

Research Article

Nonfragile Estimator Design for Fractional-Order Neural Networks under Event-Triggered Mechanism

Xiaoguang Shao, Ming Lyu , and Jie Zhang

School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China

Correspondence should be addressed to Ming Lyu; lumtz@163.com

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This paper is concerned with the nonfragile state estimation for a kind of delayed fractional-order neural network under the event-triggered mechanism (ETM). To reduce the bandwidth occupation of the communication network, the ETM is employed in the sensor-to-estimator channel. Moreover, in order to reflect the reality, the transmission delay is taken into account in the model establishment. Sufficient criteria are supplied to make sure that the augmented system is asymptotically stable by using the fractional-order Lyapunov indirect approach and the linear matrix inequality method. In the end, the theoretical result is shown by means of two numerical examples.

1. Introduction

The past several decades have witnessed that artificial neural network (ANN) has attracted particular research attention. Because of the outstanding performance, ANN has been extensively applied in image recognition, signal processing [1], fault diagnosis [2], and so on. With the rapid developments of artificial intelligence, the ANN has received considerable attention again by the scholars, which relates to synchronization, dissipativity, attractivity, stability, and state estimation (SE) for various kinds of ANNs [3–7].

As we all know, ANNs are composed of plenty of artificial neurons and the SE problem of the neurons plays a vital role in practical applications. As such, quite a lot of results have been reported on the SE issue (see [8–11] and the reference therein). For the practical systems, the parameter uncertainties are often considered. So far, a lot of research studies regarding uncertain systems have been conducted [12–14]. It is worth noting that the existing results assumed that parameter of the estimator is accurate, which, however, is unrealistic. To solve this problem, we aim to design a nonfragile estimator so as to alleviate the effects induced by the uncertainty of the estimator parameter on the system performance. Till now, some initial results have been

published on the nonfragile controller design problems [14–17].

In the networked systems, the network bandwidth is always limited which therefore may result in network congestion when a large amount of data is transmitted. Up to now, the network-induced phenomena including transmission delay, packet loss, and quantification have been discussed adequately. In recent years, much attention has been focused on the ETM and many communication protocols, which aims to avoid the occurrence of the network-induced phenomena. Based on the ETM, plenty of literature has been available on stability analysis, event-triggered condition design, controller/filter design, and so on [18]. Noting that compared with time-triggered mechanism, the ETM exhibits better performance because the necessary sampling depends on the “event” rather than the “time” [19, 20].

In addition, by applying the fractional calculus to the ANNs, the researchers have found that the performance of the fractional-order models is better than integer-order ones, especially in the aspect of memory and hereditary. Till now, some novel fractional-order theories and methods concerning the ANNs have been proposed. For example, a nonfragile nonlinear fractional-order observer is designed in

[21] and an adaptive event-triggered scheme has been developed in [22]. But these existing fractional-order systems employed ETM are introduced with single delay or without only. Especially, it is a challenge in a fractional system. However, the problem of multiple time delays in real systems is often encountered. Nevertheless, there are few related studies on the nonfragile SE for fractional-order neural network based on ETM with multiple time delays, which motivates us to shorten this gap.

Inspired by the aforementioned lines, a nonfragile state estimator is designed for a class of fractional-order neural networks (FNNs) based on ETM. The advantages in this paper are as follows: (1) compared with the existing estimators, a fractional-order nonfragile estimator is first constructed; (2) to save bandwidth resources, an ETM is applied in the SE problem of the fractional-order neural network; (3) the LMI method and the fractional Lyapunov indirect method are adopted to design the state estimator.

The remaining content is outlined as follows. In Section 2, some preliminary knowledge is recalled. In Section 3, state estimation criteria are voiced. In Section 4, two numerical examples are given with some simulation figures to support the theorems.

Notation. Throughout this paper, Z^T and the symbol $*$ in matrix Z represent matrix transposition and the

symmetric term, respectively. \mathbb{R} is the set of integers, and \mathbb{R}^n denotes the n -dimensional Euclidean space. I_n means n -dimensional identity matrix. $\mathcal{P} > 0$ ($\mathcal{P} < 0$) is defined as a positive-definite (negative-definite) matrix. $\|z\|$ is the Euclidean norm of a vector z in \mathbb{R}^n . $\lambda_{\max}(R)$ ($\lambda_{\min}(R)$) represents the maximum (minimum) eigenvalue of R and $\text{sym}(Y)$ means $Y + Y^T$.

2. Preliminaries and Problem Formulation

Some fractional definitions and model descriptions are presented firstly. In addition, some important lemmas that will be used in Section 3 are also presented.

Definition 1 (see [23, 24]). For $h(t)$, the fractional integral form is defined as

$$I^\alpha h(q) = \frac{1}{\Gamma(\alpha)} \int_{q_0}^q (q - \theta)^{\alpha-1} f(\theta) d\theta, \quad \alpha \in \mathbb{R}^+, \quad (1)$$

where $q \geq q_0$ and $\Gamma(\cdot)$ is a gamma function.

Definition 2 (see [23, 24]). Caputo's derivative of $h(q)$ is denoted by

$${}_{q_0}^C D_q^\alpha h(q) = \frac{1}{\Gamma(z - \alpha)} \int_{q_0}^q (q - \theta)^{z-\alpha-1} h^{(z)}(\theta) d\theta, \quad \alpha \in (z - 1, z), h(t) \in i^z([t_0, \infty), \mathbb{R}), \quad (2)$$

where $q \geq q_0$ and z is a positive integer.

In what follows, D^α stands for ${}_{q_0}^C D_q^\alpha$ for the convenience of presentation. In this paper, let us consider the following FNN model:

$$\begin{cases} D^\alpha v(t) = -\mathcal{C}v(t) + \mathcal{A}h(v(t)) + \mathcal{V}, \\ y(t) = Dv(t), \end{cases} \quad (3)$$

where $\alpha \in (0, 1)$ is the predetermined fractional order, $\mathcal{C} = \text{diag}\{c_1, c_2, \dots, c_n\}$ with $c_i > 0$ ($i = 1, 2, \dots, n$), $\mathcal{A} = (a_{ij})_{n \times n}$ is the connection matrix, the vector $v(t) = (v_1(t), v_2(t), \dots, v_n(t))^T \in \mathbb{R}^n$ stands for the neuron state, $h(v(t)) = (h_1(v_1), h_2(v_2), \dots, h_n(v_n))^T$ denotes the activation function of the neurons, $y(t)$ is the measurement output, \mathcal{V} is the system input, and D is a known constant matrix.

In what follows, ETM is introduced in order to reduce the communication burden. The event-triggered condition is predesigned as follows:

$$e_y^T(t) e_y(t) \leq \sigma y^T(t_k h + jh) y^T(t_k h + jh), \quad (4)$$

where $e_y(t) = y(t_k h + jh) - y(t_k h)$, σ is a given constant, jh and $t_k h$ are the sampling instant and the release instant, respectively, $y(t_k h + jh)$ stands for the latest sampled signal, and $n_i h = t_{k+1} h - t_k h$ denotes the release period.

Remark 1. The sensor is time-driven at discrete instants, which can avoid the Zeno behavior. Moreover, when $\sigma = 0$, ETM becomes a time-triggered one.

In this paper, the transmission delay $d_k \in [0, \bar{d}]$ between sensor and estimator is considered, where \bar{d} is a positive scalar. Therefore, $t_k h + d_k$ is the arrival time of the transmitted data from sensor to estimator.

