Research Article

On the Dynamics Behaviors of a Stochastic Echinococcosis Infection Model with Environmental Noise

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This paper pays main attention to the dynamics behaviors of a stochastic echinococcosis infection model with environmental noise. The existence and uniqueness of the stochastic model is showed in this paper. We obtain the sufficient condition of the ergodic stationary distribution. What is more, the condition of extinction of the stochastic model is also given.

1. Introduction

Echinococcosis is a chronic parasitic disease caused by the infection of echinococcosis larvae with human hosts and other mammals [1]. *Echinococcus* can parasitize many parts of the human body and infect many organs at the same time. At present, the cystic echinococcosis and vesicular echinococcosis are the main types of echinococcosis. Those areas, such as Eastern and Central Asia, Northern Africa, and Southern America, are mainly distributed by echinococcosis [2]. Every year, in the world, there are 18800 new cases of cystic echinococcosis, which causes about 18400 DALYs [3]. Echinococcosis infection is one of the most important public health problems in the world, seriously endangering human health and economic development.

Echinococcosis has a wide range of epidemic and serious harm, and its spread has attracted extensive attention in the world [4–8]. The research on the epidemic area, influencing factors, and transmission mechanism of echinococcosis is a hot topic in recent decades [4–6, 9]. Based on the long-term epidemic situation of echinococcosis, scholars also used a deterministic model to explore the influence of host activity and individual behavior on the transmission of echinococcosis [9, 10].

The criteria of global asymptotic stability of the traveling wave solutions of the echinococcosis model were given by Xu and Ai [11]. Wang et al. [12] gave the following model of

echinococcosis and used the results to Xinjiang of China. The variables in the model are defined in Table 1, and the parameters are defined in Table 2.

$$\begin{cases} dS_D = [A_1 - \beta_1 S_D I_L - d_1 S_D + \sigma I_D]dt, \\ dI_D = [\beta_1 S_D I_L - (d_1 + \sigma) I_D]dt, \\ dS_L = [A_2 - \beta_2 S_L I_D - d_2 S_L]dt, \\ dI_L = [\beta_2 S_L I_D - d_2 I_L]dt, \\ dS_H = [A_3 - \beta_3 S_H I_D - d_3 S_H + \gamma I_H]dt, \\ dE_H = [\beta_3 S_H I_D - (d_3 + \omega) E_H]dt, \\ dI_H = [\omega E_H - (d_3 + \mu + \gamma) I_H]dt. \end{cases} \quad (1)$$

The results showed that the dynamical properties of the model were completely determined by the basic reproduction number R_0 ($R_0^3 = (\beta_1 \beta_2 A_1 A_2 / d_1 d_2^2 (d_1 + \sigma))$). That was, if $R_0 < 1$, the disease-free equilibrium was globally asymptotically stable, and if $R_0 > 1$, the model was permanent and the endemic equilibrium was globally asymptotically stable.

Actually, in the real world, environmental noise is an inevitable important factor, so the epidemic model will be affected by environmental noise [13–20]. In this paper, we will consider a type of environmental noise (namely white noise) with the system (1). Then, the stochastic version corresponding to system (1) can be described by the following equations:

TABLE 1: Variables in the model (adapted from [12]).

Population	Definition
$S_D(t)$	Susceptible dogs at time t
$I_D(t)$	Infectious dogs at time t
$S_L(t)$	Susceptible livestock at time t
$I_L(t)$	Infectious livestock at time t
$S_H(t)$	Susceptible human at time t
$E_H(t)$	Exposed human at time t
$I_H(t)$	Infectious human at time t

TABLE 2: Parameters in the model (adapted from [12]).

Parameter	Definition
A_1	Annual recruitment rate of the dog population
d_1	Natural death rate of the dog population
σ	Recovery rate of transition from infected to noninfected dogs
A_2	Annual recruitment rate of the livestock population
d_2	Natural death rate of the livestock population
A_3	Annual recruitment rate of the human population
d_3	Natural death rate of the human population
μ	Disease-related death rate of the human population
$(1/\omega)$	Incubation period of infected individuals of the human population
γ	Recovery rate of the human population
β_1	Infection rate of susceptible dogs by infectious livestock
β_2	Infection rate of susceptible livestock by infectious dogs
β_3	Infection rate of susceptible human by infectious dogs

$$\begin{cases}
dS_D = [A_1 - \beta_1 S_D I_L - d_1 S_D + \sigma I_D]dt + \sigma_1 S_D dB_1(t), \\
dI_D = [\beta_1 S_D I_L - (d_1 + \sigma) I_D]dt + \sigma_2 I_D dB_2(t), \\
dS_L = [A_2 - \beta_2 S_L I_D - d_2 S_L]dt + \sigma_3 S_L dB_3(t), \\
dI_L = [\beta_2 S_L I_D - d_2 I_L]dt + \sigma_4 I_L dB_4(t), \\
dS_H = [A_3 - \beta_3 S_H I_D - d_3 S_H + \gamma I_H]dt + \sigma_5 S_H dB_5(t), \\
dE_H = [\beta_3 S_H I_D - (d_3 + \omega) E_H]dt + \sigma_6 E_H dB_6(t), \\
dI_H = [\omega E_H - (d_3 + \mu + \gamma) I_H]dt + \sigma_7 I_H dB_7(t),
\end{cases} \quad (2)$$

where $B_i(t)$ stands for the mutually Brownian motions independently and $B_i(0) = 0$; $\sigma_i^2 > 0$ replaces the intensities of the environmental noise, $i = 1, 2, 3, 4, 5, 6, 7$.

In Section 2 of this paper, we give the existence and uniqueness of global positive solutions of the infection model (2) by using the Lyapunov function. The result that the system (2) is ergodic stationary distribution on certain conditions is proved in Section 3. Section 4 gives the sufficient conditions for the existence of extinction of the system (2). We illustrate some simulations to demonstrate our results in Section 5.

2. Existence and Uniqueness of the Positive Solution

Defining a differential operator L associated with equation (1) using the same methods as [15], there is positive and global solution of one system if the coefficients of the system satisfy local Lipschitz conditions. Actually, for the system (2), it may explode in finite time for not satisfying the linear

growth condition. Here, we only use the Lyapunov function to proof the solution of the model (2) is positive and global.

Denote

$$\begin{aligned}
R_+^n &= \{x \in R^n: x_i > 0, \text{ for all } 1 \leq i \leq n\}, \\
\bar{R}_+^n &= \{x \in R^n: x_i \geq 0, \text{ for all } 1 \leq i \leq n\}.
\end{aligned} \quad (3)$$

We consider the general n -dimensional stochastic differential equation

$$dx(t) = f(x(t), t)dt + g(x(t), t)dB(t), \quad \text{for } t \geq t_0, \quad (4)$$

with initial value $x(t_0) = x_0 \in R^n$, where $B(t)$ denotes n -dimensional standard Brownian motions defined on the above probability space.

