

Research Article

Interest-Rate Products Pricing Problems with Uncertain Jump Processes

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Uncertain differential equations (UDEs) with jumps are an essential tool to model the dynamic uncertain systems with dramatic changes. The interest rates, impacted heavily by human uncertainty, are assumed to follow UDEs with jumps in ideal markets. Based on this assumption, two derivatives, namely, interest-rate caps (IRCs) and interest-rate floors (IRFs), are investigated. Some formulas are presented to calculate their prices, which are of too complex forms for calculation in practice. For this reason, numerical algorithms are designed by using the formulas in order to compute the prices of these structured products. Numerical experiments are performed to illustrate the effectiveness and efficiency, which also show the prices of IRCs are strictly increasing with respect to the diffusion parameter while the prices of IRFs are strictly decreasing with respect to the diffusion parameter.

1. Introduction

The stochastic differential equations have been widely applied to mathematical finance since Black and Scholes [1] derived the option pricing formulas under the assumption that the price of a stock is an exponential Brownian motion. Hull and White [2] discussed the stochastic volatilities of the option derivatives and modeled the volatility with a geometric stochastic differential equation. Amin and Jarrow [3] built a framework to price contingent claims on foreign currencies and obtained closed form solutions for currency futures. Ninomiya and Victoir [4] investigated the Asian option pricing problems of a stochastic volatility model. Recently, Wang et al. [5] developed a novel hybrid model that integrates fractal interpolation to forecast the time series of stock price indexes and proposed a method to calculate the vertical scaling factors of the fractal interpolation iterated function system. Aiming at researching the dynamic behavior of the fractional Black–Scholes model, He and Zhang [6] developed a full-discrete numerical scheme. Through investing in the measurement of interest-rate risk, Tang and

Du [7] considered the existence of embedded options under the assumption that the fluctuation of interest rate follows fuzzy stochastic process.

The abovementioned financial models are all based on the assumption that the prices of the assets are only subject to random fluctuations. However, the real financial markets often involve human uncertain factors in addition to stochastic factors. For modelling the human subjective uncertainty in the view of mathematics, Liu [8, 9] founded and refined the uncertainty theory on the basis of normality axiom, duality axiom, subadditivity axiom, and product axioms. Uncertain variables are mainly a tool of modelling quantities whose possible values are estimated according to human belief degree, and uncertain processes, which are a sequence of uncertain variables that evolve with the time, describe ever-changing uncertain systems. Frequently used uncertain processes include the Liu processes [9] and the renewal processes [10].

Driven by the uncertain processes, the UDEs are widely applied in finance. For example, the UDE was firstly introduced to the financial models by Liu [9], in which the

price of a stock is described by an exponential Liu process, and the European options of the stock are priced. Then, Sun and Su [11] modeled the price of a stock and the interest rate in long run by means of mean-reverting processes. In order to ensure the interest rate can always remain positive, Liu [12] made some improvements on the basis of Sun and Su's [11] model. Sheng and Shi [13] and Gao et al. [14] calculated the prices of Asian options and lookback options, respectively. These works were generalized by Hassanzadeh and Mehrdoust [15] and Yang et al. [16]. The UDEs were also used to model the interest rate in the uncertain environment by Chen and Gao [17], in which the zero-coupon bond is priced in an analytic form. Following that, Zhang et al. [18] investigated the interest-rate ceilings and floors based on these interest-rate models. For the purpose of deriving the European currency options, Wang [19] supposed a currency model in which interest rate and exchange rate follow stochastic process and uncertain process, respectively. By assuming that exchange rate, volatility, domestic interest rate, and foreign interest rate all follow uncertain processes, Li et al. [20] researched the pricing problem of foreign currency options and designed Runge–Kutta-99 hybrid method for solving nested uncertain differential equations. Option pricing problems based on uncertain fractional-order differential equation with Caputo type were investigated by Jin et al. [21].