In view of [25], the holding interval can be rewritten as $[t_k h + d_k, t_{k+1} h + d_{k+1}) \cup_{j=0}^{j=d_M} I_j$, where $I_j = [t_k h + jh + \bar{d}, t_k h + jh + h + \bar{d})$. For the convenience of analysis, denote $d(t) = t - t_k h - jh$, and then we have $0 \leq d(t) \leq h + \bar{d} \triangleq d_M$. Then, the measurement outputs arrived at the estimator can be rewritten as

$$\bar{y}(t) = e_y(t) + y(t - d(t)) = D e_k(t) + y(t - d(t)), \quad (5)$$

where $e_k(t)$ is the error vector.

Design a nonfragile state estimator for system (3) as follows:

$$D^\alpha \hat{v}(t) = -\mathcal{C} \hat{v}(t) + \mathcal{A} \tilde{h}(\hat{v}(t)) + \mathcal{V} + (K + \Delta K)[\bar{y}(t) - D \hat{v}(t)], \quad (6)$$

where $\hat{v}(t) \in \mathbb{R}^n$ stands for the estimate of $v(t)$, $K \in \mathbb{R}^{n \times q}$ is the gain matrix to be determined, and ΔK represents the gain variation that satisfies $\Delta K = M \mathcal{F}(t) N$, in which M and

N are known real matrices and $\mathcal{F}(t)$ is an unknown satisfying $\mathcal{F}^T(t)\mathcal{F}(t) \leq I$.

Defining $e(t) = v(t) - \hat{v}(t)$, the estimation error dynamics can be obtained from (3) and (5) as follows:

$$\begin{aligned} D^\alpha e(t) = & -[\mathcal{E} - (K + \Delta K)D]e(t) + \mathcal{A}\hat{h}(e(t)) \\ & - (K + \Delta K)De_k(t) \\ & - (K + \Delta K)Dv(t - d(t)) + (K + \Delta K)Dv(t), \end{aligned} \quad (7)$$

where $\hat{h}(e(t)) \triangleq \tilde{h}(v(t)) - \tilde{h}(\hat{v}(t))$.

For notation simplicity, we define $\eta(t) \triangleq [v^T(t) e^T(t)]^T$. An augmented system model from (3) and (8) is given in the following form:

$$D^\alpha \eta(t) = \tilde{C}\eta(t) + \tilde{A}\varphi(\eta(t)) + \tilde{E}e_k(t) + \tilde{K}\eta(t - d(t)), \quad (8)$$

where

$$\begin{aligned} \tilde{C} = & \begin{bmatrix} -C & 0 \\ (K + \Delta K)D & -C - (K + \Delta K)D \end{bmatrix}, \tilde{A} = \begin{bmatrix} \mathcal{A} & 0 \\ 0 & \mathcal{A} \end{bmatrix}, \tilde{K} = \begin{bmatrix} 0 & 0 \\ -(K + \Delta K)D & 0 \end{bmatrix}, \\ \tilde{E} = & \begin{bmatrix} 0 \\ -(K + \Delta K)D \end{bmatrix}, \varphi(\eta(t)) \triangleq [\hat{h}^T(v(t)) \tilde{h}^T(e(t))]^T. \end{aligned} \quad (9)$$

Lemma 1 (see [26]). For ξ_1 and $\xi_2 \in \mathbb{R}^n$ and any positive scale $\epsilon > 0$, one has

$$\Psi_1^T \Psi_2 + \Psi_2^T \Psi_1 \leq \epsilon \Psi_1^T \Psi_1 + \epsilon^{-1} \Psi_2^T \Psi_2. \quad (10)$$

Lemma 2 (see [27]). For $\forall \alpha \in (0, 1)$ and $t \geq 0$, if $v(t) \in \mathbb{R}^n$ is continuous and differential, then

$$D^\alpha v^T(t) Qv(t) \leq 2v^T(t) QD^\alpha v(t). \quad (11)$$

Lemma 3 (see [28]). For matrices $\omega, \mathcal{E}, \mathcal{H}$, where ω is symmetric, the inequality

$$\omega + \mathcal{E}\mathcal{F}\mathcal{H} + (\mathcal{E}\mathcal{F}\mathcal{H})^T < 0 \quad (12)$$

holds if and only if

$$\omega + \xi \mathcal{E}\mathcal{E}^T + \xi^{-1} \mathcal{H}^T \mathcal{H} < 0, \quad (13)$$

in which $\xi > 0$ refers to a scalar and $\mathcal{F}^T \mathcal{F} < I$.

Lemma 4 (see [29]). Consider a class of fractional-order nonlinear systems:

$$D^\alpha \eta(t) = h(t, \eta(t - d(t))), \quad (14)$$

and the initial condition is $\eta(t_0) = \phi \in C([t_0 - \tau, t_0], \mathbb{R}^n)$. Suppose that $\omega_i(s)$ ($i = 1, 2$): $\mathbb{R} \rightarrow \mathbb{R}$ are positive functions, and $\omega_1(0) = \omega_2(0) = 0, \omega_2(s_1) < \omega_2(s_2) (\forall 0 < s_1 < s_2)$. If there exist two constants $0 < \mu < \epsilon$ and a continuous differential function $V: \mathbb{R} \times \mathbb{R}^n$ such that $\omega_1 \leq V(t, x) \leq \omega_2$ satisfying

$$\begin{aligned} D\alpha V(t, \eta(t)) \leq & -\epsilon V(t, \eta(t)) \\ & + \mu \sup_{-d_M \leq -d(t) \leq 0} V(t + d(t), x(t + \eta(t - d(t)))), \end{aligned} \quad (15)$$

then the fractional-order system is globally uniformly asymptotically stable.

Lemma 5 (see [30]). The matrix

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{12}^T & \phi_{22} \end{bmatrix} < 0, \quad (16)$$

holds if and only if (a) $\phi_{22} < 0, \phi_{11} - \phi_{12}\phi_{22}^{-1}\phi_{12}^T < 0$, or (b) $\phi_{11} < 0, \phi_{22} - \phi_{12}^T\phi_{11}^{-1}\phi_{12} < 0$.

Lemma 6 (see [31]). V is continuous on $[t_0, +\infty)$ and bounded on $[t_0 - \rho, t_0]$. If there exist p, q such that

$$D_t^\alpha V(t) \leq -pV(t) + \sum_{k=1}^m q_k \sup_{-\rho_h \leq \omega \leq 0} V(t + \omega), \quad t \geq t_0, \quad (17)$$

where $0 < \alpha < 1, q_k > 0, p > \sum_{k=1}^m q_k$, and $\rho = \max\{\rho_1, \rho_2, \dots, \rho_m\}$, then $\lim_{t \rightarrow \infty} V(t) = 0$.

