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Research Article

Day-to-Day Dynamic Multivehicle Assignment: Deterministic Process Models

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In the near future, transportation systems modelers and planners will likely be challenged by more complex scenarios. This is due to the different types of vehicles that include different (i) powertrains (conventional, hybrid, electric, etc.), (ii) ownerships (privately-owned vs. shared vehicles), and (iii) levels of automation (from human-driven to fully autonomous). All these different vehicle types compete for the same arcs and jointly participate to congestion. Thus, existing methods for travel demand assignment to a transportation network, the main tools for transportation systems analysis to support transportation project assessment and evaluation, need to be extended to cope with mixed traffic. In this paper, deterministic process models for day-to-day dynamic multivehicle assignment are presented, including fixed-point models for equilibrium assignment as a special case. Vehicle types may be distinguished with respect to several parameters, such as flow equivalence coefficient, occupancy factor, cost equivalence coefficient, and behavioral parameters. Results of an application to a toy network are also discussed showing that advanced vehicles (AVs) may or may not have a positive effect of equilibrium stability.

1. Introduction

Technologies and prototypes for advanced vehicles (AVs), such as connected, automated, autonomous vehicles, or electric and hybrid ones, both for transportation of people or goods are fast developing. Still, the time needed to turn the existing stock of (mostly privately-owned) traditional vehicles (TVs) into (possibly shared) advanced vehicles will last several years during which mixed traffic is expected. Indeed, according to current estimates, transition to widespread fully autonomous and electric cars will require at least 20 years. Meanwhile, enhancements of existing tools of Traffic and Transportation Theory are required by transportation systems modelers and planners. Thus, methods for travel demand assignment to a transportation network, the main tools for transportation systems analysis to support transportation project assessment and evaluation, need to be extended to cope with mixed traffic.

Equilibrium assignment was introduced by Wardrop (1952) [1]; afterwards, Stochastic User Equilibrium based on random utility theory (RUT) was introduced by Daganzo and Sheffi (1977) [2]. Equilibrium assignment may effectively be formulated through fixed-point (FP) models (introduced by Daganzo, 1983 [3], through the use of inverse cost function) as shown by the general framework in Cantarella (1997) [4]. This modeling approach implemented in several commercial software packages is the most used in practical applications.

Methods for day-to-day (or interperiodic) dynamic assignment (introduced for transportation systems analysis by Horowitz, 1984 [5]) play a central role in advanced transportation system analysis, since they allow analysing and forecast equilibrium stability and fluctuations around it, as a result of past events. Indeed, even though exactly one equilibrium flow and cost patterns exist, the system state, flow and cost patterns, may evolve towards the equilibrium or towards another kind of attractor.

The stability analysis can be addressed through deterministic process (DP) models, derived from discrete-time nonlinear dynamic systems theory [6, 7]. Moreover, the equilibrium analysis does not allow analysing transients after demand and/or supply changes, or obtaining a statistical description of the system state evolution over time, i.e., means, modes, moments, and, more generally, frequency distributions, this kind of analysis requiring stochastic process (SP) models (introduced for transportation systems analysis by Cascetta, 1987 [8], 1989 [9]). A first unifying framework was proposed by Cantarella and Cascetta (1995) [10]. Additional details on SP are available (e.g., [11–13]).

A general theory of travel demand assignment including static and dynamic modeling approaches has recently been proposed in the book by Cantarella et al. (2019) [14], which is a comprehensive reference for mathematical details as well as references.

In a recent paper, Cantarella and Di Febbraro (2017) [15] showed how existing fixed-point (FP) models for equilibrium assignment can be extended to transportation networks where several types of vehicles, including, for instance, AVs and TVs, compete for the same arcs and jointly participate to congestion. Cantarella et al. [16, 17] provided sufficient conditions for fixed-point existence uniqueness, specified algorithms for large-scale applications, analysed conditions for their convergence, and discussed some large-scale examples.

This paper proposes DP models for day-to-day dynamic multivehicle assignment. Different vehicle types may be given by the following:

- Fossil fuelled vs. electrical energy
- Ownership, say privately owned vs. (operator-owned) shared vehicles
- Level of automation, from human-driven to fully autonomous
- Others

According to the proposed modeling approach, if the route choice behavior is described by applying the Random Utility Theory, vehicle type may be distinguished through different parameters regarding dispersion of perceived utility (in the choice function), weights of attributes (in the utility function), etc. Vehicle types may also be distinguished with respect to the occupancy factor, that is, the average number of users on-board, the flow equivalent factor, that is, the relative effect on capacity, and the cost equivalent factor.

In Section 2, the adopted multivehicle modeling approach is described, briefly reviewing the relevant background. Then, in Section 3, some simple examples considering two types of vehicles, TVs and AVs moving on a toy network, are discussed showing the effecting of increasing the AVs proportion. Major findings and research perspectives are discussed in Section 4.

The main original contributions of this paper are as follows:

- Deterministic process models for day-to-day dynamic multivehicle assignment, including fixed-point models for equilibrium assignment as a special case
- Vehicle types may be distinguished with respect to several parameters as shown below
- Comparison between moving average and exponential smoothing filters for modeling user memory and learning
- An application to a toy network showing that, even in very simple networks, AVs may or may not have a positive effect of equilibrium stability

2. Multivehicle Assignment

In this section, the adopted modeling approach is described, and; it is consistent with the Six Equation Assignment Modeling (SEAM) and the Two Equation Assignment Modeling approaches proposed in Cantarella et al. (2019) [14], where mathematical details are discussed. User category is not explicitly introduced in the following to simplify notations.

2.1. Assignment to Uncongested Networks.

2.1. Assignment to Uncongested Networks. Connections are described by an oriented graph. Parking facility can easily be considered. Each origin and each destination are modeled through a further node connected to the main network through connecting arcs. Users are distinguished with respect to o-d pair i they are travelling from/to and type of used vehicle m. The reference vehicle type (RVT) has index $m = 1$; usually, privately owned human-driven fossil fuelled passenger cars are used as RVT.

2.1.1. Basic Definitions and Equations. The cost per each vehicle type m is given by an affine transformation of the RVT cost:

$$\mathbf{c}_m = \chi_m * \mathbf{c} + \mathbf{c}_{\mathbf{Z}m} \forall m, \tag{1}$$

where, with reference to vehicle type m,$\mathbf{c}_{\mathbf{Z}m} \geq \mathbf{0}$ is the arc specific cost vector per vehicle type; $\mathbf{c} \geq \mathbf{0}$ is the arc cost vector, common to all vehicle types; $\chi_m > 0$ is the *cost equivalence coefficient*, modeling, for example, different on-board comfort, speeds, etc. with $\chi_1 = 1$ (for further details, see Section 2.4); and $\mathbf{c}_m \geq \mathbf{0}$ is the arc cost vector per vehicle type.

The route costs can be obtained from the corresponding arc costs through an affine transformation from the arc space to the route space defined by the transpose of arc-route incidence matrix:

$$\mathbf{w}_{im} = \mathbf{B}_{im}{}^{\mathrm{T}}.\mathbf{c}_m + \mathbf{w}_{Zim} \forall im, \tag{2}$$

where, with reference to o-d pair i and vehicle type m, $\mathbf{w}_{\mathbf{Z}im} \geq \mathbf{0}$ is the route specific or nonadditive cost vector, such as not arc-based tolls or fares; $\mathbf{B}_{im}$ is the arc-route incidence matrix; and $\mathbf{w}_{im} \geq \mathbf{0}$ is the route cost vector; arc and route (generalized transportation) costs are assumed measured by a common unit, usually travel time or money, through duly homogenization of different attributes, such as the value of time, not explicitly introduced to simplify notations.

The (systematic) utility function for o-d pair i and vehicle type m is specified through a linear transformation (almost always in research analysis as well as in practical applications):

$$\mathbf{v}_{im} = -\psi_m \mathbf{w}_{im} \forall im, \tag{3}$$

where, with reference to o-d pair i and vehicle type m,$\psi_m > 0$ is the *utility scale parameter*, such that the term $\psi_i \mathbf{w}_i$ is dimensionless, and $\mathbf{v}_{im}$ is the route systematic utility vector.

Route choice behavior for o-d pair i and vehicle type m can be modeled by applying any discrete choice modeling theory so that route choice proportions depend on route utility:

$$\mathbf{p}_{im} = \mathbf{p}_{im}(\mathbf{v}_{im}; \boldsymbol{\theta}_m)\forall im, \tag{4}$$

where, with reference to o-d pair i and vehicle type m, $\mathbf{p}_{im} > 0$ is the route choice proportion vector and $\boldsymbol{\theta}_m > 0$ is the *route choice function parameter* vector, with θ_{m1} being the scale factor (if present), as in a Logit choice model.

