Research Article

Passivity Analysis of Coupled Stochastic Neural Networks with Multiweights

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In this paper, we devote to the investigation of passivity in two types of coupled stochastic neural networks (CSNNs) with multiweights and incompatible input and output dimensions. First, some new definitions of passivity are proposed for stochastic systems that may have incompatible input and output dimensions. By utilizing stochastic analysis techniques and Lyapunov functional method, several sufficient conditions are respectively developed for ensuring that CSNNs without and with multiple delay couplings can realize passivity. Besides, the synchronization criteria for CSNNs with multiweights are established by employing the results of output-strictly passivity. Finally, two simulation examples are given to illustrate the validity of the theoretical results.

1. Introduction

In recent decades, neural networks (NNs) have potential applications in the image encryption, pseudorandom number generators, optimization, and other areas [1–3], which depend on the dynamical behaviors of NNs including stability and passivity. Therefore, the stability [4–8] and passivity [9–14] for various NNs have received special attention in recent years. Mou et al. [4] considered the asymptotic stability problem for Hopfield NNs with time delay via combining the Lyapunov functional and delay fractioning approach. Yang et al. [6] discussed the stability for a kind of NNs with time-varying delays and gave several delay-dependent stability conditions by taking advantage of integral inequality. In [9], a class of NNs with time-varying delays and parameter uncertainties was taken into account, and some exponential passivity criteria were established by

exploiting weighted integral inequalities. Xiao et al. [11] studied the passivity for a type of memristive NNs with inertial term, obtained some criteria of asymptotic stability by utilizing the passivity, and discussed the case that parameters are uncertain but bounded.

As it is known to all, stochastic perturbations are unavoidable in the implementation of NNs and may cause undesirable dynamical behaviors in NNs [15, 16]. Therefore, the dynamical behaviors including the stability [17–21] and passivity [22–27] have been widely investigated by numerous researchers for NNs with stochastic perturbations in recent years. In [18], several sufficient conditions on mean square stability for stochastic neural networks (SNNs) with local impulses were derived by using the mathematical induction method. Yang and Li [20] coped with the stability problem for switched SNNs with parameter uncertainties, and derived several conditions to guarantee the robust

stability by utilizing the state-dependent switching method. In [24], the authors took into account one type of uncertain SNNs with distributed and discrete time-varying delays and gave some passivity criteria with the help of integral inequality technique. Nagamani et al. [25], respectively, discussed the passivity and dissipativity for Markovian jump stochastic NNs with two types of time-varying delays and obtained several delay-dependent passivity and dissipativity criteria by taking a suitable Lyapunov functional.

Coupled neural networks (CNNs) comprised of a number of NNs have tremendous potential applications in many areas of engineering [28–30]. Hence, the dynamical behaviors of CNNs have attracted much attention; especially, the passivity [31–35] and synchronization [36–40] for many types of CNNs have been deeply discussed. In [34], the authors not only obtained several passivity criteria for the directed CNNs based on the developed adaptive control strategies but also discussed the case that topologies are undirected. Qi et al. [37] derived several global exponential synchronization criteria for quaternion-valued CNNs with impulses by exploiting Lyapunov functional and matrix inequalities. In [38], the authors considered CNNs with mixed impulses and developed some exponential synchronization conditions for the proposed network model through using delayed impulsive differential inequality. Particularly, a number of authors have discussed the dynamical behavior of coupled SNNs (CSNNs) in recent years [41–45]. Chen et al. [44] utilized the adaptive feedback controller to deal with the exponential synchronization for CSNNs. In [45], the authors respectively employed time-triggered and event-triggered impulsive control methods to investigate the synchronization of discrete time CSNNs with multidelay. Unfortunately, the passivity of CSNNs has not yet been investigated.

It should be pointed out that the results in [31–45] only focused on CNNs with single weight. Considering the diversity of influencing factors, some researchers discussed the synchronization and passivity for multiweighted CNNs (MWCNNs) [46–48]. In [46], the authors proposed MWCNNs without and with coupling delays and dealt with the passivity and synchronization for these network models by employing the impulsive control method. Wang and Zhao [48] not only discussed the passivity for MWCNNs by utilizing the designed proportional-integral and proportional-derivative controllers but also derived several synchronization criteria by virtue of output-strict passivity. However, the passivity and synchronization for multi-weighted CSNNs (MWCSNNs) have not been investigated.

In this paper, the passivity for two types of MWCSNNs with incompatible input and output dimensions is investigated. The main contributions have three aspects. First, we present several new definitions of passivity for stochastic systems with incompatible dimensions of output and input. Second, some sufficient conditions to ensure the passivity of MWCSNNs are obtained by taking advantage of the Lyapunov functional method and stochastic analysis techniques, and a synchronization criterion is also developed by utilizing the result of output-strictly passivity. Third, we further address the passivity and synchronization for CSNNs with multiple delay couplings (CSNNMDCs).

2. Preliminaries

2.1. Notations. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ is a complete probability space with the natural filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions. $C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$ represents the family of all nonnegative functions $V(t, \kappa(t))$ on $\mathbb{R}^+ \times \mathbb{R}^n$, which are once differentiable in t and twice continuously differentiable in $\kappa(t)$. $\text{Tr}(\cdot)$ stands for the trace of a matrix. $P \geq 0$ is used to denote a symmetric semipositive definite matrix. $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$, respectively, denote the maximum and minimum eigenvalue of a real symmetric matrix.

2.2. Lemmas

Lemma 1 (Itô formula, see [49]). *A stochastic system can be described by*

$$d\kappa(t) = f(t, \kappa(t))dt + g(t, \kappa(t))dw(t), \quad (1)$$

in which $\kappa(t) \in \mathbb{R}^n$ represents the state of system, $f(\cdot): \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is continuous nonlinear function, $g(\cdot): \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ is noise intensity function, and $w(t)$ is an n -dimensional Brownian motion (Wiener process) defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$.

For any $V(t, \kappa(t)) \in C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$, define the operator $\mathcal{L}V$ for system (1) as follows:

$$\begin{aligned} \mathcal{L}V(t, \kappa(t)) &= V_t(t, \kappa(t)) + V_{\kappa}(t, \kappa(t))f(t, \kappa(t)) \\ &\quad + \frac{1}{2}\text{Tr}(g^T(t, \kappa(t))V_{\kappa\kappa}(t, \kappa(t))g(t, \kappa(t))), \end{aligned} \quad (2)$$

where $V_t(t, \kappa(t)) = (\partial V(t, \kappa(t))/\partial t)$, $V_{\kappa}(t, \kappa(t)) = ((\partial V(t, \kappa(t))/\partial \kappa_1), (\partial V(t, \kappa(t))/\partial \kappa_2), \dots, (\partial V(t, \kappa(t))/\partial \kappa_n))$, $V_{\kappa\kappa}(t, \kappa(t)) = (\partial^2 V(t, \kappa(t))/\partial \kappa_i \partial \kappa_j)_{n \times n}$.

If $V(t, \kappa(t)) \in C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$, one has

$$\mathbb{E}V(t, \kappa(t)) = \mathbb{E}V(t_0, \kappa(t_0)) + \mathbb{E}\left\{\int_{t_0}^t \mathcal{L}V(s, \kappa(s))ds\right\}, \quad (3)$$

for all $t > t_0 \geq 0$.

