

## Research Article

# Quasi-Synchronization of Nonidentical Fractional-Order Memristive Neural Networks via Impulsive Control

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This paper investigates the quasi-synchronization of nonidentical fractional-order memristive neural networks (FMNNs) via impulsive control. Based on a newly provided fractional-order impulsive systems comparison lemma, the average impulsive interval definition, and the Laplace transform, some quasi-synchronization conditions are obtained with fractional order  $0 < \alpha < 1$ . In addition, the error convergence rates and error boundary are also obtained. Finally, one simulation example is presented to show the validity of our results.

## 1. Introduction

Memristor was predicted as the fourth circuit element describing the relationship between magnetic flux and voltage by professor Chua [1] in 1971. This component was established successfully by HP Laboratories [2, 3] in 2008. Memristors are used instead of traditional resistive elements to simulate brain neuron synapses and build the memristor neural networks (MNNs) model [4–9] because they have memory characteristics. Now, it has been widely used in the field of information processing, associative memory, and image processing [10–12].

Fractional calculus originated in the 17<sup>th</sup> century and is a generalization of integer-order calculus operations to arbitrary order calculus operations [13]. Scholars introduce fractional calculus into the study of MNNs and formed fractional-order memristive neural networks (FMNNs) model [14–16]. FMNNs can describe the memory properties of neurons more accurately and achieve many results in synchronization and stability [17–20]. The global Mittag–Leffler stabilization of a class of FMNNs with time delays was discussed under a state feedback control in [17]. Chen and Ding [20] via a sliding mode controller and fractional-order Lyapunov direct method studied projective synchronization of nonidentical FMNNs.

On the other hand, synchronization means several systems share a common dynamical process. However, connection weights of FMNNs changed according to the state dynamics and the synchronization of drive-response systems will be destroyed. When the parameter mismatches are small enough, we can control the synchronization error in a small region, which is called quasi-synchronization [21–24]. Recently, the impulsive control method has been widely used in the quasi-synchronization of chaotic systems. He et al. used a distributed impulsive control studying the quasi-synchronization problem of drive-response heterogeneous networks in [21] and the number of controlled nodes was also considered. Tang et al. [24] derived criteria for quasi-synchronization of nonidentical coupled Lur'e networks by delayed impulsive comparison principle, where synchronization errors for different impulsive effects with different functions were evaluated. Therefore, it is very meaningful to study the synchronization problem in the case of parameter mismatch [25–28].

To the best of our knowledge, the existing results about quasi-synchronization are concentrated on the integer-order systems, results about fractional-order systems are very few. Motivated by this, we use an impulsive controller to study the quasi-synchronization of FMNNs with parameter

mismatches. The main innovation points of this paper are as follows: (1) a generalized fractional-order comparison lemma is provided in this paper, which plays a central role in the prove. (2) Our results connected with the fractional-order  $\alpha$ , which more reflects the characteristics of FMNNs.

*1.1. Notations.*  $\mathbb{R}$  is the space of real number,  $\mathbb{N}$  is the space of nonnegative integers,  $\mathcal{C}$  is the space of complex number,  $\mathbb{R}^n$  denotes a dimensional Euclidean space, and  $\mathbb{R}^{m \times n}$  is the set of  $m \times n$  real matrices. For any matrix  $B$ ,  $B^T$  denotes the transpose of  $B$  and  $I$  denotes the identity matrix. For any algebraic operations, matrices are assumed to have compatible dimensions. The notation  $\|\cdot\|$  denotes the matrix 2-norm or the Euclidean vector norm. Let  $\mathcal{C}([-\infty, 0], \mathbb{R}^n)$  denotes the family of continuous functions from  $[-\infty, 0]$  to  $\mathbb{R}^n$ .  $\lambda_{\min}(A)$  and  $\lambda_{\max}(A)$  denote the minimum and maximum eigenvalue of  $A$ , respectively.  $\text{sign}(\cdot)$  is the symbolic function;  $\overline{\text{co}}\{b, c\}$  represents the closure of convex hull generated by real numbers  $b$  and  $c$ . Define  $\chi(t)$  as a continuous function except at some finite number of points  $t_k$  at which  $\chi(t_k^+) = \chi(t_k)$  and  $\chi(t_k^-)$  exist; then the set of piecewise right continuous function  $\chi(t)$  is defined as  $\text{PC}(I) = \{\chi | \chi: [-\tau, \infty) \rightarrow \mathbb{R}^l\}$ .

## 2. Preliminaries and System Description

*2.1. Caputo Fractional-Order Calculus and Mittag-Leffler Function.* Caputo fractional operator plays an important role in the fractional systems and has been more practical in physical than the Riemann-Liouville fractional operation.

Therefore, we use Caputo fractional operator as the main tool in this paper.

*Definition 1* (see [13]). The fractional integral of order  $\alpha$  for a function  $f(t)$  is defined as

$${}_0 I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad (1)$$

where  $t \geq 0$ ,  $\alpha > 0$ , and  $\Gamma(\alpha) = \int_0^{+\infty} e^{-t} t^{\alpha-1} dt$ .

*Definition 2* (see [13]). The Caputo fractional derivative of order  $\alpha$  for a function  $f(t)$  is defined by

$${}_0 D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (2)$$

where  $n-1 < \alpha < n$ ;  $n$  is a positive integer. The Laplace transform of the Caputo fractional-order derivative is

$$\mathcal{L}({}_0 D_t^\alpha f(t)) = s^\alpha \mathcal{L}(f(t)) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \quad (3)$$

where  $\alpha > 0$ ,  $n = [\alpha] + 1$ .

**Lemma 1.** Let  $Z_1(s)$  and  $Z_2(s) \in \mathbb{P}\mathbb{C}(1)$  be jumping discontinuity at  $s = s_k, s \geq 0, Z_1(s_k^+)$ , and  $Z_2(s_k^+)$  exist,  $Z_1(s_k) = Z_1(s_k^-)$ ,  $Z_2(s_k) = Z_2(s_k^-)$ . The initial value  $Z_1(s) = Z_2(s) = h(0) \geq 0$ . If there are constants  $c_1, c_2 > 0, 0 < \alpha < 1$ , and  $\mu > 0$ , such that

$$\{ {}_0 D_t^\alpha Z_1(s) \leq c_1 Z_1(s) + c_2, \quad s \neq s_k, Z_1(s_k^+) \leq \mu Z_1(s_k^-), \quad k \in \mathbb{N}, \quad (4)$$

$$\{ {}_0 D_t^\alpha Z_2(s) \geq c_1 Z_2(s) + c_2, \quad s \neq s_k, Z_2(s_k^+) = \mu Z_2(s_k^-), \quad k \in \mathbb{N}, \quad (5)$$

and  $Z_1(s) \leq Z_2(s)$  for  $s \leq 0$ , then  $Z_1(s) \leq Z_2(s)$  for  $s \geq 0$ .