Most often, the choice function $\mathbf{p}_{im}(\bullet)$ in (4) is derived from Random Utility Theory (RUT). Examples are the well-known models belonging to Logit family or Probit and the more recent Gammit and Weibit. If the parameters of the perceived utility pdf do not depend on systematic utility values, the resulting choice function, called *invariant*, is monotone increasing with symmetric semidefinite positive Jacobian.

Flow conservation ensures that flows of all routes connecting the o-d pair i using vehicle type m sum up to demand flow:

$$\mathbf{h}_{im} = d_i q_{im} \mathbf{p}_{im} \forall im, \tag{5}$$

where, with reference to o-d pair i and vehicle type m, $q_{im} \geq 0$ is the proportion of the total demand flow per vehicle type and o-d pair; $d_{im} \geq 0$ is the demand flow; and $\mathbf{h}_{im} \geq 0$ is the route flow vector.

The average number of users on board, possibly different per vehicle type, is introduced to convert demand and route flows measured in users per unit of time into arc flows measured in vehicles per unit of time. Furthermore, a flow equivalence coefficient for each vehicle type needs to be introduced to measure arc flows in RVTs per time unit (and to distinguish the effect on congestion, see Section 2.2.1).

The arc total flows are given by the sum of arc flows plus the arc base flows due to each combination of o-d pair i and vehicle type m over all o-d pairs and vehicle types:

$$\mathbf{f} = \Sigma_i \Sigma_m \eta_{im} \left(\frac{1}{\varphi_m}\right) \mathbf{B}_{im} \mathbf{h}_{im} + \mathbf{f}_z, \tag{6}$$

where, with reference to vehicle type m, $\mathbf{f}_z \geq 0$ is the arc base flow vector; $\varphi_m > 1$ is the *occupancy factor*, that is, the average number of users on board; $\eta_m > 0$ is the *flow equivalence coefficient*, with $\eta_1 = 1$; and $\mathbf{f} \geq 0$ is the arc (total) flow vector.

2.1.2. Arc Flow Function. If the vehicle type demand flows are given. the arc flow function between arc flows and (common) arc costs can be obtained by combining together equations (1)–(6):

$$\mathbf{f} = \mathbf{f}\left(\mathbf{c}; \left(\frac{\eta_m}{\varphi_m}\right), \chi_m, \psi_m, \boldsymbol{\theta}_m \forall m\right). \tag{7}$$

The arc function (7) models the effects of the route choice behavior at a macroscopic scale. It has values in the arc feasibility set S_f, which is nonempty (if the network is connected), compact, and convex. The arc flow function (7) differs from the arc flow function with one vehicle type, since several parameters have been introduced to distinguish the vehicle types; see Section 2.4 for further discussion.

The arc function (7) is a linear combination of route choice functions; thus, it is continuous and *c*. differentiable if all the route choice functions are continuous and *c*. differentiable; moreover, it is monotone nondecreasing with symmetric semi-definite negative Jacobian if all the route choice functions are invariant.

The arc flow function (7) can be computed with or without explicit route enumeration even for very large-scale applications through algorithms derived from graph theory.

2.2. Equilibrium Assignment to Congested Networks. In congested networks, arc costs are assumed depending on arc flows due to congestion. Equilibrium assignment models search for mutually consistent arc flows and costs.

2.2.1. Arc Cost Function. The arc cost function models the driving behavior at a macroscopic scale:

$$\mathbf{c} = \mathbf{c}(\mathbf{f}; \boldsymbol{\kappa}) \geq 0, \quad \forall \mathbf{f} \in S_f, \tag{8}$$

$\boldsymbol{\kappa} > \mathbf{0}$ is the vector of the *arc capacities*, with entries κ_a, say the maximum flow that may traverse arc a, measured consistently with arc flows in RVTs per time unit; in most functions, the arc cost actually depends on the ratio between the arc flow and the capacity, f_a/κ_a; in this case, the capacity plays the role of arc flow scale factor.

Other parameters of the arc cost function are not explicitly introduced.

The cost function is called *separable*; the cost of each arc a, c_a, depends on the corresponding flow, f_a only, *non-separable*, otherwise. All usually adopted cost functions are continuous and *c*. differentiable; in most cases, they are *s*. increasing monotone too; often, but not always, they have symmetric Jacobian, as it occurs for separable cost functions with diagonal Jacobian.

2.2.2. Fixed-Point Models. The above-described arc flow functions (7) can be combined with the arc cost function (8) to specify fixed-point (FP) models for equilibrium assignment:

$$\mathbf{c}^* = \mathbf{c}(\mathbf{f}^*) \in \mathbf{c}(S_\mathbf{f}), \tag{9}$$

$$\mathbf{f}^* = \mathbf{f}\left(\mathbf{c}^*; \left(\frac{\eta_m}{\varphi_m}\right), \chi_m, \psi_m, \boldsymbol{\theta}_m \forall m\right) \in S_f. \tag{10}$$

Equivalent formulations with respect to flows

$$\mathbf{f}^* = \mathbf{f}\left(\mathbf{c}(\mathbf{f}^*); \left(\frac{\eta_m}{\varphi_m}\right), \chi_m, \psi_m, \boldsymbol{\theta}_m \forall m\right) \in S_f, \tag{11}$$

or costs only

$$\mathbf{c}^* = \mathbf{c}\left(\mathbf{f}\left(\mathbf{c}^*; \left(\frac{\eta_m}{\varphi_m}\right), \chi_m, \psi_m, \boldsymbol{\theta}_m \forall m\right)\right) \in \mathbf{c}\left(S_f\right), \tag{12}$$

are often used in the literature.

The above FP models are consistent with those discussed by Cantarella and Di Febbraro (2017) [15] and Cantarella et al. [16, 17]. Existence of equilibrium flows or costs is guaranteed if both the arc cost function and the arc flow function are continuous (if the network is connected). For monotone decreasing arc flow function (7), if the arc cost function (8) is monotone *s.* increasing, uniqueness is guaranteed. The above fixed-point models can be solved for very large-scale applications through algorithms based on the Method of Successive Averages.

2.3. Day-to-Day Dynamic Assignment to Congested Networks. Day-to-day dynamic assignment models try to describe the evolution over time of arc flows and costs.

2.3.1. Further Definitions and Equations. The specification of a model for day-to-day dynamic assignment requires an extension of models for the equilibrium assignment by including submodels of the following:

(i) User memory and learning: how users forecast the level of service that they will experience today, from experience and other sources of information, such as informative systems, about previous days.

(ii) User habit and inertia to change: how users make a choice today, possibly repeating yesterday choice to avoid the effort needed to take a decision, or reconsidering it according to the forecasted level of service.

The arc cost updating relation, modeling user memory and learning, gives the today forecasted route costs with respect to previous day costs. It extends equation (9).

This relation can be specified by an exponential smoothing (ES) filter, say a convex combination of yesterday route forecasted costs and yesterday actual route costs, given by an affine transformation of the yesterday arc costs:

$$\mathbf{x}^k = \beta\, \mathbf{c}\left(\mathbf{f}^{k-1}\right) + (1-\beta)\mathbf{x}^{k-1}, \tag{13}$$

for given $\mathbf{f}^0$, and $\mathbf{x}^0 = \mathbf{c}(\mathbf{f}^0)$, where $\mathbf{x}^{k-1} \geq 0$ is the forecasted arc cost vector, common to all vehicle types, for day $k-1$; $\beta \in\,]0,1[$ is the *cost updating parameter*, that is, the weight given to yesterday actual costs; in the following, the cost updating parameter β is assumed time invariant; it can be different per vehicle type; $\mathbf{f}^{k-1} \geq 0$ is the arc flow vector for day $k-1$; $\mathbf{c}(\mathbf{f}^{k-1}; \boldsymbol{\kappa})$ is the actual arc cost vector, common to all vehicle types, for day $k-1$; and $\mathbf{x}^k \geq 0$ is the forecasted arc cost vector, common to all vehicle types, for day k.