3. Main Results

Theorem 1. For the given positive scalars $\epsilon > \mu > 0$, system (8) is globally asymptotically stable if there exist a symmetric matrix $\mathcal{P} = \text{diag}\{\mathcal{P}_1, \mathcal{P}_2\} > 0$ and four scalars β_i ($i = 1, 2, 3$) $> 0, \gamma > 0$ satisfying the following LMI:

$$\begin{aligned} \Phi = & \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ * & \Phi_{22} & 0 & 0 \\ * & * & \Phi_{33} & 0 \\ * & * & * & \Phi_{44} \end{bmatrix} < 0, \quad (18) \\ \beta_2 \sigma \tilde{I}^T \tilde{I} + \beta_3 \tilde{I}_1^T \tilde{I}_1 - \mu \mathcal{P} & < 0, \quad (19) \end{aligned}$$

where

$$\begin{aligned}
\Phi_{11} &= \begin{bmatrix} \Lambda_{11} & D^T X^T - \gamma D^T N^T N D \\ * & \Lambda_2 \end{bmatrix}, \Phi_{12} = \begin{bmatrix} \mathcal{P}_1 \mathcal{A} & 0 \\ 0 & \mathcal{P}_2 \mathcal{A} \end{bmatrix}, \Phi_{14} = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{P}_2 \overline{M} \end{bmatrix}, \\
\Phi_{13} &= \begin{bmatrix} -\gamma D^T N^T N D & -\gamma D^T N^T N D \\ -X D - \gamma D^T N^T N D & -X D - \gamma D^T N^T N D \end{bmatrix}, \Phi_{22} = \begin{bmatrix} -\beta_1 \overline{I}_1 & 0 \\ 0 & -\beta_1 \overline{I}_1 \end{bmatrix}, \\
\Phi_{33} &= \begin{bmatrix} -\beta_2 \overline{I}_1 + \gamma D^T N^T N D & \gamma D^T N^T N D \\ * & -\beta_3 \overline{I}_1 + \gamma D^T N^T N D \end{bmatrix}, \Phi_{44} = \begin{bmatrix} -\beta_3 \overline{I}_1 & 0 \\ 0 & -\gamma \overline{I}_1 \end{bmatrix}, \\
\Lambda_{11} &= -\mathcal{P}_1 C - C_1^T \mathcal{P} + \beta_1 M_{11}^T M_{11} + \epsilon \mathcal{P}_1 + \gamma D^T N^T N D, \overline{I}_1 = \text{diag}\{I, I\}, \\
\Lambda_{22} &= \text{sym}\{-\mathcal{P}_2 C - X D\} + \beta_1 M_{12}^T M_{12} + \epsilon \mathcal{P}_2 + \gamma D^T N^T N D.
\end{aligned} \tag{20}$$

Furthermore, the nonfragile estimator gain K of (8) is designed as $K = \mathcal{P}_2^{-1} X$.

Proof. First, we denote

$$\tilde{C} = \begin{bmatrix} -C & 0 \\ K D & -C - K D \end{bmatrix}, \tilde{E} = \begin{bmatrix} 0 \\ -K D \end{bmatrix}, \mathcal{P} = \begin{bmatrix} \mathcal{P}_1 & 0 \\ 0 & \mathcal{P}_2 \end{bmatrix}, \overline{K} = \begin{bmatrix} 0 & 0 \\ -K D & 0 \end{bmatrix}. \tag{21}$$

Considering system (8), design the following Lyapunov function:

$$V(t) = \eta^T(t) \mathcal{P} \eta(t). \tag{22}$$

From Lemma 2, one obtains

$$\begin{aligned}
D^\alpha V(t) &\leq 2\eta^T(t) \mathcal{P} D^\alpha \eta(t) \\
&= 2\eta^T(t) \mathcal{P} [\tilde{C} \eta(t) + \overline{A} \varphi(\eta(t)) \\
&\quad + \tilde{E} e_k(t) + \overline{K} \eta(t-d)].
\end{aligned} \tag{23}$$

It follows from Lemma 3 that

$$\begin{aligned}
2\eta^T(t) \mathcal{P} \overline{A} \varphi(\eta(t)) &\leq \beta_1^{-1} \eta^T(t) \mathcal{P} \overline{A} \overline{A}^T \mathcal{P} \eta(t) \\
&\quad + \beta_1 \eta^T(t) G^T G \eta(t),
\end{aligned} \tag{24}$$

$$\begin{aligned}
2\eta^T(t) \mathcal{P} \tilde{E} e_k(t) &\leq \beta_2^{-1} \eta^T(t) \mathcal{P} \tilde{E} \tilde{E}^T \mathcal{P} \eta(t) + \beta_2 e_k^T(t) e_k(t) \\
&\leq \beta_2^{-1} \eta^T(t) \mathcal{P} \tilde{E} \tilde{E}^T \mathcal{P} \eta(t) + \beta_2 \sigma \eta^T(t-d) \tilde{I}^T \tilde{I} \eta(t-d),
\end{aligned} \tag{25}$$

$$\begin{aligned}
2\eta^T(t) \mathcal{P} \overline{K} \eta(t-d) &\leq \beta_3^{-1} \eta^T(t) \mathcal{P} \overline{K} \overline{K}^T \mathcal{P} \eta(t) \\
&\quad + \beta_3 \eta^T(t-d) \eta(t-d),
\end{aligned} \tag{26}$$

where $\tilde{I} = [-I, I]$ and $\tilde{I} = [I, 0]$.

Combining (24)–(26), we can get

$$\begin{aligned}
D^\alpha V(t) &\leq \eta^T(t) \left[\mathcal{P} \tilde{C} + \mathcal{P} \tilde{E}_1 + \tilde{C}^T \mathcal{P} + \tilde{E}_1^T \mathcal{P} + \beta_1^{-1} \mathcal{P} \overline{A} \overline{A}^T \mathcal{P} \right. \\
&\quad \left. + \beta_1 G^T G + \beta_2^{-1} \mathcal{P} \tilde{E} \tilde{E}^T \mathcal{P} + \beta_3^{-1} \mathcal{P} \overline{K} \overline{K}^T \mathcal{P} \right] \eta(t) \\
&\quad + \eta^T(t-d) \left[\beta_2 \sigma \tilde{I}^T \tilde{I} + \beta_3 \tilde{I}_1^T \tilde{I}_1 \right] \eta(t-d).
\end{aligned} \tag{27}$$

By employing Lemma 5, $\Phi < 0$ is equivalent to

$$\overline{\Phi} + \gamma \overline{N}^T \overline{N} + \gamma^{-1} \overline{M} \overline{M}^T < 0, \tag{28}$$

where

$$\begin{aligned}
\overline{\Phi} &= \begin{bmatrix} \mathcal{P} \tilde{C} + \tilde{C}^T \mathcal{P} + \beta_1 M_1^T M_1 + \epsilon \mathcal{P}_1 & \mathcal{P} \overline{A} & \mathcal{P} \tilde{E} & \mathcal{P} \tilde{E} \tilde{I} \\ * & -\beta_1 \overline{I}_1 & 0 & 0 \\ * & * & -\beta_2 \overline{I}_1 & 0 \\ * & * & * & -\beta_3 \overline{I}_1 \end{bmatrix}, \\
\overline{M}^T &= [0, M^T \mathcal{P}_2, 0_{n \times 5n}]^T, \overline{N} = [N D, -N D, 0, 0, -N D, -N D, 0].
\end{aligned} \tag{29}$$

Defining an equation as follows:

$$\Delta \Phi = \overline{M} F(t) \overline{N} + \overline{N}^T F^T(t) \overline{M}^T, \tag{30}$$

we have

$$\Delta \Phi \leq \gamma^{-1} \overline{M} \overline{M}^T + \gamma \overline{N}^T \overline{N}. \tag{31}$$

Furthermore, from (28) and (31), we arrive at $\overline{\Phi} + \Delta \Phi < 0$.