*Proof.* From (4) and (5), we have

$$Z_1(s) \leq h(0) E_\alpha(c_1 s^\alpha) + \int_0^s (s-t)^{\alpha-1} \times E_{\alpha,\alpha}(c_1 (s-t)^\alpha) c_2 dt, \quad (6)$$

$$Z_2(s) \geq h(0) E_\alpha(c_1 s^\alpha) + \int_0^s (s-t)^{\alpha-1} \times E_{\alpha,\alpha}(c_1 (s-t)^\alpha) c_2 dt. \quad (7)$$

We first prove

$$Z_1(s) \leq Z_2(s), \quad [0, t_1]. \quad (8)$$

As  $Z_1(s), Z_2(s)$  are continuous functions and  $Z_1(s) \leq Z_2(s)$  for  $s \leq 0$ , according to (6) and (7), we have  $Z_2(s) \geq Z_1(s)$ , and therefore (8) holds. Then, assume  $Z_1(s) \leq Z_2(s)$  for  $t \in [t_{k-1}, t_k], k \leq c$ , and  $c \in \mathbb{N}$ . Then, we have  $Z_1(s_c) \leq \mu Z_1(s_c^-) \leq \mu Z_2(s_c^-) = Z_2(s_c)$ . In a similar manner to the proof of (8), we can have  $Z_1(s) \leq Z_2(s)$  for  $s \in [t_c, t_{c+1})$ . By mathematical induction, it is easy to conclude that  $Z_1(s) \leq Z_2(s)$  for  $s \geq 0$ . This completes the proof.  $\square$

**Lemma 2** (see [28]).  $x(t) \in \mathbb{R}^n$  is a derivable and continuous function, for  $t \geq t_0$ ; we have

$${}_{t_0} D_t^\alpha [x^T(t) M x(t)] \leq 2x^T(t) M {}_{t_0} D_t^\alpha x(t), \quad \alpha \in (0, 1], \quad (9)$$

where  $M \in \mathbb{R}^{n \times n}$  is a positive definite matrix.

The Mittag-Leffler function is defined in the following.

*Definition 3* (see [13]). Two-parameter Mittag-Leffler function is defined as

$$E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(k\alpha + \beta)}, \quad (10)$$

where  $\alpha > 0, \beta > 0$ , and  $t \in \mathcal{C}$ . When  $\beta = 0$ , we have

$$E_{\alpha}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(k\alpha + 1)}, \quad (11)$$

where  $\alpha > 0, t \in \mathcal{C}$ .

**Lemma 3** (see [29]).  $t \in \mathbb{R}, t > 0$ , and  $0 < \alpha < 1$ .  $E_{\alpha}(t)$  is a monotone increasing function. For any integer  $N > 1$ , and  $t \neq 0, |\arg(t)| > (\pi/2)$ , then we have the following asymptotic expansion:

$$E_{\alpha,\beta}(t) \leq \frac{1}{\alpha} t^{(1-\beta)/\alpha} e^{t^{1/\alpha}} - \sum_{p=1}^N \frac{t^{-p}}{\Gamma(\beta - \alpha p)} + O\left(\frac{1}{t^{N+1}}\right). \quad (12)$$

So, there exists  $t_0 > 0$ , when  $t > t_0$ , and we have

$$\begin{aligned} E_{\alpha}(t) &\leq \frac{1}{\alpha} e^{t^{1/\alpha}}, \\ E_{\alpha,\alpha}(t) &\leq \frac{1}{\alpha} t^{(1-\alpha)/\alpha} e^{t^{1/\alpha}}. \end{aligned} \quad (13)$$

**2.2. System Description.** Now, we consider the following FMNNs as a drive system:

$${}_0D_t^{\alpha} x_i(t) = -d_i x_i(t) + \sum_{j=1}^n b_{ij}(x_j(t)) f_j(x_j(t)) x_i(0) = x_{0i}, \quad (14)$$

where  $i, j \in \mathbb{N}$ ;  $\alpha$  is the fractional order;  $x_i(t)$  represents the state variable of the  $i$ th neuron;  $d_i > 0$  denotes the self-feedback connection weight;  $x_0 = (x_{01}, x_{02}, \dots, x_{0n})^T$  is the initial value of state vector;  $f_j(\cdot)$  denote the activation functions; and  $b_{ij}(x_j(t))$  are memristive connection weights satisfy the following conditions:

$$b_{ij}(x_j(t)) = \begin{cases} b_{ij}^*, & |x_j(t)| \leq \tau_{xj}, \\ b_{ij}^{**}, & |x_j(t)| > \tau_{xj}, \end{cases} \quad (15)$$

where the switching jumps  $\tau_{xj} > 0, b_{ij}^*$ , and  $b_{ij}^{**}$  are all constants.

The corresponding response system is described as

$$\begin{cases} {}_0D_t^{\alpha} y_i(t) = -d_i y_i(t) + \sum_{j=1}^n \tilde{b}_{ij}(y_j(t)) f_j(y_j(t)), & y_i(0) = y_{0i}, \end{cases} \quad (16)$$

where  $\theta_i(t)$  is the impulsive controller, and the memristor-based connection weights  $\tilde{b}_{ij}(y_j(t))$  satisfy the following condition:

$$\tilde{b}_{ij}(y_j(t)) = \begin{cases} \tilde{b}_{ij}^*, & |y_j(t)| \leq \tau_{yj}, \\ \tilde{b}_{ij}^{**}, & |y_j(t)| > \tau_{yj}, \end{cases} \quad (17)$$

where  $i, j \in \mathbb{N}$  and switching jumps  $\tau_{yj} > 0, \tilde{b}_{ij}^*, \tilde{b}_{ij}^{**}$  are known constants.

