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Research Article

A Theory Approach towards Tradable Rail Freight Option

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For railway freight pricing, this paper proposes a new tradable rail freight option (RFO). A multiphase trigeminal tree pricing model is established to analyze the optimal pricing decision of RFO. The main parameters of the model are identified theoretically through nonparametric Ito stochastic approach. The paper expands the theory of railway freight pricing. The aim of this paper is to construct the railway freight option theory to help railway transportation enterprise cope with fierce competition and challenge from other modes of transportation in China. The most significant feature of this method is its simplicity in analyzing the optimal pricing decision using only the algebraic category and Ito's lemma.

1. Introduction

The freight (transport by rail) is usually considered as an essential part of promoting Chinese national economic development. However, the railway freight industry has faced increasing competition from other modes of transportation year by year, as shown in Figure 1 [1]. In the process of China's fast-paced growth, the government study and formulated policies of reform, and development programs of the railway system are needed and should be carefully done. The behindhand pricing technique limits China's railway development. Reforming a national railway freight pricing, let alone one the size of Chinese Railways, is a great challenge.

At present, China's railway freight transport has achieved market-oriented operations; that is, transportation activities are accompanied by the coexistence of the contract market and spot market. In the contract market, railway transportation companies sign contract agreements with contract customers (the lead time is generally six months) to achieve the purpose of selling part of the capacity. Compared with the characteristics of poor transaction stability in the spot market, the signing of contract agreements has more advantages. It provides a stable source of bulk cargo for railway transportation and requires railway transportation companies to offer capacity guarantees for contract customers. However, during the contract period, railway transport companies are only allowed to sell the remaining capacity in the spot market and cannot obtain profits by repeatedly selling capacity when market prices fluctuate. Both parties must abide by the contract price. Therefore, it is necessary to formulate scientific, reasonable, and flexible railway freight pricing methods to facilitate for the railway transportation companies reaching transactions with customers, while enhancing the competitiveness of the railway freight market, expanding the market share of railway freight, and improving the current operating status of the railway company.

In existing research, considering that an enterprise uses both contract and spot trading channels at the same time, scholars mainly focus on different types of flexible contracts, such as quantity flexibility (QF) contract [2–5], revenue-sharing contract [6–8], and buy-back contract [9–11]. Although they are widely applied in a trade agreement for their flexibility and versatility, all of them cannot help the stakeholders make rational decisions with asymmetric information. Hence, with the advantage of financial engineering to deal with asymmetric information, investigating the role of options (contingent claims) in a buyer-supplier system has attracted significant attention from researchers

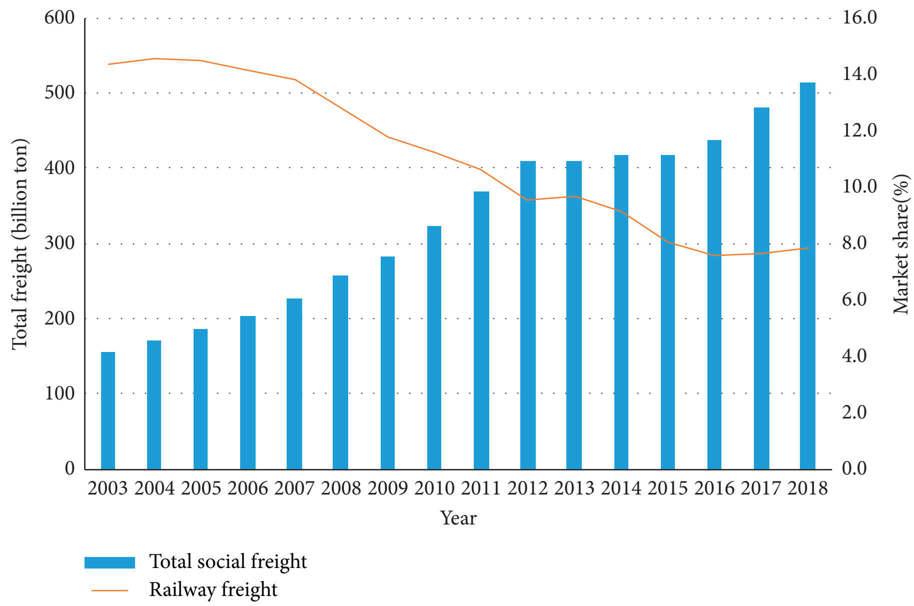

Figure 1: Total freight volume and rail freight volume in China.