Lemma 2 (see [50]). *For any matrices $M \in \mathbb{R}^{m \times n}$ and $0 \leq P \in \mathbb{R}^{m \times m}$, one obtains*

$$\text{Tr}(M^T P M) \leq \lambda_M(P)\text{Tr}(M^T M). \quad (4)$$

2.3. Definitions

Definition 1. A stochastic system with input $\beta(t) \in \mathbb{R}^p$ and output $\eta(t) \in \mathbb{R}^q$ is passive if

$$\mathbb{E}\left\{\int_{\vartheta_0}^{\vartheta_p} \eta^T(s)F\beta(s)ds\right\} \geq \mathbb{E}\{S(\vartheta_p)\} - \mathbb{E}\{S(\vartheta_0)\}, \quad (5)$$

for any $\vartheta_p, \vartheta_0 \in \mathbb{R}^+$ and $\vartheta_p \geq \vartheta_0$, in which $F \in \mathbb{R}^{q \times p}$ and S is a nonnegative function.

Definition 2. A stochastic system with input $\beta(t) \in \mathbb{R}^p$ and output $\eta(t) \in \mathbb{R}^q$ is input-strictly passive if

$$\mathbb{E} \left\{ \int_{\vartheta_0}^{\vartheta_p} \eta^T(s) F \beta(s) ds \right\} \geq \mathbb{E} \{S(\vartheta_p)\} - \mathbb{E} \{S(\vartheta_0)\} + \mathbb{E} \left\{ \int_{\vartheta_0}^{\vartheta_p} \beta^T(s) A_1 \beta(s) ds \right\}, \quad (6)$$

for any $\vartheta_p, \vartheta_0 \in \mathbb{R}^+$ and $\vartheta_p \geq \vartheta_0$, in which $F \in \mathbb{R}^{q \times p}$, $0 < A_1 \in \mathbb{R}^{p \times p}$, and S is a nonnegative function.

Definition 3. A stochastic system input $\beta(t) \in \mathbb{R}^p$ and output $\eta(t) \in \mathbb{R}^q$ is output-strictly passive if

$$\mathbb{E} \left\{ \int_{\vartheta_0}^{\vartheta_p} \eta^T(s) F \beta(s) ds \right\} \geq \mathbb{E} \{S(\vartheta_p)\} - \mathbb{E} \{S(\vartheta_0)\} + \mathbb{E} \left\{ \int_{\vartheta_0}^{\vartheta_p} \eta^T(s) A_2 \eta(s) ds \right\}, \quad (7)$$

for any $\vartheta_p, \vartheta_0 \in \mathbb{R}^+$ and $\vartheta_p \geq \vartheta_0$, in which $F \in \mathbb{R}^{q \times p}$, $0 < A_2 \in \mathbb{R}^{q \times q}$, and S is a nonnegative function.

3. Passivity for MWCSNNs

3.1. Network Model. The MWCSNNs in this paper is considered as follows:

$$d\kappa_z(t) = \left(-D\kappa_z(t) + Gf(\kappa_z(t)) + B + \sum_{n=1}^s \sum_{h=1}^M b_n C_{zh}^n \Gamma^n \kappa_h(t) + H\beta_z(t) \right) dt + \sigma(\kappa_z(t)) d\omega(t), \quad z = 1, 2, \dots, M, \quad (8)$$

where $n = 1, 2, \dots, s$, $G = (G_{zj})_{m \times m} \in \mathbb{R}^{m \times m}$, $G_{zj} \in \mathbb{R}$, represents the strength of the j th neuron on the z th neuron, $H \in \mathbb{R}^{m \times p}$ is the known matrix, $\kappa_z(t) = (\kappa_{z1}(t), \kappa_{z2}(t), \dots, \kappa_{zm}(t))^T \in \mathbb{R}^m$ is the state vector of the z th node, $\beta_z(t) \in \mathbb{R}^p$ represents the external input, $0 < D = \text{diag}(d_1, d_2, \dots, d_m) \in \mathbb{R}^{m \times m}$, $0 < d_z \in \mathbb{R}$, represents the rate with which the z th neuron will reset its potential to the resting state when disconnected from the network and external input, $B = (B_1, B_2, \dots, B_m)^T \in \mathbb{R}^m$, $f(\kappa_z(t)) = (f_1(\kappa_{z1}(t)), f_2(\kappa_{z2}(t)), \dots, f_m(\kappa_{zm}(t)))^T \in \mathbb{R}^m$, $0 < b_n \in \mathbb{R}$, denotes coupling strength, and $C^n = (C_{zh}^n)_{M \times M} \in \mathbb{R}^{M \times M}$ represents the outer coupling matrix, where C_{zh}^n satisfies the following conditions: if there exists a connection between nodes z and h , then $\mathbb{R} \ni C_{zh}^n = C_{hz}^n > 0$ ($z \neq h$), otherwise, $\mathbb{R} \ni C_{zh}^n = C_{hz}^n = 0$, and $C_{zz}^n = -\sum_{h \neq z} C_{zh}^n$, $z = 1, 2, \dots, M$; $0 < \Gamma^n = \text{diag}(\Gamma_1^n, \Gamma_2^n, \dots, \Gamma_m^n) \in \mathbb{R}^{m \times m}$ denotes the inner coupling matrix; $\omega(t) = (\omega_1(t), \omega_2(t), \dots, \omega_m(t))^T \in \mathbb{R}^m$ is an m -dimensional Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$; $\sigma(\kappa_z(t)) = \text{diag}(\sigma(\kappa_{z1}(t)), \sigma(\kappa_{z2}(t)), \dots, \sigma(\kappa_{zm}(t))) \in \mathbb{R}^{m \times m}$ is the noise intensity matrix.

In this paper, the following assumptions are made.

Assumption 1. $f_l(\cdot)$ ($l = 1, 2, \dots, m$) satisfies

$$|f_l(j_1) - f_l(j_2)| \leq \xi_l |j_1 - j_2|, \quad (9)$$

for any $j_1, j_2 \in \mathbb{R}$, where $\xi_l > 0$. Let $\xi = \text{diag}(\xi_1^2, \xi_2^2, \dots, \xi_m^2) \in \mathbb{R}^{m \times m}$.

Assumption 2. There exists a positive constant μ such that $\sigma(\cdot)$ satisfies the following inequality:

$$\text{Tr}[(\sigma(k_1) - \sigma(k_2))^T (\sigma(k_1) - \sigma(k_2))] \leq \mu^2 \|k_1 - k_2\|^2, \quad (10)$$

for any $k_1, k_2 \in \mathbb{R}^n$.

Remark 1. On the one hand, the passivity for various CNNs has been investigated and some meaningful results have been obtained [31–35]. However, the passivity of CSNNs has not yet been discussed. On the other hand, some researchers have dealt with the synchronization problem for CSNNs [41–45]. Given that passivity has been developed as a powerful tool to solve the synchronization problem of CNNs, the investigation on synchronization for CSNNs based on the passivity is apparently very valuable. Regrettably, the result about this topic has not yet been reported.

