Research Article

Reranufacturing Mode Selection regarding Technology Development

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We consider a Cournot game model between an OEM (original equipment manufacturer) and an IR (independent remanufacturer). The OEM manufactures new products and decides the quality level. IR remanufactures and obtains OEM’s technology through technology licensing or joint R&D. To prevent the cannibalization of new products by remanufactured products, the OEM may be reluctant to disclose latest technology to the IR. When the IR chooses the technology licensing mechanism, it will be in a rather disadvantaged position in the competition. In contrast, joint R&D can avoid this dilemma. The two mechanisms are comparatively analyzed under static equilibrium and complex dynamics from three aspects: (1) the output of new and remanufactured products, (2) the profits of the OEM and the IR, and (3) TEI (total environmental impact) under technology licensing mechanism and joint R&D mechanism, respectively. Based on the theoretical and numerical analysis, we derive that the joint R&D mechanism can achieve a Pareto improvement over the royalty mechanism under certain conditions. The stability, bifurcation, chaos, and largest Lyapunov exponent are analyzed in the dynamic model. Numerical examples show that chaos may cause the OEM and the IR to lose profits or even be in deficit. But from the perspective of TEI, chaos can be beneficial. Interestingly, some conclusions in the static setting are reversed in the chaotic state. We propose a feedback adjustment method to eliminate chaos.

1. Introduction

The United States Environmental Protection Agency (EPA) repeatedly promotes waste reduction and resource conservation through reusing, recycling, and remanufacturing end-of-life products. Remanufacturing is not only a requirement of environmental policies and regulations, but also a source of huge economic benefits. In the UK, a market potential of up to £5.6 billion has been identified in remanufacturing. Many companies, such as Caterpillar, Phillips, and Desso, have engaged in remanufacturing and have increased profits by creating more “circular” value [1]. Moreover, remanufactured products meet the needs of budget-constrained consumers who desire low-priced and good-quality auto parts, consumer electronics, production machinery, etc. However, the rapid growth of remanufactured products cannibalizes the market of new products.

Remanufacturing can be operated by either original equipment manufacturers (OEMs) or third-party independent remanufacturers [2]. Some OEMs are experts in manufacturing cannot operate remanufacturing profitable. For example, Ford had to abandon automotive remanufacturing because of inexperience [3]. The majority of remanufacturing activities in the United States is carried out by IRs [4]. The issue of the remanufacturing modes has aroused discussion in academia. Stamatopoulos and Tauman [5] investigated the licensing mechanisms of a quality-improving innovation in a Bertrand duopoly model: upfront fee (determined in an auction), pre-unit royalty, or their combination. Li and Ji [6] developed a differentiated duopoly model with endogenous cost-reducing R&D in the presence of technology licensing and compared the effects of a two-part tariff licensing mechanism (including a fixed fee and pre-unit royalty) between Cournot competition and Bertrand competition. Chang et al. [7] established a three-stage (R&D, technology licensing, and output) oligopoly game with one of multiple homogeneous firms undertaking a cost-reducing R&D. The technology can be licensed by a
two-part tariff contract and then encourages R&D investment and benefits social welfare. Zou et al. [8] compared the two remanufacturing modes of outsourcing and authorization from the perspective of OEM and suggested that OEM always prefers authorization. Hong et al. [9] investigated two licensing patterns, fixed fee versus royalty, from the perspective of manufacturer and showed that optimal licensing strategy is determined by a threshold of the fixed fee. Wu [10] considered two endogenous mechanisms: technology licensing and R&D joint venture from the perspective of IR. Rau et al. [11] proposed a market competition game model between IR’s supply chain systems considering technology licensing and product quality strategies. Numerical examples show that royalty licensing is the better technology licensing strategy compared to fixed-fee licensing strategy in terms of costs.

Faced with the entry threat of the IR, the OEM will use technology as a competitive tool. Orsdeemir et al. [2] found that the OEM will choose a higher quality level to weaken the IR and emphasize its advantage. For example, HP servers require special software to run, and HP’s high relicensing fees for this software limits the influence of third-party remanufacturers. To prevent competitive threats from IR, OEM will take some measures to widen the quality gap between the remanufactured products and the new products, for example, hiding latest technology information to IR. When encountering a green production disruption, Li and He [12] investigated the value of exposing quantity information. In order to meet the growing quality demands of consumers and achieve business goals, manufacturers must make continuous quality improvement. Ma and Ren [13] established a recovery master-slave game model based on customer utility expectations which the remanufacturer recycled through online and offline recyclers. Wu [10] proposed that the OEM’s choice of quality level is essential to the subsequent quantity competition between the OEM and the IR. Li et al. [14] established a stylized model that endogenize the product quality improvement decision in a remanufacturing setting to study the interaction between remanufacturing and product quality improvement. The conditions under which remanufacturing and product quality improvement are mutually beneficial or exclusive were derived. Taleizadeh et al. [15] introduced two collecting-remanufacturing scenarios to analyze the interactions among carbon reduction, quality improvement, and SC performance. Zhang et al. [16] dealt with R&D investment and technology licensing in a supply chain formed of an OEM and a contract manufacturer.