[12–14]. More precisely, some scholars initially focused on the impact of options trading decisions. For example, Wang et al. developed a news vendor model to examine the impact of customer returns on a firm's pricing and order decisions, in cases where the firm faced a price-dependent stochastic demand and had the option of purchasing option contracts [15]. Di Corato et al. studied how exit options could affect bidding behaviour and the buyer's and seller's expected payoffs in multidimensional procurement auctions [16]. It is clear from those studies that an option tool will manage peak demand and fluctuating prices in the market [17], hedge market risk [18], and promote fair trade [19].

Furthermore, many applied types of research have explored the application of options in supply chain management. Cai et al. investigated the relationship between the option contract and subsidy contract and found that supply chain coordination and Pareto improvement can be achieved by introducing the option contract [20]. Fan et al. considered option contract application in a buyer-led supply chain, where both the buyer and supplier were risk-averse [21]. Liu et al. probed the coordination of both the supplier-led and retailer-led supply chains under an option contract [22]. Sharma et al. explored the fairness concerns of the channel members in a two-echelon supply chain, composed of a single supplier and single retailer, wherein the retailer procured products from the supplier using the option contract [23]. Chen et al. examined the impact of bidirectional option contracts on a two-echelon supply chain consisting of the supplier and retailer, taking into account of service requirements [24]. By developing a two-stage model to explore the supply option contract in a two-echelon supply chain, Zhao et al. put forward some new insights into using supply option contracts in the supply chain under the condition of a stochastic spot market and demand information updating [25]. Hu et al. analyzed a conventional option contract and an option contract with a joint pricing mechanism to consider a coordination problem under option contracts in a two-echelon supply chain [26]. Aghajani et al. proposed a novel two-period option contract integrated with supplier selection and inventory prepositioning to cope with various uncertainties [27]. Hu et al. explored the characteristics of the put option contract and proved that it could provide coordination of the relief supply chain [28]. Liu et al. introduced option contracts into relief supply management by considering this system as a relief supply chain with one government and multiple suppliers and established a relief supplies purchasing (RSP) model via option contracts to derive the government's optimal order quantity and each supplier's prepositioning quantity [29]. All of those have primarily promised the potential for option contracts' applications to develop freight derivatives.

Freight option is a kind of derivative financial instrument to hedge against risk caused by freight rate movements. With the rapid development of the global economy and rapid growth in transportation demand, freight has become a changeable commodity [30]. In the field of freight derivatives, Koekebakker et al. established a theoretical framework for the valuation of the Asian options traded in the shipping market [31]. Gómez-Valle et al. provided a novel theoretical framework for pricing Asian-style options and proved the lower and upper bounds for freight options which enables us to estimate the option price [32]. Shipping freight option adopts the calculation method of an arithmetic average Asian option. Although this simple option has certain advantages in operation, its flexibility is restricted, and it is also limited by the reliability of historical data and calculation accuracy. Shi et al. used a univariate generalized autoregressive conditional heteroscedasticity model to capture the volatility characteristics of freight derivatives returns and applied a time-varying copula model to describe the nonlinear correlation between the returns of spot and freight derivatives [33]. Kavussanos et al. investigated the economic spillovers between the freight and commodity derivatives markets and found that fluctuations in the return rate of the bulk commodity market would affect the freight derivatives market through the information dissemination mechanism [34]. These studies provide theoretical support for the detailed derivation of freight option pricing formulas.

Although previous studies have used different methods to research issues related to freight options, the contracts were never traded [35]. For sea transport enterprises, the purpose of buying options is to hedge risks rather than exercise options [36, 37]. By contrast, railway transportation enterprises adopt option trading to enhance the market competitiveness of railway transportation and improve the current status of railway freight operations. Meanwhile, rail freight transport organization has its own characteristics in the industrial chain, freight products, organizational system, and so on [38]. Therefore, the basic activities of rail freight transport organization should be reflected in the option design. Besides, the transaction process of option should be consistent with the rail freight transport organization. As the railway transportation market is relatively new, not much scientific research has been done in this area. The specific options trading strategy should be designed to promote the development of railway freight transportation. To this end, proposing a new tradable RFO adapting with the market competition will be meaningful. Hence, a multiphase trigeminal tree pricing model is established to analyze the optimal pricing decision of RFO in this paper. The main contributions of this article are summarized as follows.

(1) According to the coexistence of the contract market and spot market, a new tradable RFO is designed, and its transaction process is explained in detail
(2) A multiphase trigeminal tree pricing model is proposed to achieve the optimal decision of RFO contract
(3) A nonparametric Ito's stochastic approach is applied for the estimation of the critical parameters

The rest of this paper is organized as follows. Section 2 states the main concept of RFO, required symbols, and hypotheses for our problem. Section 3 details the methodology. The main result of RFO is derived in Section 4. Section 5 will conclude our work.