For system (14), by applying the theories of set-valued maps and differential inclusions, we have the following differential inclusion form:

$${}_0D_t^{\alpha} x_i(t) \in -d_i x_i(t) + \sum_{j=1}^n K[b_{ij}(x_j(t))] f_j(x_j(t)). \quad (18)$$

Let  $\hat{b}_{ij} = \min\{b_{ij}^*, b_{ij}^{**}\}, \check{b}_{ij} = \max\{b_{ij}^*, b_{ij}^{**}\}$ , and  $co\{b_{ij}^*, b_{ij}^{**}\} = [\hat{b}_{ij}, \check{b}_{ij}]$ . The set-valued maps are defined as

$$K[b_{ij}(x_j(t))] = \begin{cases} b_{ij}^*, & |x_j(t)| < \tau_{xj}, \\ co\{b_{ij}^*, b_{ij}^{**}\}, & |x_j(t)| = \tau_{xj}, \\ b_{ij}^{**}, & |x_j(t)| > \tau_{xj}, \end{cases} \quad (19)$$

and, besides, there exist measurable functions  $\eta_{ij}(x_j(t)) \in K[b_{ij}(x_j(t))]$  such that

$${}_0D_t^{\alpha} x_i(t) = -d_i x_i(t) + \sum_{j=1}^n \eta_{ij}(x_j(t)) f_j(x_j(t)). \quad (20)$$

Similarly, there exist measurable functions  $\widetilde{\eta}_{ij}(y_j(t)) = K[\tilde{b}_{ij}(y_j(t))]$  such that

$${}_0D_t^{\alpha} y_i(t) = -d_i y_i(t) + \sum_{j=1}^n \widetilde{\eta}_{ij}(y_j(t)) f_j(y_j(t)) + \theta_i(t). \quad (21)$$

To ensure the uniqueness and existence of the solution of system (20) and (21), we make the following assumption.

**Assumption 1.** There exists a positive constant  $L_j$  such that

$$|f_j(x) - f_j(y)| \leq L_j |x - y|, \quad (22)$$

for all  $x, y \in \mathbb{R}, j \in \mathbb{N}$ .

**Lemma 4** (see [30]). Let Assumption 1 be satisfied; then the following inequalities hold:

$$\begin{aligned} &|\widetilde{\eta}_{ij}(y_j(t)) f_j(y_j(t)) - \eta_{ij}(x_j(t)) f_j(x_j(t))| \\ &\leq \tilde{b}_{ij} L_j |e_j(t)| + |\Delta b_{ij}| L_j \tau_{\max}, \end{aligned} \quad (23)$$

where  $\tilde{b}_{ij} = \max\{|b_{ij}^*|, |b_{ij}^{**}|\}, \tau_{\max} = \max\{\tau_{xj}, \tau_{yj}\}, \Delta b_{ij} = b_{ij}^* - b_{ij}^{**}$ .

**Remark 1.** To reduce the conservativeness of our results, the constraints on activation functions  $f_j(\pm \tau_{xj}) = f_j(\pm \tau_{yj}) = 0, g_j(\pm \tau_{xj}) = g_j(\pm \tau_{yj}) = 0$  are removed in this paper.

Denote  $e(t) = y(t) - x(t)$  as the synchronization error and the impulsive controller is designed as

$$\theta_i(t) = -a_i e_i(t) + \sum_{k=1}^{\infty} (\mu_k - 1) e_i(t) \delta(t - t_k), \quad (24)$$

where  $a_i$  are nonnegative constants,  $i \in \mathbb{N}$ ; impulsive intensity  $\mu_k \neq 0$ ;  $\delta(\cdot)$  denotes the Dirac impulsive function; and the time series  $\zeta = \{t_1, t_2, \dots\}$  is a strictly increasing sequence of impulsive instants which satisfies  $t_{k-1} < t_k$  and  $\lim_{k \rightarrow +\infty} t_k = +\infty$ .

$$\begin{cases} {}_0D_t^\alpha e_i(t) = -(d_i + a_i)e_i(t) + \sum_{j=1}^n [\widetilde{\eta}_{ij}(y_j(t))f_j(y_j(t)) - \eta_{ij}(x_j(t))f_j(x_j(t))], & t \neq t_k, \Delta e_i(t_k) = e_i(t_k^+) = \mu_k e_i(t_k^-), & k \in \mathbb{N}, \end{cases} \quad (25)$$

where  $e_i(t)$  is right-hand continuous at  $t = t_k$  and the initial conditions  $e_i(0) = e_{0i} \in \mathcal{C}([-\infty, 0], \mathbb{R}^n)$  for  $i \in \mathbb{N}$ .

**2.3. The Definition of Quasi-Synchronization and Average Impulsive Interval.** Now, the definitions of quasi-synchronization and average impulsive interval are given as follows.

**Definition 4** (see [24]). If there exists a compact set  $\Theta$  for any  $e_i(0) \in \mathcal{C}([-\infty, 0], \mathbb{R}^n)$ , with the error  $e(t)$  converging to  $\Theta = \{\|e(t)\| \leq \varrho\}$  as  $t$  goes to infinity, then system (14) will achieve quasi-synchronization with error bound  $\varrho > 0$ .

**Definition 5** (see [26]). Consider the impulsive sequence  $\zeta = \{t_1, t_2, \dots\}$  in the time interval  $(t, T)$ .  $\omega(t, T)$  is represented as the number of impulsive times, if there exist positive numbers  $N_0$  and  $T_a$ , such that

$$\frac{T-t}{T_a} - N_0 \leq \omega(t, T) \leq \frac{T-t}{T_a} + N_0, \quad \forall T \geq t \geq 0, \quad (26)$$

and then,  $T_a$  is greater than the average impulsive interval of the impulsive sequence  $\zeta$ .

### 3. Main Results

In this part, we explore the quasi-synchronization of system (25) by impulsive controller (24). For two different impulsive intensities  $-\alpha < \mu_k \leq \alpha$  and  $\mu_k \neq 0$  and  $\mu_k \leq -\alpha$  or  $\mu_k > \alpha$ , we have Theorems 1 and 2.