The research on the optimal technology licensing model has made some progress; Yang et al. [17] examined how the supply risk affects the supplier’s technology licensing willingness. Zhao et al. [18] studied the optimal technology licensing contract with network effects based on the Stackelberg framework. Ghosh and Saha [19] considered the optimal pricing strategy and technology licensing problem with two different countries. Yan and Yang [20] studied the licensing behavior in a Bertrand duopoly market. Takashima [21] proposed a novel method of cooperative R&D investment for reduction in greenhouse gas emission. Non-innovative companies lack innovative technology but can obtain innovation through licensing. Wu [10] found that the licensing is effective in mitigating the intensity of price competition and is beneficial for the innovative firm but not always for the non-innovative firm. It can also combine information sharing and technology licensing to conduct research on closed-loop supply chains [22–24]. Li et al. [25] studied the “gray market” by game model and combined with the technology licensing, Sim and Hong [26] analyzed the welfare implications of abatement technology licensing under taxation and emission trading schemes.

Some scholars further extended models to a dynamic setting to examine the effects of quantity adjustment on the equilibrium results [27, 28]. De Giovanni et al. [29] set up a dynamic closed-loop supply chain comprising a manufacturer and a retailer, with both players investing in a product recovery program to increase the rate of return of previously purchased products. Ma et al. [30] proposed a price game model with heterogeneous expectations and analyzed with the methods of stability domain, bifurcation diagram, and maximum Lyapunov exponent. Wu [10] extended two cooperative models with different licensing schemes to a dynamic setting to examine the effects of period and planning horizons on the equilibrium results. Peng et al. [31] formulated a Cournot duopoly remanufacturing game involving an OEM and an IR and analyzed existence, stability, and local bifurcations of the equilibrium points. Numerical simulations demonstrated that the system with varying parameters may evolve into chaos. Zhan et al. [32] built a duopoly game model of two competitive manufacturers with one manufacturer recycling used products under carbon tax policy. After solving the single-phase equilibrium solution, the model was extended to a multi-period dynamic setting. The complex characteristics, such as bifurcation, chaos, and sensitivity, are analyzed through 3D stable state diagrams, parameter basin diagrams, and bifurcation diagrams. A two-parallel model consisting of two supply chains with and without any carbon emission reduction effort was established by Lou and Ma [33]. Xie et al. [34] established the production and demand model of supply chain coordination with uncertainty, and the research results are of guiding value to the industry. In addition, the literature [28, 35–37] also discussed the complex dynamic system theory when studying supply chain problems.

This study examines OEM’s quality improvement information disclosure strategy and IR’s technology licensing schemes choice in a duopoly closed-loop supply chain. In particular, we aim to answer the following questions:

1. When granting a technology license to an IR by charging a royalty, should the OEM disclose the latest technology to the IR?
(2) When the OEM hides the latest quality improvement information, how should the IR deal with this problem?

(3) Compare the pros and cons of the licensing mechanism and the joint R&D mechanism.

This paper is structured as follows. Section 2 describes the problem. Section 3 compares OEM’s ex ante and ex post quality improvement information disclosure strategies and derives that OEM has motivation to hide information. In Section 4, the OEM and the IR competitively determine their production outputs under the joint R&D licensing scheme and their equilibrium behaviors are investigated. Section 5 first compares two cooperation schemes in a static setting and then extends to a dynamic setting for simulation. Section 6 makes a brief summary and suggestions for future research.

2. Problem Description and Notations

We consider a duopoly Cournot game model between an OEM and an IR. The OEM determines the product quality at level \( \theta_n \) (\( \geq 1 \)) and then sells new products. The IR collects used products and remanufactures them. Without R&D investment, the quality of products in the next period stays unchanged (i.e., \( \theta_n = 1 \)). The cost of technology improvement is \( \beta(\theta_n - 1)^2 \), where \( \beta \) is a scale parameter. The IR can get patented technology license from the OEM by paying royalty or investing in R&D. Consumers consider quality of remanufactured products to be inferior to new products and use \( \theta_r \) as a quality discount factor. When licensing to the IR by royalty, the OEM can choose to disclose the latest technology ex ante or ex post. The decision sequence in both cases is shown in Figure 1.