2. RFO Description

2.1. Definition of RFO. Freight has to be contracted, just like commodities. The only difference is that most commodities are real products, while freight is a service instead of a physical product. So, when freight is "bought," the service of products being transported is contracted. Due to the unstorability of freight, it should be traded in time. To protect railway transport enterprises and contract customers against market risks, a new option contract related to freight is provided according to the concept of options in the financial market, called rail freight option (RFO). Unlike other options, RFO helps railway transport enterprises sell unstorable freight in advance and also ensures stakeholders against freight rates moving beyond a specified price level.

Definition 1. An RFO is a call option contract which states that the contract customer (holder) has the right to pay/receive the average of the values of the freight rates during some period on or before the expiration date and receive/pay strike price. The railway transport enterprise (writer) then has an obligation to receive/pay this average and pay/receive the strike price when the holder decides to exercise.

In fact, an RFO is an option contract for an asset that is subject to the railway freight transport service. The railway transport enterprise as the option writer has the right, but not the obligation, to sell the option at the option price. Moreover, the contract customer is the holder who is a purchaser of option. If the execution price of the expiration date is higher than the spot market price, the contract customer will pay the execution fee and execute RFO. Otherwise, the contract customer will abandon the execution right of RFO and choose to purchase the capacity in the spot market. With an option, the contract customer has no risk of losing any money more than strike price due to freight price volatility because there is always the possibility not to exercise RFO. For the railway transportation enterprise, if the contract customer gives up the execution of RFO, they will sell the capacities in the spot market without refunding the option fees. This is a way the railway transportation enterprise can spread risk and schedule the freight train plans rationally. Under such circumstances, although the risk avoidance is realized effectively, the feasibility of the decision-making process is another issue that should be considered.

2.2. Transaction Process Description. In practice, both railway transportation enterprise and contract customer are mostly partial to risk aversion with different degrees. Therefore, the degree of risk acceptability determines the number of options purchased. The vast majority of existing studies deal with how the option is used for hedging. The optimal decision is a decentralized decision that is independent of the utility of the supply chain. Excessive pursuit of maximization of its utility based on the decentralized decision is not conducive to the long-term stability of the entire supply chain. The expectation of maximizing returns is not the optimal decision point in terms of the transportation system. Compared with the existing studies focusing on the expected returns of stakeholders, this paper studies the decision-making in the RFO trading to help maximize system utility, maintain the long-term development of the entire supply chain, and achieve Pareto optimal.

Consider a railway transportation enterprise that is looking to protect the company against a possible decrease in the freight rates. To this extent, the railway transportation enterprise writes and sells the RFO by speculating contract customer buying behaviour, to formulate reasonable option prices and circulation [39, 40]. Then, the contract customer determines the purchase amount of RFO according to the pricing announced by the railway transportation enterprise. Moreover, in the second stage, the contract customer will decide whether or not to exercise RFO. If only the total amount that the contract customer has to add up the option strike price is less than the spot freight rate at that point, the contract customer will exercise RFO, and of course the contract customer will abandon RFO in reverse. The transaction process conforms to the two-stage dynamic game model (Figure 2).

2.3. Symbol and Assumption. To better understand the model, the list of all the notations used in our work is presented in Table 1. Some notations will be more precisely defined as they appear in later sections of this paper.

In addition, in order to build the mathematical model, this study makes the following assumptions.

Assumption 1. The RFO covered in this article is a European call option, which means RFO can only be executed on the expiration date.

Assumption 2. Both railway transport enterprises and contract customers are entirely rational and risk-averse. Spot market freight rate follows the geometric Brownian motion within RFO validity. This is given as

$$\frac{\mathrm{d}S(t)}{S(t)} = \alpha \mathrm{d}t + \sigma \mathrm{d}B(t), \tag{1}$$

where $B(t)$ is standard Brownian motion that has $E[B(t)] = 0$ and $\mathrm{Var}[B(t)] = 1$.

Assumption 3. The values of α and σ are fixed within the validity period of RFO, and the information on both sides of the option contract market are symmetrical.

Assumption 4. The freight rate of the spot market in this model is an exogenous variable, which is entirely dominated by the external market economic conditions and is not affected by the railway transportation enterprises and contract customers.

Assumption 5. According to the demand curve, the market capacity demand D is generally negatively related to the spot market freight rate s_t price. This is given as

$$D = a - bs_t + \varepsilon, \tag{2}$$

where ε is a random distribution with distribution function $F(x)$ and density function $f(x)$.

3. Methodology

3.1. Model Construction. In the case where contract market coexists with spot market, two-stage Stackelberg game model is established, in which the railway transportation enterprise is the leader, and the contract customer is the follower. The game sequence is shown as follows:

Step 1. In the contract market of time T_0, the railway transport enterprise writes the RFO including the option price w_0 and the option strike price w_1.

