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Research Article

Finite-Time Passivity of Stochastic Coupled Complex Networks

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The finite-time passivity problem is, respectively, investigated for stochastic coupled complex networks (SCCNs) with and without time-varying delay. Firstly, we present several new concepts about finite-time passivity in the sense of expectation on the basis of existing passivity definition. By designing appropriate controllers, the finite-time passivity of SCCNs with and without time-varying delay is obtained. In addition, the definition of finite-time synchronization in the sense of expectation is proposed. Under some sufficient conditions and designed controllers, finite-time passivity derives finite-time synchronization. Finally, two examples are given to demonstrate the effectiveness of finite-time passive and synchronization criteria.

1. Introduction

In the real world, complex networks can be seen everywhere such as food webs, communication networks, World Wide Web, and many others [1–3]. Due to various uncertainties in the actual system, complex network systems may be affected by noise. In recent years, the stability of stochastic systems has been extensively studied. At the same time, the synchronization and stability of stochastic complex networks have gradually become a topic of widespread concern for scholars in various fields [4–8].

Passivity is one part of dissipativeness. The main property of passivity is keeping the systems internally stable. The passivity theory has been extensively applied in many fields such as stability, complexity, signal processing, chaos control, synchronization fuzzy control, and so on [9–12]. These are the main reasons why the passivity theory has been one of the most active research areas. In [13], the problem of passivity analysis was studied for discrete-time stochastic Markovian jump neural networks with both discrete and distributed delays. In [14], the problem of passivity analysis is investigated for a class of discrete-time stochastic neural networks with time-varying delays.

It is well known that passivity theory can provide a powerful tool to analyze synchronization of complex networks. However, in many existing works, synchronization is defined over the infinite time interval. Most of the theoretical methods on the synchronization of complex networks can only realize the network [15] or exponential asymptotical synchronization [16] which guarantees that error tends to 0 when t tends to infinity. That is to say, achieving asymptotically stable convergence will be in infinite time. No further consideration has been given to the time and speed of synchronization. However, in practical engineering, people usually expect faster convergence rate and predict the required convergence time. Consequently, in order to achieve better control, the idea of finite-time synchronization has been proposed, and more and more attention has been paid by researchers. This kind of method can predict the synchronization time in advance and has better robustness, anti-interference, and better control effect. It has important research significance in theory and practice. Therefore, it is more meaningful to study finite-time synchronization [17–21]. In [22], the authors study finite-time passivity of multi-weighted coupled neural networks with and without coupling delays. As far as we know, very few scholars have discussed finite-time passivity of stochastic complex networks in recent years.

Motivated by the above discussions, we will investigate finite-time passivity of stochastic coupled complex networks (SCCNs). The main novelty and contributions of this paper can be summarized as follows. Firstly, we give three concepts of finite-time passivity in the sense of expectation. Secondly, we develop several finite-time passivity criteria. Lastly, we establish the relationship between finite-time passivity and finite-time synchronization in the sense of expectation.

2. Lemmas and Definitions

In this section, we will give some lemmas and definitions.

2.1. Lemmas

Lemma 1 (see [23]). *Assume that a continuous, positive-definite function $W(t)$ satisfies the following differential inequality:*

$$\dot{W}(t) \le -\varrho W^{\mu}(t), \quad t \ge t_0, W(t_0) \ge 0, \tag{1}$$

where $\varrho > 0$ and $0 < \mu < 1$ are constants. Then, for any given t_0, $W(t)$ satisfies the following inequality:

$$\begin{aligned} W^{1-\mu}(t) &\le W^{1-\mu}(t_0) - \varrho(1-\mu)(t-t_0), \quad t_0 \le t \le t_1, \\ W(t) &\equiv 0, \quad t \ge t_1, \end{aligned} \tag{2}$$

with t_1 given by

$$t_1 = t_0 + \frac{W^{1-\mu}(t_0)}{\varrho(1-\mu)}. \tag{3}$$

Lemma 2 (see [24]). *For any $b_i \in \mathbb{R}, i = 1, \ldots, n, 0 < p \le 1$, the following inequality holds:*

$$\left(\sum_{i=1}^{n} |b_i|\right)^{p} \le \sum_{i=1}^{n} |b_i|^{p}. \tag{4}$$

Lemma 3 (see [25]). *For any vectors $x, y \in \mathbb{R}^n$ and matrix $0 < P \in \mathbb{R}^{n\times n}$, the following inequality holds:*

$$x^T y + y^T x \le x^T P x + y^T P^{-1} y. \tag{5}$$

2.2. Definitions. Next, we will give three definitions about finite-time passivity in the sense of expectation. $E\{\cdot\}$ in these definitions stands for the mathematical expectation operator with respect to the given probability.

Definition 1. A stochastic system with input $u(t) \in \mathbb{R}^n$ and output $y(t) \in \mathbb{R}^n$ is said to be finite-time passive in the sense of expectation if there exists a nonnegative function V such that

$$E\left\{u^T(t)y(t)\right\} \ge \frac{E\{\mathrm{d}V(t)\}}{\mathrm{d}t} + \beta E\{V^{\alpha}(t)\}, \tag{6}$$

for some $\alpha \in (0,1)$ and $\beta > 0$.

Definition 2. A stochastic system with input $u(t) \in \mathbb{R}^n$ and output $y(t) \in \mathbb{R}^n$ is finite-time input strictly passive in the sense of expectation if there exists a nonnegative function V such that

$$E\left\{u^T(t)y(t)\right\} - \gamma_1 E\left\{u^T(t)u(t)\right\} \ge \frac{E\{\mathrm{d}V(t)\}}{\mathrm{d}t} + \beta E\{V^{\alpha}(t)\}, \tag{7}$$

for some $\alpha \in (0,1), \beta > 0$, and $\gamma_1 > 0$.