A model of user memory and learning process with explicitly finite memory depth μ can be specified through a moving average, MA (β, μ), filter with one parameter β. At this aim, first the weights of the ES filter are only applied to μ previous days; thus at day t, the summation starts at day $t-\mu+1$; then a scaling factor ensuring that weights ζ_k sum to 1 is applied; the resulting decreasing weights ζ_j for last μ days are given by

$$\zeta_j = \left(\frac{\beta(1-\beta)^{j-1}}{(1-(1-\beta)^{\mu})}\right) \geq 0, \quad \forall j = 1, 2, \ldots \mu, \tag{14}$$

where $\mu > 1$, μ integer, is the *memory depth* and ζ_j is the weight given to the actual cost occurring in any of the μ previous days, $\sum_j \zeta_j = 1$.

The resulting (strict) convex moving average MA(β, μ) filter with one parameter is given by

$$\mathbf{x}^k = \sum_{j=1,\ldots,\mu} \zeta_j \mathbf{c}\left(\mathbf{f}^{k-j}\right), \quad \forall k > \mu, \tag{15}$$

with suitable initialization.

The arc flow updating relation, modeling user habit and inertia to change, gives the today arc flow with respect to forecasted costs and previous day flows. It extends equation (10).

This relation can be specified by an exponential smoothing (ES) filter, say a convex combination of yesterday arc flows due to users who do not reconsider their yesterday choice and today arc flows due to users who reconsider their yesterday choice:

$$\mathbf{f}^k = \alpha \mathbf{f}\left(\mathbf{x}^{k-1}; \left(\frac{\eta_m}{\varphi_m}\right), \chi_m, \psi_m, \theta_m \forall m\right) + (1-\alpha)\mathbf{f}^{k-1} \forall k, \tag{16}$$

where $\alpha \in]0,1[$ is the *choice updating parameter*, that is, the proportion of users reconsidering yesterday choice; in the following, the choice updating parameter α is assumed day-invariant; it can be different per vehicle type.

2.3.2. Deterministic Process Models. The above-described arc flow updating relation (16) can be combined with the arc cost updating relation (13) or (15) to specify deterministic process (DP) models for day-to-day dynamic assignment.

ES/ES DP models are given by the following equations:

$$\mathbf{x}^k = \beta c\left(\mathbf{f}^{k-1}\right) + (1-\beta)\mathbf{x}^{k-1}, \quad \forall k, \tag{17}$$

$$\mathbf{f}^k = \alpha \mathbf{f}\left(\mathbf{x}^{k-1}; \left(\frac{\eta_m}{\varphi_m}\right), \chi_m, \psi_m, \boldsymbol{\theta}_m \forall m\right) + (1-\alpha)\mathbf{f}^{k-1} \forall k, \tag{18}$$

for given $\mathbf{f}^0$, and $\mathbf{x}^0 = \mathbf{c}(\mathbf{f}^0)$.

The DP model (11–14) can easily be rewritten as a proper Markovian DP by putting equation (13) into (15), but still keeping equation (13). The state variables of DP model (11, 14) are ($\mathbf{x}^k$, $\mathbf{f}^k$); the updating parameters are α and β; other parameters are demand flow, d, and any other parameter in the arc flow function and in the arc cost function.

MA/ES DP models are given by the following equations:

$$\mathbf{x}^k = \sum_{j=1,\ldots,\mu}, \zeta_j \mathbf{c}\left(\mathbf{f}^{k-j}\right), \quad \forall k > \mu, \tag{19}$$

$$\mathbf{f}^k = \alpha \mathbf{f}\left(\mathbf{x}^{k-1}; \left(\frac{\eta_m}{\varphi_m}\right), \chi_m, \psi_m, \boldsymbol{\theta}_m \forall m\right) + (1-\alpha)\mathbf{f}^{k-1} \forall k, \tag{20}$$

where

$$\zeta_j = \frac{\beta(1-\beta)^{j-1}}{(1-(1-\beta)^{\mu})} \geq 0, \quad \forall j = 1, 2, \ldots, \mu, \tag{21}$$

with suitable initialization.

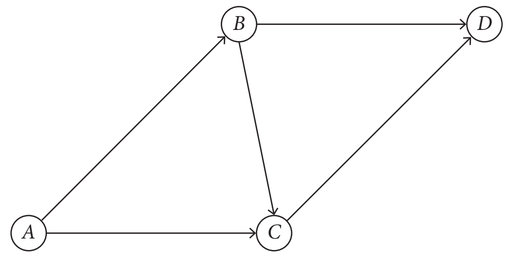


Figure 1: The toy network.

Table 1: Parameters of arc cost functions.

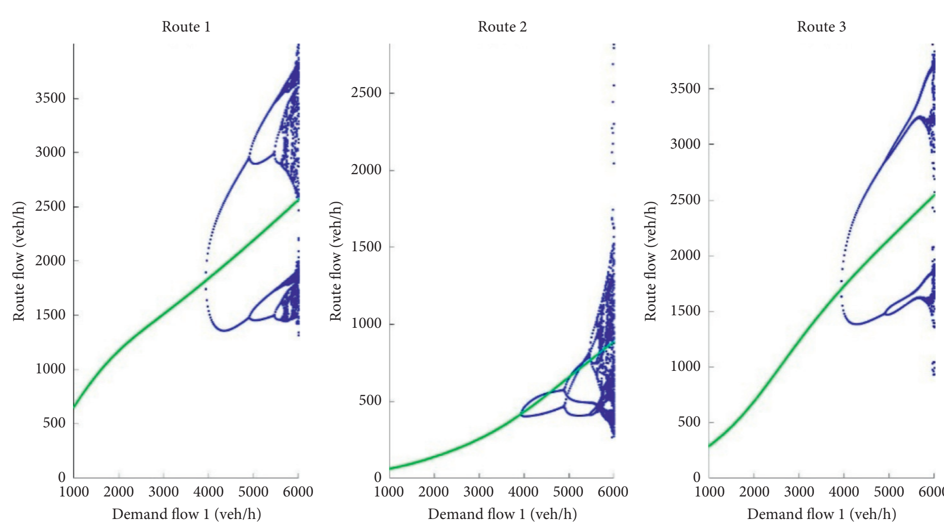
Arc	c_{0a}	κ_a	ν_1	ν_2
1	15	2400	2.5	4
2	8	3600	2.0	4
3	12	2400	1.5	4
4	24	3600	2.0	4
5	15	3600	1.5	4

Table 2: Parameters of vehicle types.

Type	(η_m/μ_m)	χ_m
1 (TVs)	1.0	1.0
2 (AVs)	0.8	0.9

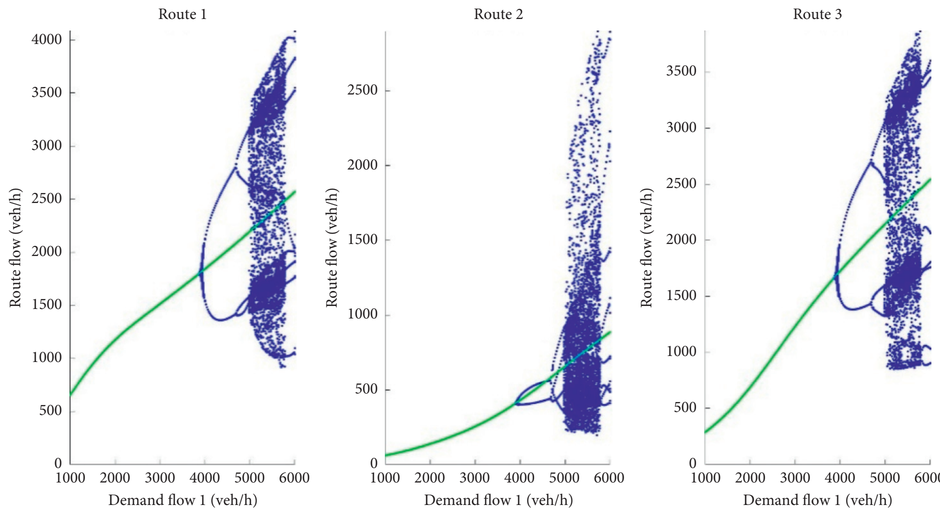
Table 3: Bifurcation demand flow, reference scenario $\theta_{TV} = 7.0$.

Scenario $\theta_{TV} = 7.0$	q_m		DP-ES/ ES	DP-MA/ES				
				$\mu = 2$	$\mu = 3$	$\mu = 4$	$\mu = 5$	$\mu = 6$
	TVs	AVs	dBif	dBif	dBif	dBif	dBif	dBif
0	1	0	3917	3875	3771	3917	3896	3917



(a)

Figure 2: Continued.