Driven by both Liu processes and renewal processes, the UDEs with jumps that were proposed by Yao [22] describe dynamic systems with uncertain sharp drifts. The stability properties of the equations were investigated by Yao [23], Gao [24], Ji and Ke [25], and Ma et al. [26]. As applications of UDEs with jumps, Ji and Wu [27] described the exchange rates between different currencies which are greatly impacted by the uncertain factors. Recently, Yu and Ning [28] presented an interest-rate model with jumps to describe the severe perturbation of interest rate and calculated the prices of zero-coupon bonds. Based on this model, we compute the price of another two widely used derivatives in this paper, namely, the IRCs and the IRFs. The remainder of this paper

is structured as follows. Section 2 introduces the uncertain interest-rate model with jumps. Then, the IRCs and the IRFs are priced in analytic but extremely complex forms in Sections 3 and 4, respectively. After that, some algorithms are designed to compute the prices numerically, and experiments are performed to illustrate the effectiveness of the algorithms in Section 5. Finally, some remarks are made in Section 6.

2. Preliminaries

The uncertainty theory is a branch of axiomatic mathematics to deal with the information involving human uncertainty. The uncertain variables are used to model the quantities whose possible values are assigned based on human experience, and the UDEs are widely applied to describe dynamic systems with human uncertainty. For more about uncertain variables and UDEs, please refer to Liu [29] and Yao [30].

In order to model the interest rate in the ideal market which may change dramatically, Yu and Ning [28] suggested

$$dX_t = \mu X_t dt + \sigma X_t dC_t + \delta X_t dN_t, \quad (1)$$

where X_t denotes the interest rate, $\mu, \sigma > 0$ and $\delta > 0$ are constant parameters, C_t is a canonical Liu process, and N_t is an uncertain renewal process. Let Ψ denote the standard normal uncertainty distribution (UD) and Φ denote the UD of interarrival times. Then, the solution,

$$X_t = X_0 \cdot \exp(\mu t + \sigma C_t) \cdot (1 + \delta)^{N_t}, \quad (2)$$

of equation (1) possesses an UD

$$\Theta_t(y) = \max_{k \geq 0} \Psi\left(\frac{\ln y - \mu t - k \ln(1 + \delta) - \ln X_0}{\sigma t}\right) \wedge \left(1 - \Phi\left(\frac{t}{k+1}\right)\right). \quad (3)$$

Yu and Ning [28] further showed that X_t possesses the inverse uncertainty distribution (IUD)

$$\Theta_t^{-1}(\beta) = X_0 \exp\left(\mu t + \sigma \Psi^{-1}(\beta)t + \ln(1 + \delta) \cdot \left(\left\lceil \frac{t}{\Phi^{-1}(1 - \beta)} \right\rceil - 1\right)\right), \quad (4)$$

where $\lceil z \rceil$ denotes the maximal integer less than or equal to z . In fact, they gave a more powerful result as follows.

Theorem 1 (see [28]). *Let Ψ denote the standard normal UD and Φ denote the UD of interarrival times. Then, the solution X_t of equation (1) satisfies*

$$\begin{aligned} M\left\{X_t \leq X_0 \exp\left(\mu t + \sigma \Psi^{-1}(\beta)t + \ln(1 + \delta) \cdot \left(\left\lceil \frac{t}{\Phi^{-1}(1 - \beta)} \right\rceil - 1\right)\right), \quad \forall t \geq 0\right\} &= \beta, \\ M\left\{X_t > X_0 \exp\left(\mu t + \sigma \Psi^{-1}(\beta)t + \ln(1 + \delta) \cdot \left(\left\lceil \frac{t}{\Phi^{-1}(1 - \beta)} \right\rceil - 1\right)\right), \quad \forall t \geq 0\right\} &= 1 - \beta. \end{aligned} \quad (5)$$

3. Interest-Rate Caps

The IRC is a contract between a lending institute and a borrower that prevents the institute charging more than a certain level of interest from the borrower. It is a benefit for the borrowers in variable interest-rate products.

Let an uncertain process X_t denote the interest rate. Then, the price of IRC, as suggested in Zhang et al. [18], is

$$f_c = 1 - E \left[\exp \left(- \int_0^T (X_t - C)^+ dt \right) \right], \quad (6)$$

where C is the maximum interest rate preset in the contract and T is the expiration date of the contract. Now, based on

the assumption of the interest rate in Yu and Ning [28], that is, it follows the UDE with jumps (1), we derive pricing formula of the IRCs.