Consumers buy products when they have a non-negative utility and choose products that offer maximum utility. The utility of consumers buying new and remanufactured products is \( u_n = \theta_n u - p_n \) and \( u_r = \theta_r u - p_r \), respectively [2, 38]. Then, according to the non-negative condition of consumer’s utility, the inverse demand function of new and remanufactured products can be obtained as \( p_n = \theta_n (1 - q_n - \theta_n q_r) \) and \( p_r = \theta_r \theta_n (1 - q_n - q_r) \). Remanufacturing is possible only if new products have been used and become cores for remanufacturing. We assume that the product can be remanufactured at most once. The IR’s remanufacturing quantity is constrained by the available cores, which is determined by the new product quantity. Thus, we have the constraint \( q_n > q_r > 0 \).

Remanufactured products consume less virgin resources by reusing some parts. We use \( a \) to describe the level of resource savings and cost advantages of the remanufactured products. The total resource (virgin material) consumption is measured as a proxy for environmental impact and given by

\[
\text{TEI} = q_n + aq_r. \tag{1}
\]

The variables and parameters to be used in this paper are listed in Table 1.

3. Technology Licensing with a Royalty

In this section, we mainly discuss the mechanism that IR obtains technology license by paying a royalty. We analyze the single-period equilibrium of the OEM under two strategies, respectively: ex ante and ex post disclose quality improvement. Then, we comprehensively compare and evaluate the two strategies. We use the superscripts \( \lambda \) and \( P \) to denote ex ante and ex post mechanism and use subscripts \( n \) and \( r \) to denote the new products and the remanufactured products.

3.1. Ex Ante Quality Improvement Disclosure. We first study the situation where the OEM has improved the quality of new products via research and development (R&D) and disclose corresponding information. IR pays royalty to OEM and gets the latest technology licenses, and then applies it to remanufacturing.

The inverse demand functions of the OEM and IR are as follows:

\[
\begin{align*}
p_n &= \theta_n (1 - q_n - \theta_n q_r), \\
p_r &= \theta_r \theta_n (1 - q_n - q_r). \tag{2}
\end{align*}
\]

The profit functions of the OEM and the IR are as follows:

\[
\begin{align*}
\pi_{\text{OEM}} &= (p_n - c_n) q_n - \beta(\theta_n - 1)^2 + r q_r, \tag{3} \\
\pi_{\text{IR}} &= (p_r - c_r - r) q_r. \tag{4}
\end{align*}
\]

\( (\partial^2 \pi_{\text{OEM}} / \partial q_n^2) = -2\theta_n < 0 \) and \( (\partial^2 \pi_{\text{IR}} / \partial q_r^2) = -2\theta_r < 0 \) indicate that the profits of OEM and IR are strictly concave with respect to \( q_n \) and \( q_r \), respectively.

According to the sequence depicted in Figure 1, we solve the game using backward induction and derive the following results.

Lemma 1. When the OEM ex ante discloses the technology improvement information to the IR, the OEM’s and the IR’s optimal decisions are obtained as follows.

(1) The optimal outputs of new and remanufactured products are

\[
\begin{align*}
q_n^\lambda &= \frac{r + ac_n + \theta_n (2 - \theta_r) - 2c_n}{\theta_n (4 - \theta_r)}, \\
q_r^\lambda &= \frac{(c_n + \theta_n) \theta_r - 2 (r + ac_n)}{\theta_n (4 - \theta_r) \theta_r}. \tag{5}
\end{align*}
\]

To satisfy \( q_n^\lambda > q_r^\lambda > 0 \), set \((3c_n + \theta_n (1 + \theta_r)) \theta_n / (2 + \theta_r) < r + c_r < (1/2) (c_n + \theta_n) \theta_r \).

(2) The maximum profits of the OEM and the IR are
3.2. Ex Post Quality Improvement Disclosure. The ex post quality improvement disclosure strategy is as shown in the second case of Figure 1. The OEM has achieved product quality improvement but did not disclose corresponding information. IR pays royalty to the OEM and gets the license of the "latest" technology to its knowledge. The corresponding inverse demand functions for the OEM and the IR are as follows:

\[
\begin{align*}
 p_n &= \theta_n (1 - q_n) - \theta_r q_r \\
 p_r &= \theta_r (1 - q_n - q_r)
\end{align*}
\]

(8)

The OEM’s and the IR’s profit functions are characterized by equations (3) and (4).

\[
(\partial^2 \pi_{\text{OEM}}/\partial q_n^2) = -2\theta < 0 \quad \text{and} \quad (\partial^2 \pi_{\text{IR}}/\partial q_r^2) = -2\theta < 0
\]

indicate that the profits of OEM and IR are strictly concave with respect to \(q_n\) and \(q_r\), respectively.