Based on Lemma 3, we can obtain

$$\begin{aligned} \mathcal{P}\tilde{C} + \mathcal{P}\tilde{E}_1 + \tilde{C}^T \mathcal{P} + \tilde{E}_1^T \mathcal{P} + \beta_1^{-1} \mathcal{P}\overline{AA}^T \mathcal{P} + \beta_1 G^T G \\ + \beta_2^{-1} \mathcal{P}\tilde{E}\tilde{E}^T \mathcal{P} + \beta_3^{-1} \mathcal{P}\tilde{K}\tilde{K}^T \mathcal{P} < -\varepsilon \mathcal{P}. \end{aligned} \quad (32)$$

Combining (27) and (32), we have

$$\begin{aligned} D^\alpha V(t) &\leq -\varepsilon \eta^T(t) \mathcal{P} \eta(t) \\ &\quad + \mu \sup_{0 \leq d(t) \leq d_M} \eta^T(t-d(t)) \mathcal{P} \eta(t-d(t)) \\ &= -\varepsilon V(t) + \mu \sup_{0 \leq d(t) \leq d_M} V(t-d(t)). \end{aligned} \quad (33)$$

$$\begin{cases} D^\alpha v(t) = -(C + \Delta C)v(t) + (A + \Delta A)f(v(t)) + (B + \Delta B)f(v(t - \tau(t))) + J, \\ y(t) = Dv(t) + \tilde{g}(v(t)), \end{cases} \quad (34)$$

where $\tau(t)$ is a time-varying delay satisfying $0 \leq \tau(t) \leq \tau_M$. Here, τ_M is a constant. $\Delta A, \Delta B, \Delta C$ are parameter uncertainties which satisfy the following condition:

$$[\Delta A \ \Delta B \ \Delta C] = \overline{M}_2 F_1(t) [N_1 \ N_2 \ N_3], \quad (35)$$

where $\overline{M}_2, N_1, N_2, N_3$ are known matrices and $F_1(t)$ is an unknown matrix function which satisfies $F_1(t)^T F_1(t) \leq I$. \square

It follows from Lemma 4 that (4) is an asymptotical estimator and system (3) is globally asymptotically stable. The proof is completed.

It is worth noting that the parameter uncertainties are often unavoidable resulting from the inaccuracy of modeling or the changing environment. In addition, the network output is composed of linear and nonlinear parts. Therefore, the following model of FNN is established:

Assumption 1. $\tilde{g}: \mathbb{R}^n \rightarrow \mathbb{R}^q$ stands for the nonlinear disturbance which satisfies the Lipschitz condition:

$$|\tilde{g}(t, a) - \tilde{g}(t, b)| \leq |F(a - b)|, \quad (36)$$

where $\tilde{g}(t, 0) = 0$ and F is a known constant matrix.

The estimator and estimation error dynamics are obtained as follows:

$$\begin{cases} D^\alpha \hat{v}(t) = -(C + \Delta C)\hat{v}(t) + (A + \Delta A)f(\hat{v}(t)) + (B + \Delta B)f(\hat{x}(t - \tau(t))) + J + (K + \Delta K)(\overline{y}(t) - D\hat{v}(t)), \\ D^\alpha e(t) = [-C - (K + \Delta K)D]e(t) + [-\Delta C + \Delta K]Dv(t) + A\tilde{f}(e(t)) + \Delta Af(v(t)) + B\tilde{f}(e(t - \tau(t))) \\ \quad + \Delta Bf(x(t - \tau(t))) - (K + \Delta K)e_y(t) - (K + \Delta K)Dx(t - d(t)) - (K + \Delta K)g(x(t - d(t))). \end{cases} \quad (37)$$

The augmented system is derived as follows:

$$\begin{aligned} D^\alpha \eta(t) &= \tilde{C}\eta(t) + \tilde{A}\varphi(\eta(t)) + \tilde{B}\varphi(\eta(t - \tau(t))) \\ &\quad + \tilde{H}\eta(t - d(t)) - \tilde{G}g(\tilde{I}\eta(t - d(t))) - \tilde{L}e_y(t), \end{aligned} \quad (38)$$

where

$$\begin{aligned} \tilde{C} &= \begin{bmatrix} -C - \Delta C & 0 \\ -\Delta C + (K + \Delta K)D & -C - (K + \Delta K)D \end{bmatrix}, \tilde{A} = \begin{bmatrix} A + \Delta A & 0 \\ \Delta A & A \end{bmatrix}, \\ \tilde{B} &= \begin{bmatrix} B + \Delta B & 0 \\ \Delta B & B \end{bmatrix}, \tilde{H} = \begin{bmatrix} 0 & 0 \\ -(K + \Delta K)D & 0 \end{bmatrix}, \tilde{I} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, \\ \tilde{G} &= \begin{bmatrix} 0 & 0 \\ K + \Delta K & 0 \end{bmatrix}, \tilde{L} = \begin{bmatrix} 0 \\ K + \Delta K \end{bmatrix}. \end{aligned} \quad (39)$$

The following theorem is given to ensure the above system is asymptotically stable.

Theorem 2. For given positive scalars $\phi, \vartheta_1, \vartheta_2$ satisfying $\phi > \vartheta_1 + \vartheta_2$, the augmented system (48) is asymptotically stable if there exist a symmetric matrix $\mathcal{P} = \text{diag}\{\mathcal{P}_1, \mathcal{P}_2\} > 0$ and seven scalars $\gamma_i (i = 1, 2, 3) > 0, \beta_i (i = 1, 2, \dots, 4) > 0$ satisfying the following LMI:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} \\ * & \Pi_{22} & 0 & \Pi_{24} & \Pi_{25} & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & \Pi_{45} & 0 \\ * & * & * & * & \Pi_{55} & 0 \\ * & * & * & * & * & \Pi_{66} \end{bmatrix} < 0, \quad (40)$$

$$\Theta = \begin{bmatrix} \Lambda 2 & 0 \\ * & \beta_1 I - \vartheta_1 \mathcal{P}_1 \end{bmatrix} < 0, \quad (41)$$

$$\Phi = \begin{bmatrix} \beta_2 M_{11}^T M_{11} - \vartheta_2 \mathcal{P}_1 & 0 \\ * & \beta_2 M_{12}^T M_{12} - \vartheta_2 \mathcal{P}_2 \end{bmatrix} < 0, \quad \text{where} \quad (42)$$

$$\begin{aligned} \Pi_{11} &= \begin{bmatrix} \Lambda_1 & D^T X^T - \gamma_3 D^T N_4^T N_4 D \\ * & \bar{Q}_2 + \gamma_3 D^T N_4^T N_4 D \end{bmatrix}, \Pi_{12} = \begin{bmatrix} -\gamma_3 D^T N_4^T N_4 D & \gamma_2 M_{41}^T N_2^T N_3 \\ -X D + \gamma_3 D^T N_4^T N_4 D & 0 \end{bmatrix}, \\ \Pi_{13} &= \begin{bmatrix} \mathcal{P}_1 B & 0 \\ 0 & \mathcal{P}_2 B \end{bmatrix}, \Pi_{14} = \begin{bmatrix} \gamma_3 D^T N_4^T N_4 & 0 \\ X - \gamma_3 D^T N_4^T N_4 D & 0 \end{bmatrix}, \Pi_{66} = \begin{bmatrix} -\gamma_2 I & 0 \\ * & -\gamma_3 I \end{bmatrix}, \\ \Pi_{15} &= \begin{bmatrix} -\gamma_3 D^T N_4^T N_4 & -\mathcal{P}_1 \bar{M}_2 \\ X - \gamma_3 D^T N_4^T N_4 D & -\mathcal{P}_2 \bar{M}_2 \end{bmatrix}, \Pi_{22} = \begin{bmatrix} -\beta_1 I + \gamma_3 D^T N_4^T N_4 D & 0 \\ * & -\beta_1 I \end{bmatrix}, \\ \Pi_{16} &= \begin{bmatrix} \mathcal{P}_1 \bar{M}_2 & 0 \\ \mathcal{P}_2 \bar{M}_2 & \mathcal{P}_2 \bar{M}_3 \end{bmatrix}, \Pi_{24} = \begin{bmatrix} -\gamma_3 D^T N_4^T N_4 & 0 \\ 0 & 0 \end{bmatrix}, \\ \Pi_{45} &= \begin{bmatrix} \gamma_3 N_4^T N_4 & 0 \\ 0 & 0 \end{bmatrix}, \Pi_{33} = \begin{bmatrix} -\beta_2 I + \gamma_2 N_3^T N_3 & 0 \\ * & -\beta_2 I \end{bmatrix}, \\ \Pi_{44} &= \begin{bmatrix} -\beta_3 I + \gamma_3 N_4^T N_4 & 0 \\ * & -\beta_3 I \end{bmatrix}, \Pi_{25} = \begin{bmatrix} -\gamma_3 D^T N_4^T N_4 & 0 \\ 0 & 0 \end{bmatrix}, \Pi_{55} = \begin{bmatrix} -\beta_4 I + \gamma_3 N_4^T N_4 & 0 \\ * & -\gamma_1 I \end{bmatrix}, \\ \Lambda_1 &= \bar{Q}_{11} + \gamma_2 M_{41}^T N_2^T N_2 M_{41} + \gamma_3 D^T N_4^T N_4 D, \\ \Lambda_2 &= \beta_1 I + \beta_3 M_{22}^T M_{22} + \beta_4 \sigma D^T D + \beta_4 \sigma \text{sym}\{D^T M_3\} + \beta_4 \sigma M_3^T M_3 - \vartheta_2 \mathcal{P}_2. \end{aligned} \quad (43)$$