3.2. Passivity Analysis. Suppose that $s(t) = (s_1(t), s_2(t), \dots, s_m(t))^T \in \mathbb{R}^m$ is an arbitrary desired solution of the isolated node of system (8), then it satisfies

$$ds(t) = (-Ds(t) + Gf(s(t)) + B)dt + \sigma(s(t))d\omega(t). \quad (11)$$

Letting $\alpha_z(t) = (\alpha_{z1}(t), \alpha_{z2}(t), \dots, \alpha_{zm}(t))^T = \kappa_z(t) - s(t)$, we can obtain from (8) and (11) that

$$d\alpha_z(t) = \left(-D\alpha_z(t) + Gf(\kappa_z(t)) - Gf(s(t)) + \sum_{n=1}^s \sum_{h=1}^M b_n C_{zh}^n \Gamma^n (\kappa_h(t) - s(t)) + H\beta_z(t) \right) dt + (\sigma(\kappa_z(t)) - \sigma(s(t)))d\omega(t). \quad (12)$$

The output vector $\eta_z(t) \in \mathbb{R}^q$ of network (12) is defined as follows:

$$\eta_z(t) = Z_1 \alpha_z(t) + Z_2 \beta_z(t), \quad (13)$$

where $Z_1 \in \mathbb{R}^{q \times m}$ and $Z_2 \in \mathbb{R}^{q \times p}$.

Define $\hat{D} = \text{diag}(D, D, \dots, D) \in \mathbb{R}^{Mm \times Mm}$, $\hat{G} = \text{diag}(G, G, \dots, G) \in \mathbb{R}^{Mm \times Mm}$, $\hat{H} = \text{diag}(H, H, \dots, H) \in \mathbb{R}^{Mm \times Mp}$, $\hat{Z}_1 = \text{diag}(Z_1, Z_1, \dots, Z_1) \in \mathbb{R}^{Mq \times Mm}$, $\hat{Z}_2 = \text{diag}(Z_2, Z_2, \dots, Z_2) \in \mathbb{R}^{Mq \times Mp}$, $\hat{\xi} = \text{diag}(\xi, \xi, \dots, \xi) \in \mathbb{R}^{Mm \times Mm}$, and $\hat{\omega}(t) = (\omega^T(t), \omega^T(t), \dots, \omega^T(t))^T \in \mathbb{R}^{Mm}$.

Take

$$\begin{aligned} \alpha(t) &= (\alpha_1^T(t), \alpha_2^T(t), \dots, \alpha_M^T(t))^T, \\ \beta(t) &= (\beta_1^T(t), \beta_2^T(t), \dots, \beta_M^T(t))^T, \\ \eta(t) &= (\eta_1^T(t), \eta_2^T(t), \dots, \eta_M^T(t))^T. \end{aligned} \quad (14)$$

According to (12), we have

$$\begin{aligned} d\alpha(t) &= \left(-\hat{D}\alpha(t) + \hat{G}(f(\kappa(t)) - f(s(t))) + \hat{H}\beta(t) + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \alpha(t) \right) dt + \hat{\sigma}(t) d\hat{\omega}(t), \\ \eta(t) &= \hat{Z}_1 \alpha(t) + \hat{Z}_2 \beta(t), \end{aligned} \quad (15)$$

where $\hat{\sigma}(t) = I_M \otimes (\sigma(\kappa(t)) - \sigma(s(t)))$.

Theorem 1. Network (15) is passive if there exist $F \in \mathbb{R}^{Mq \times Mp}$, and $0 < \hat{P} \in \mathbb{R}^{Mm \times Mm}$ satisfying

$$\begin{pmatrix} W_1 & E_1 \\ E_1^T & M_1 \end{pmatrix} \leq 0, \quad (16)$$

where $W_1 = -\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T \hat{P} + \hat{\xi} + q \mu^2 I_{Mm} + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \hat{P}$, $E_1 = \hat{P}\hat{H} - (\hat{Z}_1^T F)/2$, $M_1 = -(\hat{Z}_2^T F + F^T \hat{Z}_2)/2$, and $q = \lambda_M(\hat{P})$.

Proof. For convenience, we denote

$$y(t) = -\hat{D}\alpha(t) + \hat{G}(f(\kappa(t)) - f(s(t))) + \hat{H}\beta(t) + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \alpha(t), \quad (17)$$

then network (15) can be rewritten as

$$d\alpha(t) = y(t)dt + \hat{\sigma}(t)d\hat{\omega}(t). \quad (18)$$

Choose the following Lyapunov functional for network (15):

$$V^1(t) = \alpha^T(t) \hat{P} \alpha(t). \quad (19)$$

In light of Lemma 1, we can obtain

$$\begin{aligned}\mathcal{L}V^1(t) &= V_t^1(t) + V_\alpha^1(t)y(t) + \frac{1}{2}\text{Tr}(\hat{\sigma}^T(t)V_{\alpha\alpha}^1(t)\hat{\sigma}(t)) \\ &= 2\alpha^T(t)\hat{P}\left(-\hat{D}\alpha(t) + \hat{G}(f(\kappa(t)) - f(s(t))) + \hat{H}\beta(t) + \sum_{n=1}^s b_n(C^n \otimes \Gamma^n)\alpha(t)\right) + \text{Tr}(\hat{\sigma}^T(t)\hat{P}\hat{\sigma}(t)).\end{aligned}\quad (20)$$

Obviously,

$$2\alpha^T(t)\hat{P}\hat{G}(f(\kappa(t)) - f(s(t))) \leq \alpha^T(t)\left(\hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi}\right)\alpha(t). \quad (21)$$

From Lemma 2 and Assumption 2, we have

$$\begin{aligned}\text{Tr}(\hat{\sigma}^T(t)\hat{P}\hat{\sigma}(t)) &\leq q \sum_{z=1}^M \text{Tr}((\sigma(\kappa_z(t)) - \sigma(s(t)))^T(\sigma(\kappa_z(t)) - \sigma(s(t)))) \\ &\leq q \sum_{z=1}^M \mu^2 \|\kappa_z(t) - s(t)\|^2 \\ &= q\mu^2 \|\alpha(t)\|^2.\end{aligned}\quad (22)$$

Furthermore,

$$\begin{aligned}2\alpha^T(t)\hat{P}\left(\sum_{n=1}^s b_n(C^n \otimes \Gamma^n)\alpha(t)\right) &= \alpha^T(t)\left(\sum_{n=1}^s b_n\hat{P}(C^n \otimes \Gamma^n) + \sum_{n=1}^s b_n(C^n \otimes \Gamma^n)\hat{P}\right)\alpha(t).\end{aligned}\quad (23)$$

By (20)–(23), one obtains

$$\begin{aligned}\mathcal{L}V^1(t) &\leq \alpha^T(t)\left(-\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi} + q\mu^2 I_{Mm} + \sum_{n=1}^s b_n\hat{P}(C^n \otimes \Gamma^n) + \sum_{n=1}^s b_n(C^n \otimes \Gamma^n)\hat{P}\right)\alpha(t) \\ &\quad + 2\alpha^T(t)\hat{P}\hat{H}\beta(t).\end{aligned}\quad (24)$$

From (24), we have

$$\begin{aligned}\mathcal{L}V^1(t) - \eta^T(t)F\beta(t) &\leq \alpha^T(t)\left(-\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi} + q\mu^2 I_{Mm} \right. \\ &\quad \left. + \sum_{n=1}^s b_n\hat{P}(C^n \otimes \Gamma^n) + \sum_{n=1}^s b_n(C^n \otimes \Gamma^n)\hat{P}\right)\alpha(t) \\ &\quad + \alpha^T(t)2\hat{P}\hat{H}\beta(t) - \left(\alpha^T(t)\hat{Z}_1^T + \beta^T(t)\hat{Z}_2^T\right)F\beta(t) = \zeta^T(t)\begin{pmatrix} W_1 & E_1 \\ E_1^T & M_1 \end{pmatrix}\zeta(t),\end{aligned}\quad (25)$$

where $\zeta(t) = (\alpha^T(t), \beta^T(t))^T$.