Definition 3. A stochastic system with input $u(t) \in \mathbb{R}^n$ and output $y(t) \in \mathbb{R}^n$ is finite-time output strictly passive in the sense of expectation if there exists a nonnegative function V such that

$$E\left\{u^T(t)y(t)\right\} - \gamma_2 E\left\{y^T(t)y(t)\right\} \ge \frac{E\{\mathrm{d}V(t)\}}{\mathrm{d}t} + \beta E\{V^{\alpha}(t)\}, \tag{8}$$

for some $\alpha \in (0,1), \beta > 0$, and $\gamma_2 > 0$.

Definition 4 (see [25]). Let $A = (a_{ij})_{m\times m} \in \mathbb{R}^{m\times n}$ and $B = (b_{ij})_{p\times q} \in \mathbb{R}^{p\times q}$. Then, the Kronecker product of A and B is defined as the matrix

$$A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \cdots & a_{2n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{pmatrix} \in \mathbb{R}^{mp\times nq}. \tag{9}$$

Throughout this paper, we make the following assumptions.

(H1) (see [26]) The function $f(\cdot)$ is in the QUAD class, that is, there exist diagonal matrices $0 < P = \mathrm{diag}(p_1, p_2, \ldots, p_n) \in \mathbb{R}^{n\times n}$ and $\Delta = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_n) \in \mathbb{R}^{n\times n}$, such that

$$(x-y)^T P[f(x) - f(y) - \Delta(x-y)] \le -\lambda(x-y)^T(x-y), \tag{10}$$

for all $x, y \in \mathbb{R}^n$ and some $\lambda > 0$.

(H2) For arbitrary $u, v \in \mathbb{R}^n$, there exists a positive constant L such that the following inequality holds:

$$\mathrm{trace}[h(u) - h(v)]^T[h(u) - h(v)] \le L(u-v)^T(u-v). \tag{11}$$

Remark 1 (see [25]). It can be verified that many of the benchmark chaotic systems belong to “function class QUAD,” such as the Lorenz system, the Chen system, and the *Lü* system.

3. Finite-Time Passivity of SCCNs

3.1. Network Model. In this paper, we will consider the following stochastic coupled complex networks model:

$$\mathrm{d}z_i(t) = \left[f(z_i(t)) + a \sum_{j=1}^{N} G_{ij}\Gamma z_j(t) + u_i(t) + v_i(t) \right]\mathrm{d}t + h(z_i(t))\mathrm{d}\omega(t), \tag{12}$$

where $z_i(t) = (z_{i1}(t), z_{i2}(t), \ldots, z_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith node; N corresponds to the number of neurons; $f(z_i(t)) = (f_1(z_{i1}(t)), f_2(z_{i2}(t)), \ldots, f_n(z_{in}(t)))^T \in \mathbb{R}^n$ denotes the neuron activation function and satisfies assumption (H1); $u_i(t) = (u_{i1}(t), u_{i2}(t), \ldots, u_{in}(t))^T \in \mathbb{R}^n$ is a varying external input vector; $v_i(t) = (v_{i1}(t), v_{i2}(t), \ldots, v_{in}(t))^T \in \mathbb{R}^n$ denotes the control input; $h(\cdot) \in \mathbb{R}^{n\times n}$ satisfies assumption (H2); $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t))^T \in \mathbb{R}^n$ is a $n-$dimensional Brownian motion defined on a complete probability space (Ω, P); a is a positive real number which represents the overall coupling strength; Γ denotes the inner coupling matrix; and $G = (G_{ij})_{N\times N}$ represents the topological structure of the network, where G_{ij} is defined as follows: if there exists a connection between node i and node j, then $G_{ij} = G_{ji} > 0$; otherwise, $G_{ij} = G_{ji} = 0$, $(i \neq j)$, and the diagonal elements of matrix G are defined by

$$G_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}, \quad i = 1, 2, \ldots, N. \tag{13}$$

3.2. Finite-Time Passivity. Set synchronization function $\overline{z}(t)$ satisfies

$$\mathrm{d}\overline{z}(t) = f(\overline{z}(t))\mathrm{d}t + h(\overline{z}(t))\mathrm{d}\omega(t), \tag{14}$$

where $\overline{z}(t) = (\overline{z}_1(t), \overline{z}_2(t), \ldots, \overline{z}_n(t)) \in \mathbb{R}^n$.

Define $e_i(t) = z_i(t) - \overline{z}(t)$, $i = 1, 2, \ldots, N$. Then, we have

$$\begin{aligned}\mathrm{d}e_i(t) = {} & \Big[f(z_i(t)) - f(\overline{z}(t)) \\ & + a\sum_{j=1}^{N} G_{ij}\Gamma e_j(t) + u_i(t) + v_i(t)\Big]\mathrm{d}t \\ & + \left[h(z_i(t)) - h(\overline{z}(t))\right]\mathrm{d}\omega(t),\end{aligned} \tag{15}$$

where $i = 1, 2, \ldots, N$.