With considering the impulsive controller (24), the controlled error neural networks could be rewritten as follows:

**Theorem 1.** When impulsive intensity  $-\alpha < \mu_k \leq \alpha$  and  $\mu_k \neq 0$ , if there exist diagonal matrices  $Q > 0, P > 0$ , scalars  $\beta > 0$ , and  $d = \text{diag}(d_1, d_2, \dots, d_n)$ ,  $\overline{B} = (\overline{b}_{ij})_{n \times n}$ ,  $L = \text{diag}(L_1, L_2, \dots, L_n)$ ,  $\pi = N^T P N$ ,  $N = \Delta B L \overline{\Gamma}_{\max}$ ,  $|\Delta B| = (|\Delta b_{ij}|)_{n \times n}$ ,  $\overline{\Gamma}_{\max} = \text{diag}(\overline{\Gamma}_{\max}, \overline{\Gamma}_{\max}, \dots, \overline{\Gamma}_{\max})$ ,  $\iota = \max_{k \in \mathbb{N}} \{\mu_k^2\}$ , if

$$(1) -2Qd - 2Qa + Q\overline{B}L + \overline{L}\overline{B}^T Q + P^{-1}Q^2 - \beta Q < 0$$

$$(2) \Upsilon = \frac{\ln(1/\alpha)}{T_a} + \beta^{(1/\alpha)} < 0$$

(27)

are satisfied, then error systems (25) achieve quasi-synchronization with error bound  $\sqrt{((\beta^{(1/\alpha)-1}/\alpha)(\iota/\alpha)^{-N_0}\pi)/(\lambda_{\min}(Q) \cdot (-Y))}$  at the convergence rate  $(h/2)$ , where  $h$  is the unique solution of the equation  $h + (\ln(\iota/\alpha)/T_a) + \beta^{(1/\alpha)} = 0$ .

*Proof.* Construct the following Lyapunov function:

$$V(t) = \sum_{i=1}^n Q_i |e_i(t)|^2. \quad (28)$$

For  $t \in [t_{k-1}, t_k)$  ( $k = 1, 2, \dots$ ), according to Lemmas 2 and 4, we get

$$\begin{aligned}
 {}_0D_t^\alpha V(t) &\leq 2 \sum_{i=1}^n |e_i(t)| Q_i \text{sign}(e_i(t)) {}_0D_t^\alpha V e_i(t) \\
 &= 2 \sum_{i=1}^n |e_i(t)| Q_i \text{sign}(e_i(t)) \{-(d_i + a_i)e_i(t) \\
 &\quad + \sum_{j=1}^n [\widetilde{\eta}_{ij}(y_j(t)) f_j(y_j(t)) \\
 &\quad - \eta_{ij}(x_j(t)) f_j(x_j(t))]\} \\
 &\leq -2 \sum_{i=1}^n |e_i(t)| Q_i (d_i + a_i) |e_i(t)| \\
 &\quad + 2 \sum_{i=1}^n \sum_{j=1}^n |e_i(t)| Q_i |\widetilde{\eta}_{ij}(y_j(t)) f_j(y_j(t)) - \eta_{ij}(x_j(t)) f_j(x_j(t))| \\
 &\leq -2 \sum_{i=1}^n |e_i(t)| Q_i (d_i + a_i) |e_i(t)| + 2 \sum_{i=1}^n \sum_{j=1}^n |e_i(t)| Q_i \left( \overline{b}_{ij} \dot{L}_j |e_j(t)| + |\Delta b_{ij}| L_j \Upsilon_{\max} \right) \\
 &= -2|e(t)|^T Q(d+a)|e(t)| + 2|e(t)|^T Q \times \overline{BL}|e(t)| + 2|e(t)|^T QN \\
 &\leq -2|e(t)|^T Q(d+a)|e(t)| + 2|e(t)|^T Q \times \overline{BL}|e(t)| + |e(t)|^T P^{-1}Q^2|e(t)| + N^T PN \\
 &= |e(t)|^T \left( -2Qd - 2Qa + Q\overline{BL} + \overline{L}\overline{B}^T Q + P^{-1}Q^2 - \beta Q \right) |e(t)| + \beta |e(t)|^T Q |e(t)| + N^T PN.
 \end{aligned} \tag{29}$$

Based on (4) and (5) of Theorem 1, we get

$${}_0D_t^\alpha V(t) \leq \beta \sum_{i=1}^n Q_i |e_i(t)|^2 + \pi = \beta V(t) + \pi. \tag{30}$$

For  $t = t_k, k \in N$ , according to (25), it yields

$$\begin{aligned}
 V(t_k^+) &= \sum_{i=1}^N e_i^T(t_k^+) Q_i e_i(t_k^+) \\
 &= \mu_k^2 V(t_k^-).
 \end{aligned} \tag{31}$$

Let  $\Lambda(t)$  be the unique solution of the following delayed impulsive comparison system for any  $\gamma > 0$ :

$$\begin{cases}
 {}_0D_t^\alpha \Lambda(t) = \beta \Lambda(t) + \pi + \gamma, \Lambda(t_k^+) = \mu_k^2 \Lambda(t_k^-), & k = 1, 2, \dots, \Lambda(t) = \lambda_{\max}(Q) \|e(0)\|^2.
 \end{cases} \tag{32}$$

When  $t > 0$ , we have  $v(t) \leq \Lambda(t)$  according to Lemma 1.

For  $t \in [0, t_1)$ , using the Laplace transform on (32), one gets

$$\begin{aligned}
 \Lambda(t) &= E_\alpha(\beta t^\alpha) \Lambda(0) + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(\beta(t-s)^\alpha) \\
 &\quad \times (\pi + \gamma) ds.
 \end{aligned} \tag{33}$$

By using Lemma 3, we have

$$E_\alpha(\beta t^\alpha) \leq \frac{1}{\alpha} e^{\beta^{(1/\alpha)} t}, \tag{34}$$

$$E_{\alpha,\alpha}(\beta(t-s)^\alpha) \leq \frac{\beta^{(1/\alpha)-1}}{\alpha} (t-s)^{1-\alpha} e^{\beta^{(1/\alpha)}(t-s)}.$$

Then, we can transform (33) as

$$\Lambda(t) \leq \frac{1}{\alpha} e^{\beta^{(1/\alpha)} t} \Lambda(0) + \int_0^t \frac{\beta^{(1/\alpha)-1}}{\alpha} e^{\beta^{(1/\alpha)}(t-s)} \times (\pi + \gamma) ds. \tag{35}$$