Theorem 1. *There is a unique solution $(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t))$ of system (2) on $t \geq 0$ for any initial value $(S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), I_H(0)) \in R_+^7$, and the solution will remain in R_+^7 with probability 1, namely, $(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t)) \in R_+^7$, for all $t \geq 0$ almost surely.*

Proof. Using the same method as [17], when the local solution of the system (2) is global with the initial value

$$(S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), I_H(0)) \in R_+^7, \quad (5)$$

we only need to prove $(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t))$ on $t \in [0, \tau_e)$. Let $n_0 \geq 0$ be a large number and $S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), W(0) \in [1/n_0, n_0]$. Defining the stopping time

$$\tau_n = \inf \left\{ t \in [0, \tau_e) : \min \{ (S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t)) \} \leq \frac{1}{n} \right. \\ \left. \text{or } \max \{ (S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t)) \} \geq n \right\}, \quad (6)$$

for $n \geq n_0$. We set $\inf \phi = \infty$ (as usual ϕ denotes the empty set). According to the definition, τ_n is increasing as $n \rightarrow \infty$. Set $\tau_\infty = \lim_{n \rightarrow \infty} \tau_n$, where $\tau_\infty \leq \tau_e$ a.s. If we can prove that $\tau_\infty = \infty$ a.s., then $\tau_e = \infty$ and $(S_D(t), I_D(t), S_L(t), I_L(t), S_H(t), E_H(t), I_H(t)) \in R_+^7$ a.s., for all $t \geq 0$, and the formula $\tau_\infty = \infty$ a.s. is needed to show. Otherwise, there exists constants $S > 0$ and $\varepsilon \in (0, 1)$ such that

$$P\{\tau_\infty \leq S\} > \varepsilon. \quad (7)$$

Hence, there is an integer $n_1 \geq n_0$ such that

$$P\{\tau_\infty \leq S\} \geq \varepsilon, \quad \text{for all } n \geq n_1. \quad (8)$$

Define a C^2 -function $V: R_+^7 \rightarrow \bar{R}_+$ by

$$V = \left(S_D - a - a \ln \frac{S_D}{a} \right) + \left(I_D - b - b \ln \frac{I_D}{b} \right) + (S_L - 1 - \ln S_L) + (I_L - 1 - \ln I_L) \\ + (S_H - 1 - \ln S_H) + (E_H - 1 - \ln E_H) + (I_H - 1 - \ln I_H), \quad (9)$$

where a, b are positive constants to be determined later. The nonnegativity of this function can be seen from

$u - 1 - \log u \geq 0, \forall u > 0$. Let $n \geq n_0$ and $S > 0$ be arbitrary. Using Itô's formula, we obtain

$$dV = LV dt + \sigma_1 (S_D - a) dB_1(t) + \sigma_2 (I_D - b) dB_2(t) + \sigma_3 (S_L - 1) dB_3(t) \\ + \sigma_4 (I_L - 1) dB_4(t) + \sigma_5 (S_H - 1) dB_5(t) + \sigma_6 (E_H - 1) dB_6(t) + \sigma_7 (I_H - 1) dB_7(t), \\ LV = A_1 - d_1 S_D - a \frac{A_1}{S_D} + a \beta_1 I_L + a d_1 - a \sigma \frac{I_D}{S_D} + \frac{1}{2} a \sigma_1^2 \\ - b d_1 I_D - b \beta_1 S_D \frac{I_L}{I_D} + b \left(d_1 + \sigma + \frac{1}{2} \sigma_2^2 \right) + A_2 - d_2 S_L - \frac{A_2}{S_L} + \beta_2 I_D + d_2 + \frac{1}{2} \sigma_3^2 \\ - d_2 I_L - \beta_2 S_L \frac{I_D}{I_L} + d_2 + \frac{1}{2} \sigma_4^2 + A_3 - d_3 S_H - \frac{A_3}{S_H} + \beta_3 I_D + d_3 - \gamma \frac{I_H}{S_H} + \frac{1}{2} \sigma_5^2 \\ - d_3 E_H - \beta_3 S_H \frac{I_D}{E_H} + d_3 + \omega + \frac{1}{2} \sigma_6^2 - (d_3 + \mu) I_H - \omega \frac{E_H}{I_H} + d_3 + \mu + \gamma + \frac{1}{2} \sigma_7^2 \\ \leq (a \beta_1 - d_2) I_L + (\beta_2 + \beta_3 - b d_1) I_D + A_1 + A_2 + A_3 + [(a + b) d_1 + 2 d_2 + 3 d_3] \\ + \frac{1}{2} (a \sigma_1^2 + b \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2 + \sigma_6^2 + \sigma_7^2) + b \sigma + \omega + \mu + \gamma. \quad (10)$$

Choosing

then we get

$$\begin{aligned} a &= \frac{d_2}{\beta_1}, \\ b &= \frac{\beta_2 + \beta_3}{d_1}, \end{aligned} \quad (11)$$

$$\begin{aligned} LV &\leq A_1 + A_2 + A_3 + [(a+b)d_1 + 2d_2 + 3d_3] \\ &\quad + \frac{1}{2}(a\sigma_1^2 + b\sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2 + \sigma_6^2 + \sigma_7^2) + b\sigma + \omega + \mu + \gamma := K. \end{aligned} \quad (12)$$

The remained proof follows that in Mao et al.'s study [14]. $\square$

(ii) (B.2) there exists a nonnegative C^2 function V such that LV is negative for any $E_l \setminus U$. Then,

3. Existence of Unique and Ergodic Stationary Distribution

Lemma 1 (see [14]). *The Markov process $X(t)$ has a stationary distribution $\mu(\cdot)$ if there exists a bounded domain $U \subset E_l$ with regular boundary Γ and*

(i) (B.1) there is a positive number M such that $\sum_{i,j=1}^l a_{ij}(x)\xi_i\xi_j \geq M|\xi|^2, x \in U, \xi \in \mathbb{R}^l$.

$$P_x \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(X(t))dt = \int_{E_l} f(x)\mu(dx) \right\} = 1, \quad (13)$$

for all $x \in E_l$, where $f(\cdot)$ is a function integrable with respect to the measure μ .