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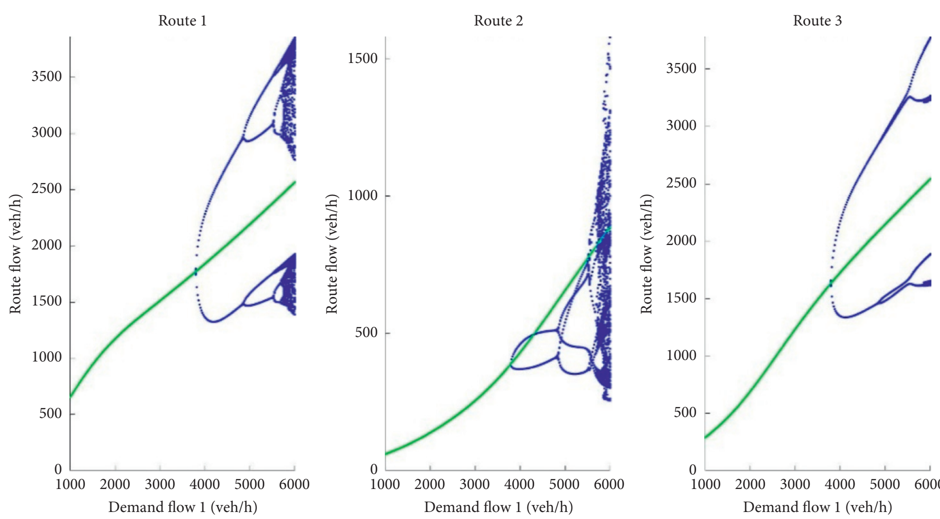


(c)

FIGURE 2: (a) Bifurcation diagrams of route flows, reference scenario, ES/ES. (b) Bifurcation diagrams of route flows, reference scenario, MA/ES $\mu = 2$. (c) Bifurcation diagrams of route flows, reference scenario, MA/ES $\mu = 3$.

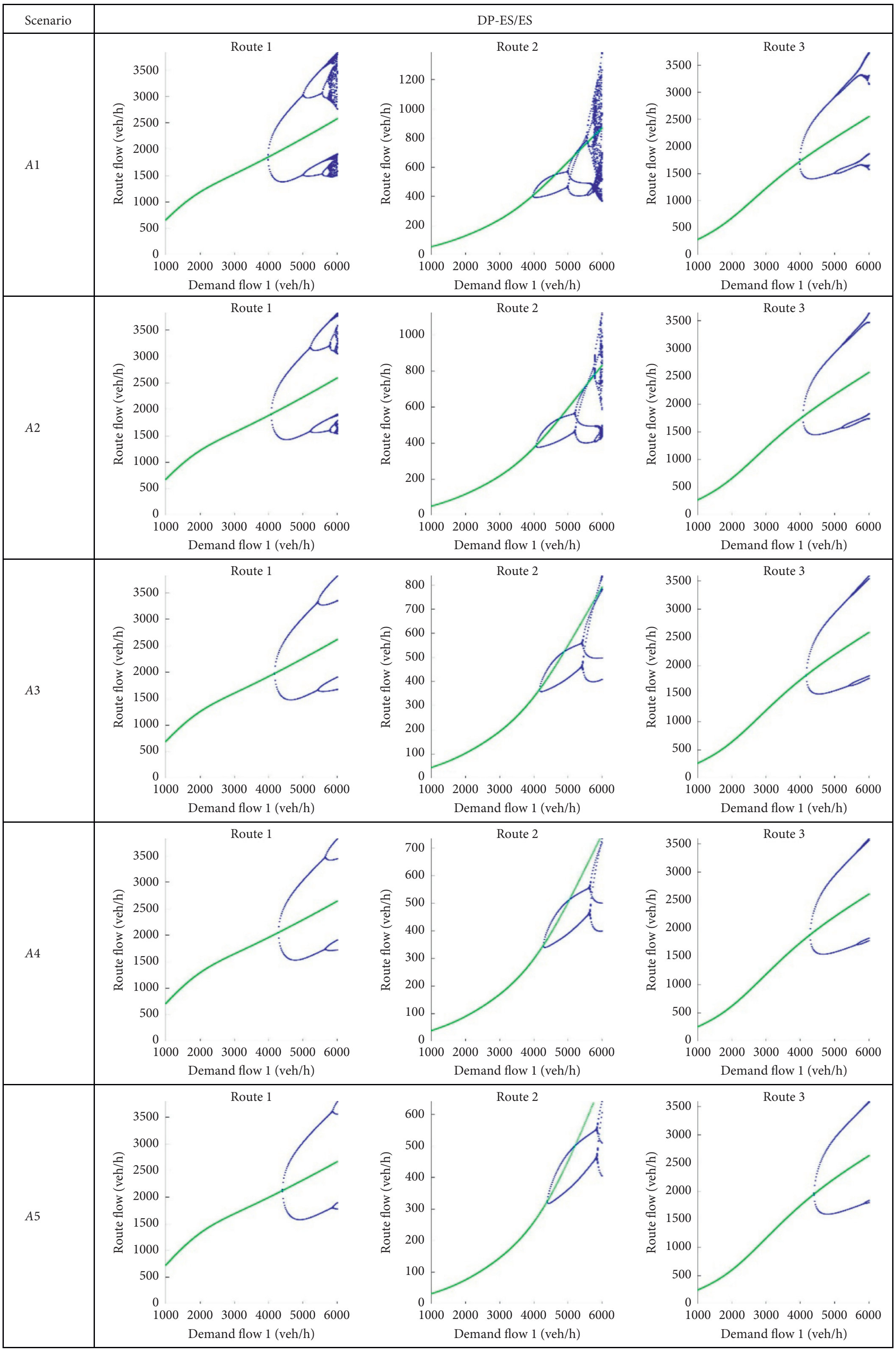
TABLE 4: Bifurcation demand flow, AVs scenarios A $\theta_{AV} = 7.0$ and $\theta_{AV} = 4.7$

Scenario $\theta_{TV} = 7.0$ $\theta_{AV} = 4.7$	q_m TVs	AVs	DP-ES/ES dBif	DP-MA/ES $\mu = 2$ dBif	$\mu = 3$ dBif	$\mu = 4$ dBif	$\mu = 5$ dBif	$\mu = 6$ dBif
A1	0.9	0.1	3958	3917	3813	3958	3938	3958
A2	0.7	0.3	4063	4000	3917	4063	4021	4063
A3	0.5	0.5	4146	4104	4000	4146	4125	4146
A4	0.3	0.7	4271	4229	4104	4271	4229	4271
A5	0.1	0.9	4375	4333	4229	4375	4354	4375

The fixed-point states of both DP model (11, 14) and (13,14) are equivalent to the equilibrium states as defined by FP model (9, 10). Applying techniques from the theory of discrete-time nonlinear dynamic systems, the above DP models can be used to study the local stability of each fixed-point state, say whether it is an attractor. Moreover, a bifurcation analysis can be carried to single out which attractor is reached by the evolution over time when an input data or a parameter is changed.