For  $t \in [t_1, t_2)$ , one obtains from (35) and the second equation of (32) that

$$\begin{aligned}
 \Lambda(t_1^+) &\leq \frac{1}{\alpha} e^{\beta^{(1/\alpha)} t_1} \Lambda(0) + \int_0^{t_1} \frac{\beta^{(1/\alpha)-1}}{\alpha} e^{\beta^{(1/\alpha)}(t_1-s)} \times (\pi + \gamma) ds, \\
 \Lambda(t) &\leq \frac{1}{\alpha} e^{\beta^{(1/\alpha)}(t-t_1)} \Lambda(t_1^+) + \int_{t_1}^t \frac{\beta^{(1/\alpha)-1}}{\alpha} e^{\beta^{(1/\alpha)}(t-s)} \times (\pi + \gamma) ds \\
 &= \frac{1}{\alpha^2} e^{\beta^{(1/\alpha)} t} \Lambda(0) + \frac{1}{\alpha^2} \int_0^{t_1} \beta^{(1/\alpha)-1} e^{\beta^{(1/\alpha)}(t-s)} \times (\pi + \gamma) ds + \frac{1}{\alpha} \int_{t_1}^t \beta^{(1/\alpha)-1} e^{\beta^{(1/\alpha)}(t-s)} \times (\pi + \gamma) ds.
 \end{aligned} \tag{36}$$

For  $t \in [t_2, t_3)$ ,

$$\begin{aligned}
 \Lambda(t_2^+) &\leq \frac{t_2^2}{\alpha^2} e^{\beta^{(1/\alpha)} t_2} \Lambda(0) + \frac{t_2^2}{\alpha^2} \int_0^{t_1} \beta^{(1/\alpha)-1} e^{\beta^{(1/\alpha)}(t_2-s)} \times (\pi + \gamma) ds + \frac{t_2}{\alpha} \int_{t_1}^{t_2} \beta^{(1/\alpha)-1} e^{\beta^{(1/\alpha)}(t_2-s)} \times (\pi + \gamma) ds, \\
 \Lambda(t) &\leq \frac{1}{\alpha} e^{\beta^{(1/\alpha)}(t-t_2)} \Lambda(t_2^+) + \int_{t_2}^t \frac{\beta^{(1/\alpha)-1}}{\alpha} e^{\beta^{(1/\alpha)}(t-s)} \times (\pi + \gamma) ds \\
 &= \frac{t^2}{\alpha^3} e^{\beta^{(1/\alpha)} t} \Lambda(0) + \frac{t^2}{\alpha^3} \int_0^{t_1} \beta^{(1/\alpha)-1} e^{\beta^{(1/\alpha)}(t-s)} \times (\pi + \gamma) ds \\
 &\quad + \frac{t}{\alpha^2} \int_{t_1}^{t_2} \beta^{(1/\alpha)-1} e^{\beta^{(1/\alpha)}(t-s)} \times (\pi + \gamma) ds \\
 &\quad + \frac{1}{\alpha} \int_{t_2}^t \beta^{(1/\alpha)-1} e^{\beta^{(1/\alpha)}(t-s)} \times (\pi + \gamma) ds.
 \end{aligned} \tag{37}$$

By induction, we can derive that, for  $t \in [t_k, t_{k+1})$ ,

$$\begin{aligned}
 \Lambda(t) &\leq \prod_{0 \leq t_k \leq t} \frac{t}{\alpha} e^{\beta^{(1/\alpha)} t} \frac{1}{\alpha} \Lambda(0) + \int_0^t \prod_{s \leq t_k \leq t} \frac{t}{\alpha} e^{\beta^{(1/\alpha)}(t-s)} \\
 &\quad \times \frac{\beta^{(1/\alpha)-1}}{\alpha} (\pi + \gamma) ds.
 \end{aligned} \tag{38}$$

According to Definition 5, we have

$$\begin{aligned}
 \Lambda(t) &\leq \left(\frac{t}{\alpha}\right)^{-N_0} \lambda_{\max}(Q) \sum_{i=1}^N \|e_i(0)\|^2 \frac{1}{\alpha} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})t} \\
 &\quad + \int_0^t \left(\frac{t}{\alpha}\right)^{-N_0} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})(t-s)} \frac{\beta^{(1/\alpha)-1}}{\alpha} \times (\pi + \gamma) ds \\
 &\leq \Omega e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})t} + \int_0^t \frac{\beta^{(1/\alpha)-1}}{\alpha} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})(t-s)} \times \left[\left(\frac{t}{\alpha}\right)^{-N_0} (\pi + \gamma)\right] ds,
 \end{aligned} \tag{40}$$

where  $\Omega = (t/\alpha)^{-N_0} (1/\alpha) \lambda_{\max}(Q) \sup \|e(0)\|^2 > 0$ . Define  $\Delta(h) = h + ((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})$ . Note that  $\Delta(h)$  is a continuous function,  $\Delta(0) = ((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)}) < 0$ , and  $\dot{\Delta}(h) = 1 > 0$ . Above all,  $\Delta(h) = 0$  has a unique solution for  $h > 0$ . Since  $\gamma > 0, h > 0, 0 < t < 1$ , we obtain

$$\begin{aligned}
 \Lambda(t) &\leq \left(\frac{t}{\alpha}\right)^{-N_0} \lambda_{\max}(Q) \|e(0)\|^2 \\
 &< \Omega e^{-ht} + \left(\left(\frac{\beta^{(1/\alpha)-1}}{\alpha}\right) (t/\alpha)^{-N_0} (\pi + \gamma)\right) / \\
 &\quad \left(-\left((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)}\right)\right),
 \end{aligned} \tag{41}$$

for  $t \leq 0$ , which implies that (41) holds for all  $t > 0$ . Assume that if (41) does not hold for all  $t > 0$ , then there at least exists a time instant  $t^* > 0$ ,

$$\Lambda(t^*) \geq \Omega e^{-ht^*} + \frac{\left(\frac{\beta^{(1/\alpha)-1}}{\alpha}\right) (t/\alpha)^{-N_0} (\pi + \gamma)}{-\left((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)}\right)}, \tag{42}$$

but, for  $t < t^*$ , we have

$$\Lambda(t) < \Omega e^{-ht} + \frac{\left(\frac{\beta^{(1/\alpha)-1}}{\alpha}\right) (t/\alpha)^{-N_0} (\pi + \gamma)}{-\left((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)}\right)}. \tag{43}$$

$$\begin{aligned}
 e^{\beta^{(1/\alpha)}(t-s)} \prod_{s \leq t_k \leq t} \frac{t}{\alpha} &\leq e^{\beta^{(1/\alpha)}(t-s)} \left(\frac{t}{\alpha}\right)^{\left((t-s/T_a) - N_0\right)} \\
 &\leq \left(\frac{t}{\alpha}\right)^{-N_0} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})(t-s)}.
 \end{aligned} \tag{39}$$