By (16), one obtains

$$\eta^T(t)F\beta(t) \geq \mathcal{L}V^1(t). \quad (26)$$

From (26), we obtain

$$\int_{t_0}^{t_p} \eta^T(t)F\beta(t)dt \geq \int_{t_0}^{t_p} \mathcal{L}V^1(t)dt. \quad (27)$$

According to (27) and Lemma 1, we can acquire

$$\mathbb{E}\left\{\int_{t_0}^{t_p} \eta^T(t)F\beta(t)dt\right\} \geq \mathbb{E}\{V^1(t_p)\} - \mathbb{E}\{V^1(t_0)\}. \quad (28)$$

□

Theorem 2. Network (15) is input-strictly passive if there exist matrices $F \in \mathbb{R}^{Mq \times Mp}$, $0 < A_1 \in \mathbb{R}^{Mp \times Mp}$, and $0 < \hat{P} \in \mathbb{R}^{Mm \times Mm}$ satisfying

$$\begin{pmatrix} W_1 & E_1 \\ E_1^T & M_2 \end{pmatrix} \leq 0, \quad (29)$$

where W_1 and E_1 have the same meanings as these in Theorem 1 and $M_2 = M_1 + A_1$.

Proof. Construct the same $V^1(t)$ as (19) for network (15), and we can easily obtain

$$\begin{aligned} & \mathcal{L}V^1(t) - \eta^T(t)F\beta(t) + \beta^T(t)A_1\beta(t) \\ & \leq \alpha^T(t) \left(-\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi} + q\mu^2 I_{Mm} + \sum_{n=1}^s b_n \hat{P}(C^n \otimes \Gamma^n) + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \hat{P} \right) \\ & \alpha(t) + 2\alpha^T(t)\hat{P}\hat{H}\beta(t) + \beta^T(t)A_1\beta(t) - \left(\alpha^T(t)\hat{Z}_1^T + \beta^T(t)\hat{Z}_2^T \right) F\beta(t) \\ & = \zeta^T(t) \begin{pmatrix} W_1 & E_1 \\ E_1^T & M_2 \end{pmatrix} \zeta(t). \end{aligned} \quad (30)$$

From (29), we have

$$\eta^T(t)F\beta(t) \geq \mathcal{L}V^1(t) + \beta^T(t)A_1\beta(t). \quad (31)$$

Similarly, we can derive

$$\mathbb{E} \left\{ \int_{t_0}^{t_p} \eta^T(t)F\beta(t) dt \right\} \geq \mathbb{E} \{ V^1(t_p) \} - \mathbb{E} \{ V^1(t_0) \} + \mathbb{E} \left\{ \int_{t_0}^{t_p} \beta^T(t)A_1\beta(t) dt \right\}. \quad (32)$$

Theorem 3. Network (15) is output-strictly passive if there exist matrices $F \in \mathbb{R}^{Mq \times Mp}$, $0 < A_2 \in \mathbb{R}^{Mq \times Mq}$, and $0 < \hat{P} \in \mathbb{R}^{Mm \times Mm}$ satisfying

$$\begin{pmatrix} W_2 & E_2 \\ E_2^T & M_3 \end{pmatrix} \leq 0, \quad (33)$$

where $W_2 = W_1 + \hat{Z}_1^T A_2 \hat{Z}_1$, $E_2 = E_1 + \hat{Z}_1^T A_2 \hat{Z}_2$, and $M_3 = M_1 + \hat{Z}_2^T A_2 \hat{Z}_2$.

Proof. We select the same $V^1(t)$ as (19) for network (15), and we can easily obtain

$$\begin{aligned} & \mathcal{L}V^1(t) - \eta^T(t)F\beta(t) + \eta^T(t)A_2\eta(t) \\ & \leq \alpha^T(t) \left(-\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi} + q\mu^2 I_{Mm} + \sum_{n=1}^s b_n \hat{P}(C^n \otimes \Gamma^n) + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \hat{P} \right) \\ & \alpha(t) + 2\alpha^T(t)\hat{P}\hat{H}\beta(t) + \left(\alpha^T(t)\hat{Z}_1^T + \beta^T(t)\hat{Z}_2^T \right) A_2 \left(\hat{Z}_1\alpha(t) + \hat{Z}_2\beta(t) \right) \\ & - \left(\alpha^T(t)\hat{Z}_1^T + \beta^T(t)\hat{Z}_2^T \right) F\beta(t) \\ & = \zeta^T(t) \begin{pmatrix} W_2 & E_2 \\ E_2^T & M_3 \end{pmatrix} \zeta(t). \end{aligned} \quad (34)$$

From (47), one has

$$\eta^T(t)F\beta(t) \geq \mathcal{L}V^1(t) + \eta^T(t)A_2\eta(t). \quad (35)$$

Similarly, we can derive

$$\mathbb{E}\left\{\int_{t_0}^{t_p} \eta^T(t) F \beta(t) dt\right\} \geq \mathbb{E}\{V^1(t_p)\} - \mathbb{E}\{V^1(t_0)\} + \mathbb{E}\left\{\int_{t_0}^{t_p} \eta^T(t) A_2 \eta(t) dt\right\}. \quad (36)$$

3.3. Synchronization in Passive MWCSNNs

Definition 4. The network (8) with input $\beta_z(t) = 0$ ($z = 1, 2, \dots, M$) achieves synchronization if

$$\lim_{t \rightarrow +\infty} \mathbb{E}\{\|\kappa_z(t) - s(t)\|\} = 0, \quad z = 1, 2, \dots, M. \quad (37)$$

Theorem 4. If network (15) is output-strictly passive with regard to storage function $K(t) = V^1(t)/2$ and $Z_1 \in \mathbb{R}^{m \times m}$ is nonsingular, then MWCSNNs (8) can achieve synchronization.