Step 2. According to the published price strategy of RFO, the contract customer decides the purchase amount of RFO to maximize the expected returns.

Step 3. In the spot market of time T_1, the contract customer will decide whether or not to exercise RFO based on the spot market freight rate s_t and the option strike price w_1. Furthermore, the execution amount of RFO and the capacity purchases amount through the spot market are determined.

Step 4. Based on the optimal RFO exercise decision of contract customer, the railway transportation enterprise will sell residual capacity in the spot market of time T_1 to maximize its expected profit.

It is worth noting how the expected profit functions of contract customer and railway transportation enterprise are given.

3.2. Contract Customer's Expected Profit Description. According to the transaction process, railway transport enterprise writes the RFO, including the option price w_0 and the option strike price w_1, first. Then, the contract customer decides the purchase amount of RFO (N) to maximize the expected returns. In the spot market of time T_1, the contract customer will determine whether or not to exercise RFO based on the spot market freight rate s_t and the option strike price w_1. Thus, the expected profit function of contract customer $E(\xi)$ is obtained as described in Theorem 1.

Theorem 1. *The expected profit function of contract customer can be expressed as follows:*

$$\begin{aligned} E(\xi) = {} & p_d \int_{-\infty}^{\infty} [(s_t - w_d)D - w_0 N] f(x)\mathrm{d}x \\ & + (p_u + p_m)\left\{ \int_{-\infty}^{N-a+bp} [(p - w_1)D - w_0 N] f(x)\mathrm{d}x + \int_{N-a+bp}^{\infty} [p\ D - (w_0 + w_1)N - w_u(D - N)] f(x)\mathrm{d}x \right\}. \end{aligned} \tag{3}$$

Proof . of Theorem 1. As mentioned, the contract customer determines the execution amount of RFO and the capacity purchases amount through spot market based on the spot market freight rate s_t and the option strike price w_1. Therefore, the expected profit function of contract customer should be discussed separately. □

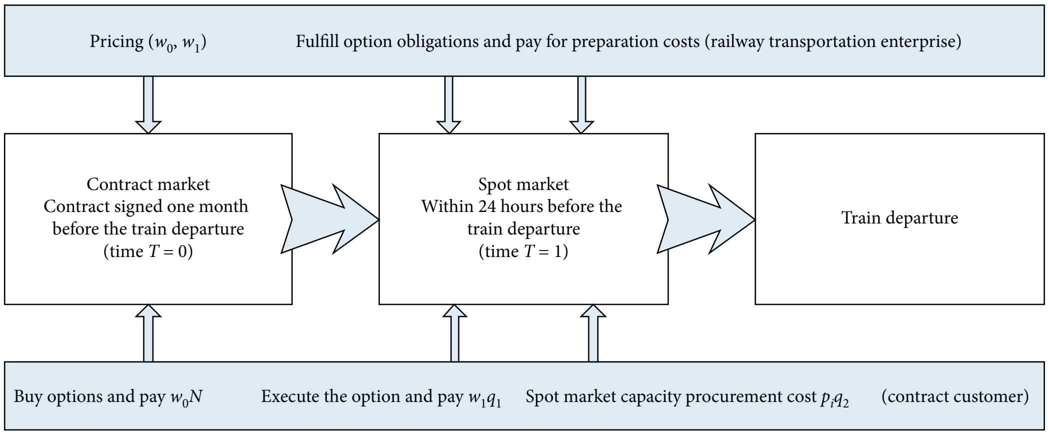

Figure 2: The transaction process of RFO.

Situation 1. The spot market freight rate s_t is lower than the option strike price w_1 at time T_1.

Under such a circumstance, the contract customer blindly purchases the capacity in the spot market, which will increase its profit. The optimal choice is to purchase the capacity from the spot market. Hence, the profit function of contract customer is constructed as follows:

$$\xi_1 = (s_t - w_d)D - w_0N, \tag{4}$$

where $(s_t - w_d)D$ stands for the cost savings of choosing purchase capacity from the spot market, and w_0N stands for the losses of ending RFO. Furthermore, the expected profit function in case of $s_t < w_1$ can be calculated as follows:

$$E\left(\xi_1\right) = \int_{-\infty}^{\infty} \left[(s_t - w_d)D - w_0N\right] f(x)\mathrm{d}x. \tag{5}$$

Situation 2. The spot market freight rate s_t is not lower than the option strike price w_1 at time T_1.

