

Research Article

Pricing Corporate Bonds with Credit Risk, Liquidity Risk, and Their Correlation

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This paper proposes a generalized bond pricing model, accounting for all the effects of credit risk, liquidity risk, and their correlation. We use an informed trading model to specify the bond liquidity payoff and analyze the sources of liquidity risk. We show that liquidity risk arises from reduced information accuracy and market risk tolerance, and it is market risk tolerance that links credit and liquidity. Then, we extend the traditional bond pricing model with only credit risk by incorporating liquidity risk into the framework in which the probabilities of the two risk events are estimated by a joint distribution. Using numerical examples, we analyze the role of the correlation between credit and liquidity in bond pricing, especially during a financial crisis. We document that the varying correlation between default and illiquidity explains the phenomenon of bond death spiral observed in a financial crisis. Finally, we take the US corporate bond market as an example to demonstrate our conclusions.

1. Introduction

Unlike government bonds, corporate bonds require risk compensation, which is referred to as yield spreads. The risk of corporate bonds is typically classified into two categories: credit risk and liquidity risk [1–5]. During the financial crisis of 2007–2008, these two risk premia increased alternately, leading to a substantial decline in bond prices, a phenomenon known as the bond death spiral. Empirical evidence shows that credit and liquidity risks interact with each other, and this interaction plays a crucial role in bond pricing. For example, some previous work documents that credit is correlated with liquidity spreads in the US bond market [6], which was particularly pronounced during the subprime crisis [7, 8]. As the correlation between the two risk factors in regression models renders the empirical results hard to interpret, researchers try to disentangle the two sources of risk in yield spreads to provide a robust analysis of yield spreads [9–12]. While this issue is empirically analyzed in

the literature, there is a lack of theoretical analysis of the way in which the correlation arises and how it impacts yield spreads.

This paper proposes a generalized bond pricing model, accounting for all the effects of credit risk, liquidity risk, and their correlation. First, we incorporate liquidity risk into the traditional bond pricing model. The two important developments in the corporate bond literature are the structural-form [13] and reduced-form models [14]. The former provides a perfect explanation of corporate bond prices, while the latter is better able to evaluate the default probability and default loss using historical data. One of problems with these models is that most traditional models deal with credit risk only and ignore the effects of liquidity risk and its correlation with credit risk. Inspired by Jarrow et al. [15] who proposed a method for pricing callable bonds by considering credit risk and call risk within a unified framework, we consider both credit and liquidity risks as a factor that leads to the potential termination of obligations.

Similar to default, liquidating a bond can also be regarded as an event that terminates a loan contract from the bondholders' perspective. Therefore, we extend the traditional one-risk model to a two-risk model in which credit risk and liquidity risk are correlated.

Second, we explore the key factor that results in the correlation between the two risks. We note that liquidity risk encompasses multiple dimensions and is more complex than credit risk. Thus, we focus on liquidity risk to gauge how the correlation arises. While there is an extensive literature on liquidity risk, there is no unified aggregate proxy for liquidity [16–18]. One of the measures of bond liquidity is the marketability discount, which is the loss due to the quick sale of a bond. Market microstructure theory on marketability discount provides distinct views of liquidity. These models consider various factors such as information asymmetry, imperfect competition, and funding constraints to interpret the determination of the marketability discount [19–22]. Some studies incorporate these factors into traditional trading models and analyze their effects on asset trading, explaining changes in asset prices in different periods [23, 24]; others examine the role of these factors in asset pricing by calculating the deviation between the asset cash flows and trading price in a single-period model [21, 25–27]. Regardless of the approach used, these models all reach similar conclusions about the factors that impact the trading price. In particular, most of these articles show a strong linkage between information asymmetry and marketability discount.

Since liquidity loss arises from the trading process, in this paper, we use an informed trading model to specify a bond's marketability discount and analyze the sources of liquidity risk. We document that liquidity risk arises from reduced information accuracy and market risk tolerance. The reduction in market risk tolerance is due to the concerns about poor firm performance and worsening market conditions. Thus, market risk tolerance is related to the incidence of default, while information accuracy is irrelevant to default. In other words, changes in the correlation between credit and liquidity are driven by changes in market risk tolerance. Moreover, to calculate the probabilities of default and trade with a nonzero correlation, we adopt a Frank Copula function to describe the joint distribution of the two events.

Third, using numerical examples, we analyze the role of the correlation between credit and liquidity in bond pricing and explain the phenomenon of bond death spiral observed in a financial crisis. We find that a positive correlation between default and trade decreases yield spreads, while a negative one increases them. In addition, we analyze both credit-leading crisis and liquidity-leading crisis by dividing the crisis time interval into sufficiently short subintervals. We explore the effect of the varying correlation, which arises from decreasing market risk tolerance, on both credit and liquidity risk premia in each subinterval. We find that, during a financial crisis, a decrease in market risk tolerance changes the correlation from positive to negative, which leads to the risk contagion, ultimately resulting in the bond death spiral.

Fourth, we take the US corporate bond market as example to illustrate how the correlation between credit risk and liquidity risk influences bond prices under different market conditions. Using a Markov-switching model to describe the changes of the relationship between yield spreads and risk factors, we show that the correlation plays an important role in bond market during financial crisis. Moreover, the changes of correlation parameters during a crisis are consistent with what our bond pricing model predicts and provide evidence in support of the numerical analysis.

The remainder of the paper is structured as follows. Section 2 describes the model framework. Section 3 discusses the method of estimating parameters in our model. Section 4 uses numerical examples to show how the correlation influences bond prices, particularly the role of the correlation in bond pricing during the crisis time. Section 5 provides the empirical analysis of the US bond market. Section 6 concludes.

2. The Model Framework

There are two alternative approaches to modeling credit risk: the structural model [13] and the reduced model [28]. The traditional structural approach assumes a stochastic process for firm value to evaluate bonds, while the reduced approach assumes an exogenous process for a firm's default time and recovery rate. From a theoretical point of view, the structural model better explains the process for bond prices. However, the perfect information assumption in this model contradicts with the theoretical and empirical evidence on credit market equilibrium [29]. Since most information used in this model is not readily available, previous research finds that the structural model is not able to accurately explain real yield spreads [30–32]. In contrast, the reduced model is better able to evaluate the probability and loss of credit risk by using historical default and trade data instead of the companies' asset value information.

In this paper, we adopt the reduced modeling approach to analyze our research issues. However, the traditional reduced model deals with credit risk only. Jarrow et al. [15] propose a reduced-form approach for valuing callable corporate bonds by characterizing the call probability via an intensity process. Following this line of thought, we develop a model framework that accommodates both credit risk and liquidity risk, in which the two risks are characterized by two events, default and trade, meaning the end of a lending relationship.

Assume that the economic uncertainty is characterized by a filtered probability space $\{\Omega, F, P\}$ satisfying the usual conditions, where Ω is the sample space, F is the set of events, and P is the statistical probability measure. We use $\{(Y, T), (Y_d, \tau_d), (Y_s, \tau_s)\}$ to denote the cash flows from a zero-coupon corporate bond under various scenarios. The first claim (Y, T) represents the obligation of the firm to pay Y dollars at maturity T . The second claim (Y_d, τ_d) represents the case of default, in which investors receive a residual value Y_d dollars at the default time τ_d . Y_d is just part of the bond's face value, and the ratio of Y_d to Y is referred to as the

recovery rate, denoted as y_d . The third claim (Y_s, τ_s) represents the case of trade, where investors sell the bond for Y_s dollars at time τ_s . The payoff of this trade is less than the present value of its future cash flow due to imperfect market conditions. In theory, the trade price is based on the bond cash flow, but is also determined by several other factors that are exogenous. We refer to $y_s = (Y_s/Y)$ as the payoff ratio. Both the default recovery rate and the liquidity payoff ratio depend on market conditions and can be stochastic.