Proof. If network (15) is output-strictly passive with respect to storage function $K(t)$, then there exist matrices $\mathbb{R}^{Mm \times Mm} \ni A_2 > 0$ and $F \in \mathbb{R}^{Mm \times Mm}$ such that

$$\mathbb{E}\left\{\int_t^{t+\delta} \eta^T(s) F \beta(s) ds\right\} \geq \mathbb{E}\{K(t+\delta)\} - \mathbb{E}\{K(t)\} + \mathbb{E}\left\{\int_t^{t+\delta} \eta^T(s) A_2 \eta(s) ds\right\}, \quad (38)$$

for any $t \in \mathbb{R}^+$ and $\delta > 0$. Then, we can easily obtain

$$\mathbb{E}\left\{\frac{K(t+\delta) - K(t)}{\varepsilon}\right\} \leq \mathbb{E}\left\{\frac{\int_t^{t+\delta} \eta^T(s) F \beta(s) ds}{\varepsilon} - \frac{\int_t^{t+\delta} \eta^T(s) A_2 \eta(s) ds}{\varepsilon}\right\}. \quad (39)$$

By taking limit $\delta \rightarrow 0$ in (39), one has

$$\mathbb{E}\{\dot{K}(t) \leq \mathbb{E}\eta^T(s) F \beta(s) - \eta^T(s) A_2 \eta(s)\}. \quad (40)$$

Letting $\beta_z(t) = 0$ ($z = 1, 2, \dots, M$), we can get from (40) that

$$\begin{aligned} \mathbb{E}\{\dot{K}(t) \leq -\mathbb{E}\alpha^T(t) \widehat{Z}_1^T A_2 \widehat{Z}_1 \alpha(t)\} \\ \leq -\lambda_m(Z_1^T Z_1) \lambda_m(A_2) \mathbb{E}\{\|\alpha(t)\|^2\}. \end{aligned} \quad (41)$$

From (41) and definition of $V^1(t)$, we can get that $\lim_{t \rightarrow +\infty} \mathbb{E}\{K(t)\}$ exists. Therefore, we can conclude that

$$\lim_{t \rightarrow +\infty} \mathbb{E}\{\|\alpha(t)\|^2\} = \theta \geq 0. \quad (42)$$

Suppose that

$$\lim_{t \rightarrow +\infty} \mathbb{E}\{\|\alpha(t)\|^2\} = \theta > 0. \quad (43)$$

Then, there obviously exists $0 < t^* \in \mathbb{R}$ such that

$$\mathbb{E}\{\|\alpha(t)\|^2\} > \frac{\theta}{2}, \quad t \geq t^*. \quad (44)$$

By (41) and (44), we have

$$\mathbb{E}\{\dot{K}(t)\} < -\frac{\lambda_m(Z_1^T Z_1) \lambda_m(A_2) \theta}{2}, \quad t \geq t^*. \quad (45)$$

From (45), we derive

$$\begin{aligned} \mathbb{E}\{-K(t^*)\} &\leq \mathbb{E}\{K(+\infty) - K(t^*)\} \\ &= \int_{t^*}^{+\infty} \mathbb{E}\{\dot{K}(t)\} dt \\ &< - \int_{t^*}^{+\infty} \frac{\lambda_m(Z_1^T Z_1) \lambda_m(A_2) \theta}{2} dt \\ &= -\infty, \end{aligned} \quad (46)$$

which results in a contradiction.

Hence, $\lim_{t \rightarrow +\infty} \mathbb{E}\{\|\alpha(t)\|\} = 0$. That is, network (8) realizes synchronization.

The following conclusion can be obtained from Theorems 3 and 4. $\square$

Corollary 1. Network (8) achieves synchronization if there exist matrices $F \in \mathbb{R}^{Mm \times Mm}$, $0 < A_2 \in \mathbb{R}^{Mm \times Mm}$, and $0 < \widehat{P} \in \mathbb{R}^{Mm \times Mm}$ satisfying

$$\begin{pmatrix} W_2 & E_2 \\ E_2^T & M_3 \end{pmatrix} \leq 0, \quad (47)$$

where $W_2 = -\widehat{P}\widehat{D} - \widehat{D}\widehat{P} + \widehat{P}\widehat{G}\widehat{G}^T\widehat{P} + \widehat{\xi} + q\mu^2 I_{Mm} + \sum_{n=1}^s b_n \widehat{P}(C^n \otimes I^n) + \sum_{n=1}^s b_n (C^n \otimes I^n) \widehat{P} + \widehat{Z}_1^T A_2 \widehat{Z}_1$, $E_2 = \widehat{P}\widehat{H} - (\widehat{Z}_1^T F)/2 + \widehat{Z}_1^T A_2 \widehat{Z}_2$, $M_3 = -(\widehat{Z}_2^T F + F^T \widehat{Z}_2)/2 + \widehat{Z}_2^T A_2 \widehat{Z}_2$, and $q = \lambda_m(\widehat{P})$.

4. Passivity for CSNNMDCs

4.1. *Network Model.* The CSNNMDCs in this paper is considered as follows:

$$d\kappa_z(t) = \left(-D\kappa_z(t) + Gf(\kappa_z(t)) + B + \sum_{n=1}^s \sum_{h=1}^M b_n C_{zh}^n \Gamma^n \kappa_h(t - \tau_n) + H\beta_z(t) \right) dt + \sigma(\kappa_z(t)) d\omega(t), \quad z = 1, 2, \dots, M, \quad (48)$$

where $\tau_n (n = 1, 2, \dots, s)$ are coupling delays and $\kappa_z(t), \beta_z(t), f(\kappa_z(t)), D, G, B, H, b_n, C_{zh}^n$, and Γ^n have the same meanings as those in Section 3.

Suppose that $s(t) = (s_1(t), s_2(t), \dots, s_m(t))^T \in \mathbb{R}^m$ is an arbitrary desired solution of the isolated node of system (48), then it satisfies

$$ds(t) = (-Ds(t) + Gf(s(t)) + B)dt + \sigma(s(t))d\omega(t). \quad (49)$$

Letting $\alpha_z(t) = (\alpha_{z1}(t), \alpha_{z2}(t), \dots, \alpha_{zm}(t))^T = \kappa_z(t) - s(t)$, we can obtain from (48) and (49) that

$$d\alpha_z(t) = \left(-D\alpha_z(t) + Gf(\kappa_z(t)) - Gf(s(t)) + \sum_{n=1}^s \sum_{j=1}^M b_n C_{zj}^n \Gamma^n \alpha_j(t - \tau_n) + H\beta_z(t) \right) dt + (\sigma(\kappa_z(t)) - \sigma(s(t)))d\omega(t). \quad (50)$$

The output vector $\eta_z(t) \in \mathbb{R}^q$ of network (48) is defined as follows:

$$\eta_z(t) = Z_1 \alpha_z(t) + Z_2 \beta_z(t), \quad (51)$$

where $Z_1 \in \mathbb{R}^{q \times m}$ and $Z_2 \in \mathbb{R}^{q \times p}$.

According to (50), we have

$$d\alpha(t) = \left(-\hat{D}\alpha(t) + \hat{G}(f(\kappa(t)) - f(s(t))) + \hat{H}\beta(t) + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \alpha(t - \tau_n) \right) dt + \hat{\sigma}(t) d\hat{\omega}(t), \quad (52)$$

$$\eta(t) = \hat{Z}_1 \alpha(t) + \hat{Z}_2 \beta(t),$$

where $\hat{\sigma}(t) = I_M \otimes (\sigma(\kappa(t)) - \sigma(s(t)))$.

Theorem 5. Network (51) is passive if there exist $F \in \mathbb{R}^{Mq \times Mp}, 0 < \hat{P} \in \mathbb{R}^{Mm \times Mm}$, and $\hat{N}_n = \text{diag}(N_n^1, N_n^2, \dots, N_n^M) \in \mathbb{R}^{Mm \times Mm}, n = 1, 2, \dots, s$, satisfying

$$\begin{pmatrix} W_3 & E_3 \\ E_3^T & M_4 \end{pmatrix} \leq 0, \quad (53)$$

where $W_3 = -\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi} + q\mu^2 I_{Mm} + \sum_{n=1}^s b_n (\hat{P}(C^n \otimes \Gamma^n))\hat{N}_n^{-1}((C^n \otimes \Gamma^n)\hat{P}) + \sum_{n=1}^s b_n \hat{N}_n, E_3 = \hat{P}\hat{H} - (\hat{Z}_1^T F)/2, M_4 = -(\hat{Z}_2^T F + F^T \hat{Z}_2)/2$, and $q = \lambda_M(\hat{P})$.