Now, it is profitable for contract customer to exercise RFO, and the contract customer can purchase the shortage capacities from the spot market. Then, the profit function of contract customer is constructed as follows:

$$\xi_2 = \begin{cases} (s_t - w_1)D - w_0N, & D \leq N, \\ (s_t - w_1)D - w_0N - w_u(D - N), & D > N, \end{cases} \tag{6}$$

where $w_u(D - N)$ stands for the losses of buying insufficient RFO at first. According to Assumption 5, the expected profit function in case of $s_t \geq w_1$ can be calculated as follows:

$$\begin{aligned} E\left(\xi_2\right) = & \int_{-\infty}^{N-a+bp} \left[(s_t - w_1)D - w_0N\right] f(x)\mathrm{d}x \\ & + \int_{N-a+bp}^{\infty} \left[(s_t - w_1)D - w_0N - w_u(D - N)\right] f(x)\mathrm{d}x. \end{aligned} \tag{7}$$

Next, combining the probability of two situations, the expected profit function of contract customer can be updated as follows:

$$\begin{aligned} E\left(\xi\right) &= p_d E\left(\xi_1\right) + \left(p_u + p_m\right) E\left(\xi_2\right) \\ &= p_d \int_{-\infty}^{\infty} \left[(s_t - w_d)D - w_0N\right] f(x)\mathrm{d}x \\ &\quad + \left(p_u + p_m\right)\left\{\int_{-\infty}^{N-a+bp} \left[(p - w_1)D - w_0N\right] f(x)\mathrm{d}x + \int_{N-a+bp}^{\infty} \left[p\ D - \left(w_0 + w_1\right)N - w_u(D - N)\right] f(x)\mathrm{d}x\right\}, \end{aligned} \tag{8}$$

which proves Theorem 1.

3.3. Railway Transportation Enterprise's Expected Profit Description. Similarly, the expected profit function of railway transportation enterprise $E(\zeta)$ is obtained as described in Theorem 2.

Theorem 2. *The expected profit function of railway transportation enterprise can be expressed as follows:*

$$\begin{aligned} E(\xi) = & p_d \int_{-\infty}^{\infty} \left[(w_d - b_2)D + w_0N(1 + r)^t - KC\right] f(x)\mathrm{d}x \\ & + \left(p_u + p_m\right)\left\{\int_{-\infty}^{N-a+bp} \left[(w_1 - b_1)D + w_0N(1 + r)^t - KC\right] f(x)\mathrm{d}x + \int_{N-a+bp}^{\infty} \left[(w_1 - b_1)N + w_0N(1 + r)^t + (w_u - b_2)(D - N) - KC\right] f(x)\mathrm{d}x\right\}. \end{aligned} \tag{9}$$

Table 1: Summary of notations.

Notation	Descriptions
w_0	Option price (unit capacity)
w_1	Option strike price (unit capacity)
s_t	Spot market freight rate (unit capacity) in time t
p_u	Probability of s_0 increase
p_d	Probability of s_0 drop
p_m	Probability of s_0 remain unchanged
w_u	Value of RFO at expiration when s_t increased
w_d	Value of RFO at expiration when s_t dropped
u	The advance ratio of s_0
d	The decline ratio of s_0
N	The purchase amount of RFO
c	Unit capacity fixed cost
q_1	The execution amount of RFO
q_2	The capacity purchases amount through spot market
b_1	Long-term preparation cost of transportation (unit capacity)
b_2	Short-term preparation cost of transportation (unit capacity)
K	Total capacity provided by railway freight transport
α	Standard deviation of railway freight market returns
D	Market capacity demand
$f(D)$	Probability density function of freight demand
r	Risk-free interest rate
α	Standard deviation of railway freight market returns
σ	Railway freight rate volatility

Proof . of Theorem 2. Although the railway transportation enterprise writes RFO, its profit depends on the contract customer's RFO execution decision. Therefore, the expected profit function of railway transportation enterprise should also be discussed separately. □

Situation 3. The spot market freight rate s_t is lower than the option strike price w_1 at time T_1.

Under such circumstance, the option price w_0 that the contract customer paid at time T_0 will not be refunded. The profit function of railway transportation enterprise is constructed as follows:

$$\zeta_1 = (w_d - b_2)D + w_0 N(1+r)^t - KC, \tag{10}$$

where $(w_d - b_2)D$ stands for the cost savings of RFO expiry without being exercised, $w_0 N(1+r)^t$ stands for the cash value of option price w_0 not refunded at time T_1, and KC stands for the fixed production cost. Furthermore, the expected profit function in case of $s_t < w_1$ can be calculated as follows:

$$E(\zeta_1) = \int_{-\infty}^{\infty} \left[(w_d - b_2)D + w_0 N(1+r)^t - KC\right] f(x)\mathrm{d}x. \tag{11}$$

Situation 4. The spot market freight rate s_t is not lower than the option strike price w_1 at time T_1.