Substituting (39) into (38) yields

Denote  $(\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)} = -r$ ; combining (40) and (43), we have

$$\begin{aligned} \Lambda(t^*) &\leq \Omega e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})t^*} + \int_0^{t^*} \frac{\beta^{(1/\alpha)-1}}{\alpha} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})(t^*-s)} \times \left(\frac{t}{\alpha}\right)^{-N_0} (\pi + \gamma) ds \\ &< e^{-rt^*} \left\{ \Omega + \frac{(\beta^{(1/\alpha)-1}/\alpha)(t/\alpha)^{-N_0}(\pi + \gamma)}{r} + \int_0^{t^*} \frac{\beta^{(1/\alpha)-1}}{\alpha} \times e^{rs} \left(\frac{t}{\alpha}\right)^{-N_0} (\pi + \gamma) ds \right\} \\ &\leq e^{-rt^*} \left\{ \Omega + \frac{(\beta^{(1/\alpha)-1}/\alpha)(t/\alpha)^{-N_0}(\pi + \gamma)}{r} + \frac{(e^{rt^*} - 1)(\beta^{(1/\alpha)-1}/\alpha)(t/\alpha)^{-N_0}(\pi + \gamma)}{r} \right\} \\ &= \Omega e^{-ht^*} + \frac{(\beta^{(1/\alpha)-1}/\alpha)(t/\alpha)^{-N_0}(\pi + \gamma)}{r}. \end{aligned} \tag{44}$$

Obviously, inequality (44) contradicts (42), which implies (41) holds for all  $t > 0$ . Letting  $\gamma \rightarrow 0$ , we have

$$\begin{aligned} \lambda_{\min}(Q) \|e(t)\|^2 &\leq V(t) \leq \Lambda(t) < \Omega e^{-ht} \\ &+ \frac{(\beta^{(1/\alpha)-1}/\alpha)(t/\alpha)^{-N_0}\pi}{-\left((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)}\right)}. \end{aligned} \tag{45}$$

Thus, according to (45), the error converges into

$$\Theta = \left\{ e(t) \in \mathbb{R}^n \mid \|e(t)\| \leq \sqrt{\frac{(\beta^{(1/\alpha)-1}/\alpha)(t/\alpha)^{-N_0}\pi}{\lambda_{\min}(Q) \cdot (-Y)}} \right\}, \tag{46}$$

when  $t \rightarrow +\infty$ . The proof for condition 1 is completed.

When impulsive intensity  $\mu_k \leq -\alpha$  or  $\mu_k > \alpha$ , we have Theorem 2.  $\square$

**Theorem 2.** When impulsive intensity  $\mu_k \leq -\alpha$  or  $\mu_k > \alpha$ , if there exist diagonal matrices  $Q > 0, P > 0$ , scalars  $\beta > 0$ , and  $d = \text{diag}(d_1, d_2, \dots, d_n)$ ,  $\bar{B} = (\bar{b}_{ij})_{n \times n}$ ,  $\dot{L} = \text{diag}(\dot{L}_1, \dot{L}_2, \dots, \dot{L}_n)$ ,  $\pi = N^T P N$ ,  $N = \Delta B \bar{L} \bar{\Gamma}_{\max}$ ,  $|\Delta B| = (|\Delta b_{ij}|)_{n \times n}$ ,  $\bar{\Gamma}_{\max} = \text{diag}(\bar{\Gamma}_{\max}, \bar{\Gamma}_{\max}, \dots, \bar{\Gamma}_{\max})$ ,  $\iota = \max_{k \in \mathbb{N}} \{\mu_k^2\}$ , if

$$(1) -2Qd - 2Qa + Q\bar{B}\dot{L} + \dot{L}\bar{B}^T Q + P^{-1}Q^2 - \beta Q < 0$$

$$(2) \bar{Y} = \frac{\ln(t/\alpha)}{T_a} + \beta^{(1/\alpha)} - \beta^{(1/\alpha)+1} < 0 \tag{47}$$

are satisfied, then error systems (25) achieve quasi-synchronization with error bound  $\sqrt{((\beta^{(1/\alpha)-1}/\alpha)(t/\alpha)^{N_0}\pi/\lambda_{\min}(Q) \cdot (-\bar{Y}))}$  at the convergence rate  $(h'/2)$ , where  $h'$  is the unique solution of the transcendental equation  $h' + (\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)} - \beta^{(1/\alpha)+1} = 0$ .

*Proof.* The same as the proof in Theorem 1, we obtain

$$\begin{aligned} e^{\beta^{(1/\alpha)}(t-s)} \prod_{s \leq t_k \leq t} \frac{t}{\alpha} &\leq e^{\beta^{(1/\alpha)}(t-s)} \left(\frac{t}{\alpha}\right)^{((t-s)/T_a) + N_0} \\ &\leq \left(\frac{t}{\alpha}\right)^{N_0} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})(t-s)}. \end{aligned} \tag{48}$$

Substituting (48) into (40) yields

$$\begin{aligned} \Lambda(t) &\leq \left(\frac{t}{\alpha}\right)^{N_0} \lambda_{\max}(Q) \sum_{i=1}^N \|e_i(0)\|^2 \frac{1}{\alpha} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})t} \\ &+ \int_0^t \left(\frac{t}{\alpha}\right)^{N_0} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})(t-s)} \frac{\beta^{(1/\alpha)-1}}{\alpha} \times (\pi + \gamma) ds \\ &\leq \bar{\Omega} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})t} + \int_0^t \frac{\beta^{(1/\alpha)-1}}{\alpha} e^{((\ln(t/\alpha)/T_a) + \beta^{(1/\alpha)})(t-s)} \times \left(\frac{t}{\alpha}\right)^{N_0} (\pi + \gamma) ds, \end{aligned} \tag{49}$$



where  $\bar{\Omega} = (\iota/\alpha)^{N_0} (1/\alpha) \lambda_{\max}(Q) \sup\|e(0)\|^2 > 0$ . Define  $\Delta(h') = h' + (\ln(\iota/\alpha)/T_a) + \beta^{(1/\alpha)} - \beta^{(1/\alpha)+1}$ . Note that  $\Delta(0) = (\ln(\iota/\alpha)/T_a) + \beta^{(1/\alpha)} - \beta^{(1/\alpha)+1} < 0$  and  $\dot{\Delta}(h') = 1 > 0$ . So, function  $\Delta(h') = 0$  has a unique solution  $h' > 0$ .