$y_i(t) \in \mathbb{R}^n$ refers to the output vector of (15) and is defined as follows:

$$y_i(t) = A_1 e_i(t) + A_2 u_i(t), \tag{16}$$

where $A_1, A_2 \in \mathbb{R}^{n\times n}$,

The controller for network (12) is defined as follows:

$$\begin{aligned}v_i(t) = {} & -Q_i\left(z_i(t) - \overline{z}(t)\right) \\ & - \beta P^{((\alpha-1)/2)}\mathrm{sign}\left(z_i(t) - \overline{z}(t)\right)\left|z_i(t) - \overline{z}(t)\right|^{\alpha},\end{aligned} \tag{17}$$

where $Q_i \in \mathbb{R}^{n\times n}$, $0 < \alpha < 1, \beta > 0, P$ is defined in (H1), and

$$\begin{aligned}\mathrm{sign}\left(z_i(t) - \overline{z}(t)\right) &= \mathrm{diag}\left(\mathrm{sign}\left(z_{i1}(t) - \overline{z}_1(t)\right), \mathrm{sign}\left(z_{i2}(t) - \overline{z}_2(t)\right), \ldots, \mathrm{sign}\left(z_{in}(t) - \overline{z}_n(t)\right)\right), \\ \left|z_i(t) - \overline{z}(t)\right|^{\alpha} &= \left(\left|z_{i1}(t) - \overline{z}_1(t)\right|^{\alpha}, \left|z_{i2}(t) - \overline{z}_2(t)\right|^{\alpha}, \ldots, \left|z_{in}(t) - \overline{z}_n(t)\right|^{\alpha}\right)^T, \\ P^{((\alpha-1)/2)} &= \mathrm{diag}\left(p_1^{((\alpha-1)/2)}, p_2^{((\alpha-1)/2)}, \ldots, p_n^{((\alpha-1)/2)}\right).\end{aligned} \tag{18}$$

Theorem 1. *Under assumptions (H1) and (H2), network model (15) is finite-time passive in the sense of expectation under controller (17) if there exists matrix $Q = diag(Q_1, Q_2, \ldots, Q_N) \in \mathbb{R}^{nN\times nN}$ such that*

$$\begin{pmatrix} K_1 & E_1 \\ E_1^T & -I_N \otimes \dfrac{A_2 + A_2^T}{2} \end{pmatrix} \leq 0, \tag{19}$$

where

$$\begin{aligned}K_1 &= I_N \otimes \left(P\Delta + \Delta P - 2\lambda I_n + \lambda_0 I_n\right) - \left(I_N \otimes P\right)Q \\ &\quad - Q^T\left(I_N \otimes P\right) + aG \otimes \left(P\Gamma + \Gamma P\right), \\ E_1 &= I_N \otimes P - \frac{I_N \otimes A_1^T}{2}.\end{aligned} \tag{20}$$

Proof. For network (15), the Lyapunov functional is chosen as follows:

$$V_1(t) = \sum_{i=1}^{N} e_i^T(t) P e_i(t). \tag{21}$$

According to Ito's lemma, we acquire from (15) and (17)

$$\mathrm{d}V_1(t) = \mathscr{L}V_1(t)\mathrm{d}t + \Phi(t)\mathrm{d}\omega(t). \tag{22}$$

Here

$$\begin{aligned}\mathscr{L}V_1(t) = {} & 2\sum_{i=1}^{N} e_i^T(t)P\Big[f(z_i(t)) - f(\overline{z}(t)) + a\sum_{j=1}^{N} G_{ij}\Gamma e_j(t) \\ & + u_i(t) - Q_i e_i(t) - \beta P^{((\alpha-1)/2)}\mathrm{sign}\left(e_i(t)\right)\left|e_i(t)\right|^{\alpha}\Big] \\ & + \sum_{i=1}^{N} \mathrm{trace}\left[h(z_i(t)) - h(\overline{z}(t))\right]^T P\left[h(z_i(t)) - h(\overline{z}(t))\right], \\ \Phi(t) = {} & 2\sum_{i=1}^{N} e_i^T(t)P\left[h(z_i(t)) - h(\overline{z}(t))\right].\end{aligned} \tag{23}$$

According to (H1), we can obtain

$$\sum_{i=1}^{N} e_i^T(t)P\left(f\left(z_i(t)\right)-f\left(\overline{z}(t)\right)\right) \leq \sum_{i=1}^{N} e_i^T(t)\left(P\Delta-\lambda I_n\right)e_i(t). \tag{24}$$

We can get the following from (H2):

$$\begin{aligned}
&\operatorname{trace}\left[h\left(z_i(t)\right)-h\left(\overline{z}(t)\right)\right]^T P\left[h\left(z_i(t)\right)-h\left(\overline{z}(t)\right)\right] \\
&\quad\leq \lambda_M(P)\operatorname{trace}\left[h\left(z_i(t)\right)-h\left(\overline{z}(t)\right)\right]^T\left[h\left(z_i(t)\right)-h\left(\overline{z}(t)\right)\right] \\
&\quad\leq L\lambda_M(P)\left[z_i(t)-\overline{z}(t)\right]^T\left[z_i(t)-\overline{z}(t)\right]=\lambda_0 e_i^T(t)e_i(t).
\end{aligned} \tag{25}$$

Here $\lambda_M(P)$ represents maximum eigenvalue of matrix P, $\lambda_0 = L\lambda_M(P)$.

Thus,

$$\begin{aligned}
\mathscr{L}V_1(t) \leq{}& \sum_{i=1}^{N} e_i^T(t)\left(P\Delta+\Delta P-2\lambda I_n+\lambda_0 I_n\right)e_i(t) \\
&+2a\sum_{i=1}^{N}\sum_{j=1}^{N} G_{ij}e_i^T(t)P\Gamma e_j(t) \\
&+2\sum_{i=1}^{N} e_i^T(t)Pu_i(t)-2\sum_{i=1}^{N} e_i^T(t)PQ_i e_i(t) \\
&-2\beta\sum_{i=1}^{N} e_i^T(t)P^{((\alpha+1)/2)}\operatorname{sign}\left(e_i(t)\right)\left|e_i(t)\right|^{\alpha} \\
={}& e^T(t)\left[I_N\otimes\left(P\Delta+\Delta P-2\lambda I_n+\lambda_0 I_n\right)\right. \\
&-\left(I_N\otimes P\right)Q-Q^T\left(I_N\otimes P\right) \\
&\left.+aG\otimes\left(P\Gamma+\Gamma P\right)\right]e(t)+2e^T(t)\left(I_N\otimes P\right)u(t) \\
&-2\beta\sum_{i=1}^{N} e_i^T(t)P^{((\alpha+1)/2)}\operatorname{sign}\left(e_i(t)\right)\left|e_i(t)\right|^{\alpha},
\end{aligned} \tag{26}$$

where $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$, $u(t) = (u_1^T(t), u_2^T(t), \ldots, u_N^T(t))^T$.