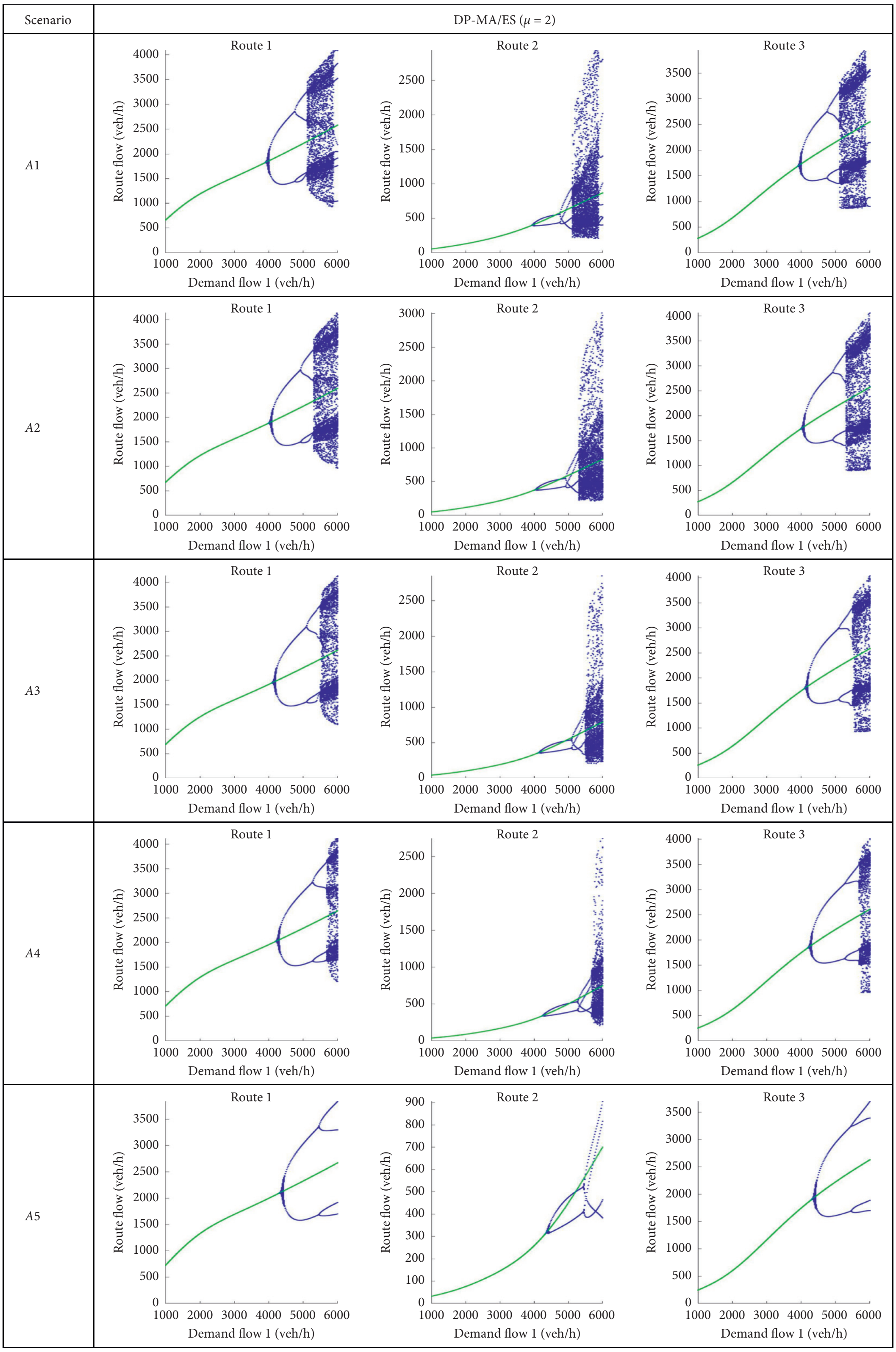
The above DP models are a generalization of those already in literature due to the many parameters introduced in the arc flow function to distinguish the vehicle types; see Section 2.4 for further discussion.

2.4. Main Parameters Suitable to Distinguish Vehicle Types. All assignment models discussed above include several parameters useful to distinguish vehicle types; they are



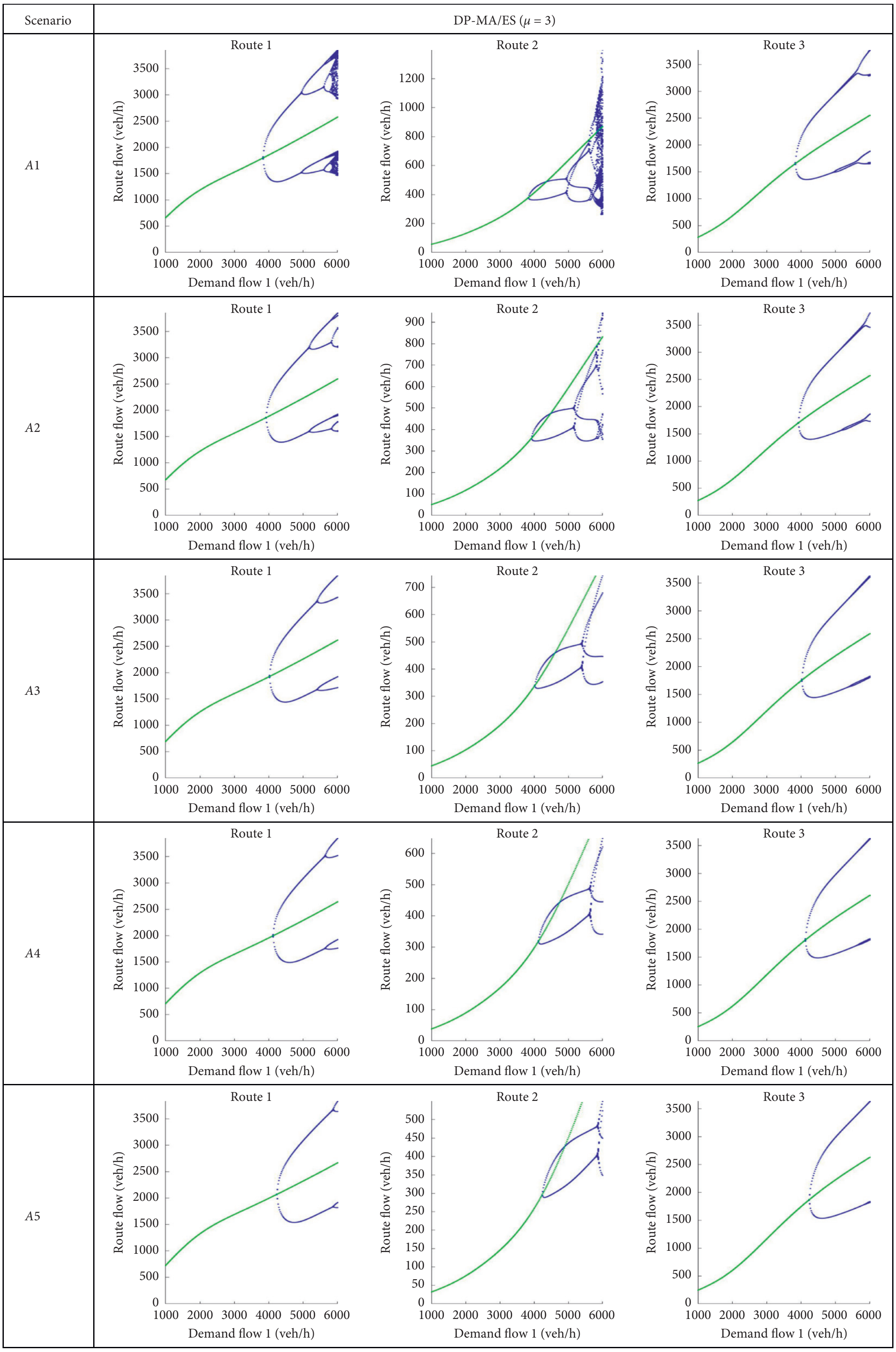
(a)

Figure 3: Continued.



(b)

Figure 3: Continued.



(c)

Figure 3: (a) Bifurcation diagrams of route flows, scenarios A, ES/ES. (b) Bifurcation diagrams of route flows, scenario A, MA/ES $\mu = 2$. (c) Bifurcation diagrams of route flows, reference scenario, MA/ES $\mu = 3$.

Table 5: Bifurcation demand flow, AVs scenarios B $\theta_{TV} = 7.0$ and $\theta_{AV} = 2.3$

Scenario $\theta_{TV} = 7.0$ $\theta_{AV} = 2.3$	q TVs	AVs	DP-ES/ES dBif
B1	0.9	0.1	3833
B2	0.7	0.3	3708
B3	0.5	0.5	3625
B4	0.3	0.7	3563
B5	0.1	0.9	3521

discussed below in detail. As already noted, the reference vehicle type (RVT) has index $m = 1$, usually privately owned human-driven fossil fuelled passenger cars.

2.4.1. Supply Parameters. Supply parameters are mainly related to congestion and level-of-service provided. Arc flows as well as arc capacities are measured consistently with arc flows in RVTs per time unit; thus

> the *flow equivalence coefficient*, $\eta_m > 0$ with $\eta_1 = 1$, is introduced to model different effects of congestion with respect to capacity, for instance, due to average length and/or width, e.g., passenger vs. vans vs. trucks, ...; it is generally assumed that AVs have less effect on congestion than TVs.

It is worth noting that the *capacity of arc a*, $\kappa_a > 0$, say the maximum flow that may traverse arc a, is measured RVTs per time unit; thus its value is independent of the traffic composition, and mostly depends on street features, such as width, slope, etc. In most cost functions, the arc cost actually depends on the ratio between the arc flow and the capacity, f_a/κ_a; in this case, the capacity plays the role of arc flow scale factor.