Accordingly, without the cost of time, the payoff from a zero-coupon bond with a face value of \$1 and maturity date of T can be expressed as follows:

$$Z = y_d \mathbf{1}_{\{\tau_d < \tau_s, \tau_d < T\}} + y_s \mathbf{1}_{\{\tau_s < \tau_d, \tau_s < T\}} + \mathbf{1}_{\{T < \tau_d, T < \tau_s\}}, \quad (1)$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function. Let r_f be the instantaneous risk-free interest rate and r be risky interest rate. Assume that the process of the risk-free rate is independent of that of the risky rate. The value of the bond at time t is given as

$$\begin{aligned} V(t, T) &= E_t^Q \left\{ e^{-\int_t^T r(u) + r_f(u) du} \right\} \\ &= E_t^Q \left\{ e^{-\int_t^{T_0} r_f(u) du} \right\} E_t^Q \left\{ e^{-\int_t^T r(u) du} \right\} \\ &= E_t^Q \left\{ e^{-\int_t^{T_0} r_f(u) du} \right\} E_t^Q \{Z\}, \end{aligned} \quad (2)$$

where V is the value of the bond, Q is the equivalent (to P) probability measure such that all discounted bond prices are martingales with respect to the information set at time t , and E^Q is the expectation operator under probability measure Q . As we can see, the value of the bond consists of two parts, $E_t^Q \left\{ e^{-\int_t^{T_0} r_f(u) du} \right\}$ and $E_t^Q \{Z\}$. The former is the discounted value at the risk-free rate (which is not the focus of this paper), and the latter is the discounted value at the risky interest rate, and the primary source of yield spreads.

The default and trade probabilities are $P_d = P[\tau_d < T, \tau_d < \tau_s]$ and $P_s = P[\tau_s < T, \tau_s < \tau_d]$, respectively. Then, the expected payoff Z from the zero-coupon bond is

$$E_t^Q \{Z\} = y_d P_d + y_s P_s + 1 P_m, \quad (3)$$

where P_m represents the probability of holding the bond until maturity. Apparently, $P_d + P_s + P_m = 1$.

Equation (3) represents a generalized model framework that accounts for the effects of the incidences of both default and trade. Importantly, in this model, default and trade are allowed to be correlated. To value a bond, we need to evaluate bondholders' losses arising from default or trade and the instantaneous probabilities of the two related events.

3. Estimating Liquidity Payoff and Probabilities of Default and Trade

3.1. Liquidity Payoff. Reduced pricing models use historical data to estimate the default loss. However, it is difficult to estimate the loss of trade using historical data, as liquidity risk is more complex than credit risk. Illiquidity can be defined as the value that bondholders must give up for bond liquidating. As discussed in Section 2, the liquidity payoff is the bond's cash flow minus its marketability discount. The payoff ratio can be expressed as

$$y_s = \gamma(1 - w), \quad (4)$$

where γ is the value of the bond with face value of \$1 considering credit risk only and w is the marketability discount. As the estimation of γ is well defined in previous bond pricing models [14, 33], y_s can be obtained if the expression of marketability discount w is known.

Equation (4) implies that liquidity risk arises from the trading process, and we can measure liquidity loss in terms of marketability discount. Note that information risk is identified as a crucial factor in the trading process [34–36]. In particular, previous studies show that information asymmetry is one of the most important factors affecting bond prices [37, 38]. Therefore, to examine the marketability discount, we consider a bond trading discount model with information asymmetry.

3.1.1. Marketability Discount. Following Lambert et al. [27], we assume that there are two types of investors in the bond market: a limited number of informed investors (such as institutional investors) and infinite uninformed investors (such as individual investors) who have no private information but can learn from market prices. The market is competitive, and all investors are risk averse and maximize their personal utilities.

At any time before maturity, let \tilde{Y} represent the present value of the bond's cash flow considering default risk and P represent its trading price. We first consider informed investors' behavior in the market. As the superior trader in the market, an informed trader owns private information about the bond value $Y_{in} = \tilde{Y} + \varepsilon_{in}$, where ε_{in} is an error with a mean of 0 and variance of σ_{in}^2 (the precision is denoted by Π_{ie}). Correspondingly, the precision of the informed investor's evaluation of the bond is $\Pi_{in} = \Pi_v + \Pi_{ie}$. Thus, the expected bond value based on his private information is

$$E[\tilde{Y} | \Omega_{in}] = E[\tilde{Y}] + \frac{\Pi_{ie}}{\Pi_{in}} (Y_{in} - E[\tilde{Y}]), \quad (5)$$

where Ω_{in} is the information set for the informed investor. Equation (5) indicates that the conditional expectation of bond value is composed of two parts: the expected bond value and the evaluation error.

We suppose the informed trader has constant absolute risk tolerance λ_{in} . Based on his belief as to how his demand affects the market, this investor chooses his demand for

bonds, D_{in} , to maximize his profits, S_{in} , which are given as follows:

$$S_{in} = \left\{ E[\tilde{Y}|\Omega_{in}] - P - \frac{D_{in}}{2\lambda_{in}\Pi_{in}} \right\} D_{in}. \quad (6)$$

Taking the partial derivative of S_{in} with respect to D_{in} and setting it equal to 0 gives

$$D_{in} = (E[\tilde{Y}|\Omega_{in}] - P)\Pi_{in}\lambda_{in}. \quad (7)$$

Similarly, for an uninformed trader, the value of the bond is $Y_{un} = \tilde{Y} + \varepsilon_{un}$, where ε_{un} is an error with mean 0, variance σ_{un}^2 , and precision Π_{ue} . The uninformed trader's constant absolute risk tolerance is λ_{un} and her demand for bonds is D_{un} . Thus, her expected value and profit function are as follows, respectively:

$$E[\tilde{Y}|\Omega_{un}] = E[\tilde{Y}] + \frac{\Pi_{ue}}{\Pi_{un}} (Y_{un} - E[\tilde{Y}]), \quad (8)$$

$$S_{un} = \left\{ E[\tilde{Y}|\Omega_{un}] - P - \frac{D_{un}}{2\lambda_{un}\Pi_{un}} \right\} D_{un}. \quad (9)$$

Taking the derivative of equation (9) with respect to D_{un} and setting it equal to 0 yields

$$D_{un} = (E[\tilde{Y}|\Omega_{un}] - P)\Pi_{un}\lambda_{un}. \quad (10)$$

Let L be the supply in the bond market and N (M) represent the number of informed (uninformed) traders. The market clearing condition is as follows:

$$ND_{in} + MD_{un} = L. \quad (11)$$

As assumed, there are infinite uninformed investors in the market with limited wealth. They do not have any private information about the bond value, but they can analyze market prices to infer the private information owned by informed investors. For simplicity, we assume that M is large (i.e., $M \rightarrow +\infty$) and λ_{un} is small (i.e., $\lambda_{un} \rightarrow 0$), and the product of M and λ_{un} converges to a nonnegative constant ω , or $M\lambda_{un} \rightarrow \omega$.

We can prove that the value the uninformed trader learns from market prices can be expressed as follows:

$$Y_{learn} = \tilde{Y} + \varepsilon_{in} - (N\lambda_{in}\Pi_{ie})^{-1}L. \quad (12)$$

See the proof in Appendix A.

Similar to equation (8), we have the following:

$$E[\tilde{Y}|\Omega_{un}] = E[\tilde{Y}] + \frac{\Pi_{learn}}{\Pi_{un}} (Y_{learn} + E[Y_{learn}]), \quad (13)$$

where Π_{learn} is the precision of ε_{learn} and $\Pi_{un} = \Pi_v + \Pi_y$ is the precision of a uninformed investor's evaluation of the bond.

Thus, we can derive the marketability discount w , as follows:

$$w = E[\tilde{Y}] - E[P] = \Pi^{-1}E(L)\lambda^{-1}, \quad (14)$$

where Π and λ are given as follows:

$$\Pi = \frac{N\lambda_{in}\Pi_{in} + M\lambda_{un}\Pi_{un}}{N\lambda_{in} + M\lambda_{un}}, \quad (15)$$

$$\lambda = N\lambda_{in} + M\lambda_{un}. \quad (16)$$

The proofs of equations (14) to (16) are provided in Appendix B. Equation (14) is the gap between the trading price of a bond and its future cash flow.

3.1.2. Decomposition of Marketability Discount. The trading discount model provides a method to measure the marketability discount, and equations (14) to (16) show that the marketability discount depends on a variety of variables, such as participants' information accuracy, risk tolerance, number of participants, and amount of supply. Now, we simplify these expressions and explore the economic implications of the model.