Define a parameter

$$R_0^s = \frac{\beta_1\beta_2A_1A_2}{(d_1 + (1/2)\sigma_1^2)(d_1 + \sigma + (1/2)\sigma_2^2)(d_2 + (1/2)\sigma_3^2)(d_2 + (1/2)\sigma_4^2)}. \quad (14)$$

Theorem 2. *Assume that $R_0^s > 1$, then there is a unique stationary distribution $\mu(\cdot)$ for system (2) and it has ergodic property.*

Proof

(i) Step 1: verify that (B.1) holds. Apparently, the diffusion matrix of system (2) is

$$\Lambda(S_D, I_D, S_L, I_L, S_H, E_H, I_H) = \begin{pmatrix} \sigma_1^2 S_D^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 I_D^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 S_L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 I_L^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 S_H^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_6^2 E_H^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_7^2 I_H^2 \end{pmatrix}. \quad (15)$$

Besides, there is $R = \min_{(S_D, I_D, S_L, I_L, S_H, E_H, I_H)} \in \mathbb{R}_+^7$ $\{\sigma_1^2 S_D^2, \sigma_2^2 I_D^2, \sigma_3^2 S_L^2, \sigma_4^2 I_L^2, \sigma_5^2 S_H^2, \sigma_6^2 E_H^2, \sigma_7^2 I_H^2\} > 0$ such that

$$\sum_{i,j=1}^7 \lambda_{ij}(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \xi_i \xi_j = \sigma_1^2 S_D^2 + \sigma_2^2 I_D^2 + \sigma_3^2 S_L^2 + \sigma_4^2 I_L^2 + \sigma_5^2 S_H^2 + \sigma_6^2 E_H^2 + \sigma_7^2 I_H^2 \geq R|\xi|^2, \quad (16)$$

for all $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in \bar{U}$, $\xi \in R^7$, which implies condition (B.1) is satisfied.

(ii) Step 2: verify that (B.2) holds. Now, we will construct a nonnegative C^2 -function V and a closed set $U \in \Sigma$ such that

$$\sup_{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 \setminus U} LV(S_D, I_D, S_L, I_L, S_H, E_H, I_H) < 0, \quad (17)$$

which can assure that (B.2) is satisfied. For the convenience of calculation, let

$$\begin{aligned} \bar{S}_D &= \frac{A_1}{d_1 + (1/2)\sigma_1^2}, \\ \bar{S}_L &= \frac{A_2}{d_2 + (1/2)\sigma_3^2}. \end{aligned} \quad (18)$$

$\bar{I}_D$ and $\bar{I}_L$ satisfy the following equations:

$$\beta_3 \bar{S}_L \bar{I}_D = \left(d_2 + \frac{1}{2}\sigma_4^2\right) \bar{I}_L. \quad (19)$$

Beyond that

$$\begin{aligned} \hat{S}_D &= \frac{S_D}{\bar{S}_D}, \\ \hat{I}_D &= \frac{I_D}{\bar{I}_D}, \\ \hat{S}_L &= \frac{S_L}{\bar{S}_L}, \\ \hat{I}_L &= \frac{I_L}{\bar{I}_L}. \end{aligned} \quad (20)$$

Considering a C^2 -function $R_+^7 \rightarrow R$

$$\Phi(S_D, I_D, S_L, I_L, S_H, E_H, I_H) = MV_1 + V_2 + V_3, \quad (21)$$

we assume that $\tilde{\Phi}$ is the minimum value of Φ . Then, we define a nonnegative C^2 -function V ,

$$V = \Phi - \tilde{\Phi}, \quad (22)$$

where

$$\begin{aligned} V_1 &= -\bar{I}_D \ln I_D - c_1 \bar{S}_D \ln S_D - c_2 \bar{S}_L \ln S_L - c_3 \bar{I}_L \ln I_L + cI_L, \\ V_2 &= -\ln S_D - \ln S_L - \ln I_L - \ln S_H - \ln E_H - \ln I_H, \\ V_3 &= \frac{1}{m+2} (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2}, \end{aligned} \quad (23)$$

and c, c_1, c_2, c_3 are positive constants to be determined later; $m > 0$ is a sufficiently small number such that

$$m < \frac{2(d_1 \wedge d_2 \wedge d_3)}{\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2 \vee \sigma_7^2} - 1. \quad (24)$$

M is a sufficiently large number such that

$$AM\bar{I}_D + E \leq -2, \quad (25)$$

where

$$\begin{aligned} E &= \sup_{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7} \left\{ \beta_1 I_L + (\beta_2 + \beta_3) I_D + Mc\beta_2 \varepsilon (S_L^{m+2} + 1) \right. \\ &\quad \left. - \frac{1}{2} \rho (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} + B + C \right\}, \end{aligned} \quad (26)$$

where ε is a sufficiently small number such that

$$Mc\beta_2 \varepsilon - \frac{1}{2} \rho \leq 0, \quad (27)$$

and B, C will be defined in (36) and (38), respectively, then

$$L(-\ln S_D) = -\frac{A_1}{\bar{S}_D} + \beta_1 I_L - \sigma \frac{I_D}{\bar{S}_D} + d_1 + \frac{1}{2} \sigma_1^2. \quad (28)$$

By applying the inequality $x - 1 \geq \ln x$ ($x > 0$), we obtain

$$\begin{aligned} L(-\bar{S}_D \ln S_D) &= -\frac{A_1}{\bar{S}_D} + \beta_1 \bar{S}_D I_L - \sigma \bar{S}_D \frac{I_D}{\bar{S}_D} + \bar{S}_D \left(d_1 + \frac{1}{2} \sigma_1^2\right) \\ &\leq -A_1 \left(\frac{1}{\bar{S}_D} - 1\right) + \beta_1 \bar{S}_D \leq A_1 \ln \hat{S}_D + \beta_1 \bar{S}_D I_L. \end{aligned} \quad (29)$$

From (25), we can obtain

$$\frac{\bar{I}_L}{\bar{I}_D} = \frac{\beta_2 \bar{S}_L}{d_2 + (1/2)\sigma_4^2}. \quad (30)$$

Then,

$$\begin{aligned} L(-\ln I_D) &= -\beta_1 S_D \frac{I_L}{\bar{I}_D} + \left(d_1 + \sigma + \frac{1}{2}\sigma_2^2\right) \\ &= -\beta_1 \frac{\bar{S}_D \bar{I}_L}{\bar{I}_D} \frac{\hat{S}_D \hat{I}_L}{\hat{I}_D} + \left(d_1 + \sigma + \frac{1}{2}\sigma_2^2\right) \\ &= -\beta_1 \frac{\bar{S}_D \bar{I}_L}{\bar{I}_D} \left(\frac{\hat{S}_D \hat{I}_L}{\hat{I}_D} - 1\right) - \beta_1 \frac{\bar{S}_D \bar{I}_L}{\bar{I}_D} + \left(d_1 + \sigma + \frac{1}{2}\sigma_2^2\right) \\ &\leq -\beta_1 \bar{S}_D \frac{\beta_2 \bar{S}_L}{d_2 + (1/2)\sigma_4^2} + \left(d_1 + \sigma + \frac{1}{2}\sigma_2^2\right) - \beta_1 \frac{\bar{S}_D \bar{I}_L}{\bar{I}_D} \ln \frac{\hat{S}_D \hat{I}_L}{\hat{I}_D} \\ &= -\left(d_1 + \sigma + \frac{1}{2}\sigma_2^2\right) \left(\frac{\beta_1 \bar{S}_D \beta_2 \bar{S}_L}{(d_1 + \sigma + (1/2)\sigma_2^2)(d_2 + (1/2)\sigma_4^2)} - 1\right) - \beta_1 \frac{\bar{S}_D \bar{I}_L}{\bar{I}_D} \ln \frac{\hat{S}_D \hat{I}_L}{\hat{I}_D} \\ &= -(R_0^s - 1) \left(d_1 + \sigma + \frac{1}{2}\sigma_2^2\right) - \beta_1 \frac{\bar{S}_D \bar{I}_L}{\bar{I}_D} \ln \frac{\hat{S}_D \hat{I}_L}{\hat{I}_D} \\ &:= -A - \beta_1 \frac{\bar{S}_D \bar{I}_L}{\bar{I}_D} \ln \frac{\hat{S}_D \hat{I}_L}{\hat{I}_D}, \end{aligned} \quad (31)$$