Theorem 2. Assume the interest rate X_t follows an UDE with jumps

$$dX_t = \mu X_t dt + \sigma X_t dC_t + \delta X_t dN_t. \quad (7)$$

Let Ψ denote the standard normal UD and Φ denote the UD of interarrival times. Then, the IRC, which is of an expiration date T and a maximum interest rate C , is priced by

$$f_c = 1 - \int_0^1 \exp \left(- \int_0^T \left(X_0 \exp \left(\mu t + \sigma \Psi^{-1}(\beta)t + \ln(1 + \delta) \cdot \left(\lceil \frac{t}{\Phi^{-1}(1 - \beta)} \rceil - 1 \right) \right) - C \right)^+ dt \right) d\beta. \quad (8)$$

Proof. Denote the IUD of X_t with $\Theta_t^{-1}(\beta)$, i.e.,

$$\Theta_t^{-1}(\beta) = X_0 \exp \left(\mu t + \sigma \Psi^{-1}(\beta)t + \ln(1 + \delta) \cdot \left(\lceil \frac{t}{\Phi^{-1}(1 - \beta)} \rceil - 1 \right) \right). \quad (9)$$

Then, according to Theorem 1, we have

$$\begin{aligned} M \{ X_t \leq \Theta_t^{-1}(\beta), \quad \forall t \geq 0 \} &= \beta, \\ M \{ X_t > \Theta_t^{-1}(\beta), \quad \forall t \geq 0 \} &= 1 - \beta. \end{aligned} \quad (10)$$

Since

$$\left\{ \int_0^T (X_t - C)^+ dt \leq \int_0^T (\Theta_t^{-1}(\beta) - C)^+ dt \right\} \{ X_t \leq \Theta_t^{-1}(\beta), \quad \forall t \in [0, T] \}, \quad (11)$$

we have

$$M \left\{ \int_0^T (X_t - C)^+ dt \leq \int_0^T ((\Theta_t^{-1}(\beta) - C)^+ dt) \right\} \geq M \{ X_t \leq \Theta_t^{-1}(\beta), \forall t \in [0, T] \} \geq \beta. \quad (12)$$

Similarly, we have

$$M \left\{ \int_0^T (X_t - C)^+ dt > \int_0^T (\Theta_t^{-1}(\beta) - C)^+ dt \right\} \geq M \{ X_t > \Theta_t^{-1}(\beta), \quad \forall t \in [0, T] \} \geq 1 - \beta. \quad (13)$$

Furthermore, since

$$M\left\{\int_0^T (X_t - C)^+ dt \leq \int_0^T (\Theta_t^{-1}(\beta) - C)^+ dt\right\} + M\left\{\int_0^T (X_t - C)^+ dt > \int_0^T (\Theta_t^{-1}(\beta) - C)^+ dt\right\} = 1, \quad (14)$$

we get

$$M\left\{\int_0^T (X_t - C)^+ dt \leq \int_0^T (\Theta_t^{-1}(\beta) - C)^+ dt\right\} = \beta, \quad (15)$$

that is,

$$\int_0^T (X_t - C)^+ dt, \quad (16)$$

has an IUD

$$\int_0^T (\Theta_t^{-1}(\beta) - C)^+ dt. \quad (17)$$

Hence,

$$\exp\left(-\int_0^T (X_t - C)^+ dt\right), \quad (18)$$

has an IUD

$$\exp\left(-\int_0^T (\Theta_t^{-1}(1 - \beta) - C)^+ dt\right), \quad (19)$$

as the function $\exp(-z)$ is decreasing with z . Then,

$$\begin{aligned} f_c &= 1 - E\left[\exp\left(-\int_0^T (X_t - C)^+ dt\right)\right] \\ &= 1 - \int_0^1 \exp\left(-\int_0^T (\Theta_t^{-1}(1 - \beta) - C)^+ dt\right) d\beta \\ &= 1 - \int_0^1 \exp\left(-\int_0^T (\Theta_t^{-1}(\beta) - C)^+ dt\right) d\beta. \end{aligned} \quad (20)$$

The theorem is proved. \square

4. Interest-Rate Floors

The IRF is a contract between a lending institute and a borrower that prevents the borrower repaying less than a certain level of interest to the lending institute. It is a benefit for the lending institute in variable interest-rate products.