Proof. For convenience, we denote

$$y(t) = -\hat{D}\alpha(t) + \hat{G}(f(\kappa(t)) - f(s(t))) + \hat{H}\beta(t) + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \alpha(t - \tau_n), \quad (54)$$

then network (51) can be rewritten as

$$d\alpha(t) = y(t)dt + \hat{\sigma}(t)d\hat{\omega}(t). \quad (55)$$

Choose the following Lyapunov functional for network (51):

In light of Lemma 1, we can obtain

$$V^2(t) = \alpha^T(t) \hat{P} \alpha(t) + \sum_{n=1}^s b_n \int_{t-\tau_n}^t \alpha^T(h) \hat{N}_n \alpha(h) dh. \quad (56)$$

$$\begin{aligned} \mathcal{L}V^2(t) &= V_t^2(t) + V_\alpha^2(t) \gamma(t) + \frac{1}{2} \text{Tr}(\hat{\sigma}^T(t) V_{\alpha\alpha}^2(t) \hat{\sigma}(t)) \\ &= \sum_{n=1}^s b_n \alpha^T(t) \hat{N}_n \alpha(t) + 2\alpha^T(t) \hat{P} \left(-\hat{D}\alpha(t) + \hat{H}\beta(t) + \hat{G}(f(\kappa(t)) - f(s(t))) + \sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \alpha(t - \tau_n) \right) \\ &\quad - \sum_{n=1}^s b_n \alpha^T(t - \tau_n) \hat{N}_n \alpha(t - \tau_n) + \text{Tr}(\hat{\sigma}^T(t) \hat{P} \hat{\sigma}(t)). \end{aligned} \quad (57)$$

Moreover,

$$\begin{aligned} 2\alpha^T(t) \hat{P} \left(\sum_{n=1}^s b_n (C^n \otimes \Gamma^n) \alpha(t - \tau_n) \right) &\leq \sum_{n=1}^s b_n \alpha^T(t) \left((\hat{P}(C^n \otimes \Gamma^n)) \hat{N}_n^{-1} ((C^n \otimes \Gamma^n) \hat{P}) \right) \alpha(t) \\ &\quad + \sum_{n=1}^s b_n \alpha^T(t - \tau_n) \hat{N}_n \alpha(t - \tau_n). \end{aligned} \quad (58)$$

From (57) and (58), we have

$$\begin{aligned} \mathcal{L}V^2(t) &\leq \alpha^T(t) \left(-\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi} + \mu^2 q I_{Mm} + \sum_{n=1}^s b_n (\hat{P}(C^n \otimes \Gamma^n)) \hat{N}_n^{-1} ((C^n \otimes \Gamma^n) \hat{P}) + \sum_{n=1}^s b_n \hat{N}_n \right) \\ &\quad \alpha(t) + 2\alpha^T(t) \hat{P} \hat{H} \beta(t). \end{aligned} \quad (59)$$

From (59), we have

$$\begin{aligned} \mathcal{L}V^2(t) - \eta^T(t) F \beta(t) &\leq \alpha^T(t) \left(-\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi} + \mu^2 q I_{Mm} + \sum_{n=1}^s b_n (\hat{P}(C^n \otimes \Gamma^n)) \hat{N}_n^{-1} ((C^n \otimes \Gamma^n) \hat{P}) + \sum_{n=1}^s b_n \hat{N}_n \right) \\ &\quad \alpha(t) + 2\alpha^T(t) \hat{P} \hat{H} \beta(t) - \left(\alpha^T(t) \hat{Z}_1^T + \beta^T(t) \hat{Z}_2^T \right) F \beta(t) \\ &= \zeta^T(t) \begin{pmatrix} W_3 & E_3 \\ E_3^T & M_4 \end{pmatrix} \zeta(t). \end{aligned} \quad (60)$$

By (53), we can acquire

$$\mathbb{E} \left\{ \int_{t_0}^{t_p} \eta^T(t) F \beta(t) dt \right\} \geq \mathbb{E} \{ V^2(t_p) \} - \mathbb{E} \{ V^2(t_0) \}. \quad (61)$$

By employing similar proof methods in Theorem 5, we can get the following conclusions. $\square$

Theorem 6. Network (51) is input-strictly passive if there exist matrices $F \in \mathbb{R}^{Mq \times Mp}$, $0 < A_1 \in \mathbb{R}^{Mp \times Mp}$, $0 < \hat{P} \in \mathbb{R}^{Mm \times Mm}$, and $\hat{N}_n = \text{diag}(N_n^1, N_n^2, \dots, N_n^M) \in \mathbb{R}^{Mm \times Mm}$, $n = 1, 2, \dots, s$, satisfying

$$\begin{pmatrix} W_3 & E_3 \\ E_3^T & M_5 \end{pmatrix} \leq 0, \quad (62)$$

where W_3 and E_3 have the same meanings as those in Theorem 5 and $M_5 = A_1 + M_4$.

Theorem 7. Network (51) is output-strictly passive if there exist matrices $F \in \mathbb{R}^{Mq \times Mp}$, $0 < A_2 \in \mathbb{R}^{Mq \times Mq}$, $0 < \hat{P} \in \mathbb{R}^{Mm \times Mm}$, and $\hat{N}_n = \text{diag}(N_n^1, N_n^2, \dots, N_n^M) \in \mathbb{R}^{Mm \times Mm}$, $n = 1, 2, \dots, s$, satisfying

$$\begin{pmatrix} W_4 & E_4 \\ E_4^T & M_6 \end{pmatrix} \leq 0, \quad (63)$$

where $W_4 = W_3 + \hat{Z}_1^T A_2 \hat{Z}_1$, $E_4 = E_3 + \hat{Z}_1^T A_2 \hat{Z}_2$, and $M_6 = M_4 + \hat{Z}_2 A_2 \hat{Z}_2$.

4.2. Synchronization in Passive CSNNMDCs

Theorem 8. The CSNNMDCs (48) can achieve synchronization if network (51) is output-strictly passive with regard to

storage function $K(t) = V^2(t)/2$, and $Z_1 \in \mathbb{R}^{m \times m}$ is nonsingular.

Proof. The results can be easily obtained by employing similar proof method in the Theorem 4.

The following conclusion can be obtained from Theorems 7 and 8. $\square$

Corollary 2. Network (48) achieves synchronization if there exist matrices $F \in \mathbb{R}^{Mm \times Mm}$, $0 < A_2 \in \mathbb{R}^{Mm \times Mm}$, $0 < \hat{P} \in \mathbb{R}^{Mm \times Mm}$, and $\hat{N}_n = \text{diag}(N_n^1, N_n^2, \dots, N_n^M) \in \mathbb{R}^{Mm \times Mm}$, $n = 1, 2, \dots, s$, satisfying

$$\begin{pmatrix} W_4 & E_4 \\ E_4^T & M_6 \end{pmatrix} \leq 0, \quad (64)$$

where $W_4 = -\hat{P}\hat{D} - \hat{D}\hat{P} + \hat{P}\hat{G}\hat{G}^T\hat{P} + \hat{\xi} + q\mu^2 I_{Mm} + \sum_{n=1}^s b_n (\hat{P}(C^n \otimes \Gamma^n))\hat{N}_n^{-1}((C^n \otimes \Gamma^n)\hat{P}) + \sum_{n=1}^s b_n \hat{N}_n + \hat{Z}_1^T A_2 \hat{Z}_1$, $E_4 = \hat{P}\hat{H} - (\hat{Z}_1^T F)/2 + \hat{Z}_1^T A_2 \hat{Z}_2$, $M_6 = -(\hat{Z}_2^T F + F^T \hat{Z}_2)/2 + \hat{Z}_2^T A_2 \hat{Z}_2$, and $q = \lambda_M(\hat{P})$.