$$L(-\bar{I}_D \ln I_D) \leq -A \bar{I}_D - \beta_1 \bar{S}_D \bar{I}_L \ln \frac{\hat{S}_D \hat{I}_L}{\hat{I}_D},$$

$$L(-\bar{S}_L \ln S_L) = -\frac{A_2}{\bar{S}_L} + \beta_2 \bar{S}_L I_D + \bar{S}_L \left(d_2 + \frac{1}{2}\sigma_3^2\right) \leq A_2 \ln \hat{S}_L + \beta_2 \bar{S}_L I_D,$$

$$\begin{aligned} L(-\bar{I}_L \ln I_L) &= -\beta_2 \bar{S}_L \bar{I}_D \frac{\hat{S}_L \hat{I}_D}{\hat{I}_L} + \bar{I}_L \left(d_2 + \frac{1}{2}\sigma_4^2\right) = -\beta_2 \bar{S}_L \bar{I}_D \left(\frac{\hat{S}_L \hat{I}_D}{\hat{I}_L} - 1\right) \\ &\leq -\beta_2 \bar{S}_L \bar{I}_D \ln \frac{\hat{S}_L \hat{I}_D}{\hat{I}_L}. \end{aligned}$$

Then,

$$\begin{aligned} LV_1 &\leq -A \bar{I}_D + (c_1 \beta_1 \bar{S}_D - c d_2) I_L + c_2 \beta_2 \bar{S}_L I_D + c \beta_2 S_L I_D \\ &\quad + (c_1 A_1 - \beta_1 \bar{S}_D \bar{I}_L) \ln \hat{S}_D + (\beta_1 \bar{S}_D \bar{I}_L - c_3 \beta_2 \bar{S}_L \bar{I}_D) \ln \hat{I}_D \\ &\quad + (c_2 A_2 - c_3 \beta_2 \bar{S}_L \bar{I}_D) \ln \hat{S}_L + (-\beta_1 \bar{S}_D \bar{I}_L + c_3 \beta_2 \bar{S}_L \bar{I}_D) \ln \hat{I}_L. \end{aligned} \quad (32)$$

Let

$$\begin{cases} c_1 A_1 - \beta_1 \bar{S}_D \bar{I}_L = 0, \\ \beta_1 \bar{S}_D \bar{I}_L - c_3 \beta_2 \bar{S}_L \bar{I}_D = 0, \\ c_2 A_2 - c_3 \beta_2 \bar{S}_L \bar{I}_D = 0, \\ -\beta_1 \bar{S}_D \bar{I}_L + c_3 \beta_2 \bar{S}_L \bar{I}_D = 0, \end{cases} \quad (33)$$

then the unique solution of above equations is given as follows:

$$\begin{aligned} c_1 &= \frac{\beta_1 \bar{S}_D \bar{I}_L}{A_1}, \\ c_2 &= \frac{\beta_1 \bar{S}_D \bar{I}_L}{A_2}, \\ c_3 &= \frac{\beta_1 \bar{S}_D \bar{I}_L}{\beta_2 \bar{S}_L \bar{I}_D}. \end{aligned} \quad (34)$$

Choosing $c = (c_1 \beta_1 \bar{S}_D / d_2)$, then

$$L(MV_1) \leq -AM\bar{I}_D + Mc_2\beta_2\bar{S}_L I_D + Mc\beta_2 S_L I_D, \quad (35)$$

$$\begin{aligned} LV_2 &= L(-\ln S_D - \ln S_L - \ln I_L - \ln S_H - \ln E_H - \ln I_H) \\ &= -\frac{A_1}{S_D} + \beta_1 I_L - \sigma \frac{I_D}{S_D} + d_1 + \frac{1}{2}\sigma_1^2 - \frac{A_2}{S_L} + \beta_2 I_D + d_2 + \frac{1}{2}\sigma_3^2 - \beta_2 S_L \frac{I_D}{I_L} + \left(d_2 + \frac{1}{2}\sigma_4^2\right) \\ &\quad - \frac{A_3}{S_H} + \beta_3 I_D - \gamma \frac{I_H}{S_H} + d_3 + \frac{1}{2}\sigma_5^2 - \beta_3 S_H \frac{I_D}{E_H} + \left(d_3 + \omega + \frac{1}{2}\sigma_6^2\right) - \omega \frac{E_H}{I_H} + \left(d_3 + \mu + \gamma + \frac{1}{2}\sigma_7^2\right) \\ &\leq -\frac{A_1}{S_D} - \frac{A_2}{S_L} - \beta_2 S_L \frac{I_D}{I_L} - \frac{A_3}{S_H} - \beta_3 S_H \frac{I_D}{E_H} - \omega \frac{E_H}{I_H} + \beta_1 I_L + (\beta_2 + \beta_3)I_D + B, \end{aligned} \quad (36)$$

where

$$B = d_1 + 2 \sum_{i=2}^3 d_i + \frac{1}{2} \left(\sigma_1^2 + \sum_{i=3}^7 \sigma_i^2 \right) + \omega + \mu + \gamma, \quad (37)$$

$$\begin{aligned} LV_3 &= L\left(\frac{1}{m+2}(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2}\right) \\ &= (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+1} \\ &\quad [A_1 + A_2 + A_3 - d_1(S_D + I_D) - d_2(S_L + I_L) - d_3(S_H + E_H) - (d_3 + \mu)I_H] \\ &\quad + \frac{m+1}{2}(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^m (\sigma_1^2 S_D^2 + \sigma_2^2 I_D^2 + \sigma_3^2 S_L^2 + \sigma_4^2 I_L^2 + \sigma_5^2 S_H^2 + \sigma_6^2 E_H^2 + \sigma_7^2 I_H^2) \\ &\leq (A_1 + A_2 + A_3)(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+1} \\ &\quad - (d_1 \wedge d_2 \wedge d_3)(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \frac{m+1}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2 \vee \sigma_7^2)(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\leq C - \frac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2}, \end{aligned} \quad (38)$$

where

$$\begin{aligned} C &= \sup_{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7} \left\{ (A_1 + A_2 + A_3)(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+1} \right. \\ &\quad \left. - \frac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \right\}, \\ \rho &= (d_1 \wedge d_2 \wedge d_3) - \frac{m+1}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2 \vee \sigma_4^2 \vee \sigma_5^2 \vee \sigma_6^2 \vee \sigma_7^2) > 0. \end{aligned} \quad (39)$$