Based on equation (15), Π can be considered as the weighted average value of Π_{ie} and Π_{un} , where the weights are $N\lambda_{in}(N\lambda_{in} + M\lambda_{un})^{-1}$ and $M\lambda_{un}(N\lambda_{in} + M\lambda_{un})^{-1}$, respectively. $N\lambda_{in}$ ($M\lambda_{un}$) measures all informed (uninformed) traders' risk tolerance or wealth. Thus, Π measures the market average information accuracy level, and it does not change with wealth but can be affected by information transparency. Since λ is the sum of two risk tolerances of informed and uninformed investors as can be seen in equation (16), it measures total risk tolerance of all investors, which is referred to as the market risk tolerance. Unlike Π , λ can be affected by wealth.

The changes of these two factors, information accuracy Π or the market risk tolerance λ , are closely related to bond supply. Accordingly, in equation (14), L can be divided into two parts: L_{Π} and L_{λ} . L_{Π} is caused by information asymmetry, and L_{λ} is caused by reduced market risk tolerance.

Similarly, marketability discount is also influenced by market average information accuracy level and market risk tolerance, and thus, the marketability discount in equation (14) can be rewritten as follows:

$$w = (\Pi^{-1}E(L_{\Pi})) (E(L_{\lambda})\lambda^{-1}) = \Pi^*\lambda^*. \quad (17)$$

Equation (17) says that an increase in Π^* leads to an increase in liquidity risk. This is because a decrease in the average information accuracy of the market Π , which can be due to the inaccuracy of firms' information disclosure or the lack of investors' capability of gathering and processing information, will have a negative effect on trading, as shown in dramatic crashes in the US stock market, most notably the 1929 and 1987 crashes. In a market where uninformed investors are unable to distinguish hedging activity from information-based trades, large numbers of such investors may revise downward their expectations when there are, what appear to be, infinitesimal shifts in information or other small shocks that lead to lower prices [39]. Thus, equation (17) implies that the liquidity loss can be due to the reduction in information accuracy.

In addition, an increase in λ^* can also result in higher liquidity risk. This is because the total risk tolerance λ

declines as a result of the deterioration of a firm's own performance or poor macroeconomic conditions, which might lead to bond default. In the case of firms' poor performance, lack of confidence in the firms' capability of meeting their obligations makes investors more cautious when trading to avoid losses, reducing λ_x ($x = \text{in or un}$). In the case of poor economic conditions, traders may not have sufficient funds to absorb trading losses, leading to decreased risk tolerance, λ_x . The former fear of loss arises directly from the deteriorating financial health of firms, while the latter is a macroeconomic factor that indirectly increases traders' fear of default via the aggregate market liquidity. These two types of fear make investors more cautious when making trading decisions and more sensitive to default risk, leading to a lower value of λ^* . Thus, equation (17) implies that the liquidity loss can also be due to the reduced market risk tolerance.

Identifying the sources of liquidity risk has several implications. First, it provides an explanation of liquidity risk in bond pricing and a method to calculate liquidity payoff. Plugging the expression of marketability discount in equations (17) into (4) gives the payoff ratio as follows:

$$\gamma_s = \gamma - \Pi^* \lambda^*. \quad (18)$$

Second, it provides a theoretical explanation about the correlation between credit and liquidity risks in bond markets. Previous studies [8, 9] show that liquidity risk is related to default risk, and their impacts on yield spreads are not independent of one another, but these studies are not able to explain the causes of the correlation. We find that bond liquidity risk arises from reduced information accuracy and market risk tolerance. The first factor reflects the opacity of information and investors' ability to capture the information, which are determined by exogenous factors such as market regulations and the overall quality of market investors. This factor is independent of the credit level of bonds. In contrast, reduced market risk tolerance arises from deterioration of the firm's performance and worsening economic conditions, both of which contribute significantly to corporate default and make investors more concerned about the loss of their investments. In conclusion, there are significant differences between these two factors. The former is independent of default risk, while the latter is related to default risk. Moreover, a decrease in market risk tolerance not only aggregates the marketability discount but also influences the correlation between credit and liquidity risks.

3.2. Probabilities of Default and Trade. The traditional reduced-form model provides a framework for us to price corporate bonds, whereas the trading discount model specifies liquidity loss as a trading process. Given our discussions in Section 3.1, we can obtain a generalized bond pricing model by combining the traditional and trading methods. Plugging equation (18) into (3), we obtain the expected payoff of a zero-coupon bond as follows:

$$E_t^Q\{Z\} = \gamma_d P_d + (\gamma - \Pi^* \lambda^*) P_s + P_m. \quad (19)$$

In this section, we turn our attention to estimating the probabilities of default and trade. Following Jarrow et al. [15] who characterized both the call and default as a point process, we assume that both default and trade arrive with an intensity process, where their intensities are $h_d(t)$ and $h_s(t)$, respectively. For a sufficiently small number Δ , the intensity process can be expressed as

$$h_d(t)\Delta = P(t < \tau_d < t + \Delta | \tau_d > t) = \frac{F'_d(t)}{1 - F_d(t)}, \quad (20)$$

$$h_s(t)\Delta = P(t < \tau_s < t + \Delta | \tau_s > t) = \frac{F'_s(t)}{1 - F_s(t)}.$$

Then, for $x \in [t, t + \Delta]$, the marginal distributions can be calculated as follows:

$$\begin{aligned} F_d(x) &= 1 - e^{-h_d(x-t)t} \\ F_s(x) &= 1 - e^{-h_s(x-t)t} \end{aligned} \quad (21)$$

Since the incidences of default and trade are not independent, we use a joint distribution function to describe the probabilities of the two events. To this end, we assume time τ_d and τ_s have a joint probability density function $f(\tau_d, \tau_s)$ and a joint probability distribution function $F(\tau_d, \tau_s)$. Therefore, in the interval $[t, t + \Delta]$, the probabilities of the two events, $P_d(\Delta)$ and $P_s(\Delta)$, can be expressed as

$$P_d(\Delta) = \int_t^{t+\Delta} \left(\frac{\partial F}{\partial x} \Big|_{y=+\infty} - \frac{\partial F}{\partial x} \Big|_{y=x} \right) dx, \quad (22a)$$

$$P_s(\Delta) = \int_t^{t+\Delta} \left(\frac{\partial F}{\partial y} \Big|_{x=+\infty} - \frac{\partial F}{\partial y} \Big|_{x=y} \right) dy. \quad (22b)$$

Given that risk varies with macroeconomic and market conditions, we divide the interval $[0, T]$ into N subintervals $[t_i, t_{i+1}]$ ($i = 1, \dots, N$), where N is big enough so that each subinterval is very short. In each subinterval, the probabilities can be written as $P_{d,i} = P[\tau_d \leq t_{i+1}, \tau_d \leq \tau_s | \Phi_i]$ and $P_{s,i} = P[\tau_s \leq t_{i+1}, \tau_s \leq \tau_d | \Phi_i]$, where Φ_i is the information set at time t_i in which $\tau_d > t_i$ and $\tau_s > t_i$.

Proposition 1. *The probabilities of the two events in $[0, T]$ are given by*

$$P_d = \sum_{i=1}^n P_{d,i} \prod_{j=1}^i (1 - P_{d,j} - P_{s,j}), \quad (23a)$$

$$P_s = \sum_{i=1}^n P_{s,i} \prod_{j=1}^i (1 - P_{d,j} - P_{s,j}). \quad (23b)$$

Proof. See Appendix C

By specifying the joint distribution function $F(x, y)$, the probabilities of the two events can be estimated using equations 23a and 23b. With the estimates of the losses of default and trade, we are able to price bonds using equation (19), which accounts for liquidity risk, credit risk, and their correlation. We use the Frank Copula function as the cumulative distribution function in our numerical analysis, which is as follows:

$$F(x, y) = C(u, v) = \frac{1}{\alpha} \ln \left\{ 1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{e^{-\alpha} - 1} \right\}, \quad (24)$$

where $\alpha \neq 0$ and u and v represent the distribution functions of default and trade, respectively. The marginal distributions are given as follows:

$$\begin{aligned} F_x &= \frac{h_d e^{-h_d x} \Psi_y (\Psi_x + 1)}{\Psi_y \Psi_x + (e^{-\alpha} - 1)}, \\ F_y &= \frac{h_s e^{-h_s y} \Psi_x (\Psi_y + 1)}{\Psi_y \Psi_x + (e^{-\alpha} - 1)}, \end{aligned} \quad (25)$$

where

$$\begin{aligned} \Psi_x &= \exp\{-\alpha(1 - e^{-h_d x})\} - 1, \\ \Psi_y &= \exp\{-\alpha(1 - e^{-h_s y})\} - 1. \end{aligned} \quad (26)$$

As noted, the impacts of reduced information accuracy and market risk tolerance on bond pricing differ from each other. Information accuracy influences $h_s(t)$ and y_s , while market risk tolerance impacts $h_s(t)$, y_s , and α . \square

4. Numerical Analysis

The correlation between credit and liquidity risks has an effect on the estimated probabilities of default and trade. In this section, we analyze the influence of the correlation on bond pricing and investigate the role it plays in explaining bond death spiral in a financial crisis. For simplicity, we assume that all parameters in our model are constants in each subinterval $[t_i, t_{i+1}]$.