Remark 2. In this paper, two types of network models are proposed (see (8) and (48)), some sufficient conditions for ensuring the passivity of networks (8) and (48) are acquired by employing the stochastic analysis techniques and Lyapunov functional method (see Theorems 1–3 and Theorems 5–7), and several synchronization criteria for networks (8) and (48) are established in view of the output-strictly passivity (see Corollaries 1 and 2).

5. Numerical Examples

Example 1. Take the following MWCSNNs into consideration:

$$d\kappa_z(t) = \left(-D\kappa_z(t) + Gf(\kappa_z(t)) + B + 0.4 \sum_{h=1}^6 C_{zh}^1 \Gamma^1 \kappa_h(t) + 0.2 \sum_{h=1}^6 C_{zh}^2 \Gamma^2 \kappa_h(t) + 0.1 \sum_{h=1}^6 C_{zh}^3 \Gamma^3 \kappa_h(t) + H\beta_z(t) \right) dt + \sigma(\kappa_z(t)) d\omega(t), \quad (65)$$

where $z = 1, 2, \dots, 6$, $f_i(j) = 0.25(|j+1| - |j-1|)$, $i = 1, 2, 3$, $D = \text{diag}(0.5, 0.2, 0.4)$, $B = (0, 0, 0)^T$, $\Gamma^1 = \text{diag}(0.9, 0.4, 0.3)$, Γ^2

$= \text{diag}(0.8, 0.3, 0.4)$, $\Gamma^3 = \text{diag}(0.6, 0.3, 0.8)$, $\sigma(\kappa_z(t)) = \text{diag}(0.2\kappa_{z1}(t), 0.4\kappa_{z2}(t), 0.3\kappa_{z3}(t))$,

$$\begin{aligned}
G &= \begin{pmatrix} 0.3 & 0.5 & 0.1 \\ 0.2 & 0.1 & 0.2 \\ 0.12 & 0.1 & 0.4 \end{pmatrix}, \\
H &= \begin{pmatrix} 0.4 & 0.6 \\ 0.8 & 0.3 \\ 0.5 & 0.7 \end{pmatrix}, \\
C^1 &= \begin{pmatrix} 0.8 & 0.2 & 0 & 0.1 & 0.2 & 0.3 \\ 0.2 & -0.6 & 0.1 & 0.2 & 0.1 & 0 \\ 0 & 0.1 & -0.5 & 0 & 0.3 & 0.1 \\ 0.1 & 0.2 & 0 & -0.8 & 0.2 & 0.3 \\ 0.2 & 0.1 & 0.3 & 0.2 & -0.9 & 0.1 \\ 0.3 & 0 & 0.1 & 0.3 & 0.1 & -0.8 \end{pmatrix}, \\
C^2 &= \begin{pmatrix} -0.5 & 0.2 & 0 & 0.1 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.1 & 0.1 & 0.2 & 0 \\ 0 & 0.1 & -0.8 & 0 & 0.3 & 0.4 \\ 0.1 & 0.1 & 0 & -0.6 & 0.1 & 0.3 \\ 0.1 & 0.2 & 0.3 & 0.1 & -0.8 & 0.1 \\ 0.1 & 0 & 0.4 & 0.3 & 0.1 & -0.9 \end{pmatrix}, \\
C^3 &= \begin{pmatrix} -0.9 & 0.4 & 0 & 0.2 & 0.2 & 0.1 \\ 0.4 & -0.8 & 0.2 & 0.1 & 0.1 & 0 \\ 0 & 0.2 & -0.5 & 0 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0 & -0.8 & 0.4 & 0.1 \\ 0.2 & 0.1 & 0.1 & 0.4 & -0.9 & 0.1 \\ 0.1 & 0 & 0.2 & 0.1 & 0.1 & -0.5 \end{pmatrix}.
\end{aligned} \tag{66}$$

Apparently, $s(t) = (0, 0, 0)^T \in \mathbb{R}^3$ is a desired solution of an isolated node of network (65), and $f_i(\cdot)$ meets Assumption 1 with $\xi_l = 0.5$.

The $\eta_z(t) \in \mathbb{R}^3$ is selected as

$$\eta_z(t) = Z_1 \alpha_z(t) + Z_2 \beta_z(t), \quad z = 1, 2, \dots, 6, \tag{67}$$

where $\beta_z(t) = (0.6z * \sin(t), 0.4z * \sin(t))^T$,

$$\begin{aligned}
Z_1 &= \begin{pmatrix} 0.3 & 0.1 & 0.5 \\ 0.8 & 0.5 & 0.4 \\ 0.4 & 0.2 & 0.7 \end{pmatrix}, \\
Z_2 &= \begin{pmatrix} 0.7 & 0.6 \\ 0.8 & 0.5 \\ 0.2 & 0.3 \end{pmatrix}.
\end{aligned} \tag{68}$$

Case 1: the following matrices F and $\hat{P}$ can be obtained:

$$\begin{aligned}
F &= I_6 \otimes \begin{pmatrix} -0.4836 & 0.3568 \\ 1.5425 & -0.9371 \\ 0.0897 & 2.7229 \end{pmatrix}, \\
\hat{P} &= I_6 \otimes \begin{pmatrix} 0.6443 & -0.0057 & -0.0834 \\ -0.0057 & 0.7983 & -0.2136 \\ -0.0834 & -0.2136 & 1.1224 \end{pmatrix}.
\end{aligned} \tag{69}$$

From Theorem 1, network (65) is passive and Figures 1 and 2 display the simulation results.

Case 2: the following matrices F , $\hat{P}$, and A_2 that satisfy the condition of Theorem 3 can be obtained:

$$\begin{aligned}
F &= I_6 \otimes \begin{pmatrix} 41.6646 & 36.6249 \\ 4.1455 & -8.3129 \\ -30.5560 & -11.0012 \end{pmatrix}, \\
\hat{P} &= I_6 \otimes \begin{pmatrix} 1.9525 & 0.3124 & -1.6051 \\ 0.3124 & 3.8069 & -3.2750 \\ -1.6051 & -3.2750 & 6.8604 \end{pmatrix}, \\
A_2 &= I_6 \otimes \begin{pmatrix} 20.0009 & -0.5782 & -12.8691 \\ -0.5782 & 1.0321 & -0.9658 \\ -12.8691 & -0.9658 & 10.1673 \end{pmatrix}.
\end{aligned} \tag{70}$$

From Theorem 4, network (65) is synchronized, Figure 3 demonstrates the effectiveness and correctness of the obtained results.