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**Table 1: Notations.**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_n, q_r)</td>
<td>Total demand/quantities for new products and remanufactured products (q_n &gt; q_r)</td>
</tr>
<tr>
<td>(P_n, P_r)</td>
<td>Unit prices of the new and remanufactured products</td>
</tr>
<tr>
<td>(\pi_{\text{OEM}}, \pi_{\text{IR}})</td>
<td>Profits of OEM and IR</td>
</tr>
<tr>
<td>TEI</td>
<td>Abbreviation of the total environmental impact</td>
</tr>
<tr>
<td>(c_n, c_r)</td>
<td>Unit production cost of new products and remanufactured products</td>
</tr>
<tr>
<td>(u_n, u_r)</td>
<td>Net utility of a consumer buying the new product and remanufactured product</td>
</tr>
<tr>
<td>(U)</td>
<td>Consumer’s willingness-to-pay for new products without quality improvement</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Controlling parameter of (\pi_{\text{IR}})</td>
</tr>
<tr>
<td>(\theta_n)</td>
<td>Quality level of OEM, (\theta_n &gt; 1) and (\theta_n = 1) for with and without quality improvement</td>
</tr>
<tr>
<td>(\theta_r)</td>
<td>Valuation discount for remanufactured products, (\theta_r \in (0, 1))</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Remanufacturing cost advantage owing to the savings of raw materials, i.e., (c_r = \alpha c_n, \alpha \in (0, 1))</td>
</tr>
<tr>
<td>(\phi)</td>
<td>The scale parameter of total investment in quality improvement</td>
</tr>
<tr>
<td>(r)</td>
<td>Unit royalty fee paid by the IR to the OEM for the production and remanufactured products</td>
</tr>
<tr>
<td>(\psi_1, \psi_2)</td>
<td>The output adjustment parameters of new and remanufactured products under mechanism of joint R&amp;D</td>
</tr>
<tr>
<td>(k_1, k_2)</td>
<td>The output adjustment parameters of new and remanufactured products under mechanism of licensing royalty</td>
</tr>
</tbody>
</table>

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**Figure 1:** The decision sequence under OEM’s two information disclosure strategies based on technology licensing mechanism.
Lemma 2. When the OEM ex post discloses the technology improvement information to the IR, the OEM’s and the IR’s optimal decisions are obtained as follows:

1. The optimal outputs of new and remanufactured products are

\[
q_n^p = \frac{2\theta_n - \theta_r + r + ac_n - 2c_n}{4\theta_n - \theta_r},
\]

\[
q_r^p = \frac{(c_n + \theta_n)\theta_r - 2\theta_n(r + ac_n)}{(4\theta_n - \theta_r)\theta_r}.
\]

(9)

To satisfy \(q_n^p > q_r^p > 0\), set \((\theta_r(3c_n - \theta_n + \theta_r)/2\theta_n + \theta_r) < r + c_r < ((c_n + \theta_n)\theta_r/2\theta_n)\).

2. The maximum profits of the OEM and the IR are

\[
r_n^p = \frac{\theta_n(2\theta_n - \theta_r + r + ac_n - 2c_n)^2}{(4\theta_n - \theta_r)^2},
\]

\[
r_r^p = \frac{r(\theta_n + \theta_r)\theta_r - 2\theta_n(r + ac_n) - \beta(\theta_n - 1)^2}{(4\theta_n - \theta_r)\theta_r}.
\]

(10)

3. The total environmental impacts of the OEM and the IR are

\[
TEI^p = \frac{\theta_n(2 + a) - (2 - a)c_n)\theta_r + \theta_n(\theta_r - 2a(r + ac_n)) - \theta_r^2}{(4\theta_n - \theta_r)\theta_r}.
\]

(11)

3.3. Comparative Analysis

Proposition 1

(1) When the IR obtains the technology license by paying royalty to OEM, the OEM will choose ex post strategy, i.e., hide the quality improvement information for greater profits.

(2) The OEM’s ex post strategy will hurt the IR’s profits.

(3) Ex ante strategy has less total environment impact when remanufacturing cost is lower. As the cost increases, TEI under the ex ante strategy will increase and even exceed that under the ex post strategy.

Proof

(1) \(q_n^p > q_n^A\), \(q_r^p < q_r^A\).

(2) When \(r + c_r > 2c_n - 2\theta_n + (2\theta_n\theta_r(2 + 2\theta_n - \theta_r)/8\theta_n - \theta_n + \theta_r)\) (sufficient and unnecessary conditions), \(n_n^A > n_r^A > 0\); \(n_r^p < n_r^A\).