In this case, it is profitable for contract customer to exercise the RFO purchased, and the railway transportation enterprise charges option strike price w_1. Besides, if the demand for the contract customer is more than the RFO purchased, the contract customer will purchase the shortage capacities from the spot market. Thus, the profit function of railway transportation enterprise is constructed as follows:

$$\zeta_2 = \begin{cases} (w_1 - b_1)D + w_0 N(1+r)^t - KC, & D \le N, \\ (w_1 - b_1)N + w_0 N(1+r)^t + (w_u - b_2)(D-N) - KC, & D > N, \end{cases} \tag{12}$$

where $(w_1 - b_1)N$ stands for the incomes of exercising RFO purchased, and $(w_u - b_2)(D-N)$ stands for the incomes of selling the remaining capacity in the spot market. According to Assumption 5, the expected profit function in case of $s_t \ge w_1$ can be calculated as follows:

$$\begin{aligned} E(\zeta_2) = &\int_{-\infty}^{N-a+bp} \left[(w_1 - b_1)D + w_0 N(1+r)^t - KC\right] f(x)\mathrm{d}x \\ &+ \int_{N-a+bp}^{\infty} \left[(w_1 - b_1)N + w_0 N(1+r)^t + (w_u - b_2)(D-N) - KC\right] f(x)\mathrm{d}x. \end{aligned} \tag{13}$$

Next, combining the probability of two situations, the expected profit function of railway transportation enterprise can be updated as follows:

$$\begin{aligned}E(\zeta) &= p_d E(\zeta_1) + (p_u + p_m)E(\zeta_2)\\ &= p_d \int_{-\infty}^{\infty} \left[(w_d - b_2)D + w_0 N(1+r)^t - KC\right] f(x)\mathrm{d}x\\ &+ (p_u + p_m)\left\{\int_{-\infty}^{N-a+bp} \left[(w_1 - b_1)D + w_0 N(1+r)^t - KC\right] f(x)\mathrm{d}x + \int_{N-a+bp}^{\infty} \left[(w_1 - b_1)N + w_0 N(1+r)^t + (w_u - b_2)(D-N) - KC\right] f(x)\mathrm{d}x\right\},\end{aligned} \tag{14}$$

which proves Theorem 2.

4. Main Result

4.1. Parameter Solution. A significant point that is believed to be more critical in the model solving processes is determining the parameters. This research proposes a nonparametric Ito stochastic approach for the estimation of the critical parameters.

As is well known, the trinomial tree is derived from the extension of the binomial tree. Although the binomial tree effectively simplifies price changes and facilitates calculations, it only roughly considers the two cases of up and down. Therefore, the binomial tree only has $n + 1$ cases after n step leading to low accuracy. The trinomial tree adds a new case; that is, the price does not change after a while. Under such a circumstance, $2n + 1$ results will be obtained after n step leading to high accuracy. At the same time, the single-period trinomial tree has limitations in price evaluation for RFO. To end this, this article intends to divide the expiration time into multiple parts and establish a multiperiod trinomial tree to solve the optimal pricing decision of RFO. The basic trinomial tree is shown in the following Figure 3.

As the number of periods increases, the trinomial tree model faces the issue to determine parameters such as the magnitude and probability of s_0 increase and drop in the spot market. It is necessary to adjust the parameters to ensure that the standard deviation of the standard deviation of railway freight market returns remains unchanged. First of all, discretize the continuously changing underlying asset price (s_t).

Dividing time $[0, T]$ into n equal parts, according to Assumption 5, s_t is subject to geometric Brownian motion within time $[t_i, t_{i+\Delta t}]$. This equation can be defined by

$$\frac{\Delta S}{S} = \alpha \Delta t + \sigma \left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right). \tag{15}$$

Using the Ito's stochastic integral, formula (15) can be updated as follows:

$$S(t) = S\left(t_i\right)e^{\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) + \left(\alpha - \left(\sigma^2/2\right)\right)\left(t - t_i\right)}, \tag{16}$$

where $e^{\sigma B(t) - 0.5\sigma^2}$ is the equivalence of martingale measure. Using the fact that $e^{\sigma B(t) - 0.5\sigma^2}$ is a martingale and $E[e^{\sigma B(t) - 0.5\sigma^2}] = 1$, the following is concluded:

$$\begin{aligned}E[S(t)] &= E\left[S\left(t_i\right)e^{\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) + \left(\alpha - \left(\sigma^2/2\right)\right)\left(t - t_i\right)}\right]\\ &= S\left(t_i\right)E\left[e^{\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) - \left(\sigma^2/2\right)\left(t - t_i\right)}e^{\alpha\left(t - t_i\right)}\right] = S\left(t_i\right)e^{\alpha\left(t - t_i\right)}.\end{aligned} \tag{17}$$