Different types of vehicles may experience different costs on the same arc, thus

> the *cost equivalence coefficient*, $\chi_m > 0$ with $\chi_1 = 1$, is introduced to model, for example, different speeds, etc. According to the literature, it can be assumed that advanced vehicles may experience smaller generalized costs than TVs, e.g., energy consumption, travel time, and value of time [18–22].

Different types of vehicles may have different arc access allowance, e.g., lanes or areas reserved to e-vehicles, and/or experience different tolls, e.g., congestion tools; thus

> the *arc specific cost* of traversing arc a per vehicle type m, $\mathbf{c}_{Zma}$ collected in vector $\mathbf{c}_{Zm} \geq 0$, is summed to the generic cost affected by congestion, say $\chi_m\ \mathbf{c}_a$.

2.4.2. Demand Parameters. Demand parameters are mainly related to user behavior. Users moving with different types of vehicles along the same route may experience different (dis) utility with respect to the same route cost; thus in the utility function

> the *utility scale parameter*, $\psi_m > 0$ such that the term $\psi_i\ \mathbf{w}_i$ is dimensionless, is introduced to model on-board comfort, privacy, etc.; it is generally assumed that AVs experience smaller disutility than TVs since other activities than driving may be carried out when on-board.

Any route choice function, usually but not necessarily derived from Random Utility Theory (RUT), contains some parameters modeling dispersion and other features of the perceived utility distribution that may be different per vehicle type; thus

> the *route choice function parameter*, vector, $\boldsymbol{\theta}_m > 0$, is introduced; in some models, as the Logit, a scale factor θ_{m1} is present, often proportional to the perceived utility dispersion.

As far as calibration is concerned, the utility scale parameter θ_{m1} may not be distinguished from the utility parameter ψ_m; thus their ratio plays the role of a single parameter; on the other hand, the meanings of the two parameters are different and this condition should carefully be taken into account in scenario design.

To convert demand and route flows measured in users per unit of time into arc flows measured in vehicles per unit of time,

> the *occupancy factor*, say the average number of users on-board $\varphi_m \geq 1$, is introduced; for example, it is expected that advanced vehicles may allow a higher occupancy factor than other types, for instance, in large-scale ride-sharing with autonomous vehicles.

From the mathematical point of view, each ratio (η_m/μ_m) plays the role of a single parameter; on the other hand, the meanings of the two parameters are different and this condition should carefully be taken into account in scenario design.

Vehicle types may also be differentiated with respect to the route choice function $\mathbf{p}_{im}(\bullet)$.

2.4.3. Dynamic Updating Parameters. Dynamic updating parameters, the choice updating parameter, the cost updating parameter, and the memory depth in MA filters, are related to memory and learning modeling; they might be differentiated per vehicle type. But, in this case the fixed-point stability conditions and bifurcation analysis already in literature (see Cantarella et al. (2019) [14] for details) no longer apply; extension of these results may easily be conceived from the theoretical point of view, but can hardly be derived in practice since involving the closed form solution of high-degree polynomial equations with complex coefficients.

3. Numerical Examples

This section discusses the results of the application of the ES/ES and MA/ES DP models to a toy network to study the effects on fixed-point state stability of introducing AVs.

3.1. Input Data and Assumptions. A toy network with 4 nodes A, B, C, D, and 5 arcs, 1 = (A, C), 2 = (B, D), 3 = (B, C), 4 = (A, B), 5 = (C, D), is considered (Figure 1).

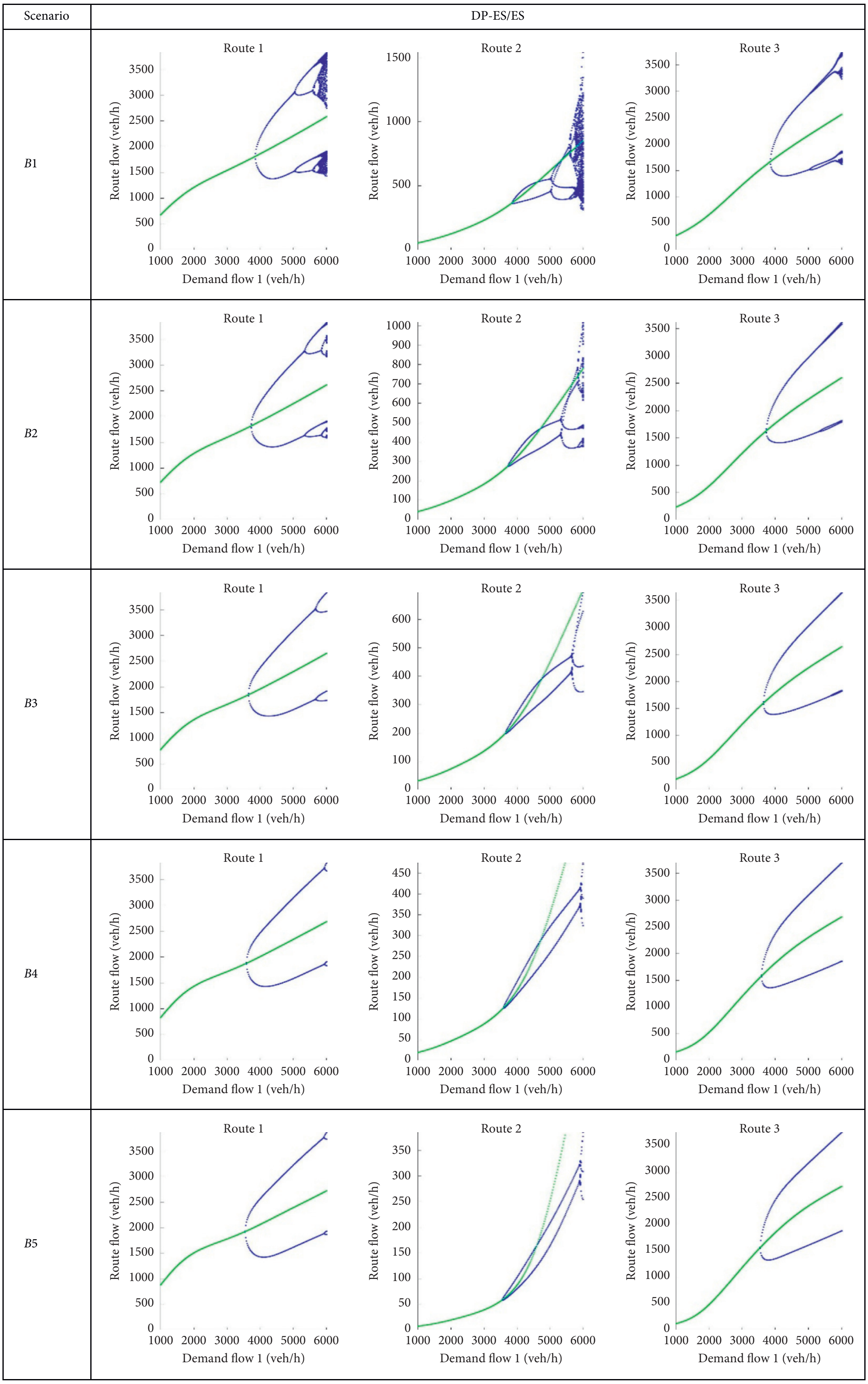


Figure 4: Bifurcation diagrams of route flows, scenarios A, ES/ES with $\theta_{TV} = 7.0$ and $\theta_{AV} = 2.3$.

Table 6: Parameters of vehicle types.