Furthermore, the nonfragile estimator gain K is designed as $K = \mathcal{P}_2^{-1} X$.

Proof. Construct the following Lyapunov functional:

$$V(t) = \eta^T(t) \mathcal{P} \eta(t). \quad (44)$$

From Lemma 2, the following inequality is obtained:

$$\begin{aligned} D^\alpha V(t) &\leq 2\eta^T(t) \mathcal{P} D^\alpha \eta(t) \\ &= 2\eta^T(t) \mathcal{P} [\tilde{C}\eta(t) + \tilde{H}\eta(t-d(t)) + \tilde{A}\varphi(\eta(t)) \\ &\quad + \tilde{B}\varphi(\eta(t-\tau(t))) - \tilde{G}_1 g(\bar{I}\eta(t-d(t))) - \tilde{L}e_y(t)]. \end{aligned} \quad (45)$$

By using Lemma 1 and Lipschitz condition, one gets

$$\begin{aligned} 2\eta^T(t) \mathcal{P} \tilde{H}\eta(t-d(t)) &\leq \beta_1^{-1} \eta^T(t) \mathcal{P} \tilde{H}\tilde{H}^T \mathcal{P} \eta(t) \\ &\quad + \beta_1 \eta(t-d(t))^T \eta(t-d(t)), \end{aligned} \quad (46)$$

$$\begin{aligned} 2\eta^T(t) \mathcal{P} \tilde{B}\varphi(\eta(t-\tau(t))) &\leq 2\eta^T(t) \mathcal{P} \tilde{B} M_1 \eta(t-\tau(t)) \\ &\leq \beta_2^{-1} \eta^T(t) \mathcal{P} \tilde{B}\tilde{B}^T \mathcal{P} \eta + \beta_2 \eta^T(t-\tau(t)) M_1^T M_1 \eta(t-\tau(t)), \end{aligned} \quad (47)$$

$$\begin{aligned} -2\eta^T(t) \mathcal{P} \tilde{G} g(\bar{I}\eta(t-d(t))) &\leq \beta_3^{-1} \eta^T(t) \mathcal{P} \tilde{G}\tilde{G}^T \mathcal{P} \eta(t) \\ &\quad + \beta_3 \eta^T(t-d(t)) \bar{I}^T M_2^T M_2 \bar{I} \eta(t-d(t)), \end{aligned} \quad (48)$$

$$-2\eta^T(t) \mathcal{P} \tilde{L}e_y(t) \leq \beta_4^{-1} \eta^T(t) \mathcal{P} \tilde{L}\tilde{L}^T \mathcal{P} \eta(t) + \beta_4 e_y(t)^T e_y(t). \quad (49)$$

From event-triggered condition (5), we can obtain and therefore

$$\begin{aligned}
e_y(t)^T e_y(t) &\leq \sigma y^T(t_k h + jh) y(t_k h + jh) \\
&\leq \sigma y^T(t-d(t)) y(t-d(t)) \\
&\leq \sigma \eta^T(t-d(t)) \bar{D}^T \bar{D} \eta(t-d(t)) \\
&\quad + \sigma \eta^T(t-d(t)) \left[\bar{D}^T M_3 \bar{I} + \sigma \bar{I}^T M_3^T \bar{D} \right] \eta(t-d(t)) \\
&\quad + \sigma \eta^T(t-d(t)) \bar{I}^T M_3^T M_3 \bar{I} \eta(t-d(t)).
\end{aligned} \tag{50}$$

Combining (45)–(50), one can get

$$\begin{aligned}
D^\alpha V(t) &\leq 2\eta^T(t) \mathcal{P} D^\alpha \eta(t) \\
&\leq \eta^T(t) \left[\mathcal{P} \tilde{C} \eta(t) + \tilde{C}^T \mathcal{P} + \mathcal{P} \tilde{A} M_4 + M_4^T \tilde{A}^T \mathcal{P} \right. \\
&\quad + \beta_1^{-1} \mathcal{P} \tilde{H} \tilde{H}^T \mathcal{P} + \beta_2^{-1} \mathcal{P} \tilde{B} \tilde{B}^T \mathcal{P} + \beta_3^{-1} \mathcal{P} \tilde{G} \tilde{G}^T \mathcal{P} \\
&\quad \left. + \beta_4^{-1} \mathcal{P} \tilde{L} \tilde{L}^T \mathcal{P} \right] \eta(t) \\
&\quad + \eta^T(t-d(t)) \left[\beta_1 \tilde{I} + \beta_3 \bar{I}^T M_2^T M_2 \bar{I} + \beta_4 \sigma \bar{D}^T M_3 \tilde{I} \right. \\
&\quad \left. + \beta_4 \sigma \bar{I}^T M_3^T \bar{D} + \bar{I}^T M_3^T M_3 \tilde{I} \right] \eta(t-d(t)) \\
&\quad + \eta^T(t-\tau(t)) \left[\beta_2 M_1^T M_1 \right] \eta^T(t-\tau(t)),
\end{aligned} \tag{51}$$

$$\begin{aligned}
D^\alpha V(t) + \phi V(t) - \vartheta_1 \sup_{-\tau_M \leq \omega \leq 0} V(t+\omega, x(t+\omega)) - \vartheta_2 \sup_{-d_M \leq \omega \leq 0} V(t+\omega, x(t+\omega)) \\
\leq \eta^T(t) \left[\mathcal{P} \tilde{C} \eta(t) + \tilde{C}^T \mathcal{P} + \mathcal{P} \tilde{A} M_4 + M_4^T \tilde{A}^T \mathcal{P} + \beta_1^{-1} \mathcal{P} \tilde{H} \tilde{H}^T \mathcal{P} + \beta_2^{-1} \mathcal{P} \tilde{B} \tilde{B}^T \mathcal{P} + \beta_3^{-1} \mathcal{P} \tilde{G} \tilde{G}^T \mathcal{P} + \beta_4^{-1} \mathcal{P} \tilde{L} \tilde{L}^T \mathcal{P} \right] \\
\cdot \eta(t) + \eta^T(t-d(t)) \\
\left[\beta_1 \tilde{I} + \beta_3 \bar{I}^T M_2^T M_2 \bar{I} + \beta_4 \sigma \bar{D}^T M_3 \tilde{I} + \beta_4 \sigma \bar{I}^T M_3^T \bar{D} + \bar{I}^T M_3^T M_3 \tilde{I} \right] \eta(t-d(t)) \\
+ \eta^T(t-\tau(t)) \left[\beta_2 M_1^T M_1 \right] \eta^T(t-\tau(t)) + \phi V(t) - \vartheta_1 \eta^T(t-\tau(t)) \mathcal{P} \eta(t-\tau(t)) - \vartheta_2 \eta^T(t-d(t)) \mathcal{P} \eta(t-d(t)).
\end{aligned} \tag{52}$$