Similar to (41), we can prove the following inequality is true for all  $t > 0$ :

$$\Lambda(t) < \bar{\Omega} e^{-h't} + \frac{(\beta^{(1/\alpha)-1}/\alpha)(\iota/\alpha)^{N_0}(\pi + \gamma)}{-((\ln(\iota/\alpha)/T_a) + \beta^{(1/\alpha)}) + \beta^{(1/\alpha)+1}}. \quad (50)$$

Letting  $\gamma \rightarrow 0$ , we have

$$\lambda_{\min}(Q)\|e(t)\|^2 \leq V(e(t)) \leq \Lambda(t) < \bar{\Omega} e^{-h't} + \frac{(\beta^{(1/\alpha)-1}/\alpha)(\iota/\alpha)^{N_0}\pi}{-((\ln(\iota/\alpha)/T_a) + \beta^{(1/\alpha)}) + \beta^{(1/\alpha)+1}}. \quad (51)$$

Then, according to (51), the error converges into

$$\bar{\Theta} = \left\{ e(t) \in \mathbb{R}^n \mid \|e(t)\| \leq \sqrt{\frac{(\beta^{(1/\alpha)-1}/\alpha)(\iota/\alpha)^{N_0}\pi}{\lambda_{\min}(Q) \cdot (-\bar{Y})}} \right\}, \quad (52)$$

with the convergence rate  $(h'/2)$ . So far, all proofs are over.  $\square$

*Remark 2.* In impulsive dynamical networks or impulsive control mechanism, the impulsive effect  $\mu_k$  is essential to the final results of the whole networks. Some previous works have discussed the impulsive effects in the synchronization of complex dynamical networks. Specifically, the impulsive effect  $\mu_k$  is always assumed to satisfy  $(-1, 0)$  or  $(-2, 0)$ . However, when a fractional-order system exchanges information with another system, the impulsive effect could either play positive roles or negative roles. In this paper, the impulsive gain  $\mu_k$  is talked over under the value ranges: (1)  $-\alpha < \mu_k \leq \alpha$  and  $\mu_k \neq 0$ , (2)  $\mu_k \leq -\alpha$  or  $\mu_k > \alpha$ , where  $\mu_k$  only have a relation with variable  $\alpha$ . The range of gain  $\mu_k$  values was extended.

*Remark 3.* In this paper, the controller (24) contains the impulsive control part and the feedback control part. For the case  $-\alpha < \mu_k \leq \alpha$  and  $\mu_k \neq 0$ , the impulsive effect  $\mu_k$  plays a positive role in the quasi-synchronization. In this case, the feedback control gain  $a_i$  could be set as zero in order to save control costs and manufacture. However, for the case  $\mu_k \leq -\alpha$  or  $\mu_k > \alpha$ , the impulsive effect  $\mu_k$  could hinder the synchronization of neural networks or even cause the instability of the coupled neural networks. In this case, the feedback control part could control effectively reduce the adverse effects brought by the disadvantageous impulse. In addition, if  $\mu_k = -1$ , it implies  $e_i(t_k^+) = -e_i(t_k^-)$ , which is impossible for impulsive control. If  $\mu_k = 0$ ,  $e_i(t_k^+) = e_i(t_k^-) = 0$ , which means that there is no impulsive control effect.

## 4. Illustrative Examples

In this part, one example is given to show the effectiveness of our methods.

*Example 1.* Consider 2-dimension drive system (14) with  $\alpha = 0.9$ ,  $f_j(s) = \tanh(s)$ ,  $j \in \mathbb{N}$ . Choose  $d_1 = 3.4$ ,  $d_2 = 1.5$ ,  $\bar{\Gamma}_{x_j} = 1L_j = 1$ ,

$$\begin{aligned} b_{11}(x_1) &= \begin{cases} 1.3, & |x_1| \leq 1, \\ 1.6, & |x_1| > 1, \end{cases} \\ b_{12}(x_2) &= \begin{cases} -2, & |x_2| \leq 1, \\ -2.45, & |x_2| > 1, \end{cases} \\ b_{21}(x_1) &= \begin{cases} -0.3, & |x_1| \leq 1, \\ -0.6, & |x_1| > 1, \end{cases} \\ b_{22}(x_2) &= \begin{cases} 2.3, & |x_2| \leq 1, \\ 2.6, & |x_2| > 1. \end{cases} \end{aligned} \quad (53)$$

The switching jumps and memristive connection weights of the response system (16) are the same as system (14). The initial conditions of the drive-response system are  $x_{0i} = (1.5, 1)^T$ ,  $y_{0i} = (0.5, -0.5)^T$ . The average impulsive interval is less than  $T_a = 0.02$  and the positive constant  $N_0 = 1$ .

Case 1: when  $-0.9 < \mu_k \leq 0.9$  and  $\mu_k \neq 0$ , for Theorem 1, considering disadvantageous impulsive effects here, set  $a_i = 8$  ( $i \in \mathbb{N}$ ),  $\mu_k = -0.7$ , so we get  $\iota = \mu_k^2 = 0.49$ . Denoting  $\beta = 7$ , then we have  $\bar{Y} = (\ln(\iota/\alpha)/T_a) + \beta^{(1/\alpha)} = -21.7099 < 0$ . By solving conditions (4) and (5) in Theorem 1, we drive  $P = \text{diag}\{15.0648, 14.9541\}$ ,  $Q = \text{diag}\{22.5648, 21.4627\}$ ,  $N = \Delta BL \bar{\Gamma}_{\max} = (0.75, 0.6)^T$ ,  $\pi = \|N^T P N\| = 13.8574$  and the corresponding error bound could be calculated as

$$\|e(t)\| \leq \sqrt{\frac{(\beta^{(1/\alpha)-1}/\alpha)(\iota/\alpha)^{-N_0}\pi}{\lambda_{\min}(Q) \cdot (-\bar{Y})}} = 0.2677. \quad (54)$$

In Figure 1, we plot the chaotic attractors of the drive-response system with initial conditions. In Figure 2, one can find that the error is under the bound 0.1752, which is less than the error bound  $\|e(t)\|$  derived from the theorem. It implies the systems achieve quasi-synchronization with a given error bound.