Type	(η_m/μ_m)	χ_m
1 (TVs)	1.0	1.0
2 (AVs)	0.7	0.8

Table 7: Bifurcation demand flow, AVs scenarios C: $\theta_{\mathrm{TV}} = 7.0$ and $\theta_{\mathrm{AV}} = 4.7$; $(\eta_m/\mu_m) = 0.7$ and $\chi_m = 0.8$

Scenario $\theta_{\mathrm{TV}} = 7.0$ $\theta_{\mathrm{AV}} = 4.7$ $(\eta_m/\mu_m) = 0.7$ $\chi_m = 0.8$	q		DP-ES/ES
	TVs	AVs	dBif
*C*1	0.9	0.1	4021
*C*2	0.7	0.3	4250
*C*3	0.5	0.5	4500
*C*4	0.3	0.7	4771
*C*5	0.1	0.9	5104

Table 8: Bifurcation demand flow, AVs scenarios *D*: $\theta_{\mathrm{TV}} = 7.0$ and $\theta_{\mathrm{AV}} = 2.3$; $(\eta_m/\mu_m) = 0.7$ and $\chi_m = 0.8$

Scenario $\theta_{\mathrm{TV}} = 7.0$ $\theta_{\mathrm{AV}} = 2.3$ $(\eta_m/\mu_m) = 0.7$ $\chi_m = 0.8$	q		DP-ES/ES
	TVs	AVs	dBif
*D*1	0.9	0.1	3917
*D*2	0.7	0.3	3938
*D*3	0.5	0.5	3979
*D*4	0.3	0.7	4042
*D*5	0.1	0.9	4125

The pair of nodes (A, D) is the only o-d pair, connected by 3 routes: $1 = (\mathrm{ACD})$, $2 = (\mathrm{ABD})$, and $3 = (\mathrm{ABCD})$ (paths are numbered after arcs 1, 2, and 3, respectively)

BPR-like separable arc cost functions are considered (no arc specific cost is considered):

$$c_a = c_{0\mathrm{a}}\left(1 + \nu_1\left(\frac{f_a}{\kappa_a}\right)^{\nu_2}\right), \tag{22}$$

where $\nu_1 > 0$ is the congestion multiplier: how much greater the arc cost is when flow is equal to capacity with respect to zero flow cost; this parameter is to be calibrated against data, in urban applications, 2 is an often used value; $\nu_2 > 0$ is the congestion exponent: how fast the arc cost increases against flow; this (integer) parameter too is to be calibrated against data; in urban applications, 2 is an often used value; as this value increases, the shape of the function tends to a vertical asymptote; $c_{0\mathrm{a}} > 0$ is the cost when flow is zero.

Costs are given by travel times and measured in minutes; all flows and capacities are measured in vehicles per hour. Values of parameters are given in Table 1.

Given the values of capacities in Table 1, the max flow that can traverse the network from A to D is 6000. Given the zero flow arc costs, the cost of the shortest path from A to D with is $w_{\mathrm{MINo}} = 30$.

User may travel through either of two types of vehicles: TVs (1) and AVs (2). In both cases, the route choice behavior for user travelling is modeled by a Logit choice function:

$$p_r = \frac{\exp\left(\nu_r/\theta_m\right)}{\sum_{k\in Ri}\exp\left(\mathrm{v}_k/\theta_m\right)}, \quad \text{for any route r}, \tag{23}$$

where the dispersion parameter $\theta_m > 0$ per vehicle type m is proportional to the standard deviation σ of the route perceived utility modeled as a random variable, according to the Random Utility Theory. It includes the utility scale parameter ψ_m.

Vehicle types are also distinguished with respect to ratio (η_m/μ_m) and cost equivalent χ_m parameters, with TVs (1) being the RVT, as given in Table 2, where AVs are assumed having a beneficial effect on congestion and being faster.

3.2. Results of a Bifurcation Analysis. To study the effects of introducing of AVs on fixed-point state stability, the results of a bifurcation analysis with respect to the value of demand flow d are presented below varying the vehicle type proportions q_m as well as the dispersion parameters θ_m.

The evolution over time of the system is described by the DP models in Section 2.3, say ES/ES and MA/ES with $\mu = 2, 3, 4, 5, 6$ to compare the effect of different memory and learning filters. In all scenarios analyzed below, $\alpha = 0.5$ and, $\beta = 0.6$.

Since the arc cost functions are continuous and monotone increasing and the Logit choice functions are continuous and invariant (dispersion parameter θ_m does not depend on the systematic utility v_{im}), there exists exactly one fixed-point state, equivalent to the equilibrium pattern.

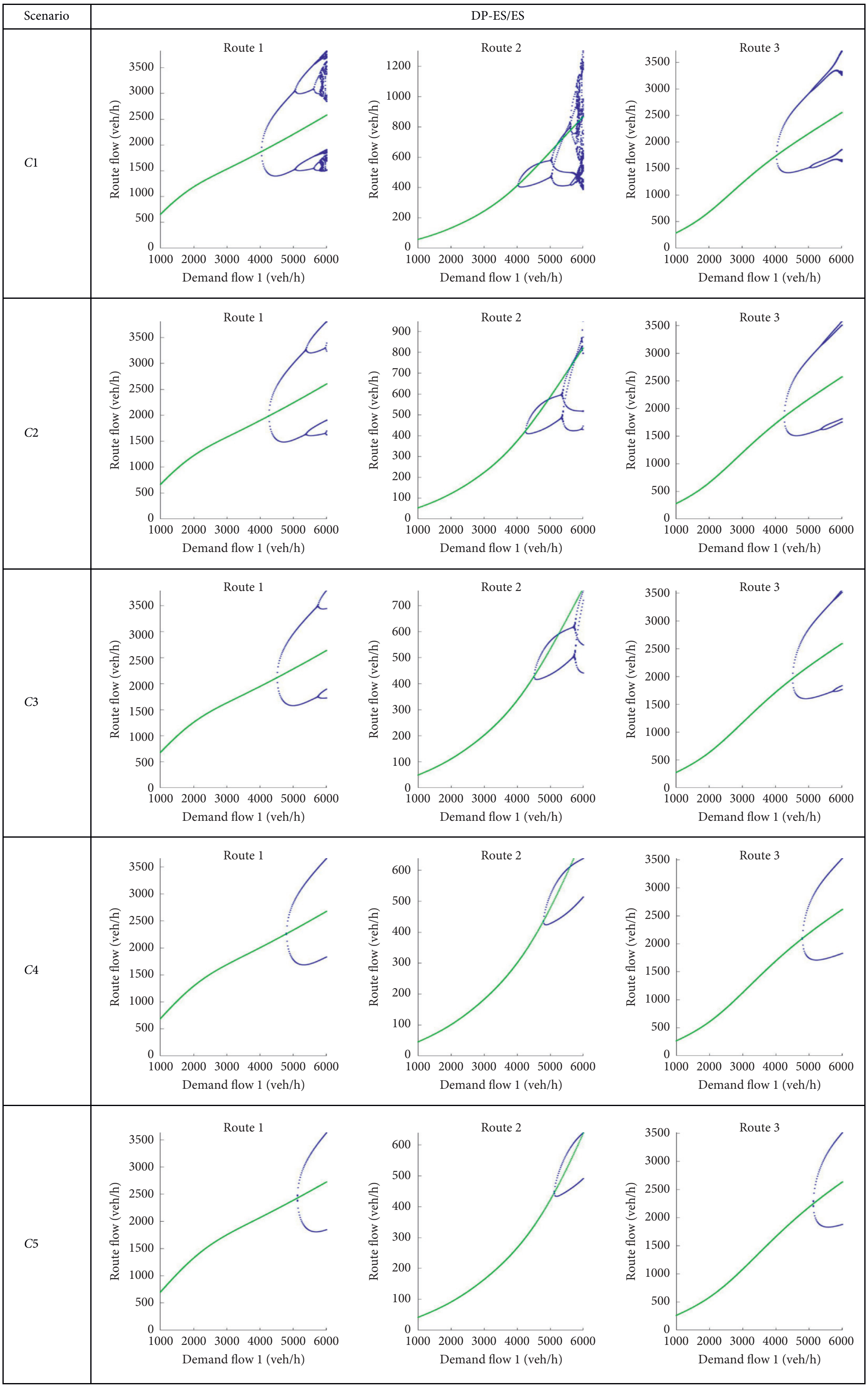


Figure 5: Bifurcation demand flow, AVs scenarios C: $\theta_{TV} = 7.0$ and $\theta_{AV} = 4.7$; $(\eta_m/\mu_m) = 0.7$ and $\chi m = 0.8$.