4.1. Correlation and Bond Pricing. The correlation between credit and liquidity risks varies over time and thus parameter α changes. In general, a slight decline in the credit level in a company makes investors more sensitive to bond prices and increases the turnover rates, resulting in a positive correlation parameter α . As $\alpha > 0$ means that an increase in credit risk reduces liquidity risk, a positive α represents a negative correlation between credit and liquidity. However, if the credit level drops drastically, investors' fear of default increases sharply, thereby decreasing the market risk tolerance substantially. Then, the demand for bonds in the market is reduced, leading to fewer transactions, and thus a negative α . $\alpha < 0$ means that an increase in credit risk increases liquidity risk and thus represents a positive correlation between credit and liquidity. As stated in Section 3, the correlation between

credit risk and liquidity risk is influenced by the percentage of market risk tolerance λ^* in trading cost w . Then, $\alpha \propto (-\lambda^*/w)$. Here, we assume that α is linearly related to (λ^*/w) , and $\alpha = 10 - 20(\lambda^*/w)$.

In this analysis, we consider various values of α to gauge the influence of the correlation on bond pricing. To this end, we consider bonds with a face value of 100 dollars and maturities ranging from 1 to 30 years. We use the average annualized yield spreads of these bonds as a measure of risk impact on bond prices. The model parameters are specified as follows: $h_d = 0.1$, $h_s = 0.2$, $y_d = 0.5$, $y_s = \gamma(1 - w)$, and $w = 0.3$, λ^* ranges from 0 to 0.3, and α ($\alpha = 10 - 20(\lambda^*/w)$) ranges from -10 to 10 ($\alpha = 0$ means no correlation). The results are plotted in Figure 1(a).

Figure 1(a) shows that the yield spreads of bonds with different maturities vary with the value of the correlation parameter α . As noted, a positive α implies active trading in the market, which means that it is easy for investors to trade bonds to reduce their holding risk, resulting in lower yield spreads. On the contrary, in the case of a negative α , it is difficult for investors to sell their bonds, as the probability of default is high. Thus, investors have to lower the prices to sell or continue to hold their bonds with high credit risk, which decreases the payoff of bonds directly or indirectly, leading to a higher yield spreads. Mathematically, a positive correlation parameter α reduces the probability of both default and trade, or $P_d = P[\tau_d < T, \tau_d < \tau_s]$ and $P_s = P[\tau_s < T, \tau_s < \tau_d]$. Conversely, a negative correlation parameter α can lead to a higher probability of both risk events, or $P_d + P_s$, for both short- and long-term bonds. Moreover, the changes in yield spreads become less pronounced when the absolute value of the correlation parameter, $|\alpha|$, is higher.

To further analyze the correlation effects, we consider the α effects of investment grade and speculative grade bonds. To illustrate, we use parameter values for investment grade bonds as follows: $h_d = 0.05$, $h_s = 0.4$, $y_d = 0.75$, $y_s = \gamma(1 - w)$, $w = 0.3$, λ^* ranges from 0 to 0.3, and α ($\alpha = 10 - 20(\lambda^*/w)$) ranges from -10 to 10 ($\alpha = 0$ means no correlation). For speculative grade bonds, these values are $h_d = 0.35$, $h_s = 0.4$, $y_d = 0.35$, $y_s = \gamma(1 - w)$, $w = 0.3$, and α ranges from -10 to 10. We use $((r_\alpha - r_0)/r_0)$, where r_α is the average yield spread of bonds with different maturities when $\alpha \neq 0$ and r_0 is the yield spread when $\alpha = 0$, as a proxy for the correlation effect. Figure 1(b) plots the correlation effect as a function of α . The figure shows that speculative grade and short-term bonds are more sensitive to α , regardless of whether α is positive or negative.

4.2. Correlation and Yield Spreads during a Financial Crisis. A financial crisis can be driven by either a sharp drop in the credit level of bonds or the lack of liquidity as a result of poor macroeconomic conditions. Accordingly, there are two types of crisis: credit-leading crisis and liquidity-leading crisis. While the market reacts quickly to credit or liquidity shocks, it takes time for the market to fully absorb the information. In this section, we analyze various stages of the two types of crisis to show the role that correlation plays in

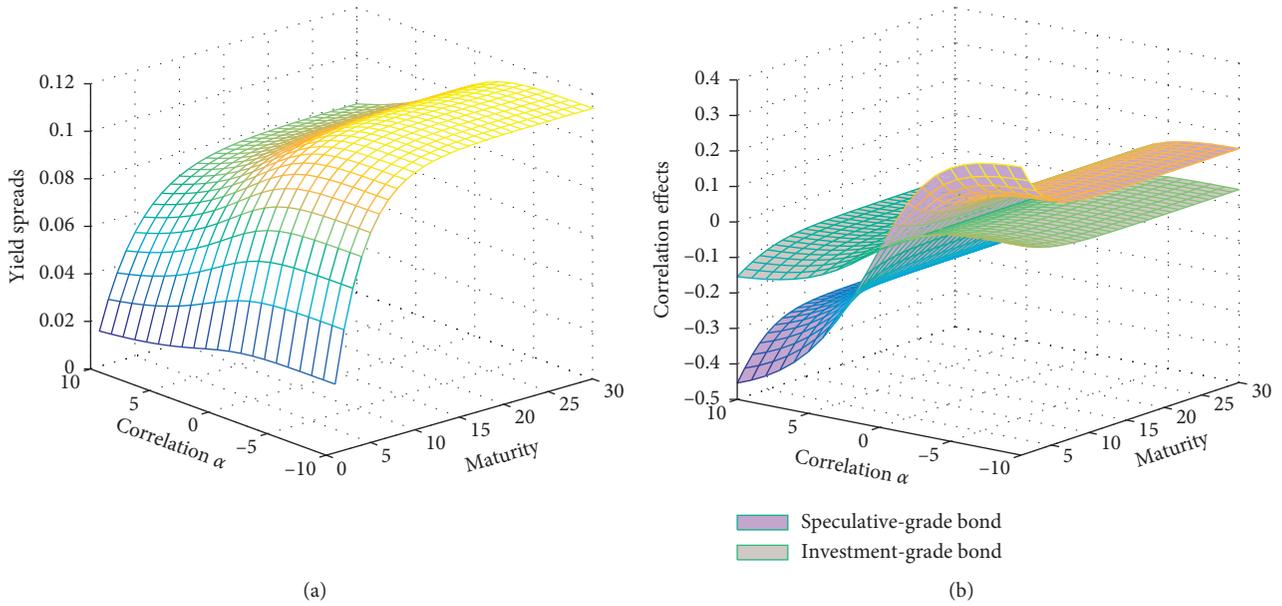


FIGURE 1: The influence of correlation parameter α on yield spreads.

explaining the changes in yield spreads and risk premia in different stages and interpret the way in which bond death spiral forms during a financial crisis.

4.2.1. Credit-Leading Crisis. Credit-leading crisis is caused by a sudden drop in the credit level of bonds, which may be due to significant news events or reported changes in fundamentals. This drop decreases investors' risk tolerance, leading to a substantial decrease in market risk tolerance. The sudden and disruptive re-pricing of Euro area sovereign credit risk in 2008–2012 is a vivid example. To avoid crisis, Euro area governments announced a set of rescue packages to increase confidence in their banking systems [39]. As noted in Section 4.1, in this case correlation parameter, α changes from positive to negative. On the contrary, as we see in Section 3, a decrease in market risk tolerance results in a lower liquidity payoff. Investors tend to sell their bonds as quickly as possible in the market to reduce their risk. To alleviate price shocks, in this case, investors tend to split a sell order into several trades rather than executing in a single trade [1]. For this reason, we assume that reduced market risk tolerance does not change the parameters of trade intensity. Additionally, due to the deterioration of market conditions, companies are prone to conceal information from investors, which reduces information accuracy.