Then, combining with (32), (33), and (35), we get the value of LV finally:

$$\begin{aligned} LV &\leq -AM\bar{I}_D + c_2 M\beta_2 \bar{S}_L I_D + Mc\beta_2 S_L I_D - \frac{A_1}{S_D} - \frac{A_2}{S_L} - \beta_2 S_L \frac{I_D}{I_L} \\ &\quad - \frac{A_3}{S_H} - \beta_3 S_H \frac{I_D}{E_H} - \omega \frac{E_H}{I_H} - \frac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (\beta_2 + \beta_3)I_D + B + C. \end{aligned} \quad (40)$$

Then, defining a closed set

$$U_\varepsilon = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : \varepsilon \leq S_D \leq \frac{1}{\varepsilon}, \varepsilon \leq I_D \leq \frac{1}{\varepsilon}, \varepsilon \leq S_L \leq \frac{1}{\varepsilon}, \right. \\ \left. \varepsilon^3 \leq I_L \leq \frac{1}{\varepsilon^3}, \varepsilon \leq S_H \leq \frac{1}{\varepsilon}, \varepsilon^3 \leq E_H \leq \frac{1}{\varepsilon^3}, \varepsilon^4 \leq I_H \leq \frac{1}{\varepsilon^4} \right\}, \quad (41)$$

where ε is a sufficiently small positive number such that

$$-\frac{\theta}{\varepsilon} + D \leq -1, \quad (42)$$

$$-AM\bar{I}_D + Mc_2\beta_2\bar{S}_L\varepsilon + E \leq -1, \quad (43)$$

$$-\frac{\theta}{\varepsilon} + F \leq -1, \quad (44)$$

$$-\frac{1}{4}\rho\frac{1}{\varepsilon^{m+2}} + G \leq -1, \quad (45)$$

$$-\frac{1}{4}\rho\frac{1}{\varepsilon^{3(m+2)}} + G \leq -1, \quad (46)$$

$$-\frac{1}{4}\rho\frac{1}{\varepsilon^{4(m+2)}} + G \leq -1, \quad (47)$$

where $\theta = A_1 \wedge A_2 \wedge A_3 \wedge \beta_2 \wedge \beta_3 \wedge \omega$, D , E , F are positive constants which can be found from inequations. Divide $R_+^7 U_\varepsilon$ to fourteen domains:

$$U_1^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : 0 < S_D < \varepsilon \right\}, \\ U_2^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : 0 < I_D < \varepsilon \right\}, \\ U_3^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : 0 < S_L < \varepsilon \right\}, \\ U_4^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : 0 < I_L < \varepsilon^3, S_L > \varepsilon, I_D > \varepsilon \right\}, \\ U_5^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : 0 < S_H < \varepsilon \right\}, \\ U_6^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : 0 < E_H < \varepsilon^3, S_H > \varepsilon, I_D > \varepsilon \right\}, \\ U_7^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : 0 < I_H < \varepsilon^4, E_H > \varepsilon^3 \right\}, \\ U_8^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : S_D > \frac{1}{\varepsilon} \right\}, \\ U_9^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : I_D > \frac{1}{\varepsilon} \right\}, \\ U_{10}^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : S_L > \frac{1}{\varepsilon} \right\}, \\ U_{11}^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : I_L > \frac{1}{\varepsilon^3} \right\}, \\ U_{12}^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : S_H > \frac{1}{\varepsilon} \right\}, \\ U_{13}^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : E_H > \frac{1}{\varepsilon^3} \right\}, \\ U_{14}^c = \left\{ (S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7 : I_H > \frac{1}{\varepsilon^4} \right\}. \quad (48)$$

Obviously, $R_+^7 U_\varepsilon = \cup_{1 \leq i \leq 14} U_i^c$. Next, we will show that $LV \leq -1$ on $R_+^7 U_\varepsilon$, which is equivalent to verifying it on the above fourteen domains. $\square$

Case 1. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_1^c$, in view of (39) and (41), thus

$$\begin{aligned} LV &\leq -\frac{A_1}{S_D} - \frac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\frac{A_1}{\varepsilon} + D \leq -\frac{\theta}{\varepsilon} + D \leq -1, \end{aligned} \quad (49)$$

where

$$\begin{aligned} D = \sup_{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7} &\left\{ -\frac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \right. \\ &\left. + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \right\}. \end{aligned} \quad (50)$$

Case 2. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_2^c$, in view of (24), (39), and (42), thus

$$\begin{aligned} LV &\leq -AM\bar{I}_D + Mc_2\beta_2\bar{S}_L I_D + \beta_1 I_L + (\beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D \\ &\quad - \frac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} + B + C \\ &\leq -AM\bar{I}_D + Mc_2\beta_2\bar{S}_L \varepsilon + E \leq -1. \end{aligned} \quad (51)$$

Case 3. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_3^c$, in view of (39) and (41), thus

$$\begin{aligned} LV &\leq -\frac{A_2}{S_L} - \frac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \leq -\frac{A_2}{\varepsilon} + D \leq -\frac{\theta}{\varepsilon} + F \leq -1, \end{aligned} \quad (52)$$

where

$$\begin{aligned} F = \sup_{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7} &\left\{ \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 \varepsilon (I_D^{m+2} + 1) \right. \\ &\left. - \frac{1}{2}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} + B + C \right\}. \end{aligned} \quad (53)$$

Case 4. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_4^c$, in view of (39) and (41), thus

$$\begin{aligned} LV &\leq -\beta_2 S_L \frac{I_D}{I_L} - \frac{1}{2} \rho (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2 \beta_2 \bar{S}_L + \beta_2 + \beta_3) I_D + Mc \beta_2 S_L I_D + B + C \\ &\leq -\frac{\beta_2}{\varepsilon} + D \leq -\frac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \quad (54)$$

Case 5. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_5^c$, in view of (39) and (41), thus

$$\begin{aligned} LV &\leq -\frac{A_3}{S_H} - \frac{1}{2} \rho (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2 \beta_2 \bar{S}_L + \beta_2 + \beta_3) I_D + Mc \beta_2 S_L I_D + B + C \\ &\leq -\frac{A_3}{\varepsilon} + D \leq -\frac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \quad (55)$$

Case 6. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_6^c$, in view of (39) and (41), thus

$$\begin{aligned} LV &\leq -\beta_3 S_H \frac{I_D}{E_H} - \frac{1}{2} \rho (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2 \beta_2 \bar{S}_L + \beta_2 + \beta_3) I_D + Mc \beta_2 S_L I_D + B + C \\ &\leq -\frac{\beta_3}{\varepsilon} + D \leq -\frac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \quad (56)$$

Case 7. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_7^c$, in view of (39) and (41), thus

$$\begin{aligned} LV &\leq -\omega \frac{E_H}{I_H} - \frac{1}{2} \rho (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2 \beta_2 \bar{S}_L + \beta_2 + \beta_3) I_D + Mc \beta_2 S_L I_D + B + C \leq -\frac{\omega}{\varepsilon} + D \leq -\frac{\theta}{\varepsilon} + D \leq -1. \end{aligned} \quad (57)$$