Based on the above inequations, we have

$$\begin{aligned}
D^\alpha V(t) + \phi V(t) - \vartheta_1 \sup_{-\tau_M \leq \omega \leq 0} V(t+\omega, x(t+\omega)) - \vartheta_2 \sup_{-d_M \leq \omega \leq 0} V(t+\omega, x(t+\omega)) \\
\leq \eta^T(t) Q_1 \eta^T(t) + \eta^T(t-d(t)) Q_2 \eta^T(t-d(t)) + \eta^T(t-\tau(t)) Q_3 \eta^T(t-\tau(t)),
\end{aligned} \tag{53}$$

where

$$\begin{aligned}
Q_1 &= \text{sym}\{\mathcal{P}\tilde{C} + \mathcal{P}\tilde{A}M_4\} + \beta_1\mathcal{P}\tilde{H}\tilde{H}^T\mathcal{P} + \beta_2\mathcal{P}\tilde{B}\tilde{B}^T\mathcal{P} \\
&\quad + \beta_1\mathcal{P}\tilde{G}\tilde{G}^T\mathcal{P} + \beta_4\mathcal{P}\tilde{L}\tilde{L}^T\mathcal{P} + \phi\mathcal{P}, \\
Q_2 &= \beta_1\check{I} + \beta_3\check{I}^T M_2^T M_2 \check{I} + \beta_4\sigma\check{D}^T \check{D} + \text{sym}\{\beta_4\sigma\check{D}^T M_3 \check{I}\} \\
&\quad + \beta_4\sigma\check{I}^T M_3^T M_3 \check{I} - \vartheta_1\mathcal{P}, \\
Q_3 &= \beta_2 M_1^T M_1 - \vartheta_2\mathcal{P}.
\end{aligned} \tag{54}$$

By Lemma 5, $\Pi < 0$ is equivalent to

$$(\bar{Q} + \gamma_1 S_1^T S_1 + \gamma_2 S_2^T S_2 + \gamma_3 S_3^T S_3) + \gamma_1^{-1} R_1 R_1^T + \gamma_2^{-1} R_2 R_2^T + \gamma_3^{-1} R_3 R_3^T < 0, \tag{55}$$

where

$$\begin{aligned}
\bar{Q} &= \begin{bmatrix} \text{sym}\{\mathcal{P}C_1 + \mathcal{P}A_1 M_4\} & \mathcal{P}H_1 & \mathcal{P}B_1 & \mathcal{P}G_1 & \mathcal{P}L_1 \\ * & -\beta_1\check{I} & 0 & 0 & 0 \\ * & * & -\beta_2\check{I} & 0 & 0 \\ * & * & * & -\beta_3\check{I} & 0 \\ * & * & * & * & -\beta_4\check{I} \end{bmatrix}, \\
R_1 &= \begin{bmatrix} -\mathcal{P}_1 \bar{M}_2 \\ -\mathcal{P}_2 \bar{M}_2 \\ 0_{7n \times n} \end{bmatrix}, R_2 = \begin{bmatrix} \mathcal{P}_1 \bar{M}_2 \\ \mathcal{P}_2 \bar{M}_2 \\ 0_{7n \times n} \end{bmatrix}, \\
R_3 &= \begin{bmatrix} 0 \\ \mathcal{P}_2 \bar{M}_2 \\ 0_{7n \times n} \end{bmatrix}, S_1 = [N_1, 0_{n \times 8n}], \\
S_2 &= [N_2 M_{41}, 0_{n \times 3n}, N_3, 0_{n \times 4n}], S_3 = [N_4 D, -N_4 D, -N_4 D, 0_{n \times 3n}, N_4, 0, N_4].
\end{aligned} \tag{56}$$

Let

$$\begin{aligned}
\bar{Q}' &= R_1 F_1(t) S_1 + R_2 F_1(t) S_2 + R_3 F_2(t) S_3 + S_1^T F_1^T(t) R_1^T \\
&\quad + S_2^T F_1^T(t) R_2^T + S_3^T F_2^T(t) R_3^T \\
&\leq \gamma_1 S_1^T S_1 + \gamma_2 S_2^T S_2 + \gamma_3 S_3^T S_3 + \gamma_1^{-1} R_1 R_1^T \\
&\quad + \gamma_2^{-1} R_2 R_2^T + \gamma_3^{-1} R_3 R_3^T.
\end{aligned} \tag{57}$$

Then, one gets $\bar{Q} + \bar{Q}' < 0$. Based on Lemma 5, (55) and (57) imply that $Q_1 \leq 0$. According to (40) and (42), we know that $Q_2 < 0$, and $Q_3 < 0$. Therefore, we can obtain

$$\begin{aligned}
D^\alpha V(t) + \phi V(t) - \vartheta_1 \sup_{-\tau_M \leq \omega \leq 0} V(t + \omega, v(t + \omega)) \\
- \vartheta_2 \sup_{-d_M \leq \omega \leq 0} V(t + \omega, v(t + \omega)) \leq 0.
\end{aligned} \tag{58}$$

Therefore, system (37) is an asymptotical estimator of (34) by using Lemma 6. This completes the proof. \square

4. Numerical Example

To illustrate the theoretical results, two numerical examples are shown in this section.

Example 1.

$$\mathcal{A} = \begin{bmatrix} -1.2 & -0.5 & 1 \\ 1 & -0.2 & 1.6 \\ 0.2 & 0.8 & -1 \end{bmatrix}, \mathcal{E} = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, D = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.28 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \tag{59}$$

where $\bar{M} = \text{diag}\{0.4, 0.4, 0.4\}$, $N = \text{diag}\{0.3, -0.3, 0.3\}$, and $M_{11} = M_{12} = \text{diag}\{0.2, 0.2, 0.2\}$.

The other parameters are given as $\alpha = 0.98$, $\tau = 1$, $\mathcal{V} = [0.5, 0.2, -0.8]^T$. The activation function is given as follows:

$$\mathfrak{h}(v_1, v_2, v_3) = \{\sin(0.4\pi v_1), \cos(0.2\pi v_2), \sin(0.6\pi v_3)\}. \tag{60}$$

By using Matlab to solve LMIs (26) and (27), the gain matrix K can be obtained as

$$K = \begin{bmatrix} 1.5663 & 0.1752 & -1.0655 \\ 0.1475 & 1.9113 & -0.8569 \\ -0.7003 & -0.7138 & 1.6980 \end{bmatrix}. \tag{61}$$

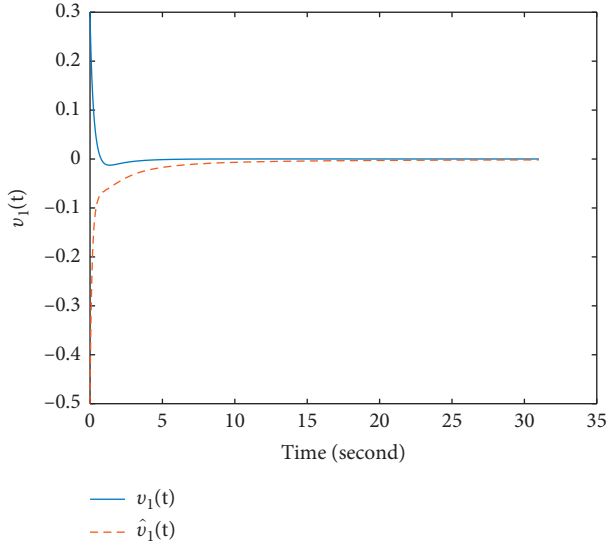


FIGURE 1: Trajectories of $v_1(t)$ and $\hat{v}_1(t)$.