(3) When \(r + c_r < ((2 - \alpha)(c_n + \theta_n)/2\alpha\theta_n(4 - \theta_r) + (\theta_r - 2\alpha)\theta_r)\), \(TEI^p > TEI^A\).

When \(r + c_r > ((2 - \alpha)(c_n + \theta_n)/2\alpha\theta_n(4 - \theta_r) + (\theta_r - 2\alpha)\theta_r)\), \(TEI^A > TEI^P\).

When the OEM does not authorize the latest technology to IR, the quality gap between new products and remanufactured products is greater, so consumers’ demand for new products will increase, while demand for remanufactured products will decrease. The OEM obtains maximum profits in the ex post quality disclosure strategy whereas the IR’s profits are lower in this strategy. Increasing the output of new products and decreasing the output of remanufactured products will lead to an increase in total resource consumption, which will have a greater negative impact on the environment. However, the overall market demand for the product (whether new or remanufactured) is reduced; the total consumption of resources will be less than the ex ante strategy.

□

4. Joint R&D

In this section, we study the joint R&D technology licensing mechanism, in which the IR obtains technology license by sharing the OEM’s technology development cost. The IR shares a portion \(\phi\) of the R&D cost and cooperates in developing quality technology before the OEM’s quality level is determined. Thus, the model with the R&D joint venture is regarded as an ex ante licensing mechanism. The mechanism of IR’s R&D joint venture can avoid the problem of OEM hiding product quality improvement information under the technology licensing mechanism. We use superscript \(J\) to denote the joint R&D mechanism.

The inverse demand function under the joint R&D mechanism is the same as equation (2) under the royalty mechanism when the OEM adopts the ex ante strategy. Under the joint R&D mechanism, the profit functions of OEM and IR are as follows:

\[
\pi_{OEM} = (p_n - c_n)q_n(1 - \phi)\theta_n - 1,\]

\[
\pi_{IR} = (p_r - c_r)q_r - \phi\theta_n - 1.
\]

(12)

\(\theta_n\pi_{OEM}/\partial q_n^J = -2\theta_n\beta > 0\) and \(\theta_n\pi_{IR}/\partial q_r^J = -2\theta_n\beta < 0\) indicate that the profits of OEM and IR are strictly concave with respect to and , respectively.

Lemma 3. When IR adopts joint R&D mechanism to obtain technology license by sharing the OEM’s fixed cost of technology research and development, the OEM’s and the IR’s optimal decisions are obtained as follows.

1. The optimal outputs of new and remanufactured products are

\[
q_n^J = \frac{ac_n + \theta_n(2\theta_r - 2c_n)}{\theta_n(4 - \theta_r)}.
\]

\[
q_r^J = \frac{(c_n + \theta_n)\theta_r - 2ac_n}{\theta_n(4 - \theta_r)\theta_r}.
\]

To ensure \(q_n^J > q_r^J > 0\), set \((3c_n + \theta_n(-1 + \theta_r)\theta_r/2 + \theta_r) < c_r < (1/2)(c_n + \theta_n)\theta_r\).
5. Technology Licensing Mechanism Comparison

In this section, we perform a static and dynamic comparative analysis of the two technology licensing mechanisms. We use the ex post strategy to represent licensing royalty mechanism, which is denoted by the superscript R; the joint R&D mechanism is still represented by the superscript J.

5.1. Static Equilibrium Comparison

Proposition 2

1. Demand for remanufactured products increases as the quality gap narrows and, accordingly, demand for new products shrinks.

2. If \( \phi \beta (\theta_n - 1)^2 \leq M(r) \), then joint R&D is the better mechanism for IR; otherwise, royalty is the better mechanism. When \( \phi \beta (\theta_n - 1)^2 \geq N(r) \), joint R&D delivers Pareto improvement. The results can be seen in Figure 2.

Proof

1. \( q^R_n < q^p_n, q^J_n > q^p_n \).

2. Let \( \pi^J_n \geq \pi^p_n \); we derive that

\[
\phi \beta (\theta_n - 1)^2 \leq \frac{(-2ac_n + (c_n + \theta_n) \theta_n)^2}{\theta_n(-4 + \theta_n)^2 \theta_r} - \frac{(-2(r + ac_n) \theta_n + (c_n + \theta_n) \theta_r)^2}{\theta_r(-4 + \theta_r)^2 \theta_r} = M(r).
\]

These two thresholds are represented by \( M(r) \) and \( N(r) \), respectively, and the corresponding regions are drawn in Figure 2.

Proposition 2 part (2) implies that whether the joint R&D mechanism increases or decreases the IR’s maximum profits compared with the license royalty mechanism. The results are decided by the relationship between the share of investment in R&D (\( \phi \)) and the royalty (\( r \)). This relationship is illustrated in Figure 2, which is divided into three regions. The characteristics of each region will be discussed next.