In summary, credit crash influences the credit level, market risk tolerance, payoff of liquidity, and information accuracy, which in turn impact yield spreads. To illustrate, we consider these in four stages. The changes in model parameters in each stage are described as follows:

- (1) In the first stage, credit risk rises. In other words, default payoff y_d declines and default intensity h_d rises. Thus, the value of the bond's cash flow

decreases, and the marketability discount w increases slightly. Then, liquidity payoff y_s falls, given that y_s equals $\gamma(1-w)$.

- (2) In the second stage, the market risk tolerance decrease significantly, namely, (λ^*/w) rises. As a result, the correlation parameter α changes from positive to negative.
- (3) In the third stage, the payoff of liquidity drops. As part of liquidity, reduced market risk tolerance decreases the payoff of liquidity.
- (4) Payoff of liquidity continues to decline due to worsening information transparency.

The changes in these parameters in these stages are plotted in Figure 2.

Figure 3 plots the average values of yield spreads as well as credit and liquidity premia for bonds with various maturities considered in different stages. In each period, yield spreads increase over time. From Figures 2 and 3, we see that increases in yield spreads are caused not only by increases in liquidity and credit risks but also by changes in the correlation parameter. In stage 2, while both default and trade parameters remain unchanged, the yield spreads increase significantly due to the sharp drop in market risk tolerance.

The credit and liquidity risk premia presented in Figure 3 also show an up-trend during all the periods. At the first two stages, credit risk plays a dominant role in the changes in yield spreads, as default parameters change substantially. During these two periods, liquidity risk moves up mildly. The increase in (λ^*/w) in stage 2 is the primary reason for the increase in liquidity risk in stage 3. A significant drop in trade payoff caused by transaction dilemma brings a notable increase in liquidity premium, which accounts for a large part of the yield spreads.

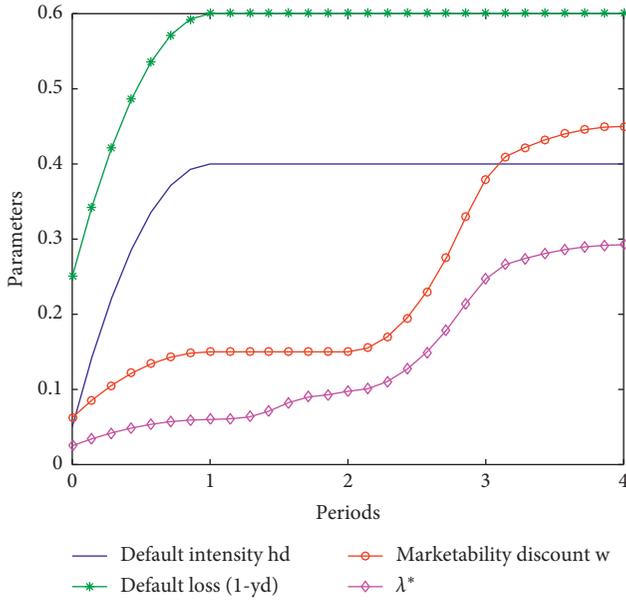


FIGURE 2: Parameter settings for credit-leading crisis.

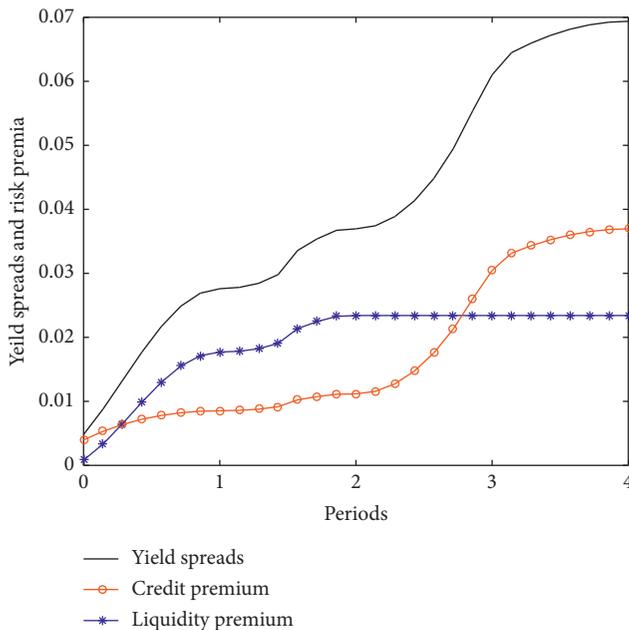


FIGURE 3: Yield spreads and risk premium in credit-leading crisis.

4.2.2. Liquidity-Leading Crisis. Liquidity-leading crisis is caused by a sudden drop in market liquidity. During such a crisis, investors become more sensitive to market risk, which triggers flight-to-liquidity. Wegener et al.'s [40] study points out that the global financial crisis caused a traditional liquidity crisis that also affected the German covered bond market. Wegener et al. [12] discuss the reason of the increase of sovereign credit risk in Europe and find that the financial crisis in the US is a trigger for the EMU debt crisis. The bursting US home price bubble lead to flight-to-quality effects to Germany and a loss of investors confidence in the fiscal situation of the peripheral countries. In other words,

bond investors' risk tolerance declines, lowering market risk tolerance. Consequently, the correlation parameter changes from positive to negative, as noted in Section 4.1. The lack of capital not only results in flight-to-liquidity but also tightens corporations' capital constraints, which in turn increases the likelihood of default. Thus, credit risk increases correspondingly.

Similar to the credit-leading crisis analyzed in Section 4.2.1, liquidity-leading crisis also has effects on yield spreads through its impacts on the payoff of liquidity, correlation level, credit level, and degree of information accuracy. As the deterioration of information accuracy does not impact credit risk and the correlation between credit and liquidity, as shown in Section 4.2.1, we demonstrate the effects of liquidity crisis in only three stages:

- (1) Liquidity risk rises substantially. Due to the increase of marketability discount w , liquidity payoff y_s drops quickly.
- (2) The market risk tolerance drops, and (λ^*/w) grows up. Then, the correlation parameter α changes quickly from positive to negative.
- (3) Credit level declines due to tightened capital constraints, as we can see that the default payoff declines and default intensity increases in this period.

In Section 4.1, we note that the correlation effect can be influenced by the credit level. To provide a full picture of the role of correlation in the liquidity-leading crisis, we consider both investment and speculative bonds. The parameters for these two types of bonds are as follows: $h_d = 0.05$, $h_s = 0.4$, $y_d = 0.75$, $y_s = 0.875$, and $\alpha = 2$ represent the initial state of an investment grade bond and $h_d = 0.35$, $h_s = 0.4$, $y_d = 0.35$, $y_s = 0.775$, and $\alpha = 2$ represent the initial state of a speculative grade bond. The corresponding parameters are presented separately in Figures 4(a) and 4(b).

Figures 5(a) and 5(b) plot the yield spreads and risk premia of these two types of bonds in three different periods. Similar to the conclusion in the analysis of crisis-leading risk, yield spreads rise over time in every step in a liquidity-leading crisis. However, for different bonds, the correlation plays a different role during the crisis. In particular, in the second period, yield spreads are relatively stable in Figure 5(a), but rise significantly in Figure 5(b). Thus, for speculative grade bonds, the effect of correlation is much stronger. The risk premia plotted in Figures 5(a) and 5(b) also corroborate this. For investment grade bonds, the credit risk premium changes mildly in the first two periods and then shoots up, while for speculative grade bonds, it begins to increase in period 2 and rises further thereafter.

The numerical results show that speculative grade bonds can be easily crashed by liquidity-leading crisis, and the crash is worse and earlier than investment grade bonds. This is consistent with the US market data in the financial crisis of 2008 [1].