Considering $\operatorname{sign}x \cdot x = |x|$, $(\forall x \in \mathbb{R})$ and Lemma 2, we can easily conclude that

$$\begin{aligned}
&\sum_{i=1}^{N} e_i^T(t)P^{((\alpha+1)/2)}\operatorname{sign}\left(e_i(t)\right)\left|e_i(t)\right|^{\alpha} \\
&\quad=\sum_{i=1}^{N}\sum_{j=1}^{n} p_j^{((\alpha+1)/2)}\left|e_{ij}(t)\right|^{\alpha+1} \\
&\quad\geq\sum_{i=1}^{N}\left(\sum_{j=1}^{n} p_j e_{ij}^2(t)\right)^{((\alpha+1)/2)} \\
&\quad=\sum_{i=1}^{N}\left(e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)}.
\end{aligned} \tag{27}$$

Set $y(t) = (y_1^T(t), y_2^T(t), \ldots, y_N^T(t))^T$; consequently,

$$\begin{aligned}
u^T(t)y(t) &= \sum_{i=1}^{N} u_i^T(t)y_i(t) \\
&= \sum_{i=1}^{N} u_i^T(t)\left[A_1 e_i(t)+A_2 u_i(t)\right] \\
&= u^T(t)\left(I_N\otimes A_1\right)e(t) \\
&\quad+u^T(t)\left(I_N\otimes A_2\right)u(t).
\end{aligned} \tag{28}$$

From (19) and (26)–(28),

$$\begin{aligned}
&\mathscr{L}V_1(t)-u^T(t)y(t) \\
&\quad\leq -2\beta\sum_{i=1}^{N}\left(e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)} \\
&\qquad+\zeta^T(t)\begin{pmatrix} K_1 & E_1 \\ E_1^T & -I_N\otimes\dfrac{A_2+A_2^T}{2}\end{pmatrix}\zeta(t) \\
&\quad\leq -2\beta\sum_{i=1}^{N}\left(e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)} \\
&\quad\leq -2\beta\left(\sum_{i=1}^{N} e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)} \\
&\quad= -2\beta V_1^{((\alpha+1)/2)}(t),
\end{aligned} \tag{29}$$

where $\zeta(t) = (e^T(t), u^T(t))^T$.

Considering $E\{\mathrm{d}V_1(t)\} = E\{\mathscr{L}V_1(t)\mathrm{d}t\}$, consequently

$$\begin{aligned}
&E\left\{\mathrm{d}V_1(t)-u^T(t)y(t)\mathrm{d}t\right\} \\
&\quad=E\left\{\left[\mathscr{L}V_1(t)-u^T(t)y(t)\right]\mathrm{d}t\right\} \\
&\quad\leq -2\beta E\left\{V_1^{((\alpha+1)/2)}(t)\mathrm{d}t\right\}.
\end{aligned} \tag{30}$$

Then, we can obtain

$$E\left\{u^T(t)y(t)\right\} \geq \frac{E\left\{\mathrm{d}V_1(t)\right\}}{\mathrm{d}t}+2\beta E\left\{V_1^{((\alpha+1)/2)}(t)\right\}. \tag{31}$$

Therefore, network (15) is finite-time passive in the sense of expectation under controller (17). □

Theorem 2. *Under assumptions (H1) and (H2), network model (15) is finite-time input strictly passive in the sense of expectation under controller (17) if there exist matrix* $Q = \operatorname{diag}(Q_1, Q_2, \ldots, Q_N) \in \mathbb{R}^{nN\times nN}$ *and a positive real number* γ_1 *such that*

$$\begin{pmatrix} K_1 & E_1 \\ E_1^T & \gamma_1 I_{nN}-I_N\otimes\dfrac{A_2+A_2^T}{2}\end{pmatrix}\leq 0, \tag{32}$$

where K_1, E_1 have the same meanings as in Theorem 1.

Proof. We will choose the same $V_1(t)$ as (21) for network (15).

By (26)–(28), one can get

$$\begin{aligned}
&\mathscr{L}V_1(t) - u^T(t)y(t) + \gamma_1 u^T(t)u(t) \\
&\le e^T(t)[I_N \otimes (P\Delta + \Delta P - 2\lambda I_n + \lambda_0 I_n) \\
&\quad - (I_N \otimes P)Q - Q^T(I_N \otimes P) \\
&\quad + aG \otimes (P\Gamma + \Gamma P)]e(t) \\
&\quad + 2e^T(t)(I_N \otimes P)u(t) + \gamma_1 u^T(t)u(t) \\
&\quad - 2\beta \sum_{i=1}^{N} e_i^T(t)P^{((\alpha+1)/2)}\operatorname{sign}(e_i(t))|e_i(t)|^\alpha \\
&\quad - u^T(t)(I_N \otimes A_1)e(t) - u^T(t)(I_N \otimes A_2)u(t) \\
&= -2\beta \sum_{i=1}^{N} e_i^T(t)P^{((\alpha+1)/2)}\operatorname{sign}(e_i(t))|e_i(t)|^\alpha \\
&\quad + \zeta^T(t)\begin{pmatrix} K_1 & E_1 \\ E_1^T & \gamma_1 I_{nN} - I_N \otimes \dfrac{A_2 + A_2^T}{2} \end{pmatrix}\zeta(t) \\
&\le -2\beta\left(\sum_{i=1}^{N} e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)} = -2\beta V_1^{((\alpha+1)/2)}(t).
\end{aligned} \tag{33}$$