Case 8. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_8^c$, in view of (39) and (43), thus

$$\begin{aligned} LV &\leq -\frac{1}{4} \rho (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad - \frac{1}{4} \rho (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2 \beta_2 \bar{S}_L + \beta_2 + \beta_3) I_D + Mc \beta_2 S_L I_D + B + C \\ &\leq -\frac{1}{4} \rho (S_D)^{m+2} + G \leq -\frac{1}{4} \rho \frac{1}{\varepsilon^{m+2}} + G \leq -1, \end{aligned} \quad (58)$$

where

$$\begin{aligned} G &= \sup_{(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in R_+^7} \left\{ -\frac{1}{4} \rho (S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \right. \\ &\quad \left. + \beta_1 I_L + (Mc_2 \beta_2 \bar{S}_L + \beta_2 + \beta_3) I_D + Mc \beta_2 S_L I_D + B + C \right\}. \end{aligned} \quad (59)$$

Case 9. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_9^c$, in view of (39) and (43), thus

$$\begin{aligned} LV &\leq -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \leq -\frac{1}{4}\rho(I_D)^{m+2} + G \leq -\frac{1}{4}\rho\frac{1}{\epsilon^{m+2}} + G \leq -1. \end{aligned} \quad (60)$$

Case 10. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{10}^c$, in view of (39) and (43), thus

$$\begin{aligned} LV &\leq -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} - \frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\frac{1}{4}\rho(S_L)^{m+2} + G \leq -\frac{1}{4}\rho\frac{1}{\epsilon^{m+2}} + G \leq -1. \end{aligned} \quad (61)$$

Case 11. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{11}^c$, in view of (39) and (44), thus

$$\begin{aligned} LV &\leq -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\frac{1}{4}\rho(I_L)^{m+2} + G \leq -\frac{1}{4}\rho\frac{1}{\epsilon^{3(m+2)}} + G \leq -1. \end{aligned} \quad (62)$$

Case 12. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{12}^c$, in view of (39) and (43), thus

$$\begin{aligned} LV &\leq -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\frac{1}{4}\rho(S_H)^{m+2} + G \leq -\frac{1}{4}\rho\frac{1}{\epsilon^{m+2}} + G \leq -1. \end{aligned} \quad (63)$$

Case 13. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{13}^c$, in view of (39) and (44), thus

$$\begin{aligned} LV &\leq -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\frac{1}{4}\rho(E_H)^{m+2} + G \leq -\frac{1}{4}\rho\frac{1}{\epsilon^{3(m+2)}} + G \leq -1. \end{aligned} \quad (64)$$

Case 14. For any $(S_D, I_D, S_L, I_L, S_H, E_H, I_H) \in U_{14}^c$, in view of (39) and (45), thus

$$\begin{aligned} LV &\leq -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad -\frac{1}{4}\rho(S_D + I_D + S_L + I_L + S_H + E_H + I_H)^{m+2} \\ &\quad + \beta_1 I_L + (Mc_2\beta_2\bar{S}_L + \beta_2 + \beta_3)I_D + Mc\beta_2 S_L I_D + B + C \\ &\leq -\frac{1}{4}\rho(I_H)^{m+2} + G \leq -\frac{1}{4}\rho\frac{1}{\epsilon^{4(m+2)}} + G \leq -1. \end{aligned} \quad (65)$$

Through the discussion of the above ten cases, we can satisfy condition (B.2) of Lemma 1. We complete the proof of Theorem 2.

4. Extinction

Define a parameter

$$\hat{R}_0 = \frac{\beta_1 S_D^0 + \beta_2 \int_0^\infty x \pi(x) dx + \beta_3 S_H^0 + \omega}{(1/8)(\sigma_2^2 \wedge \sigma_4^2 \wedge \sigma_6^2 \wedge \sigma_7^2)}, \quad (66)$$

where

$$\pi(x) = Q x^{-2-(2d_2/\sigma_3^2)} e^{-(2/\sigma_3^2)(A_2/x)}, \quad x \in (0, \infty). \quad (67)$$

Theorem 3. Let $(S_D, I_D, S_L, I_L, S_H, E_H, I_H)$ be the solution of system (2) with any initial value

$$(S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), I_H(0)) \in R_+^7. \quad (68)$$

If $\hat{R}_0 < 1$, then the solution $(S_D, I_D, S_L, I_L, S_H, E_H, I_H)$ of system (2) satisfies

$$\limsup_{t \rightarrow \infty} \frac{\ln(I_D + I_L + E_H + I_H)}{t} \leq \beta_1 S_D^0 + \beta_2 \int_0^\infty x \pi(x) dx + \beta_3 S_H^0 + \omega - \frac{1}{8}(\sigma_2^2 \wedge \sigma_4^2 \wedge \sigma_6^2 \wedge \sigma_7^2) < 0 \text{ a.s.} \quad (69)$$

Namely,

$$\limsup_{t \rightarrow \infty} I_D = \limsup_{t \rightarrow \infty} I_L = \limsup_{t \rightarrow \infty} E_H = \limsup_{t \rightarrow \infty} I_H = 0. \quad (70)$$

Also, the distribution of $S_L(t)$ converges weakly to the measure which has the density

$$\pi(x) = Q x^{-2-(2d_2/\sigma_3^2)} e^{-(2/\sigma_3^2)(A_2/x)}, \quad x \in (0, \infty), \quad (71)$$

where Q is a constant such that $\int_0^\infty \pi(x) dx = 1$.

Proof. Consider the following auxiliary logistic equation with random perturbation

$$dX(t) = (A_2 - d_2 X)dt + \sigma_3 X dB_3(t), \quad (72)$$

with the initial value $X(0) = S_L(0) > 0$. Setting

$$\int_0^\infty \frac{1}{\sigma^2(x)} e^{\int_1^x (2b(\tau)/\sigma^2(\tau)) d\tau} = C \int_0^\infty x^{-2-(2d_2/\sigma_3^2)} e^{-(2/\sigma_3^2)(A_2/x)} dx < \infty. \quad (76)$$

Consequently, the condition of Theorem 1.16 in [14] follows clearly from above. Thus, system (72) has the ergodic property, and the invariant density is given by

$$\pi(x) = Q x^{-2-(2d_2/\sigma_3^2)} e^{-(2/\sigma_3^2)(A_2/x)} dx, \quad x \in (0, \infty), \quad (77)$$

where Q is a constant such that $\int_0^\infty \pi(x) dx = 1$. From the ergodic theorem, it follows that

$$\begin{aligned} b(x) &= A_2 - d_2 x, \\ \sigma(x) &= \sigma_3 x, \quad x \in (0, \infty), \end{aligned} \quad (73)$$

we compute that

$$\int \frac{b(u)}{\sigma^2(u)} du = \frac{1}{\sigma_3^2} \int \left(\frac{A_2}{u^2} - \frac{d_2}{u} \right) du = \frac{1}{\sigma_3^2} \left(-\frac{A_2}{x} - d_2 \ln x \right) + Q. \quad (74)$$

Therefore,

$$\int (b(u)/\sigma^2(u)) du = e^Q x^{-(d_2/\sigma_3^2)} e^{-(1/\sigma_3^2)(A_2/x)}. \quad (75)$$

Clearly, we have

$$\lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t x(s) ds = \int_0^\infty x \pi(x) dx \text{ a.s.} \quad (78)$$

Let $X(t)$ be the solution of SDE (72) with the initial value $X(0) = S_L(0) > 0$, then we can see that

$$S_L(t) \leq X(t) \text{ a.s.} \quad (79)$$

On the other hand, integrating from 0 to t and then dividing by t on both sides of (2) lead to

TABLE 3: List of parameters.