Moreover, the function of an Ito process is still an Ito process according to the Ito lemma, and then the nonparametric Ito stochastic approach can be applied to generate

$$S^2(t) = S^2\left(t_i\right)e^{2\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) + \left(2\alpha - \sigma^2\right)\left(t - t_i\right)}, \tag{18}$$

$$S^3(t) = S^3\left(t_i\right)e^{3\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) + \left(3\alpha - \left(3\sigma^2/2\right)\right)\left(t - t_i\right)}, \tag{19}$$

where $e^{2\sigma B(t) - 2\sigma^2}$ and $e^{3\sigma B(t) - 4.5\sigma^2}$ are the equivalence of martingale measure. Here, using the fact that $E[e^{2\sigma B(t) - 2\sigma^2}] = 1$ and $E[e^{3\sigma B(t) - 4.5\sigma^2}] = 1$, the following is concluded:

$$\begin{aligned}E\left[S^2(t)\right] &= E\left[S^2\left(t_i\right)e^{2\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) + \left(2\alpha - \sigma^2\right)\left(t - t_i\right)}\right]\\ &= S^2\left(t_i\right)E\left[e^{2\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) - 2\sigma^2\left(t - t_i\right)}e^{\left(2\alpha + \sigma^2\right)\left(t - t_i\right)}\right]\\ &= S^2\left(t_i\right)e^{\left(2\alpha + \sigma^2\right)\left(t - t_i\right)},\end{aligned} \tag{20}$$

$$\begin{aligned}E\left[S^3(t)\right] &= E\left[S^3\left(t_i\right)e^{3\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) + \left(3\alpha - \left(3\sigma^2/2\right)\right)\left(t - t_i\right)}\right]\\ &= S^3\left(t_i\right)E\left[e^{3\sigma\left(B\left(t_{i+\Delta}\right) - B\left(t_i\right)\right) - \left(9\sigma^2/2\right)\left(t - t_i\right)}e^{\left(3\alpha + 3\sigma^2\right)\left(t - t_i\right)}\right]\\ &= S^3\left(t_i\right)e^{\left(3\alpha + 3\sigma^2\right)\left(t - t_i\right)}.\end{aligned} \tag{21}$$

As mentioned above, there are three cases of changes in transportation prices, rising (p_u), unchanged (p_m), and falling (p_d), respectively. According to formulas (15)–(21), the system of parameter equations can be constructed as follows:

$$\begin{cases} p_u + p_d + p_m = 1, \\ p_u u + p_m + p_d d = e^{\alpha\Delta t}, \\ p_u u^2 + p_m + p_d d^2 = e^{\left(2\alpha+\sigma^2\right)\Delta t}, \\ p_u u^3 + p_m + p_d d^3 = e^{\left(3\alpha+3\sigma^2\right)\Delta t}, \\ u\, d = 1, \end{cases} \tag{22}$$

and the parameters are solved as follows:

$$\begin{cases} p_u = \dfrac{(1+d)e^{\alpha\Delta t} - e^{\left(2\alpha+\sigma^2\right)\Delta t} - d}{(d-u)(u-1)}, \\ p_m = \dfrac{(u+d)e^{\alpha\Delta t} - e^{\left(2\alpha+\sigma^2\right)\Delta t} - 1}{(1-d)(u-1)}, \\ p_d = \dfrac{(1+u)e^{\alpha\Delta t} - e^{\left(2\alpha+\sigma^2\right)\Delta t} - u}{(d-u)(1-d)}, \\ u = e^{\sqrt{\left(\sigma^2-\lambda\right)\Delta t}}, \\ d = e^{-\sqrt{\left(\sigma^2-\lambda\right)\Delta t}}, \\ \lambda = \dfrac{e^{\alpha\Delta t} + e^{\left(3\alpha+3\sigma^2\right)\Delta t} - e^{\left(2\alpha+\sigma^2\right)\Delta t} - 1}{2\left[e^{\left(2\alpha+\sigma^2\right)\Delta t} - e^{\alpha\Delta t}\right]}. \end{cases} \tag{23}$$

Generally speaking, the probability of freight rate changes is low in a short time. In order to be more realistic, a strong assumption $p_m = (2/3)$ is imposed here. The parameters can be recalculated as

$$\begin{aligned} p_u &= \frac{e^{\left(2\alpha+\sigma^2\right)\Delta t} - (2/3) - \left(e^{\alpha\Delta t} - (2/3)\right)d}{u(u-d)}, \\ p_d &= \frac{1}{3} - p_u. \end{aligned} \tag{24}$$

4.2. The Optimal Pricing Decision of RFO. According to the expected profit function of contract customer $E\ (\xi)$ and railway transportation enterprise $E\ (\zeta)$ obtained, the optimal pricing decision of RFO is obtained as described in Theorem 3.