Taking the mathematical expectation on both sides above, one can derive that

$$\begin{aligned}
&E\{u^T(t)y(t)\} - \gamma_1 E\{u^T(t)u(t)\} \\
&\ge \frac{E\{\mathrm{d}V_1(t)\}}{\mathrm{d}t} + 2\beta E\{V_1^{((\alpha+1)/2)}(t)\}.
\end{aligned} \tag{34}$$

Therefore, network (15) is finite-time input strictly passive in the sense of expectation under controller (17). □

Theorem 3. *Under assumptions (H1) and (H2), network model (15) is finite-time output strictly passive in the sense of expectation under controller (17) if there exist matrix $Q = diag(Q_1, Q_2, \ldots, Q_N) \in \mathbb{R}^{nN\times nN}$ and a positive real number γ_2 such that*

$$\begin{pmatrix} K_2 & E_2 \\ E_2^T & K_3 \end{pmatrix} \le 0, \tag{35}$$

where

$$\begin{aligned}
K_2 &= K_1 + \gamma_2(I_N \otimes A_1^T)(I_N \otimes A_1), \\
E_2 &= E_1 + \gamma_2(I_N \otimes A_1^T)(I_N \otimes A_2), \\
K_3 &= \gamma_2(I_N \otimes A_2^T)(I_N \otimes A_2) - I_N \otimes \frac{A_2 + A_2^T}{2},
\end{aligned} \tag{36}$$

K_1, E_1 have the same meanings as in Theorem 1.

Proof. Firstly we calculate the following equality:

$$\begin{aligned}
y^T(t)y(t) &= \sum_{i=1}^{N} y_i^T(t)y_i^T(t) \\
&= \sum_{i=1}^{N} [A_1 e_i(t) + A_2 u_i(t)]^T [A_1 e_i(t) + A_2 u_i(t)] \\
&= \sum_{i=1}^{N} [e_i^T(t)A_1^T + u_i^T(t)A_2^T][A_1 e_i(t) + A_2 u_i(t)] \\
&= \sum_{i=1}^{N} [e_i^T(t)A_1^T A_1 e_i(t) + e_i^T(t)A_1^T A_2 u_i(t) \\
&\quad + u_i^T(t)A_2^T A_1 e_i(t) + u_i^T(t)A_2^T A_2 u_i(t)] \\
&= e^T(t)(I_N \otimes A_1^T A_1)e(t) + e^T(t)(I_N \otimes A_1^T A_2)u(t) \\
&\quad + u^T(t)(I_N \otimes A_2^T A_1)e(t) + u^T(t)(I_N \otimes A_2^T A_2)u(t) \\
&= e^T(t)(I_N \otimes A_1^T)(I_N \otimes A_1)e(t) \\
&\quad + e^T(t)(I_N \otimes A_1^T)(I_N \otimes A_2)u(t) \\
&\quad + u^T(t)(I_N \otimes A_2^T)(I_N \otimes A_1)e(t) \\
&\quad + u^T(t)(I_N \otimes A_2^T)(I_N \otimes A_2)u(t).
\end{aligned} \tag{37}$$

For the last step, we utilize the important properties of the Kronecker product:

$$(M_1 \otimes M_2)(M_3 \otimes M_4) = (M_1 M_3) \otimes (M_2 M_4). \tag{38}$$

Select the same $V_1(t)$ as (21) for network (15). We can obtain

$$\begin{aligned}
&\mathscr{L}V_1(t) - u^T(t)y(t) + \gamma_2 y^T(t)y(t) \\
&\le e^T(t)[I_N \otimes (P\Delta + \Delta P - 2\lambda I_n + \lambda_0 I_n) \\
&\quad - (I_N \otimes P)Q - Q^T(I_N \otimes P) \\
&\quad + aG \otimes (P\Gamma + \Gamma P)]e(t) + 2e^T(t)(I_N \otimes P)u(t) \\
&\quad - u^T(t)(I_N \otimes A_1)e(t) - u^T(t)(I_N \otimes A_2)u(t) \\
&\quad + \gamma_2 e^T(t)(I_N \otimes A_1^T)(I_N \otimes A_1)e(t) \\
&\quad + \gamma_2 e^T(t)(I_N \otimes A_1^T)(I_N \otimes A_2)u(t) \\
&\quad + \gamma_2 u^T(t)(I_N \otimes A_2^T)(I_N \otimes A_1)e(t) \\
&\quad + \gamma_2 u^T(t)(I_N \otimes A_2^T)(I_N \otimes A_2)u(t) \\
&\quad - 2\beta \sum_{i=1}^{N} e_i^T(t)P^{((\alpha+1)/2)}\operatorname{sign}(e_i(t))|e_i(t)|^\alpha \\
&= -2\beta \sum_{i=1}^{N} e_i^T(t)P^{((\alpha+1)/2)}\operatorname{sign}(e_i(t))|e_i(t)|^\alpha \\
&\quad + \zeta^T(t)\begin{pmatrix} K_2 & E_2 \\ E_2^T & K_3 \end{pmatrix}\zeta(t) \\
&\le -2\beta\left(\sum_{i=1}^{N} e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)} = -2\beta V_1^{((\alpha+1)/2)}(t).
\end{aligned} \tag{39}$$

Taking the mathematical expectation on both sides above, one can derive that

$$\begin{aligned} &E\{u^T(t)y(t)\} - \gamma_2 E\{y^T(t)y(t)\} \\ &\geq \frac{E\{\mathrm{d}V_1(t)\}}{\mathrm{d}t} + 2\beta E\{V_1^{((\alpha+1)/2)}(t)\}. \end{aligned} \quad (40)$$

Therefore, network (15) is finite-time output strictly passive in the sense of expectation under controller (17). □

3.3. Finite-Time Synchronization. In this section, we will verify finite-time synchronization in the sense of expectation for SCCNs (12). Firstly, the definition of finite-time synchronization is given as follows.