Region I specifies that the conditions under licensing royalty are the better mechanism for IR. The royalty is low and share of R&D investment is high, where royalty mechanism has a cost advantage over co-development. In region II and region III, joint R&D is the optimal choice for
IR. Region II specifies the condition under which IR earns more while OEM earns fewer profits by investing R&D than paying royalty. In this region II, the OEM, who suffers profit loss, has no incentive to continue the co-development relationship. A Pareto improvement can only be realized in region III, in which both parties’ maximum profits under joint R&D mechanism are greater than those under the licensing royalty mechanism. Consequently, such a cooperative relationship will be embraced by both the OEM and the IR.

5.2. Dynamic Complexity Analysis

5.2.1. Dynamic System and Local Stability. It is almost impossible for OEM and IR to get complete information from the market. So we assume that both parties make output decisions with bounded rationality based on current marginal profit. If the profit margin is positive (negative), the OEM or the IR will increase (decrease) the quantity. Therefore, the dynamic output decision process under the two technology license mechanisms is described below.

Under the royalty mechanism, the marginal benefits of OEM and IR are given by

\[
\begin{align*}
\frac{\partial \pi_{\text{OEM}}}{\partial q_n} &= \theta_n(1 - 2q_n) - q_r \theta_r - c_n, \\
\frac{\partial \pi_{\text{IR}}}{\partial q_r} &= \theta_r(1 - q_n - 2q_r) - r - \alpha c_n.
\end{align*}
\]

The corresponding output adjustment mechanism can be modified as follows:

\[
\begin{align*}
q_n^{R}(t + 1) &= q_n^{R}(t) + v_1 q_n^{R}(t) \left( \theta_n(1 - 2q_n^{R}(t)) - \theta_r q_r^{R}(t) - c_n \right), \\
q_r^{R}(t + 1) &= q_r^{R}(t) + v_2 q_r^{R}(t) \left( \theta_r(1 - q_n^{R}(t) - 2q_r^{R}(t)) - r - \alpha c_n \right).
\end{align*}
\]

When the IR adopts joint R&D mechanism, the marginal profits of the OEM and the IR are

\[
\begin{align*}
\frac{\partial \pi_{\text{OEM}}}{\partial q_n} &= \theta_n(1 - 2q_n - q_r \theta_r - c_n), \\
\frac{\partial \pi_{\text{IR}}}{\partial q_r} &= \theta_r \theta_r(1 - q_n - 2q_r - \alpha c_n).
\end{align*}
\]

The corresponding dynamic output adjustment system is

\[
\begin{align*}
q_n^{J}(t + 1) &= q_n^{J}(t) + k_1 q_n^{J}(t) \left( \theta_n(1 - 2q_n^{J}(t)) - \theta_r q_r^{J}(t) - c_n \right), \\
q_r^{J}(t + 1) &= q_r^{J}(t) + k_2 q_r^{J}(t) \left( \theta_r(1 - q_n^{J}(t) - 2q_r^{J}(t)) - r - \alpha c_n \right).
\end{align*}
\]

In order to investigate the local stability of the two systems, we return to system (19) and system (21). The Jacobian matrix of the systems can be given by

\[
J(R) = \begin{pmatrix}
1 + v_1 \left( \theta_n(1 - 4q_n^{R}) - \theta_r q_r^{R} - c_n \right) & -\theta_r v_1 q_r^{R} \\
-\theta_r v_2 q_r^{R} & 1 + v_2 \left( \theta_r(1 - 4q_r^{R} - 4q_r^{J}) - r - \alpha c_n \right)
\end{pmatrix},
\]

\[
J(J) = \begin{pmatrix}
1 + k_1 \left( \theta_n(1 - 4q_n^{J}) - \theta_r q_r^{J} - c_n \right) & -\theta_r k_1 q_r^{J} \\
-\theta_r k_2 q_r^{J} & 1 + k_2 \left( \theta_r(1 - q_n^{J} - 4q_r^{J}) - r - \alpha c_n \right)
\end{pmatrix}.
\]

According to Jury stability criterion, the condition of asymptotic stability of equilibrium of static model is to satisfy the following conditions:

\[
\begin{align*}
1 - \text{Tr}(J) + \text{Det}(J) &< 0, \\
1 + \text{Tr}(J) + \text{Det}(J) &> 0, \\
\text{Det}(J) &> 0.
\end{align*}
\]

5.2.2. Numerical Example. We use the method of numerical simulation to analyze the dynamical behaviors of system (19) and system (21). The values of the parameters are set as follows: \( \theta_r = 0.7, \theta_n = 1.3, c_n = 0.3, \alpha = 0.5, r = 0.01, \beta = 0.5, \phi = 0.35, v_1 = 2, v_2 = 4, k_1 = 2, \) and \( k_2 = 4. \) Unless otherwise specified, the following parts of the numerical simulation remain unchanged. Then, the equilibrium outputs of the OEM and the IR under two mechanisms are \( E^R = (0.324, 0.223) \) and \( E^J = (0.289, 0.273). \) Substituting the parameter values into the Jacobian matrix (22) and the Jury criterion (24), the stable regions of \( E^R \) and \( E^J \) are shown in Figure 3.