Let an uncertain process X_t denote the interest rate. Then, Zhang et al. [18] suggested computing the price of the IRF via

$$f_l = E\left[\exp\left(\int_0^T (L - X_t)^+ dt\right)\right] - 1, \quad (21)$$

where L is the minimum interest rate preset in the contract and T is the expiration date of the contract. Now, based on the assumption of the interest rate in Yu and Ning [28], that is, it follows the UDE with jumps (1), we derive the pricing formula of the IRFs.

Theorem 3. Assume the interest rate X_t follows an UDE with jumps

$$dX_t = \mu X_t dt + \sigma X_t dC_t + \delta X_t dN_t. \quad (22)$$

Let Ψ denote the standard normal UD and Φ denote the UD of interarrival times. Then, the IRF, which is of an expiration date T and a minimum interest rate L , is priced by

$$f_l = \int_0^1 \exp\left(\int_0^T \left(L - X_0 \exp\left(\mu t + \sigma \Psi^{-1}(\beta)t + \ln(1 + \delta) \cdot \left(\left\lceil \frac{t}{\Phi^{-1}(1 - \beta)} \right\rceil - 1\right)\right)\right)^+ dt\right) d\beta - 1. \quad (23)$$

Proof. Denote the IUD of X_t with $\Theta_t^{-1}(\beta)$, i.e.,

$$\Theta_t^{-1}(\beta) = X_0 \exp\left(\mu t + \sigma \Psi^{-1}(\beta)t + \ln(1 + \delta) \cdot \left(\left\lceil \frac{t}{\Phi^{-1}(1 - \beta)} \right\rceil - 1\right)\right). \quad (24)$$

Then, according to Theorem 1, we have

$$\begin{aligned} M\{X_t < \Theta_t^{-1}(1 - \beta), \quad \forall t \geq 0\} &= 1 - \beta, \\ M\{X_t \geq \Theta_t^{-1}(1 - \beta), \quad \forall t \geq 0\} &= \beta. \end{aligned} \tag{25}$$

Since

$$\left\{ \int_0^T (L - X_t)^+ dt \leq \int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt \right\} \{X_t \geq \Theta_t^{-1}(1 - \beta), \quad \forall t \in [0, T]\}, \tag{26}$$

we have

$$M\left\{ \int_0^T (L - X_t)^+ dt \leq \int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt \right\} \geq M\{X_t \geq \Theta_t^{-1}(1 - \beta), \quad \forall t \in [0, T]\} \geq \beta. \tag{27}$$

Similarly, we have

$$M\left\{ \int_0^T (L - X_t)^+ dt > \int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt \right\} \geq M\{X_t < \Theta_t^{-1}(1 - \beta), \quad \forall t \in [0, T]\} \geq 1 - \beta. \tag{28}$$

Furthermore, since

$$M\left\{ \int_0^T (L - X_t)^+ dt \leq \int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt \right\} + M\left\{ \int_0^T (L - X_t)^+ dt > \int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt \right\} = 1, \tag{29}$$

we get

$$M\left\{ \int_0^T (L - X_t)^+ dt \leq \int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt \right\} = \beta, \tag{30}$$

that is,

$$\int_0^T (L - X_t)^+ dt, \tag{31}$$

has an IUD

$$\int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt. \tag{32}$$

Hence,

$$\exp\left(\int_0^T (L - X_t)^+ dt\right), \tag{33}$$

has an IUD

$$\exp\left(\int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt\right), \tag{34}$$

as the function $\exp(z)$ is increasing with z . Then,

$$\begin{aligned} f_l &= E\left[\exp\left(\int_0^T (L - X_t)^+ dt\right)\right] - 1 \\ &= \int_0^1 \exp\left(\int_0^T (L - \Theta_t^{-1}(1 - \beta))^+ dt\right) d\beta - 1 \\ &= \int_0^1 \exp\left(\int_0^T (L - \Theta_t^{-1}(\beta))^+ dt\right) d\beta - 1. \end{aligned} \tag{35}$$

The theorem is proved. □

5. Numerical Algorithms

The numerical algorithms to compute the prices of the IRCs and IRFs are designed based on formulas (8) and (23) and illustrated via some experiments in the subsections.