Definition 5. The SCCN (12) is finite-time synchronized in the sense of expectation if there exists a constant $T > 0$ such that

$$\begin{aligned} &\lim_{t \longrightarrow T^-} E\{\|z_i(t) - \bar{z}(t)\|_2\} = 0, \\ &E\{\|z_i(t) - \bar{z}(t)\|_2\} \equiv 0, \quad t \geq T, \end{aligned} \quad (41)$$

for $i = 1, 2, \ldots, N$, where $u_i(t) = 0, i = 1, 2, \ldots, N$.

Theorem 4. *Assume that a continuous, positive-definite function $V(t)$ satisfies the following inequality:*

$$\varphi_1(E\|e(t)\|_2) \leq V(t), \quad (42)$$

where φ_1: $[0, +\infty) \longrightarrow [0, +\infty)$ is continuous and strictly monotonically increasing function and $\varphi_1(s)$ is positive for $s > 0$ with $\varphi_1(0) = 0$. If network (15) is finite-time passive (finite-time input strictly passive, finite-time output strictly passive) in the sense of expectation with respect to $V(t)$, then SCCN (12) is finite-time synchronized in the sense of expectation under controller (17).

Proof. The network model (15) is finite-time passive in the sense of expectation with respect to $V(t)$ under controller (17), that is to say, there exist $\alpha \in (0, 1)$ and $\beta > 0$ such that

$$E\{u^T(t)y(t)\} \geq \frac{E\{\mathrm{d}V(t)\}}{\mathrm{d}t} + \beta E\{V^\alpha(t)\}. \quad (43)$$

Considering $u(t) = 0$, one obtains

$$\frac{E\{\mathrm{d}V(t)\}}{\mathrm{d}t} + \beta E\{V^\alpha(t)\} \leq 0. \quad (44)$$

According to the property of mathematical expectation,

$$\dot{V}(t) \leq -\beta V^\alpha(t). \quad (45)$$

Choosing $t_0 = 0$ in Lemma 1, we can obtain $V(t) \equiv 0$ for $t \geqslant t_1$, where $t_1 = (V^{1-\alpha}(0)/\beta(1-\alpha))$. On the one hand, since

$$\varphi_1[E(\|e(t)\|_2)] \leq V(t), \quad (46)$$

one has

$$\varphi_1[E(\|e(t)\|_2)] \leq V(t) \equiv 0, \quad (47)$$

for $t \geq t_1$. Since $\varphi_1(s) = 0$ if and only if $s = 0$. Then, we can conclude that

$$E(\|e(t)\|_2) \equiv 0, \quad t \geq t_1. \quad (48)$$

On the other hand, $V(t)$ is continuous, so

$$\lim_{t \longrightarrow t_1^-} V(t) = \lim_{t \longrightarrow t_1^+} V(t) = 0. \quad (49)$$

Taking the limit $t \longrightarrow t_1^-$ on both sides of (46), we will get

$$\lim_{t \longrightarrow t_1^-} \varphi_1[E(\|e(t)\|_2)] = 0. \quad (50)$$

Namely, SCCN (12) is finite-time synchronized in the sense of expectation under controller (17).

Similarly, it is easy to prove that SCCN (12) is also finite-time synchronized in the sense of expectation under controller (17) if network model (15) is finite-time input strictly passive or finite-time output strictly passive in the sense of expectation. □

4. Finite-Time Passivity of SCCNs with Time-Varying Delay

4.1. Network Model. In this section, the network model is described by

$$\begin{aligned} \mathrm{d}z_i(t) = \Bigg[& f(z_i(t)) + a \sum_{j=1}^{N} G_{ij}\Gamma z_j(t - \tau(t)) \\ & + u_i(t) + v_i(t) \Bigg]\mathrm{d}t + h(z_i(t))\mathrm{d}\omega(t), \end{aligned} \quad (51)$$

where $\tau(t)$ is the time delay and satisfies $0 \leq \tau(t) \leq \tau, 0 \leq \dot{\tau}(t) \leq d < 1$.

4.2. Finite-Time Passivity. Let $\bar{z}(t)$ also satisfy

$$\mathrm{d}\bar{z}(t) = f(\bar{z}(t))\mathrm{d}t + h(\bar{z}(t))\mathrm{d}\omega(t). \quad (52)$$

Define $e_i(t) = z_i(t) - \bar{z}(t)$, $i = 1, 2, \ldots, N$. Then, we have

$$\begin{aligned} \mathrm{d}e_i(t) = \Bigg[& f(z_i(t)) - f(\bar{z}(t)) \\ & + a \sum_{j=1}^{N} G_{ij}\Gamma e_j(t - \tau(t)) + u_i(t) + v_i(t) \Bigg]\mathrm{d}t \\ & + [h(z_i(t)) - h(\bar{z}(t))]\mathrm{d}\omega(t). \end{aligned} \quad (53)$$

The output vector $y_i(t)$ of network (53) is defined as follows:

$$y_i(t) = B_1 e_i(t) + B_2 u_i(t). \quad (54)$$

Design the following controller for network (51):

$$v_i(t) = -\beta P^{-1}\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t}(z_i(h)-\overline{z}(h))^T M_i (z_i(h) - \overline{z}(h))\mathrm{d}h\right)^{((\alpha+1)/2)} \frac{z_i(t)-\overline{z}(t)}{\|z_i(t)-\overline{z}(t)\|_2^2} - \beta P^{((\alpha-1)/2)}\operatorname{sign}(z_i(t)-\overline{z}(t))|z_i(t)-\overline{z}(t)|^{\alpha} - Q_i(z_i(t)-\overline{z}(t)), \tag{55}$$

where $0 < M_i \in \mathbb{R}^{n\times n}$, $M = \operatorname{diag}(M_1, M_2, \ldots, M_N)$, Q_i, α, P, β, $P^{((\alpha-1)/2)}$, $\operatorname{sign}(z_i(t)-\overline{z}(t))$, $|z_i(t)-\overline{z}(t)|^{\alpha}$ have the same meanings as in (17).