Parameter	Value	Parameter	Value
A_1	0.9	A_2	1.0
A_3	0.7	d_1	0.02
d_2	0.01	d_3	0.05
β_1	0.08	β_2	0.02
β_3	0.08	σ	0.6
μ	0.03	ω	0.3
γ	0.8		

$$\left\{ \begin{array}{l} \frac{S_D(t) - S_D(0)}{t} = A_1 - \beta_1 \langle S_D I_L \rangle - d_1 \langle S_D \rangle + \sigma \langle I_D \rangle + \frac{\sigma_1}{t} \int_0^t S_r dB_1(r), \\ \frac{I_D(t) - I_D(0)}{t} = \beta_1 \langle S_D I_L \rangle - (d_1 + \sigma) \langle I_D \rangle + \frac{\sigma_2}{t} \int_0^t I_D(r) dB_2(r), \\ \frac{S_H(t) - S_H(0)}{t} = A_3 - \beta_3 \langle S_H I_D \rangle - d_3 \langle S_H \rangle + \gamma \langle I_H \rangle + \frac{\sigma_5}{t} \int_0^t S_H(r) dB_5(r), \\ \frac{E_H(t) - E_H(0)}{t} = \beta_3 \langle S_H I_D \rangle - (d_3 + \omega) \langle E_H \rangle + \frac{\sigma_6}{t} \int_0^t E_H(r) dB_6(r), \\ \frac{I_H(t) - I_H(0)}{t} = \omega \langle E_H \rangle - (d_3 + \mu + \gamma) \langle I_H \rangle + \frac{\sigma_7}{t} \int_0^t I_H(r) dB_7(r). \end{array} \right. \quad (80)$$

We can obtain that

$$\frac{S_D(t) - S_D(0)}{t} + \frac{I_D(t) - I_D(0)}{t} = A_1 - d_1 \langle S_D \rangle_t - d_1 \langle I_D \rangle_t, \quad (81)$$

then

$$\langle S_D \rangle_t = \frac{A_1 - d_1 \langle I_D \rangle_t}{d_1} + \varphi_1(t) = \frac{A_1}{d_1} - \langle I_D \rangle_t + \varphi_1(t) = S_D^0 - \langle I_D \rangle_t + \varphi_1(t), \quad (82)$$

where

$$\varphi_1(t) = -\frac{1}{d_1} \left(\frac{S_D(t) - S_D(0)}{t} + \frac{I_D(t) - I_D(0)}{t} \right), \quad (83)$$

$$\frac{S_H(t) - S_H(0)}{t} + \frac{E_H(t) - E_H(0)}{t} + \frac{I_H(t) - I_H(0)}{t} = A_3 - d_3 \langle S_H \rangle_t - d_3 \langle E_H \rangle_t - (d_3 + \mu) \langle I_H \rangle_t,$$

then

$$\varphi_2(t) = -\frac{1}{d_3} \left(\frac{S_H(t) - S_H(0)}{t} + \frac{E_H(t) - E_H(0)}{t} + \frac{I_H(t) - I_H(0)}{t} \right). \quad (85)$$

$$\langle S_H \rangle_t = S_H^0 - \langle E_H \rangle_t - \frac{d_3 + \mu}{d_3} + \varphi_2(t), \quad (84)$$

From [9], we can obtain that

where

$$\lim_{t \rightarrow \infty} \varphi_i(t) = 0, \quad \text{a.s. } i = 1, 2. \quad (86)$$

FIGURE 1: The red lines describe the solution of system (2); the green lines stand for the solution of the corresponding system (1). The right pictures are the histogram of the density function for S_D , I_D , and S_L populations.

Let $P(t) = I_D + I_L + E_H + I_H$. Applying Itô's formula, we can obtain that

$$\begin{aligned}
 d \ln P(t) = & \left\{ \frac{1}{P} [\beta_1 S_D I_L - (d_1 + \sigma) I_D + \beta_2 S_L I_D - d_2 I_L] \right. \\
 & + \beta_3 S_H I_D - (d_3 + \omega) E_H + \omega E_H - (d_3 + \mu + \gamma) I_H \Big] \\
 & - \frac{1}{2P^2} (\sigma_2^2 I_D^2 + \sigma_4^2 I_L^2 + \sigma_6^2 E_H^2 + \sigma_7^2 I_H^2) \Big\} dt \\
 & + \frac{1}{P} [\sigma_2 I_D dB_2(t) + \sigma_4 I_L dB_4(t) + \sigma_6 E_H dB_6(t) + \sigma_7 I_H dB_7(t)] \\
 \leq & \left\{ \beta_1 S_D + \beta_2 S_L + \beta_3 S_H + \omega - \frac{1}{8} (\sigma_2^2 \wedge \sigma_4^2 \wedge \sigma_6^2 \wedge \sigma_7^2) \right\} dt \\
 & + \sigma_2 dB_2(t) + \sigma_4 dB_4(t) + \sigma_6 dB_6(t) + \sigma_7 dB_7(t).
 \end{aligned} \tag{87}$$

Integrating (87) from 0 to t and then dividing by t on both sides, one can see that

$$\begin{aligned}
 \frac{\ln P(t)}{t} - \frac{\ln P(0)}{t} \leq & \beta_1 \langle S_D \rangle_t + \frac{\beta_2}{t} \int_0^t S_L(s) ds + \beta_3 \langle S_H \rangle_t \\
 & + \omega - \frac{1}{8} (\sigma_2^2 \wedge \sigma_4^2 \wedge \sigma_6^2 \wedge \sigma_7^2) + \frac{\sigma_2}{t} B_2(t) + \frac{\sigma_4}{t} B_4(t) + \frac{\sigma_6}{t} B_6(t) + \frac{\sigma_7}{t} B_7(t).
 \end{aligned} \tag{88}$$

As application of strong law of large numbers [13], one has

$$\lim_{t \rightarrow \infty} \frac{B_i(t)}{t} = 0, \quad \text{a.s. } i = 2, 4, 6, 7. \tag{89}$$

FIGURE 2: The red lines describe the solution of system (2); the green lines stand for the solution of the corresponding system (1). The right pictures are the histogram of the probability density function for I_L, S_H populations.