Figure 3 shows that the stable range of \( v_1, k_1 \) is larger than that of \( v_2, k_2. \) The management significance can be considered as follows: the quantity adjustment speed of new products has more influence on the stability of the system than that of remanufactured products. New product output adjustment decisions need to be more prudent than remanufactured products.

In Figure 4, we simulate the process of the decision variables \( q_n \) and \( q_r \) of system (19) and system (21) from equilibrium to period-doubling bifurcation path and then entering a chaotic state. If we keep the other parameters fixed and only vary one, bifurcations and chaos occur, and this phenomenon is detected.
by the Lyapunov exponents. In the numerical example, blue represents system (19), i.e., the royalty mechanism, and red represents system (21), i.e., the joint R&D mechanism. The thicker line connected by “+” in the middle of the bifurcation diagram connected by “-” is the average value of outputs.

Figure 5(a) shows the bifurcation diagram and the mean of new product quantities under royalty mechanism and joint R&D mechanism. The average value of new product output under the royalty mechanism is always above the output under the joint R&D. Figure 5(b) shows the bifurcation diagram and the mean of remanufactured product quantities under royalty mechanism and joint R&D mechanism. The average value of the output of remanufactured products under the joint R&D mechanism is first above and then, after falling into chaos, below the output under the royalty mechanism, which is the opposite result of the stable state. It can be seen from Figure 5 that the value range of \( k_1, k_2 \) is smaller than \( v_1, v_2 \) (blue), which means that as output adjustment parameters increase, the OEM and the IR under the joint R&D mechanism will exit the market before that under the royalty mechanism. The largest Lyapunov exponent plotted in Figures 6(a) and 6(b) corresponds to the parameters of Figure 5. When LLE is equal to 0, it means that bifurcation occurred at the point. When LLE is greater than 0, it indicates that the system is in a chaotic state. System (19) and system (21) lose stability through a flip bifurcation and enter chaos.

**Proposition 3**

1. The numerical simulation results of the new product output are the same as Proposition 2 part (1) \( q_1^r > q_1^R \) in the stable state, but \( q_2^r < q_2^R \) in the chaotic state.

2. As output adjustment parameters increase, the OEM and IR under the joint R&D mechanism will exit the market before under the royalty mechanism.

Figure 6 depicts the bifurcation diagrams and the mean of profits of the OEM (a) and the IR (b) as \( v_1, k_1 \) and \( v_2, k_2 \) increase, respectively. Under the joint R&D mechanism, the profits of OEM and IR are higher than those under the royalty mechanism. In this case, the joint R&D mechanism achieves a Pareto improvement over the royalty mechanism. In Figure 6(a), as \( v_1, k_1 \) increases, the profits of the OEM enter chaos and then exit the market. The average profit in chaos has also declined. It is worth noting that the profits of the OEM under the royalty mechanism will be less than 0 under chaotic conditions. The same result also appears in Figure 6(b); under the joint R&D mechanism, the profit of IR will also be lower than 0. In the sense of economics, firm is in deficit.

**Proposition 4.** With the increase of output adjustment parameters, OEM’s and IR’s profits will enter a chaotic state from a stable state and then exit the market. In a state of chaos, the profits will reduce or even run into deficit.

Figures 7(a) and 7(b), respectively, describe the total environmental impacts under the royalty mechanism and the joint R&D mechanism when the output adjustment parameters of the new and remanufactured products change simultaneously. Figures 7(c) and 7(d) plot the average value of TEI with the increase of both output adjustment parameters under two mechanisms. The red line represents the royalty mechanism and the blue line represents the joint R&D mechanism. It can be seen from Figure 7(c) that the red line is below the blue line, which means \( \text{TEI}^r > \text{TEI}^J \). However, in Figure 7(d) where \( \theta_r = 0.5 \), the red line is first above and then below the blue line after entering the chaotic state. The phenomenon of the red line below the blue line in Figure 7(d) is opposite to the steady state result. The TEI under both mechanisms are reduced after entering chaos. From the perspective of environmental impact, chaos is beneficial.