Theorem 5. *Under assumptions (H1) and (H2), network model (53) is finite-time passive in the sense of expectation under controller (55) if there exist matrices $Q = diag(Q_1, Q_2, \ldots, Q_N) \in \mathbb{R}^{nN\times nN}$, $0 < M = diag(M_1, M_2, \ldots, M_N) \in \mathbb{R}^{nN\times nN}$ such that*

$$\begin{pmatrix} W_1 & \Omega_1 \\ \Omega_1^T & -I_N \otimes \frac{B_2 + B_2^T}{2} \end{pmatrix} \leq 0, \tag{56}$$

where

$$W_1 = I_N \otimes (P\Delta + \Delta P - 2\lambda I_n + \lambda_0 I_n) - (I_N \otimes P)Q - Q^T(I_N \otimes P) + \frac{a}{1-d}M + a[G\otimes(P\Gamma)]M^{-1}[G\otimes(\Gamma P)], \tag{57}$$

$$\Omega_1 = I_N \otimes P - \frac{I_N \otimes B_1^T}{2}.$$

Proof. Choose the following Lyapunov functional for network (51):

$$V_2(t) = \sum_{i=1}^{N} e_i^T(t) P e_i(t) + \frac{a}{1-d}\int_{t-\tau(t)}^{t} e^T(h) M e(h)\mathrm{d}h, \tag{58}$$

where $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$.

According to Ito's lemma, we acquire from (53) and (55)

$$\mathrm{d}V_2(t) = \mathscr{L}V_2(t)\mathrm{d}t + \Phi(t)\mathrm{d}\omega(t). \tag{59}$$

Here

$$\begin{aligned}\mathscr{L}V_2(t) &= 2\sum_{i=1}^{N} e_i^T(t)P\Bigg[f(z_i(t)) - f(\overline{z}(t)) + a\sum_{j=1}^{N} G_{ij}\Gamma e_j(t-\tau(t)) + u_i(t) - Q_i e_i(t) - \beta P^{((\alpha-1)/2)}\operatorname{sign}(e_i(t))|e_i(t)|^{\alpha}\Bigg] \\ &+ \sum_{i=1}^{N}\operatorname{trace}[h(z_i(t)) - h(\overline{z}(t))]^T P[h(z_i(t)) - h(\overline{z}(t))] \\ &- 2\beta\sum_{i=1}^{N}\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t} e_i^T(h) M_i e_i(h)\mathrm{d}h\right)^{((\alpha+1)/2)} \\ &- 2\beta\sum_{i=1}^{N} e_i^T(t) P^{((\alpha+1)/2)}\operatorname{sign}(e_i(t))|e_i(t)|^{\alpha} \\ &+ \frac{a}{1-d}\left[e^T(t)Me(t) - e^T(t-\tau(t))Me(t-\tau(t))(1-\dot{\tau}(t))\right], \\ \Phi(t) &= 2\sum_{i=1}^{N} e_i^T(t)P[h(z_i(t)) - h(\overline{z}(t))]. \end{aligned} \tag{60}$$

According to Lemma 3, we can take

$$x = e(t-\tau(t)), \quad y = [G\otimes(\Gamma P)]e(t). \tag{61}$$

It is not difficult to obtain

$$2ae^T(t)[G\otimes(P\Gamma)]e(t-\tau(t)) \leq ae^T(t)[G\otimes(P\Gamma)]M^{-1}[G\otimes(\Gamma P)]e(t) + ae^T(t-\tau(t))Me(t-\tau(t)). \tag{62}$$

From the above, one has

$$\begin{aligned}\mathscr{L}V_2(t) &\leq e^T(t)[I_N \otimes (P\Delta + \Delta P - 2\lambda I_n + \lambda_0 I_n) - (I_N\otimes P)Q - Q^T(I_N \otimes P)]e(t) \\ &+ 2ae^T(t)[G\otimes(P\Gamma)]e(t-\tau(t)) \\ &- 2\beta\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t} e^T(h)Me(h)\mathrm{d}h\right)^{((\alpha+1)/2)} \\ &- 2\beta\left(\sum_{i=1}^{N} e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)} \\ &- ae^T(t-\tau(t))Me(t-\tau(t)) \\ &+ \frac{a}{1-d}e^T(t)Me(t) + 2e^T(t)(I_N\otimes P)u(t)\end{aligned}$$

$$\leq e^T(t)\Bigg[I_N\otimes\left(P\Delta+\Delta P-2\lambda I_n+\lambda_0 I_n\right)$$
$$-(I_N\otimes P)Q-Q^T(I_N\otimes P)+\frac{a}{1-d}M$$
$$+a[G\otimes(P\Gamma)]M^{-1}[G\otimes(\Gamma P)]\Bigg]e(t)$$
$$-2\beta\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t}e^T(h)Me(h)\mathrm{d}h\right)^{((\alpha+1)/2)}$$
$$-2\beta\left(\sum_{i=1}^{N}e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)}$$
$$+2e^T(t)(I_N\otimes P)u(t). \tag{63}$$