FIGURE 3: The red lines describe the solution of system (2); the green lines stand for the solution of the corresponding system (1). The right pictures are the histogram of the probability density function for E_H, I_H populations.

Taking the superior limit on both sides of (88) and note that $\tilde{R}_0 < 1$ leads to

$$\limsup_{t \rightarrow \infty} \frac{\ln P(t)}{t} \leq \beta_1 S_D^0 + \beta_2 \int_0^\infty x \pi(x) dx + \beta_3 S_H^0 + \omega - \frac{1}{8} (\sigma_2^2 \wedge \sigma_4^2 \wedge \sigma_6^2 \wedge \sigma_7^2) < 0 \text{ a.s.}, \quad (90)$$

which implies that

$$\limsup_{t \rightarrow \infty} I_D = \limsup_{t \rightarrow \infty} I_L = \limsup_{t \rightarrow \infty} E_H = \limsup_{t \rightarrow \infty} I_H = 0 \text{ a.s.} \quad (91)$$

Thus, for any small $\varepsilon > 0$, there exist t_0 and a set $\Omega_\varepsilon \subset \Omega$ such that $\mathbf{P}(\Omega_\varepsilon) > 1 - \varepsilon$ and $\beta_2 S_L I_D \leq \varepsilon S_L$ for $t \geq t_0$ and $\omega \in \Omega_\varepsilon$. Now, from

TABLE 4: List of parameters.

Parameter	Value	Parameter	Value
A_1	0.5	A_2	0.6
A_3	0.5	d_1	0.1
d_2	0.08	d_3	0.05
β_1	0.02	β_2	0.02
β_3	0.02	σ	0.8
μ	0.03	ω	0.1
γ	0.8		

$$[A_2 - \beta_2 S_L I_D - d_2 S_L]dt + \sigma_3 S_L dB_3(t) \leq dS_L(t) \leq [A_2 - d_2 S_L]dt + \sigma_3 S_L dB_3(t), \quad (92)$$

we can obtain that the process $S_L(t)$ converges to the measure π . We finish the proof. $\square$

5. Numerical Simulation

By using Milstein's higher order method in [12], some numerical simulations are used to illustrate our results. We choose the initial values

$$(S_D(0), I_D(0), S_L(0), I_L(0), S_H(0), E_H(0), I_H(0)) = (0.9, 0.5, 1.2, 0.6, 1.2, 0.3, 0.3). \quad (93)$$

The corresponding discretizing equations of model (2) are as follows:

$$\left\{ \begin{aligned} S_D(k+1) &= [A_1 - \beta_1 S_D(k)I_L(k) - d_1 S_D(k) + \sigma I_D(k)]\Delta t + \sigma_1 S_D(k)\sqrt{\Delta t}\xi_{1k} + \frac{\sigma_1^2 S_D(k)}{2}\Delta t(\xi_{1k}^2 - 1), \\ I_D(k+1) &= [\beta_1 S_D(k)I_L(k) - (d_1 + \sigma)I_D(k)]\Delta t + \sigma_2 I_D(k)\sqrt{\Delta t}\xi_{2k} + \frac{\sigma_2^2 I_D(k)}{2}\Delta t(\xi_{2k}^2 - 1), \\ S_L(k+1) &= [A_2 - \beta_2 S_L(k)I_D(k) - d_2 S_L(k)]\Delta t + \sigma_3 S_L(k)\sqrt{\Delta t}\xi_{3k} + \frac{\sigma_3^2 S_L(k)}{2}\Delta t(\xi_{3k}^2 - 1), \\ I_L(k+1) &= [\beta_2 S_L(k)I_D(k) - d_2 I_L(k)]\Delta t + \sigma_4 I_L(k)\sqrt{\Delta t}\xi_{4k} + \frac{\sigma_4^2 I_L(k)}{2}\Delta t(\xi_{4k}^2 - 1), \\ S_H(k+1) &= [A_3 - \beta_3 S_H(k)I_D(k) - d_3 S_H(k) + \gamma I_H(k)]\Delta t + \sigma_5 S_H(k)\sqrt{\Delta t}\xi_{5k} + \frac{\sigma_5^2 S_H(k)}{2}\Delta t(\xi_{5k}^2 - 1), \\ E_H(k+1) &= [\beta_3 S_H(k)I_D(k) - (d_3 + \omega)E_H(k)]\Delta t + \sigma_6 E_H(k)\sqrt{\Delta t}\xi_{6k} + \frac{\sigma_6^2 E_H(k)}{2}\Delta t(\xi_{6k}^2 - 1), \\ I_H(k+1) &= [\omega E_H(k) - (d_3 + \mu + \gamma)I_H(k)]\Delta t + \sigma_7 I_H(k)\sqrt{\Delta t}\xi_{7k} + \frac{\sigma_7^2 I_H(k)}{2}\Delta t(\xi_{7k}^2 - 1), \end{aligned} \right. \quad (94)$$

where the time increment Δt is positive and ξ_{ik} are the Gaussian random variables which follow the distribution $N(0, 1)$, $i = 1, 2, 3, 4, 5, 6, 7$.

Example 1. Firstly, we choose the values of system parameters to test the existence of ergodic stationary distribution of the system (94) $\sigma_1^2 = 10^{-4}$, $\sigma_2^2 = 3.6 \times 10^{-3}$,

FIGURE 4: The red line, blue line, and magenta line represent S_D , S_L , and S_H , respectively; the green line, cyan line, yellow line, and black line represent I_D , I_L , E_H , and I_H , respectively.

$\sigma_3^2 = 10^{-2}$, $\sigma_4^2 = 4 \times 10^{-4}$, $\sigma_5^2 = 10^{-4}$, $\sigma_6^2 = 4 \times 10^{-4}$, $\sigma_7^2 = 2.5 \times 10^{-3}$. For other values of the system parameters, see Table 3. Direct calculation leads to $R_0^s = 754.928752 > 1$, where R_0^s is defined before Theorem 2. In other words, the conditions of Theorem 2 hold. In view of Theorem 2, there is an ergodic stationary distribution $\mu(\cdot)$ of system (2). Figures 1–3 illustrate this.

Example 2. In order to obtain the extinction of the infection, in (2), we choose the values of the system parameters as follows: $\sigma_1^2 = 10^{-2}$, $\sigma_2^2 = 9 \times 10^{-2}$, $\sigma_3^2 = 10^{-2}$, $\sigma_4^2 = 1.6 \times 10^{-1}$, $\sigma_5^2 = 4 \times 10^{-4}$, $\sigma_6^2 = 9 \times 10^{-2}$; $\sigma_7^2 = 1.6 \times 10^{-1}$. For other values of the system parameters, see Table 4. By Theorem 3, (I_D, I_L, E_H, I_H) will tend to be zero exponentially with probability one. We give the simulations to support our results in Figure 4.

Data Availability

All data in this article are accessible.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors carried out the proof and conceived the study. All authors read and approved the final manuscript.

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