**Proposition 5**

1. From the perspective of TEI, chaos is beneficial.

2. In chaos, conclusions of TEI under static equilibrium may be reversed.

5.2.3. Feedback Adjustment. It can be seen from the results of numerical example that chaos can lead to profits loss of the OEM and IR or even deficit. In order to avoid chaos, it is necessary to choose a suitable output adjustment scale so that it does not exceed the stability region. But when chaos has already appeared, to resort the system to stability, we consider using a feedback control method to control chaos. Use the output gap between the previous period and the current period as feedback information to modify the output decision of the next cycle.

We change system (19) into the controlled system (25).
\[
\begin{align*}
q_n^{\text{R}}(t + 1) &= q_n^{\text{R}}(t) + v_1 q_n^{\text{R}}(t)(\theta_n(1 - 2q_n^{\text{R}}(t)) - \theta_n q_n^{\text{R}}(t) - c_n) + v(q_n^{\text{R}}(t) - x(t)), \\
q_r^{\text{R}}(t + 1) &= q_r^{\text{R}}(t) + v_2 q_r^{\text{R}}(t)(\theta_r(1 - q_r^{\text{R}}(t) - 2q_r^{\text{R}}(t)) - r - \alpha c_n), \\
x(t + 1) &= q_n^{\text{R}}(t),
\end{align*}
\]

where \( v > 0 \) is the controlling factor, indicating the extent of feedback adjustment. We set \( v_1 = 2.9, v_2 = 4, \) and the other parameters are the same as above. The bifurcation diagram of the system (25) with respect to \( v \) is shown in Figure 8.

Figure 8 describes the process that after feedback adjustment, as the control coefficient changes, \( q_n \) enters the periodic solution from chaos and then reaches a fixed point. This means that as the control parameter increases, the chaos of system (25) is successfully controlled.
Figure 6: The bifurcation diagrams and the mean of profits of OEM (a) and IR (b) with respect to $v_1, k_1$ and $v_2, k_2$, respectively.

Figure 7: The bifurcation diagram and the mean (c, d) of TEI under (a) royalty mechanism, (b) joint R&D mechanism, (c) $\theta_r = 0.7$, and (d) $\theta_r = 0.5$. 
6. Conclusion

In this paper, we built a duopoly Cournot game model of an OEM (original equipment manufacturer) and an IR (independent remanufacturer). The OEM manufactures new products and continuously improves product quality. IR remanufactures and needs to obtain technology licenses from OEM through royalty mechanism or joint R&D mechanism. Through a comparative analysis of the OEM's strategy to ex ante and ex post disclose product quality improvement information under the royalty mechanism, we find the OEM concealing the latest technology from IR will hurt the profits of IR. To circumvent this behavior, we consider the case where IR adopts a joint R&D mechanism and comparatively analyze the two mechanisms under static equilibrium and complex dynamics. We mainly analyze from the three aspects: (1) output of new and remanufactured products, (2) profits of the OEM and the IR, and (3) TEI (total environmental impact) of royalty mechanism and joint R&D mechanism.

In the static setting, we get the following results:

(1) Demand for remanufactured products increases as the quality gap narrows and, accordingly, demand for new products shrinks.

(2) If \( \phi \beta (\theta - 1)^2 \leq M(r) \), then joint R&D is the better mechanism for IR; otherwise, royalty is the better mechanism. When \( \phi \beta (\theta - 1)^2 \geq N(r) \), joint R&D delivers Pareto improvement.

Under dynamic setting, the results of numerical simulation show the following:

(1) The results of the new product output are the same as Proposition 2 part (1) \( q^a_1 < q^r_n \). The simulation results of the remanufactured product output are the same as Proposition 2 part (1) \( q^l > q^R \) in the stable state and opposite in the chaotic state \( q^l < q^R \).

(2) As output adjustment parameters increase, the OEM and IR under the joint R&D mechanism will exit the market before the royalty mechanism.

(3) With the increase of output adjustment parameters, OEM’s and IR’s profits will enter a chaotic state from a stable state and then exit the market. In a state of chaos, the profits will fall or even run into deficit.

(4) In chaos, TEI concludes that reverse static equilibrium will appear.

We consider a feedback control method to eliminate chaos and restore stability.

This paper can be extended from the following directions:

(1) Study the case when the quality level of new products is an endogenous variable of OEM. If the quality level is too low, products will be eliminated by the market; if too high, the sales of existing products will be affected and the difficulty of upgrading the next generation products will be increased. When quality level is endogenous, the OEM can choose the most appropriate quality level to maximize their profits.

(2) Consider that the OEM charges different royalty for technology license of different quality level.

(3) When carbon regulation policy is considered, discuss how the conclusions about these two technology licensing mechanisms will change.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request. The questionnaire data were acquired mainly through e-mail and filling paper out.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References