Case 2: when  $\mu_k > 0.9$ , for Theorem 2, impulsive effect  $\mu_k = 1.3$ , so  $\iota = 1.1^2 = 1.69$ ; we set the control gain  $a_i = 0$ ,  $i \in \mathbb{N}$ . As a similar process in Example 1, we have  $\bar{Y} = (\ln(\iota/\alpha)/T_a) + \beta^{(1/\alpha)} - \beta^{(1/\alpha)+1} = -28.6372 < 0$  and

$$\|e(t)\| \leq \sqrt{\frac{(\beta^{(1/\alpha)-1}/\alpha)(\iota/\alpha)^{N_0}\pi}{\lambda_{\min}(Q) \cdot (-\bar{Y})}} = 0.2356. \quad (55)$$



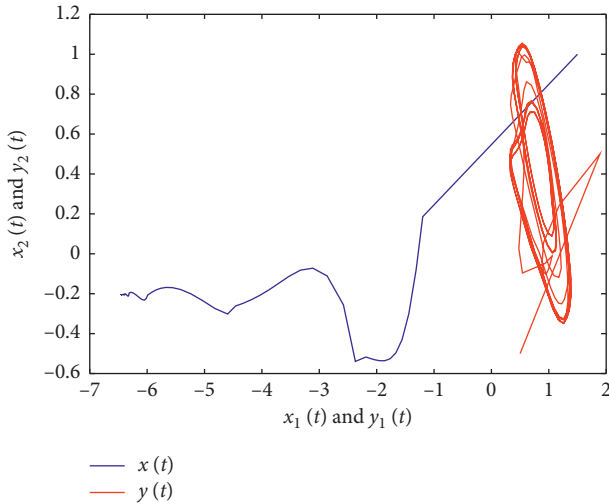


FIGURE 1: Chaotic attractor of the drive-response system when  $\mu_k = -0.7$ .

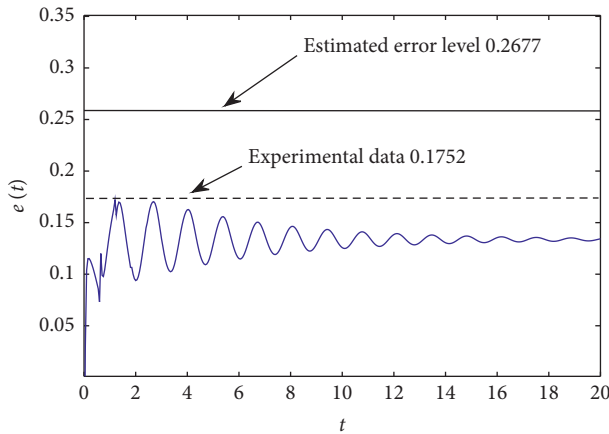


FIGURE 2: The synchronization error  $\|e(t)\|$  when  $\mu_k = -0.7$ .

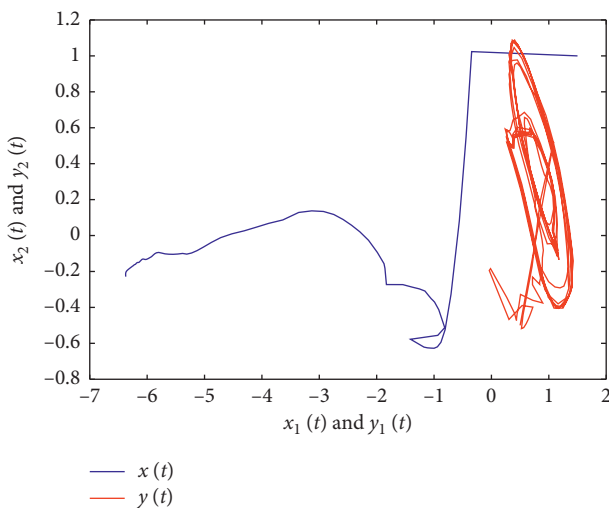


FIGURE 3: Chaotic attractor of the drive-response system when  $\mu_k = 1.3$ .

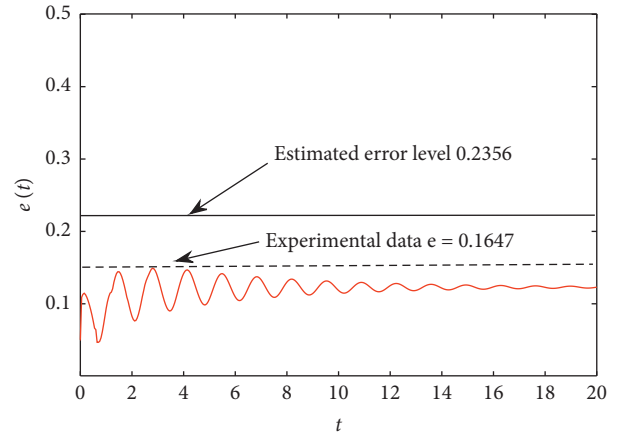


FIGURE 4: The synchronization error  $\|e(t)\|$  when  $\mu_k = 1.3$ .

The chaotic attractors of the drive-response system with initial conditions are shown in Figure 3. In Figure 4, one can find that the error is under the bound 0.1647, which is less than the error bound  $\|e(t)\|$  derived from the theorem. It can be obviously seen that the drive-response systems achieved quasi-synchronization.

### 5. Conclusion

In this paper, the problem of quasi-synchronization of delayed FMNNs is investigated. By designing an effective impulsive controller and a new fractional comparison lemma, some effective criteria for quasi-synchronization ( $0 < \alpha < 1$ ) are established. In the future, we will consider the quasi-synchronization problem of delayed FMNNs.

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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