Theorem 3. *Based on the key parameters that are obtained through nonparametric Ito stochastic approach, the railway transportation enterprise decides the optimal pricing decision of RFO which satisfies the following formula:*

$$w_1 = w_u - \frac{b_1 - b_2}{(1+r)^t - 1}. \tag{25}$$

Proof . of Theorem 3. The expected profit function of contract customer $E\ (\xi)$ and railway transportation enterprise $E\ (\zeta)$ are affected by the purchase amount of RFO N. The first-order partial derivatives of N for formulas (3) and (9) are obtained on the basis of optimization theory as follows:

$$\begin{aligned} \frac{\partial E(\xi)}{\partial N} &= p_d \frac{\partial E\left(\xi_1\right)}{\partial N} + \left(p_u + p_m\right)\frac{\partial E\left(\xi_2\right)}{\partial N} \\ &= -w_0 F(N) + \left(p_u + p_m\right)\left[-w_0 + \left(w_u - w_1\right)\right][1 - F(N)], \\ \frac{\partial E(\zeta)}{\partial N} &= p_d \frac{\partial E\left(\zeta_1\right)}{\partial N} + \left(p_u + p_m\right)\frac{\partial E\left(\zeta_2\right)}{\partial N} \\ &= w_0 F(N)(1+r)^t + \left(p_u + p_m\right)\left[w_0 (1+r)^t + \left(w_1 - b_1 - w_u + b_2\right)\right][1 - F(N)]. \end{aligned} \tag{26}$$

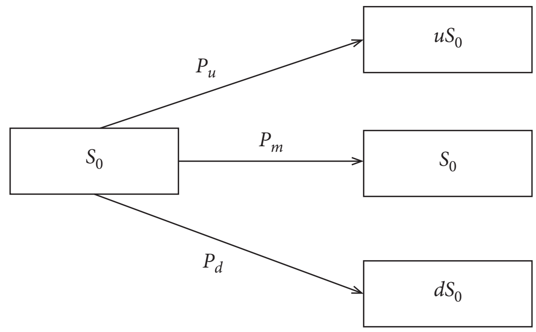

Figure 3: The basic trinomial tree.

The optimal RFO order quantity N^* of RFO can be calculated when the differential equations are equal to 0. In other words, N^* is defined by the following equation:

$$\begin{aligned} &-w_0 F\left(N^*\right) + \left(p_u + p_m\right)\left[-w_0 + \left(w_u - w_1\right)\right]\left[1 - F\left(N^*\right)\right] = 0, \\ &w_0 F\left(N^*\right)(1+r)^t + \left(p_u + p_m\right)\left[w_0(1+r)^t + \left(w_1 - b_1 - w_u + b_2\right)\right]\left[1 - F\left(N^*\right)\right] = 0. \end{aligned} \tag{27}$$

Furthermore, the optimal RFO order quantity N^* of RFO satisfies the following formula:

$$1 - \frac{1}{F\left(N^* - a + bs_t\right)} = \frac{-w_0}{\left(1 - p_u\right)\left(w_u - w_0 - w_1\right)}. \tag{28}$$

Next, combined with formula (24), solving the simultaneous equations of formulas (19) and (20), the optimal option strike price w_1^* of RFO can be easily gained as follows:

$$w_1^* = w_u - \frac{b_1 - b_2}{(1+r)^t - 1}, \tag{29}$$

which proves Theorem 3.

According to the aforementioned analysis, when the railway transportation enterprise writes RFO, the railway transportation enterprise should pay more attention to the value of RFO at expiration when s_t increased (w_u), the long-term preparation cost of transportation b_1, the short-term preparation cost of transportation b_2, and the risk-free interest rate r. □

5. Conclusion

This paper puts forward the theory of tradable rail freight option (RFO). The transaction process is designed as shown in Figure 2. The expected profit functions of contract customer and railway transportation enterprise are obtained as shown in formulas (3) and (9). The nonparametric Ito stochastic approach is applied to calculate key parameters as shown in formula (24), and an advantage of this method over the quantitative and stochastic methods is that it simplifies the parameters' solution. The most important result of this paper is Theorem 3. It is observed that the optimal option strike price w_1^* of RFO is decided by the value of RFO at expiration when s_t increased (w_u), the long-term preparation cost of transportation b_1, the short-term preparation cost of transportation b_2, and the risk-free interest rate r.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and publication of this paper.

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