Our analysis demonstrates two types of crisis in the bond market. As noted, default and liquidity risks are correlated, which widens risk spreads. High credit risk leads to deterioration of liquidity and vice versa. These two types of risk

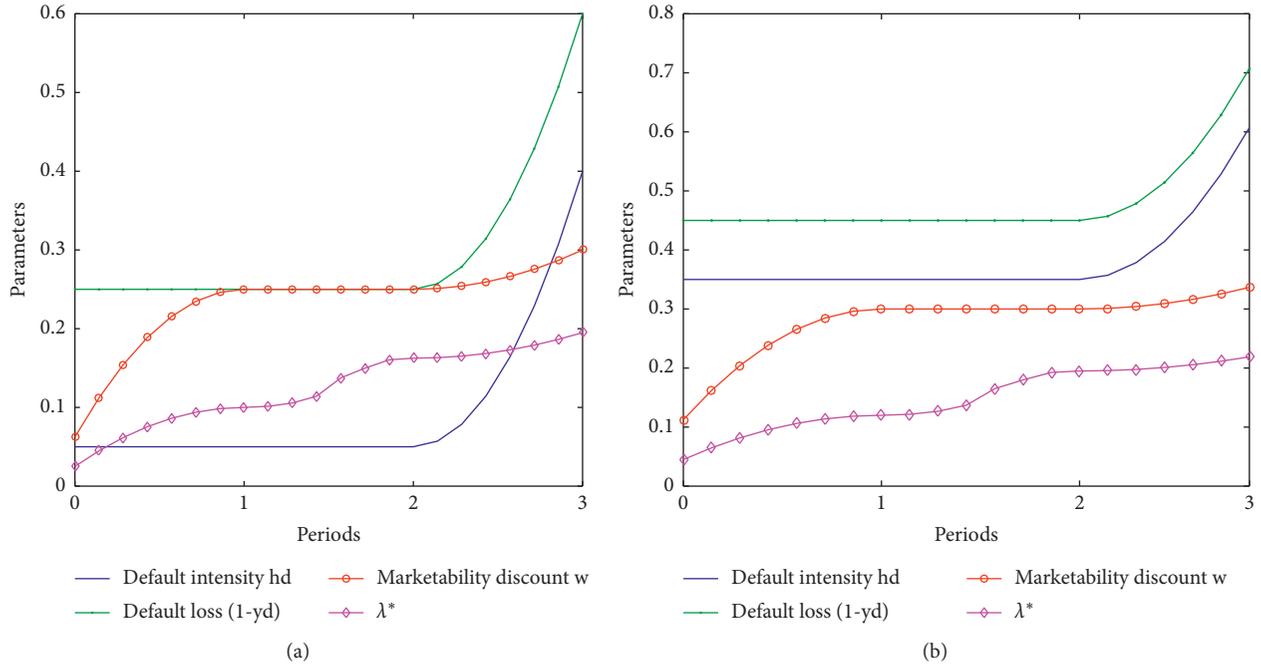


FIGURE 4: Parameter settings for liquidity-leading crisis (investment-grade/speculative-grade bond).

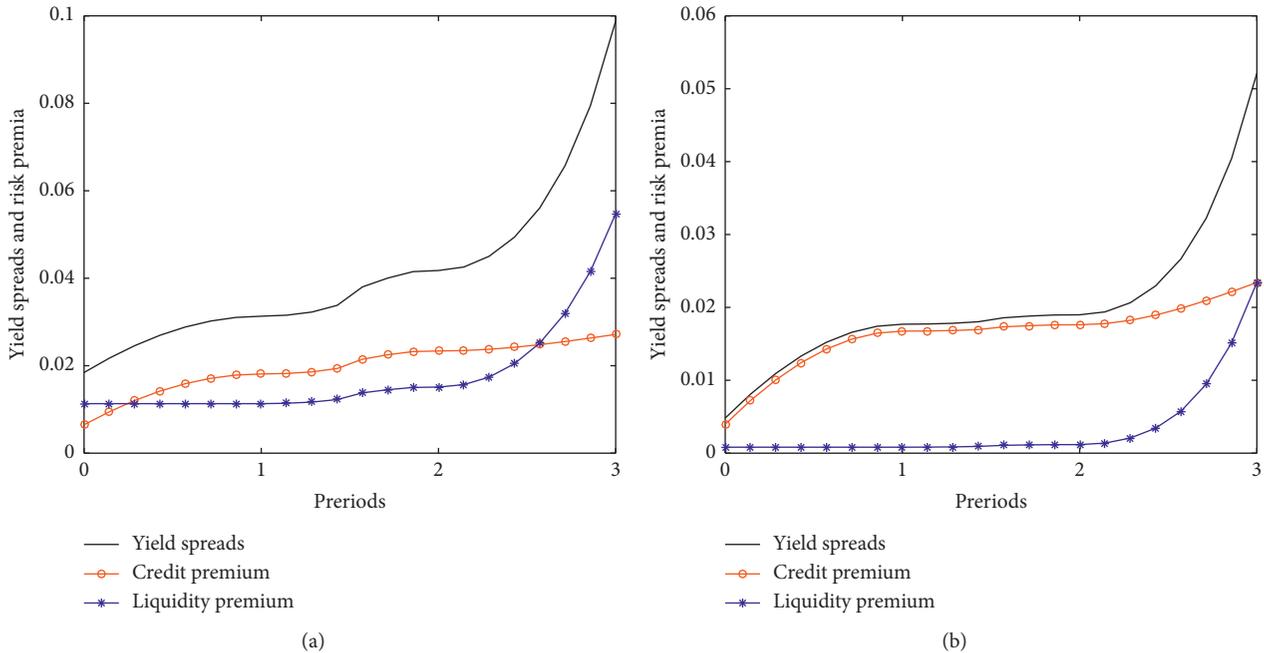


FIGURE 5: Yield spreads and risk premium in liquidity-leading crisis (investment-grade/speculative-grade bond).

in the bond market influence one another, forming a continuous cycle of risk. A decreasing credit level leads to reduced investors' risk tolerance, which leads to a greater and negative correlation parameter α and higher liquidity risk. The lack of liquidity further lowers investors' risk tolerance. As a result, credit and liquidity risks become more correlated, and credit risk increases further. This process continues, which can trigger a bond death spiral in the bond

market. In such a death spiral, the reduction in information accuracy acts as a catalyst.

5. An Illustration with US Data

We take the US corporate bond market. We remove the following types of bonds from our sample: bonds that are not listed or traded in the US public market, bonds with a

TABLE 1: Estimated coefficients for the Markov-switching model.

	C	$\gamma \times SP$	Expected duration	Transition probabilities	
				State1	State2
State1	2.008*** (0.00)	0.008*** (0.00)	131.92	0.99 (0.00)	0.06 (0.32)
State2	4.582*** (0.00)	0.011*** (0.00)	17.32	0.01 (0.32)	0.94 (0.00>)

Note. This table provides the estimated coefficients for the Markov-switching model: $y_{s_t} = \alpha_i + \beta_i \gamma_t \times SP_t + \varepsilon_{i,t}$, where $i = 1$ or 2 represents different states. We rely on the transaction records in Bloomberg for the sample period from January 2006 to December 2018. The liquidity risk is proxied by γ proposed by Bao et al. [41], based on the bond price trading deviation theory, to make a comparative study. The credit risk is proxied by the current S&P rating, and all ratings are assigned a number to facilitate the analysis; for example, 22 refers to a D rating, ... and 1 refers to AAA. The first two columns show the values of α and β for the two regimes, where the second row of each regression result reports the p -values for the HAC statistics calculated by Newey–West standard errors. *** indicates the significance at the 1% level. The third column is the expected duration of each regime. The last two columns report the transition probabilities between different states and the corresponding p -values for HAC statistics calculated by Newey–West standard errors.

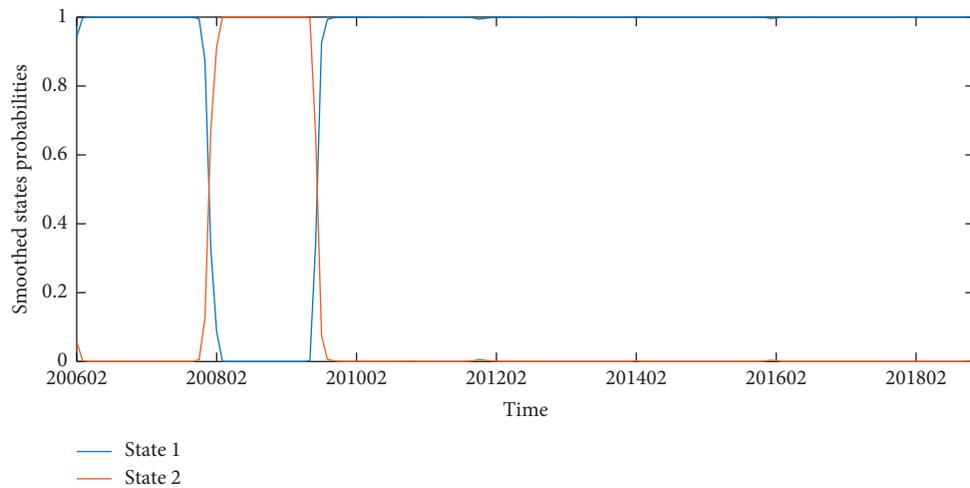


FIGURE 6: Smoothed states' probabilities.