Example 2. Take the following CSNNMDCs into consideration:

$$\begin{aligned}
d\kappa_z(t) &= \left(-D\kappa_z(t) + Gf(\kappa_z(t)) + B + 0.2 \sum_{h=1}^6 C_{zh}^1 \Gamma^1 \kappa_h(t-0.5) \right. \\
&\quad \left. + 0.1 \sum_{h=1}^6 C_{zh}^2 \Gamma^2 \kappa_h(t-0.2) + 0.5 \sum_{h=1}^6 C_{zh}^3 \Gamma^3 \kappa_h(t-0.3) + H\beta_z(t) \right) dt \\
&\quad + \sigma(\kappa_z(t)) d\omega(t),
\end{aligned} \tag{71}$$

where $z = 1, 2, \dots, 6$, $f_i(j) = 0.25(|j+1| - |j-1|)$, $i = 1, 2, 3$, $D = \text{diag}(0.8, 0.9, 0.7)$, $B = (0, 0, 0)^T$, $\Gamma^1 = \text{diag}(0.5, 0.3, 0.2)$, Γ^2

$= \text{diag}(0.2, 0.5, 0.3)$, $\Gamma^3 = \text{diag}(0.3, 0.2, 0.5)$, $\sigma(\kappa_z(t)) = \text{diag}(0.4\kappa_{z1}(t), 0.3\kappa_{z2}(t), 0.5\kappa_{z3}(t))$,

FIGURE 1: $\alpha_{z1}(t)$, $\alpha_{z2}(t)$, and $\alpha_{z3}(t)$, $z = 1, 2, \dots, 6$. FIGURE 2: $\|\eta_z(t)\|$, $z = 1, 2, \dots, 6$.

FIGURE 3: $\alpha_{z1}(t)$, $\alpha_{z2}(t)$, and $\alpha_{z3}(t)$, $z = 1, 2, \dots, 6$.

FIGURE 4: Continued.

FIGURE 4: $\alpha_{z1}(t)$, $\alpha_{z2}(t)$, and $\alpha_{z3}(t)$, $z = 1, 2, \dots, 6$.

$$\begin{aligned}
 G &= \begin{pmatrix} 0.1 & 0.3 & 0.1 \\ 0.2 & 0.1 & 0.3 \\ 0.1 & 0.2 & 0.1 \end{pmatrix}, & Z_1 &= \begin{pmatrix} 0.5 & 0.7 & 0.9 \\ 0.6 & 0.4 & 0.6 \\ 0.2 & 0.5 & 0.1 \end{pmatrix}, \\
 H &= \begin{pmatrix} 0.2 & 0.1 \\ 0.5 & 0.3 \\ 0.1 & 0.4 \end{pmatrix}, & Z_2 &= \begin{pmatrix} 0.8 & 0.6 \\ 0.5 & 0.9 \\ 0.7 & 0.4 \end{pmatrix}.
 \end{aligned} \tag{74}$$

The matrices $\hat{N}_1, \hat{N}_2$, and $\hat{N}_3$ are chosen as, respectively,

$$\begin{aligned}
 \hat{N}_1 &= I_6 \otimes \text{diag}(0.5, 0.6, 0.3), \\
 \hat{N}_2 &= I_6 \otimes \text{diag}(0.3, 0.4, 0.2), \\
 \hat{N}_3 &= I_6 \otimes \text{diag}(0.2, 0.1, 0.4).
 \end{aligned} \tag{75}$$

Case 1: the following matrices F and $\hat{P}$ can be obtained:

$$\begin{aligned}
 F &= I_6 \otimes \begin{pmatrix} 1.7842 & -0.9653 \\ -2.6726 & 3.2707 \\ 3.1171 & -0.5937 \end{pmatrix}, \\
 \hat{P} &= I_6 \otimes \begin{pmatrix} 1.4460 & -0.0495 & -0.0150 \\ -0.0495 & 1.3677 & -0.0831 \\ -0.0150 & -0.0831 & 1.6143 \end{pmatrix}.
 \end{aligned} \tag{76}$$

From Theorem 5, network (67) is passive, and Figures 4 and 5 display the simulation results.

Case 2: the following matrices $F, \hat{P}$, and A_2 that satisfy the condition of Theorem 7 can be obtained.

$$\begin{aligned}
 F &= I_6 \otimes \begin{pmatrix} 1.6654 & -7.2114 \\ -5.4735 & 10.4853 \\ 12.4571 & 2.6646 \end{pmatrix}, \\
 \hat{P} &= I_6 \otimes \begin{pmatrix} 1.8950 & -0.2464 & 0.1908 \\ -0.2464 & 2.3726 & -0.2827 \\ -0.1908 & -0.2827 & 1.5491 \end{pmatrix}, \\
 A_2 &= I_6 \otimes \begin{pmatrix} 1.4514 & -1.7789 & -0.2735 \\ -1.7789 & 2.5172 & -0.2573 \\ -0.2735 & -0.2573 & 1.7342 \end{pmatrix}.
 \end{aligned} \tag{77}$$

Apparently, $s(t) = (0, 0, 0)^T \in \mathbb{R}^3$ is a desired solution of an isolated node of network (67), and $f_i(\cdot)$ satisfies Assumption 1 with $\xi_i = 0.5$.

The $\eta_z(t) \in \mathbb{R}^3$ is selected as

$$\eta_z(t) = Z_1 \alpha_z(t) + Z_2 \beta_z(t), \quad z = 1, 2, \dots, 6, \tag{73}$$

where $\beta_z(t) = (0.4z * \sin(t), 0.5z * \sin(t))^T$,

From Theorem 8, network (67) is synchronized, and Figure 6 demonstrates the effectiveness and correctness of the obtained results.

FIGURE 5: $\|\eta_z(t)\|$, $z = 1, 2, \dots, 6$. FIGURE 6: $\alpha_{z1}(t)$, $\alpha_{z2}(t)$, and $\alpha_{z3}(t)$, $z = 1, 2, \dots, 6$.

Remark 3. More recently, some research results on the dynamical behaviors of CSNNs have been obtained, but they all discussed the single weighted network models [41–45]. In this paper, we respectively consider the passivity and synchronization of MWCSNNs (65) and CSNNMDCs (67), which are apparently different from these network models considered in [41–45]. Figures 1 and 2 (Figures 4 and 5) respectively show the change tendencies of $\alpha_{z1}(t)$, $\alpha_{z2}(t)$, $\alpha_{z3}(t)$, $z = 1, 2, \dots, 6$ and $\|\eta_z(t)\|$, $z = 1, 2, \dots, 6$ for networks (65) and (67). From Figures 3 and 6, we can explicitly see that $\alpha_{z1}(t)$, $\alpha_{z2}(t)$, and $\alpha_{z3}(t)$, $z = 1, 2, \dots, 6$, in networks (65) and (67), respectively, converge to 0 after 12s and 5s, which verify the correctness of the obtained synchronization criteria.

6. Conclusion

Two kinds of MWCSNNs models have been proposed, in which the dimension of output is incompatible with input. On the one hand, we have analyzed the passivity, input-strict passivity, and output-strict passivity for MWCSNNs by employing stochastic analysis techniques. Moreover, two synchronization criteria for MWCSNNs and CSNNMDCs have been derived on the basis of output-strict passivity. Finally, the correctness of the passivity and synchronization criteria has been verified through two numerical examples.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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