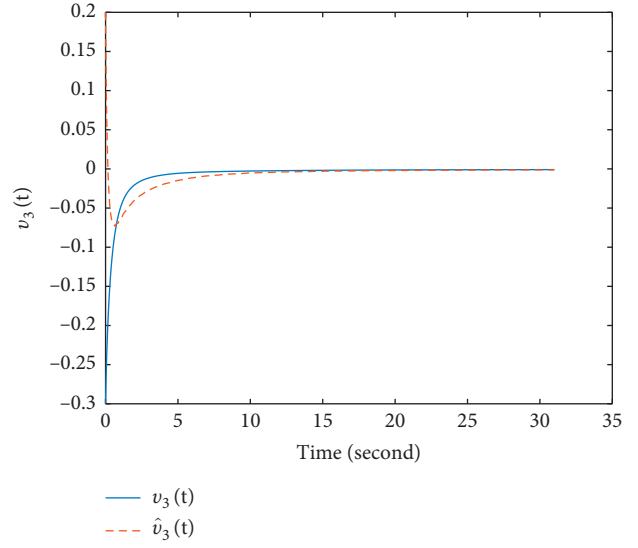


FIGURE 3: Trajectories of $v_3(t)$ and $\hat{v}_3(t)$.

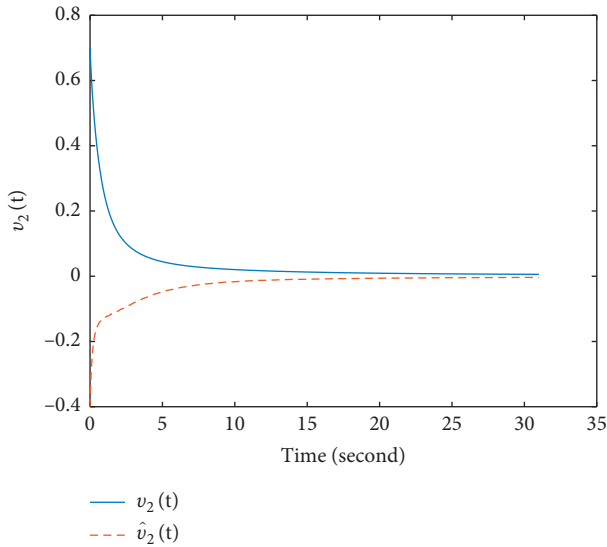


FIGURE 2: Trajectories of $v_2(t)$ and $\hat{v}_2(t)$.

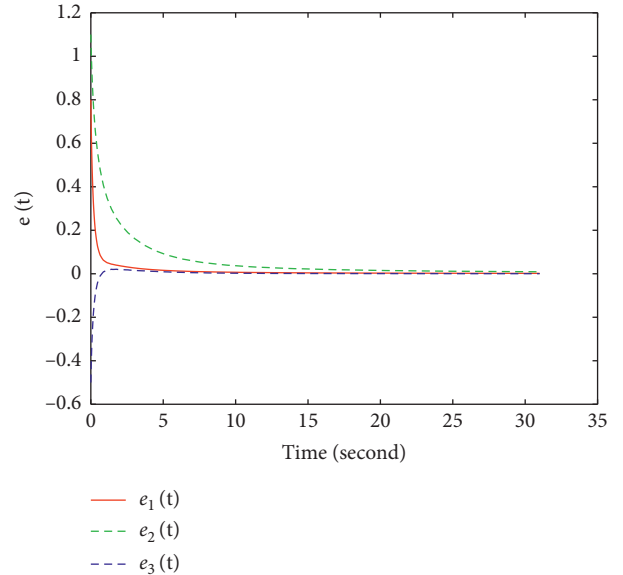


FIGURE 4: The estimation errors.

The simulation results are shown in Figures 1–3, where $v_1(t), v_2(t), v_3(t)$ represent the true states and their estimates $\hat{v}_1(t), \hat{v}_2(t), \hat{v}_3(t)$ and the initial conditions are $(\forall t \in [-1, 0]): v(t) = [0.3, 0.7, -0.3]^T, \hat{v}(t) = [-0.5, -0.4, 0.2]^T$.

Figure 4 shows the estimate error $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$. According to the simulation results, we can see the effectiveness of the estimator design method. Figure 5 shows the release instants and intervals with the threshold parameter $\sigma = 0.06$.

Example 2. To verify that the estimator contains uncertain terms in (34), the following fractional-order model is shown, and the corresponding parameters are as follows:

$$A = \begin{bmatrix} -0.6 & -0.5 & 1 \\ 0.1 & -0.2 & 1 \\ 0.2 & 0.3 & -1 \end{bmatrix}, B = \begin{bmatrix} 0.2 & -0.2 & 0.1 \\ 0.3 & -0.2 & 0.1 \\ -0.4 & -0.1 & 0.3 \end{bmatrix}, C = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, \quad (62)$$

where $\overline{M}_2 = \text{diag}\{0.4, 0.2, 0.4\}$, $\overline{M}_3 = \text{diag}\{0.1, 0.1, 0.1\}$, $M_{11} = M_{12} = M_{41} = M_{42} = \text{diag}\{0.2, 0.2, 0.2\}$, $N_1 = N_2 = \text{diag}\{0.1, 0.1, 0.1\}$, and $N_3 = N_4 = \{0.2, 0.2, 0.2\}$. Besides, $\alpha = 0.92$, and the time delays are set as $\tau_M = 1, d_M = 0.1, J = [0.01, 0.02, -0.01]^T$. The measurement output is $\tilde{g}(t, v) = [\tanh(0.4\pi v_1), \tanh(0.2\pi v_2), \tanh(0.6\pi v_3)]$. Let the activation function be $\tilde{h}(v_1, v_2, v_3) = [\tanh(0.4\pi v_1), \tanh(0.3\pi v_2), \tanh(0.2\pi v_3)]$.

By using Matlab to solve the LMI (37), the gain matrix K can be obtained as

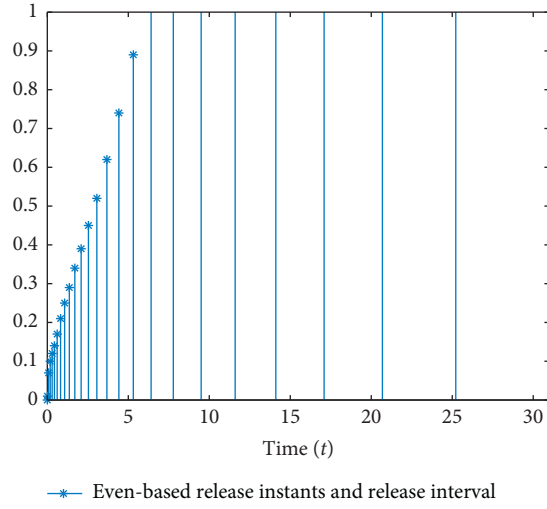


FIGURE 5: Event-triggered release instants and intervals of Theorem 1.