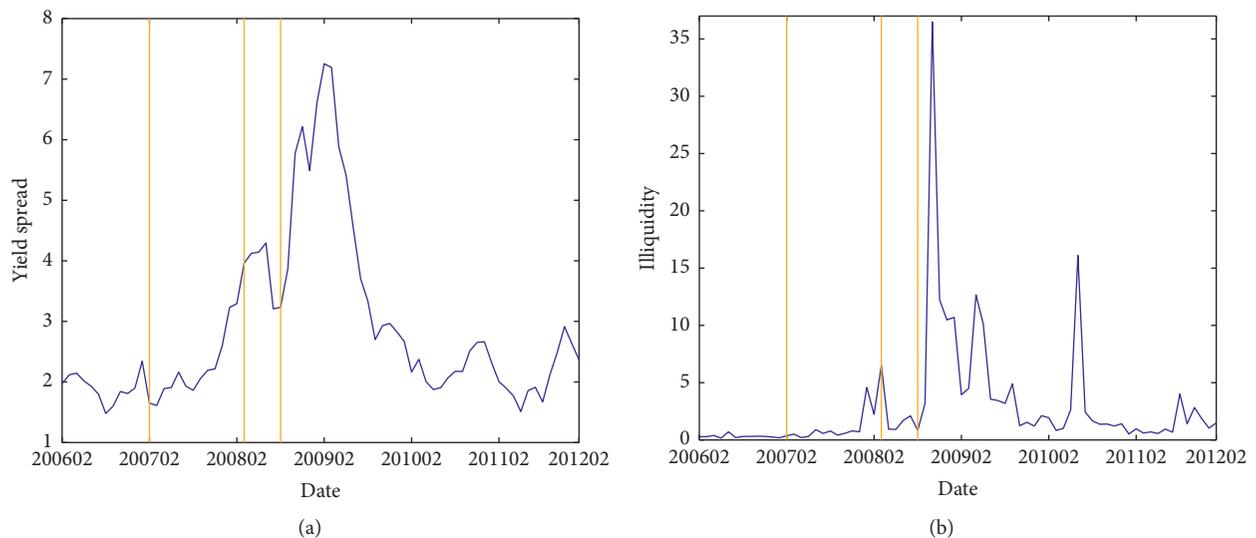
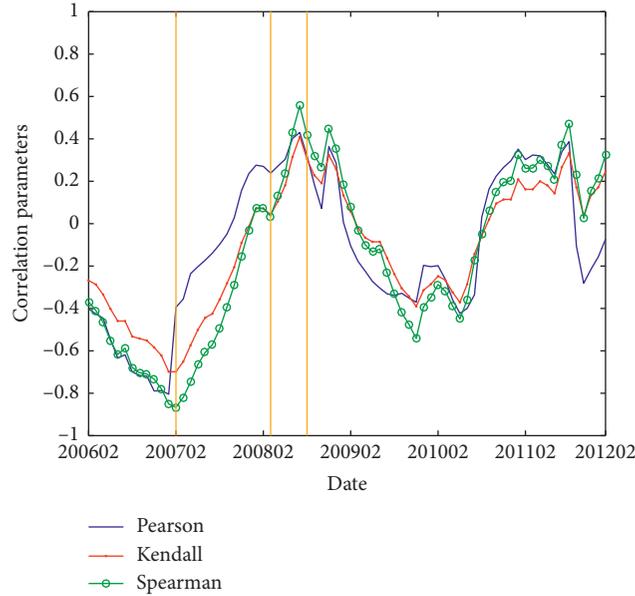


FIGURE 7: Continued.



(c)

FIGURE 7: Yield spreads, liquidity risk, and the correlation parameters during financial crisis.

maturity of less than one year, convertible bonds, bonds that trade under \$10 or above \$1000, bonds with floating rate, bonds with a floating coupon rate, bonds with less than one year to maturity, bonds that nonzero-trading months are less than 20, government-related or financial institution-related bonds, and bonds without Standard and Poor’s ratings. Our final sample includes 9972 corporate bonds. As an example to document how the correlation between credit and liquidity affects bond prices. We use the transaction records in Bloomberg for the sample period from January 2006 to December 2018.

We model the yield spread with a two-state Markov-switching regime to analyze the changes in yield spread during different time periods as follows:

$$ys_t = \alpha_i + \beta_i \gamma_t \times SP_t + \varepsilon_{i,t}, \quad (27)$$

where $i = 1$ or 2 represent different regimes. The dependent variable ys_t is the corporate bond yield spread, and the independent variable $SP_t \times \gamma_t$ is the product of credit risk and liquidity risk, which are measured by the S&P rating level and illiquidity proxy proposed by Bao et al. [41], respectively. Our ADF test on the two variables shows they are both stationary, and the ADF-statistics of ys_t and $SP_t \times \gamma_t$ are -6.64 and -6.57 , respectively.

Table 1 provides the results for equation (27). Our model distinguishes regimes which are highly persistent. The probability of remaining in the present state is 99% and 94%, respectively. The smoothed probabilities are given in Figure 6. For most of the time, the correlation between credit risk and liquidity risk affects the yield spread slightly and this state is more stable. However, at the end of 2007, the regime of the regression model changes, and the correlation plays a

more important role in bond pricing, that is, the deterioration of market leads to changes in the premia of credit risk, liquidity risk, and their correlation.

The Markov-switching model confirms that the correlation between credit risk and liquidity risk can significantly influence the yield spread, especially during the financial crisis.

To analyze how the correlation changes during crisis, we plot three correlation parameters for different months in Figure 7. In early 2007, default risk in subprime mortgages increased dramatically, leading to the so-called subprime crisis. This crisis deteriorated the overall credit of the corporate bond market, which caused great losses in bond prices in Figure 7(a). Following the credit crisis, the liquidity premium increased slightly in 2007, as shown in Figure 7(b), similar to the analysis in Section 4.2.1. Meanwhile, the correlation in Figure 7(c) between credit risk and liquidity risk kept climbing during this period. The increased correlation eventually triggered a series of defaults in the market, including the bankruptcy of Lehman. Due to this high credit risk, the liquidity risk and yield spreads further increased. The combination of rising credit and liquidity risks constituted an ongoing spiral, in which market conditions constantly deteriorated, as shown in our analysis. During the bond death spiral, lower information accuracy also caused greater market volatility around the time of the Federal Reserve bailout of Bear Stearn, which had an impact on the liquidity risk and bond prices, but did not influence correlation parameters, as shown in Figure 7(c).

Note: Figure 7 plots the yield spreads, liquidity risk, and correlation parameters of the U.S. corporate bonds; the sample period is from February, 2006 to February, 2012. Figure (c) plots Pearson, Kendall, and Spearman correlation

parameters with 20-month rolling window. The three vertical lines represent the subprime crisis, downfall of Bear Stearns, and Lehman bankrupt.

6. Conclusion

In this paper, we propose a generalized bond pricing model that incorporates credit risk, liquidity risk, and their correlation. We analyze the way in which the correlation between the two risks arises and explore the role of the correlation in explaining bond pricing.

Using the trading model, we specify the liquidity payoff and show that liquidity risk arises from two sources: information accuracy and market risk tolerance. It is the market risk tolerance rather than information accuracy that links credit and liquidity. By adopting a joint probability distribution, Frank Copula function, we calculate the probabilities of default and trade. Using numerical examples, we show how the correlation determines yield spreads under various market conditions. We show that a credit-leading crisis and a liquidity-leading crisis interact with each other and can result in a drastic decline in bond prices. Our model provides an explanation for the bond death spiral observed in a financial crisis. Moreover, we analyze the influence of the correlation in the US corporate bond market in different periods, and the empirical results provide evidence in support of our numerical analysis.

Appendix

A

Proof. of equation (12).