5.1. Algorithm for Interest-Rate Caps

Step 1: according to the precision degree, preset two large numbers K and J . Set $\beta_k = k/K$ and $t_j = j \cdot T/J$, $k = 1, 2, \dots, K - 1$, $j = 1, 2, \dots, J$.

Step 2: set $k = 0$.

Step 3: set $k \leftarrow k + 1$.

Step 4: set $j = 0$.

Step 5: set $j \leftarrow j + 1$.

Step 6: compute

$$H_{k,j} = \max\left(0, X_0 \exp\left(\mu t_j + \sigma \Psi^{-1}(\beta_k) t_j + \ln(1 + \delta) \cdot \left(\left\lceil \frac{t_j}{\Phi^{-1}(1 - \beta_k)} \right\rceil - 1\right)\right) - C\right). \quad (36)$$

If $j < J$, return to Step 5.

Step 7: compute

$$\exp\left(-\frac{T}{J} \sum_{j=1}^J H_{k,j}\right). \quad (37)$$

If $k < K - 1$, return to Step 3.

Step 8: compute

$$f_c = 1 - \frac{1}{K-1} \sum_{k=1}^{K-1} \exp\left(-\frac{T}{J} \sum_{j=1}^J H_{k,j}\right). \quad (38)$$

Example 1. Consider the uncertain interest-rate model $dX_t = \mu X_t dt + \sigma X_t dC_t + \delta X_t dN_t$ with an initial interest rate $X_0 = 0.04$. Assume the parameters are $\mu = 0.05$, $\sigma = 0.03$, and $\delta = 0.01$; the standard normal UD is

$$\Psi(z) = \left(1 + \exp\left(-\frac{\pi z}{\sqrt{3}}\right)\right)^{-1}, \quad (39)$$

and the UD of interarrival times is

$$\Phi(z) = \left(1 + \exp\left(\frac{\pi(2 - \ln z)}{\sqrt{3}}\right)\right)^{-1}. \quad (40)$$

Then, the IRC with an expiration date $T = 4$ and a maximum value $C = 0.05$ has a price $f_c = 0.0017$.

By using the algorithm mentioned above, we can numerically investigate the relationships of the IRC price f_c

with respect to the parameters in the pricing formula (8) such as μ , σ , δ , T , and C . The next example plots the function of f_c with respect to μ when the other parameters are given.

Example 2. Consider the uncertain interest-rate model $dX_t = \mu X_t dt + \sigma X_t dC_t + \delta X_t dN_t$ with an initial interest rate $X_0 = 0.04$. Assume the parameters are $\sigma = 0.03$ and $\delta = 0.01$; the standard normal UD is

$$\Psi(z) = \left(1 + \exp\left(-\frac{\pi z}{\sqrt{3}}\right)\right)^{-1}, \quad (41)$$

and the UD of interarrival times is

$$\Phi(z) = \left(1 + \exp\left(\frac{\pi(2 - \ln z)}{\sqrt{3}}\right)\right)^{-1}. \quad (42)$$

The IRC has an expiration date $T = 4$ and a maximum value $C = 0.05$. Then, the function of the IRC price f_c with respect to the parameter μ is shown in Figure 1. Apparently, the price f_c is a strictly increasing function with respect to μ .

5.2. Algorithm for Interest-Rate Floors

Step 1: according to the precision degree, preset two large numbers K and J . Set $\beta_k = k/K$ and $t_j = j \cdot T/J$, $k = 1, 2, \dots, K - 1$, $j = 1, 2, \dots, J$.

Step 2: set $k = 0$.

Step 3: set $k \leftarrow k + 1$.

Step 4: set $j = 0$.

Step 5: set $j \leftarrow j + 1$.

Step 6: compute

$$H_{k,j} = \max\left(0, L - X_0 \exp\left(\mu t_j + \sigma \Psi^{-1}(\beta_k) t_j + \ln(1 + \delta) \cdot \left(\left\lceil \frac{t_j}{\Phi^{-1}(1 - \beta_k)} \right\rceil - 1\right)\right)\right). \quad (43)$$

If $j < J$, return to Step 5.

