \sum_{j=1}^r h_j(\theta(\alpha)) \cdot (A_i x(\alpha) + A_{di} x(\alpha - \tau) + B_i K_j x(\alpha)) \right] d\alpha. \quad (37)$$

$$\rho(t) = \nu + \sum_{i=1}^2 h_i(\theta(t)) (0.14 \|\mathcal{G}(x)\| \|x(t)\| + 0.15 \|\mathcal{G}(x)\| \|x(t - \tau)\| + \gamma_{1i} \|\mathcal{G}(x) \times \mathcal{B}(x)\| \|x(t)\| + \gamma_{2i} \|\mathcal{G}(x)\mathcal{B}(x)\| \|x(t - \tau)\| + \|\mathcal{G}(x)B_{wi}\| \|w(t)\|), \quad (39)$$

with the adjustable parameters $\nu = 0.5$.

The simulation results are shown in the above figures. Among them, Figures 1 and 2 show the state response of the system. Figure 3 shows the response of the sliding surface. Figure 4 depicts the response of the control input. From these figures, we can see that, under the designed FISMC law, the system is asymptotically stable.

6. Conclusion

The dissipation issue for a class of nonlinear stochastic systems has been studied by developing a new fuzzy integral sliding mode control. For adapting to the characteristics of T-S fuzzy model, a new fuzzy integral switching surface is designed by introducing an integral term of state-dependent projection matrices. It has been shown that, in the corresponding sliding motion, the matched uncertainty is completely rejected and the unmatched one and external disturbance are not amplified in the sliding motion. The main advantage of this novel fuzzy integral sliding mode controller is that different input matrices are allowed to be used, and the strict requirements for local input matrices are eliminated. Consequently, it enlarges the range of its applications. It is worth noting that the uncertainties proposed in this paper are usually unknown. For this case, a new sliding mode controller may need to be designed.

Data Availability

The data used to support the findings of this study are included within the article.

The FISMC law in (30) can be computed as

$$u(t) = \sum_{j=1}^2 h_j(\theta(t)) \left[K_j x(t) - \rho(t) (\mathcal{G}(x)\mathcal{B}(x))^{-1} \frac{s(t)}{\|s(t)\|} \right], \quad (38)$$

where

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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