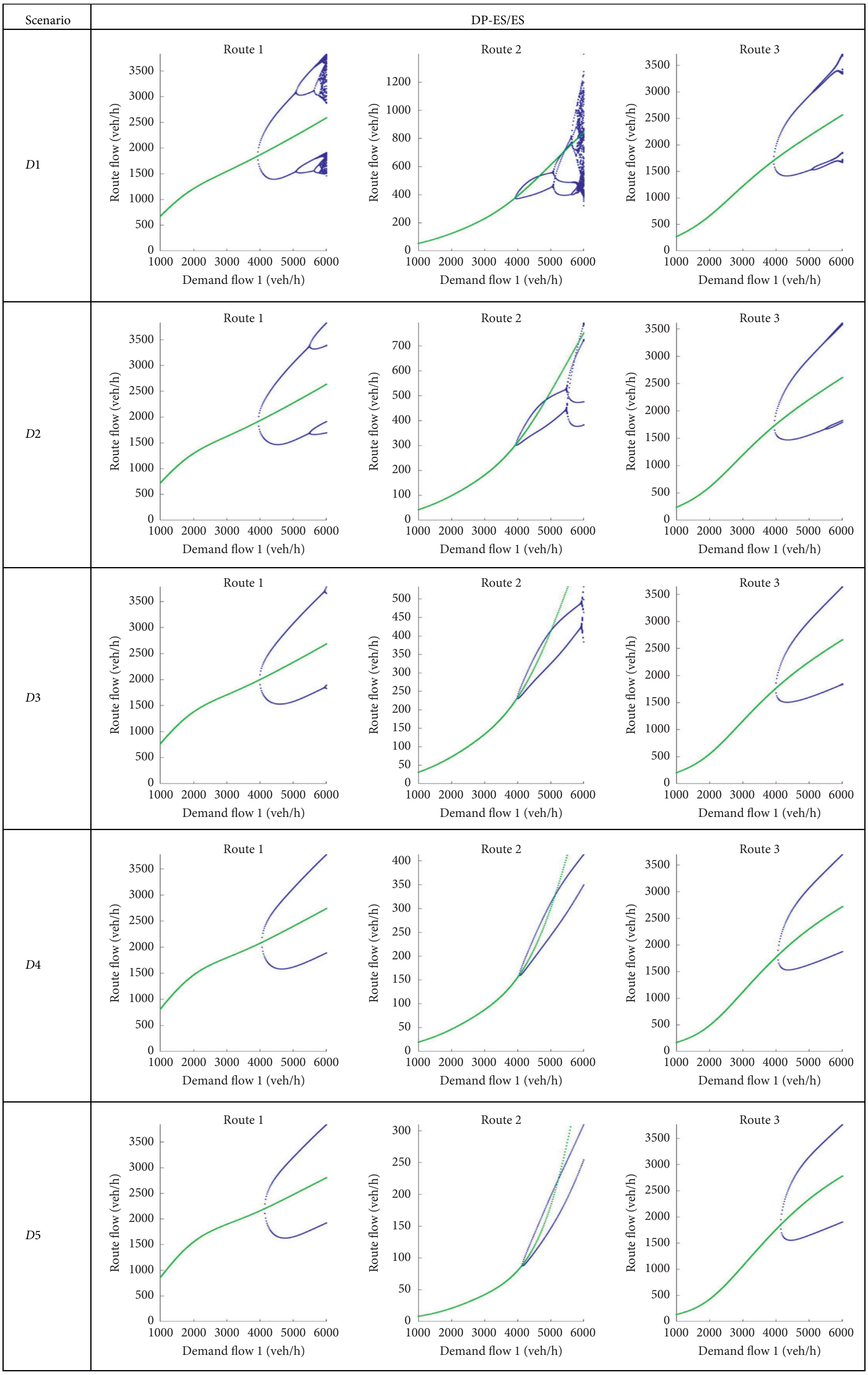


FIGURE 6: Bifurcation demand flow, AVs scenarios C: $\theta_{\mathrm{TV}} = 7.0$ and $\theta_{\mathrm{AV}} = 2.3$; $(\eta_m/\mu_m) = 0.7$ and $\chi\mathrm{m} = 0.8$.

Moreover, it can be proven that flip bifurcations only may occur since the arc cost functions are monotone increasing and separable (see Cantarella et al. 2019 [14]).

3.2.1. Reference Scenario: All TVs. In the reference scenario, only TVs are considered. The dispersion parameter is $\theta_{\mathrm{TV}} = 7.0$, corresponding to 30% variation coefficient with respect to w_{MINo}. Table 3 shows the values of demand flow, dBif, at which the flip bifurcation occurs: the unique fixed-point stable becomes unstable. The values of dBif have been found through numerical search.

They have also been verified through the stability conditions based on eigenvalues of the Jacobian of the ES/ES DP model (details in Cantarella et., 2019 [14]). Analogous conditions for the MA/ES DP model are not yet available.

Regarding the memory and learning filters, greater memory depth μ has a stabilizing effect, as expected, and results for MA tend to ES as the μ increases. Rather surprisingly, the convergence of the results from the MA filter to those from the ES filter as the memory depth μ increases is not smooth but dBif oscillates between odd and even values, dBif for an odd value of μ being smaller than the dBif for the two near even values. This condition is worth further research work.

For values of demand flow greater than dBif, the system evolves towards a 2-periodic attractor as shown by bifurcation diagrams (Figure 2) with respect to values of demand flow up to 6000 for ES and MA with $\mu = 2$ or 3 only, the green line showing the equilibrium route flow.

3.2.2. Scenarios A and B: Increasing Proportion of AVs. In scenarios A, the effect of increasing the AVs proportion is analyzed. The dispersion parameter of TVs is still $\theta_{\mathrm{TV}} = 7.0$, but the dispersion parameter of AVs is $\theta_{\mathrm{AV}} = 4.7$, corresponding to 20% variation coefficient with respect to w_{MINo}, since AVs may have access to information reducing uncertainty both of users and modelers. Table 4 shows the values of demand flow, dBif, at which the flip bifurcation occurs. Bifurcation diagrams are reported in Figure 3; as expected, flip bifurcations occur towards periodic and aperiodic attractors.

Increasing the AVs proportion has a positive effect on fixed-point stability, since the bifurcation demand flow increases reducing the set of values of demand flow for which the fixed point is not stable. Indeed, AVs have less effect on congestion as modeled by the flow equivalent flow coefficient (including the occupancy factor) and smaller costs as modeled by the cost equivalent parameter. In all scenarios, great memory depth μ has a stabilizing effect.

The beneficial effect of AVs may vanish if the dispersion among users is reduced to $\theta_{\mathrm{AV}} = 2.3$, corresponding to 10% variation coefficient with respect to w_{MINo}. Table 5 shows, for ES/ES DP only, that the values of demand flow dBif at which the flip bifurcation occurs reduce as the AVs proportion increases, increasing the values of demand flow for which the fixed point is not stable. Indeed, reducing dispersion may lead fixed-point states towards instability. Bifurcation diagrams are reported in Figure 4.

3.2.3. Scenarios C and D: Decreasing AVs Vehicle Type Parameters. In these scenarios, the vehicle types parameters for AVs have been reduced. In Table 6, the values of the AVs parameters adopted have been reported.

The DP-ES-ES cases have been analyzed with these new parameters both for $\theta\mathrm{AV} = 4.7$ (Scenario C) that for $\theta\mathrm{AV} = 2.3$ (Scenario D). Results are shown in Tables 7 and 8 for Scenario C and Scenario D, respectively.

In Scenario C, the same general trend observed in Scenario A (where the same $\theta_{\mathrm{AV}} = 4.7$ is used) can be observed (i.e., increasing the AVs proportion has a positive effect on fixed-point stability, since the bifurcation demand flow increases reducing the set of values of demand flow for which the fixed point is not stable). In comparison with Scenario A, higher values of dBif can be observed in this Scenario C; thus, reducing the AVs parameters of vehicle types, a positive effect can be observed on the system. Similar results are highlighted comparing Scenario D (see Table 8) with Scenario B, where the same $\theta_{\mathrm{AV}} = 2.3$ is adopted.

Bifurcation diagrams, for Scenarios C and D, are reported in Figures 5 and 6.

4. Conclusions

In this paper, deterministic process models for day-to-day dynamic multivehicle assignment, including fixed-point models for equilibrium assignment as a special case, have been analyzed.

Furthermore, results of an application to a toy network are discussed and a comparison has also been carried out between moving average and exponential smoothing filters for modeling user memory and learning. Similar results may be obtained through bifurcation analysis with respect to other parameters, such the occupancy factor or the updating parameters α and β.

Generally, AVs have less impact on congestion than TVs do, but, even on a very simple network application resulted in that AVs may not have a positive effect of equilibrium stability since negative effect of reducing dispersion may be greater than positive effects on congestion.

To apply the proposed methodological framework to a real case-study (network and vehicle characteristics), a parameter calibration against real data is required. However, as of today, proper data are not available in the literature.

Worthy of further research efforts is the application of the proposed framework to real-world cases that will become increasingly important in the near future to evaluate the impact of new vehicle types, and stability conditions for MA/ES DP with $\mu = 2$. Moreover, a detailed and effective modeling of the effects of shared vehicles with public access, so-called robotaxi, is still an open issue.

Data Availability

No real data have been used in the paper. Theoretical findings are grounded on simulation experiments in a laboratory network.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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