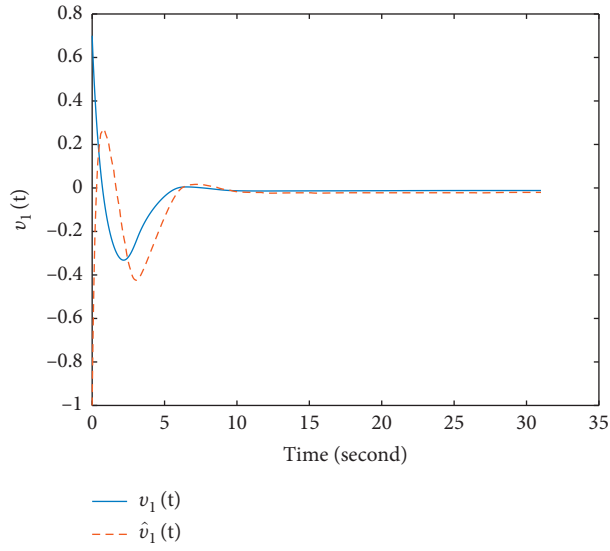


FIGURE 6: Trajectories of $v_1(t)$ and $\hat{v}_1(t)$.

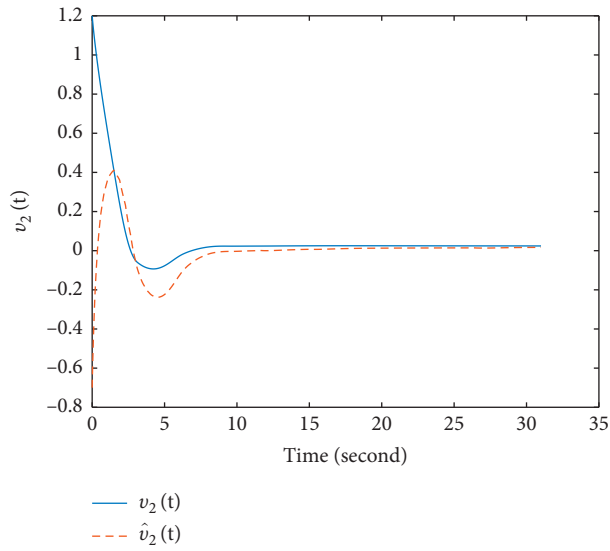


FIGURE 7: Trajectories of $v_2(t)$ and $\hat{v}_2(t)$.

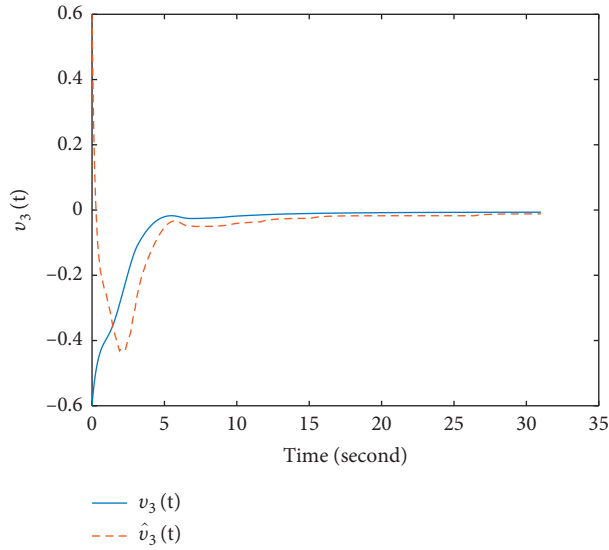
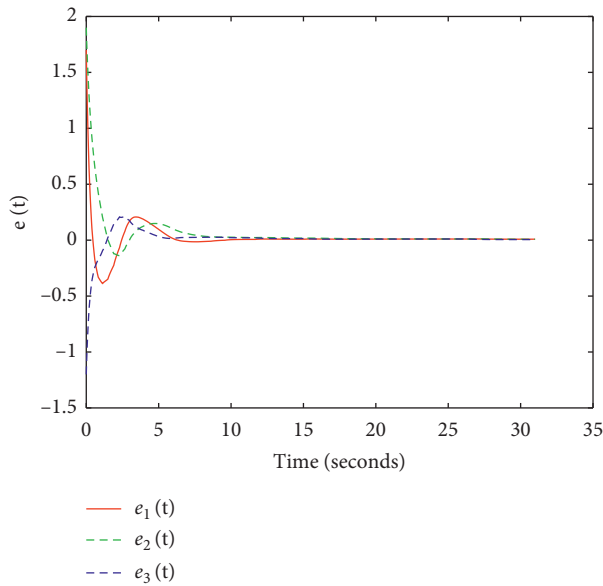

 FIGURE 8: Trajectories of $v_3(t)$ and $\hat{v}_3(t)$.


FIGURE 9: The estimation errors.

$$K = \begin{bmatrix} 1.4199 & 0.2566 & -1.4064 \\ 0.0808 & 1.2616 & -0.6868 \\ -0.3186 & -0.3723 & 2.8109 \end{bmatrix}. \quad (63)$$

The simulation results are shown in Figures 6–8, where $v_1(t), v_2(t), v_3(t)$ represent the true states and their estimates $\hat{v}_1(t), \hat{v}_2(t), \hat{v}_3(t)$ are depicted, respectively, with the initial condition ($\forall t \in [-1, 0]$): $v(t) = [0.7; 1.2; -0.6]$, $\hat{v}(t) = [-1; -0.7; 0.6]$.

Figure 9 shows the estimate error $e_1(t), e_2(t), e_3(t)$, as $t \rightarrow \infty$. In Figure 8, we can see clearly the error states $e_i(t) \rightarrow 0$. Figure 10 shows the release instants and intervals with $\sigma = 0.06$. According to the figures, we can see that the simulation results voiced the effectiveness of the estimator design.

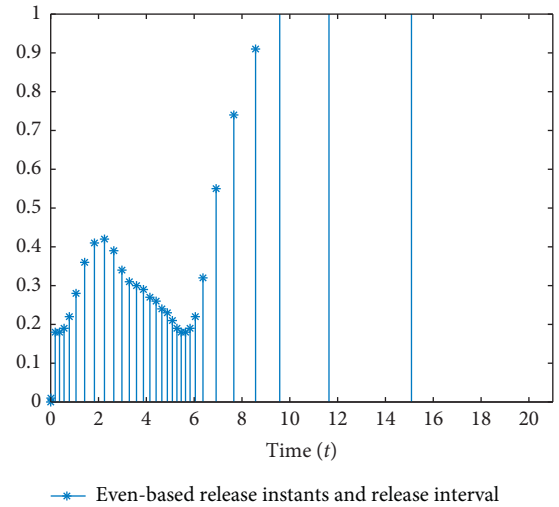


FIGURE 10: Event-triggered release instants and intervals of Theorem 2.

5. Conclusions

This paper has investigated the nonfragile SE issue under the ETM for the FNNs with time delays. Sufficient conditions have been obtained to ensure the asymptotic stability of the considered system by means of the fractional-order Lyapunov functions and the LMI method. The gain matrix of the nonfragile estimator has been characterized by a LMI. At last, two numerical results have confirmed the validity of the designed estimator. In addition, the results could be extended to the SE issue of discrete FNNs with fading measurements and so on.

Data Availability

In this paper, the initial conditions have been given. As a control system, the data in the Numerical Example is sufficient to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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