Step 7: compute

$$\exp\left(-\frac{T}{J} \sum_{j=1}^J H_{k,j}\right). \quad (44)$$

If $k < K - 1$, return to Step 3.

Step 8: compute

$$f_l = \frac{1}{K-1} \sum_{k=1}^{K-1} \exp\left(-\frac{T}{J} \sum_{j=1}^J H_{k,j}\right) - 1. \quad (45)$$

Example 3. Consider the uncertain interest-rate model $dX_t = \mu X_t dt + \sigma X_t dC_t + \delta X_t dN_t$ with an initial interest rate $X_0 = 0.04$. Assume the parameters are $\mu = 0.02$, $\sigma = 0.03$, and $\delta = 0.01$; the standard normal UD is

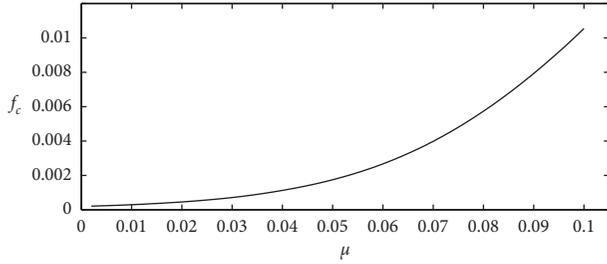


FIGURE 1: Relationship between f_c and μ in Example 2.

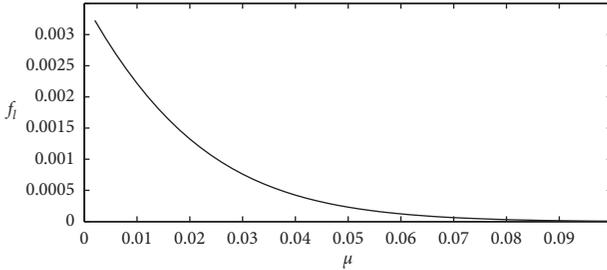


FIGURE 2: Relationship between f_l and μ in Example 4.

$$\Psi(z) = \left(1 + \exp\left(\frac{-\pi z}{\sqrt{3}}\right) \right)^{-1}, \quad (46)$$

and the UD of interarrival times is

$$\Phi(z) = \left(1 + \exp\left(\frac{\pi(2 - \ln z)}{\sqrt{3}}\right) \right)^{-1}. \quad (47)$$

Then, the IRF with an expiration date $T = 4$ and a minimum value $L = 0.04$ has a price $f_l = 0.0013$.

By using the algorithm mentioned above, we can numerically investigate the relationships of the IRF price f_l with respect to the parameters in the pricing formula (23) such as μ, σ, δ, T , and L . The next example plots the function of f_l with respect to μ when the other parameters are given.

Example 4. Consider the uncertain interest-rate model $dX_t = \mu X_t dt + \sigma X_t dC_t + \delta X_t dN_t$ with an initial interest rate $X_0 = 0.04$. Assume the parameters are $\sigma = 0.03$ and $\delta = 0.01$; the standard normal UD is

$$\Psi(z) = \left(1 + \exp\left(\frac{-\pi z}{\sqrt{3}}\right) \right)^{-1}, \quad (48)$$

and the UD of interarrival times is

$$\Phi(z) = \left(1 + \exp\left(\frac{\pi(2 - \ln z)}{\sqrt{3}}\right) \right)^{-1}. \quad (49)$$

The IRF has an expiration date $T = 4$ and a minimum value $L = 0.05$. Then, the function of the IRF price f_l with respect to the parameter μ is shown in Figure 2. Apparently, the price f_l is a strictly decreasing function with respect to μ .

6. Conclusions

Based on an interest-rate model described by an UDE with jumps, pricing problems of IRCs and IRFs were considered. Some pricing formulas were obtained in analytic but extremely complex forms. Numerical algorithms were designed based on the pricing formulas, and numerical experiments were performed to test the effectiveness of the presented algorithms. Further research may consider the structured product pricing problems based on interest-rate models with both positive and negative jumps.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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