Substituting equation (5) into $D_{\text{in}} = (E[\tilde{Y}|\Omega_{\text{in}}] - P)\Pi_{\text{in}}$ λ_{in} gives the following:

$$D_{\text{in}} = \lambda_{\text{in}}\Pi_{\text{v}}E[\tilde{Y}] + \lambda_{\text{in}}\Pi_{\text{ie}}Y_{\text{in}} - \lambda_{\text{in}}\Pi_{\text{in}}P. \quad (\text{A.1})$$

Equation (A.1) shows that D_{in} is a linear function of private information and price.

Substituting equations (A.1) and (10) into equation (11) yields

$$P = \Delta(N\lambda_{\text{in}}\Pi_{\text{v}} + N\lambda_{\text{in}}\Pi_{\text{ie}}Y_{\text{in}} + \omega\Pi_{\text{un}}E[\tilde{Y}|\Omega_{\text{un}}] - L), \quad (\text{A.2})$$

where $\Delta = (N\lambda_{\text{in}}\Pi_{\text{in}} + \omega\Pi_{\text{un}})^{-1}$. Equation (A.2) suggests that, for the uninformed trader, P is a linear function of the informed trader's information and the supply of bonds.

Solving for Y_{in} from equation (A.2), we obtain

$$Y_{\text{in}} = (N\Delta\lambda_{\text{in}}\Pi_{\text{ie}})^{-1}(P - \Delta(N\lambda_{\text{in}}\Pi_{\text{v}} + \omega\Pi_{\text{un}}E[\tilde{Y}|\Omega_{\text{un}}] - L)). \quad (\text{A.3})$$

Because the market bond supply is not observable, to get the private information of the informed trader, the uninformed trader can infer the private information Y_{in} as follows:

$$Y_{\text{learn}} = (N\Delta\lambda_{\text{in}}\Pi_{\text{ie}})^{-1}(P - \Delta(N\lambda_{\text{in}}\Pi_{\text{v}} + \omega\Pi_{\text{un}}E[\tilde{Y}|\Omega_{\text{un}}])). \quad (\text{A.4})$$

Thus, there is a deviation between Y_{learn} and Y_{in} , and Y_{learn} can be rewritten as

$$Y_{\text{learn}} = Y_{\text{in}} - (N\Delta\lambda_{\text{in}}\Pi_{\text{ie}})^{-1}L. \quad (\text{A.5})$$

B

Proof. of equations (14)–(16).

Based on equation (5), the expected value of cash flows for informed investors $E[\tilde{Y}|\Omega_{\text{in}}]$ is

$$E[E[\tilde{Y}|\Omega_{\text{in}}]] = E[\tilde{Y}] + \frac{\Pi_{\text{ie}}}{\Pi_{\text{in}}}(E[Y_{\text{in}}] - E[\tilde{Y}]) = E[\tilde{Y}]. \quad (\text{B.1})$$

Plugging $Y_{\text{in}} = \tilde{Y} + \varepsilon_{\text{in}}$ into equation (A.5), we obtain the cash flows for uninformed investors:

$$Y_{\text{learn}} = \tilde{Y} + (\varepsilon_{\text{in}} - (N\lambda_{\text{in}}\Pi_{\text{ie}})^{-1}L) = \tilde{Y} + \varepsilon_{\text{learn}}. \quad (\text{B.2})$$

We denote the precision of $\varepsilon_{\text{learn}}$ as Π_{learn} ; then, the expected value for uninformed investors is

$$E[\tilde{Y}|\Omega_{\text{un}}] = E[\tilde{Y}] + \frac{\Pi_{\text{learn}}}{\Pi_{\text{un}}}(Y_{\text{learn}} - E[Y_{\text{learn}}]). \quad (\text{B.3})$$

The expected value of $E[\tilde{Y}|\Omega_{\text{un}}]$ is

$$\begin{aligned} E[E[\tilde{Y}|\Omega_{\text{un}}]] &= E[\tilde{Y}] + \frac{\Pi_{\text{learn}}}{\Pi_{\text{un}}}(E[Y_{\text{learn}}] - E[Y_{\text{learn}}]) \\ &= E[\tilde{Y}]. \end{aligned} \quad (\text{B.4})$$

Substituting equations (7) and (10) into equation (11) gives

$$N(E[\tilde{Y}|\Omega_{\text{in}}] - P)\Pi_{\text{in}}\lambda_{\text{in}} + M(E[\tilde{Y}|\Omega_{\text{un}}] - P)\Pi_{\text{un}}\lambda_{\text{un}} = L. \quad (\text{B.5})$$

Solving for P from equation (B.5) gives

$$P = (NE[\tilde{Y}|\Omega_{\text{in}}]\Pi_{\text{in}}\lambda_{\text{in}} + ME[\tilde{Y}|\Omega_{\text{un}}]\Pi_{\text{un}}\lambda_{\text{un}} - L)(N\Pi_{\text{in}}\lambda_{\text{in}} + M\Pi_{\text{un}}\lambda_{\text{un}})^{-1}. \quad (\text{B.6})$$

Given that $E[\tilde{Y}|\Omega_{in}] = E[\tilde{Y}]$ and $E[E[\tilde{Y}|\Omega_{un}]] = E[\tilde{Y}]$, the expected value of equation (B.6) is

$$E[P] = (NE[\tilde{Y}]\Pi_{in}\lambda_{in} + ME[\tilde{Y}]\Pi_{un}\lambda_{un} - E[L])(N\Pi_{in}\lambda_{in} + M\Pi_{un}\lambda_{un})^{-1}. \quad (B.7)$$

Rearranging equation (B.7) gives

$$E[\tilde{Y}] - E[P] = E[L](N\Pi_{in}\lambda_{in} + M\Pi_{un}\lambda_{un})^{-1}. \quad (B.8)$$

Namely,

$$E[\tilde{Y}] - E[P] = \left(\frac{N\lambda_{in}\Pi_{in} + M\lambda_{un}\Pi_{un}}{N\lambda_{in} + M\lambda_{un}}\right)^{-1} E[L] \quad (B.9)$$

$$(N\lambda_{in} + M\lambda_{un})^{-1}. \quad \square$$

C

Proof. of Proposition 1.

$P_{d,i}$ and $P_{s,i}$ represent the probabilities of default and trade, respectively, in each subinterval $[t_i, t_{i+1}]$. As $P_{d,i} = P[\tau_d < \tau_s, \tau_d < t_{i+1} | \tau_s > t_i, \tau_d > t_i]$ and $P_{s,i} = P[\tau_s < \tau_d, \tau_s < t_{i+1} | \tau_s > t_i, \tau_d > t_i]$, following equations 22a and 22b, we obtain

$$P_{d,i} = \int_{t_i}^{t_{i+1}} \left(\frac{\partial F_i}{\partial x} \Big|_{y=+\infty} - \frac{\partial F_i}{\partial x} \Big|_{y=x} \right) dx, \quad (C.1a)$$

$$P_{s,i} = \int_{t_i}^{t_{i+1}} \left(\frac{\partial F_i}{\partial y} \Big|_{x=+\infty} - \frac{\partial F_i}{\partial y} \Big|_{x=y} \right) dy, \quad (C.1b)$$

where F_i is the joint distribution in subinterval $[t_i, t_{i+1}]$. Let $P_i = P_{d,i} + P_{s,i}$ represents the probability of any event occurring in subinterval $[t_i, t_{i+1}]$ and $p^k = P[\tau \leq t_{k+1}]$ is the probability of any event occurring in the first k periods. Then, p^k can be written as

$$p^k = 1 - \prod_{i=1}^k (1 - P_i). \quad (C.3)$$

Using the formula of total probability, we get the probabilities of default and trade in interval $[0, T]$ as follows:

$$P_d = \sum_{i=1}^n P_{d,i} (1 - p^i), \quad (C.4a)$$

$$P_s = \sum_{i=1}^n P_{s,i} (1 - p^i). \quad (C.4b)$$

Plugging equation (C.3) into equations C.4a and C.4b, we obtain

$$P_d = \sum_{i=1}^n P_{d,i} \prod_{1}^i (1 - P_i), \quad (C.5a)$$

$$P_s = \sum_{i=1}^n P_{s,i} \prod_{1}^i (1 - P_i). \quad (C.5b)$$

□

Data Availability

The data used to support the findings of this study are available from Bloomberg Terminal at <https://www.bloomberg.net/> with the permission of Bloomberg. Restrictions apply to the availability of these data, which were used under license for this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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