Thus,

$$\mathscr{L}V_2(t)-u^T(t)y(t)$$
$$\leq\zeta^T(t)\begin{pmatrix}W_1 & \Omega_1\\ \Omega_1^T & -I_N\otimes\dfrac{B_2+B_2^T}{2}\end{pmatrix}\zeta(t)$$
$$-2\beta\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t}e^T(h)Me(h)\mathrm{d}h\right)^{((\alpha+1)/2)}$$
$$-2\beta\left(\sum_{i=1}^{N}e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)}$$
$$\leq-2\beta\left\{\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t}e^T(h)Me(h)\mathrm{d}h\right)^{((\alpha+1)/2)}\right. \tag{64}$$
$$\left.+\left(\sum_{i=1}^{N}e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)}\right\}$$
$$\leq-2\beta\left(\sum_{i=1}^{N}e_i^T(t)Pe_i(t)\right.$$
$$\left.+\frac{a}{1-d}\int_{t-\tau(t)}^{t}e^T(h)Me(h)\mathrm{d}h\right)^{((\alpha+1)/2)}$$
$$=-2\beta V_2^{((\alpha+1)/2)}(t),$$

where $u(t)=(u_1^T(t),u_2^T(t),\ldots,u_N^T(t))^T$, $y(t)=(y_1^T(t), y_2^T(t),\ldots,y_N^T(t))^T$, $\zeta(t)=(e^T(t),u^T(t))^T$.

Taking the mathematical expectation on (59), we can obtain

$$E\left\{u^T(t)y(t)\right\}\geq\frac{E\{\mathrm{d}V_2(t)\}}{\mathrm{d}t}+2\beta E\left\{V_2^{((\alpha+1)/2)}(t)\right\}. \tag{65}$$

Consequently, network model (53) is finite-time passive in the sense of expectation under controller (55). □

Theorem 6. *Under assumptions (H1) and (H2), network model (53) is finite-time input strictly passive in the sense of expectation under controller (55) if there exist matrices* $Q=diag(Q_1,Q_2,\ldots,Q_N)\in\mathbb{R}^{nN\times nN}$, $0<M=diag(M_1,M_2,\ldots,M_N)\in\mathbb{R}^{nN\times nN}$ *and a positive real number* γ_3 *such that*

$$\begin{pmatrix}W_1 & \Omega_1\\ \Omega_1^T & \gamma_3 I_{nN}-I_N\otimes\dfrac{B_2+B_2^T}{2}\end{pmatrix}\leq 0, \tag{66}$$

where W_1,Ω_1 *have the same meanings as in Theorem 5.*

Proof. We also select the same $V_2(t)$ as (58) for network (53). By (64), we get

$$\mathscr{L}V_2(t)-u^T(t)y(t)+\gamma_3u^T(t)u(t)$$
$$\leq\zeta^T(t)\begin{pmatrix}W_1 & \Omega_1\\ \Omega_1^T & \gamma_3 I_{nN}-I_N\otimes\dfrac{A_2+A_2^T}{2}\end{pmatrix}\zeta(t)$$
$$-2\beta\left\{\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t}e^T(h)Me(h)\mathrm{d}h\right)^{((\alpha+1)/2)}\right.$$
$$\left.+\left(\sum_{i=1}^{N}e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)}\right\}$$
$$\leq-2\beta\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t}e^T(h)Me(h)\mathrm{d}h\right.$$
$$\left.+\sum_{i=1}^{N}e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)}=-2\beta V_2^{((\alpha+1)/2)}(t). \tag{67}$$

Taking the mathematical expectation on (59), we can obtain

$$E\left\{u^T(t)y(t)\right\}-\gamma_3E\left\{u^T(t)u(t)\right\}$$
$$\geq\frac{E\{dV_2(t)\}}{dt}+2\beta E\left\{V_2^{((\alpha+1)/2)}(t)\right\}. \tag{68}$$

Therefore, network (53) is finite-time input strictly passive in the sense of expectation under controller (55). □

Theorem 7. *Under assumptions (H1) and (H2), network model (53) is finite-time output strictly passive in the sense of expectation under controller (55) if there exist matrices* $Q=diag(Q_1,Q_2,\ldots,Q_N)\in\mathbb{R}^{nN\times nN}$, $0<M=diag(M_1,M_2,\ldots,M_N)\in\mathbb{R}^{nN\times nN}$ *and a positive real number* γ_4 *such that*

$$\begin{pmatrix} W_2 & \Omega_2 \\ \Omega_2^T & W_3 \end{pmatrix} \leq 0, \quad (69)$$

where

$$W_2 = W_1 + \gamma_4 \left(I_N \otimes B_1^T\right)\left(I_N \otimes B_1\right),$$
$$\Omega_2 = \Omega_1 + \gamma_4 \left(I_N \otimes B_1^T\right)\left(I_N \otimes B_2\right),$$
$$W_3 = \gamma_4 \left(I_N \otimes B_2^T\right)\left(I_N \otimes B_2\right) - I_N \otimes \frac{B_2 + B_2^T}{2}, \quad (70)$$

W_1, Ω_1 have the same meanings as in Theorem 5.

Proof. Select the same $V_2(t)$ as (58) for network (53). By (37) and (64), we get

$$\begin{aligned}
&\mathscr{L}V_2(t) - u^T(t)y(t) + \gamma_4 y^T(t)y(t) \\
&\leq \zeta^T(t)\begin{pmatrix} W_2 & \Omega_2 \\ \Omega_2^T & W_3 \end{pmatrix}\zeta(t) \\
&\quad - 2\beta\left[\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t} e^T(h)Me(h)\mathrm{d}h\right)^{((\alpha+1)/2)} + \left(\sum_{i=1}^{N} e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)}\right] \\
&\leq -2\beta\left(\frac{a}{1-d}\int_{t-\tau(t)}^{t} e^T(h)Me(h)\mathrm{d}h + \sum_{i=1}^{N} e_i^T(t)Pe_i(t)\right)^{((\alpha+1)/2)} \\
&= -2\beta V_2^{((\alpha+1)/2)}(t).
\end{aligned} \quad (71)$$

Taking the mathematical expectation on (59), we can obtain

$$E\left\{u^T(t)y(t)\right\} - \gamma_4 E\left\{y^T(t)y(t)\right\} \geq \frac{E\{\mathrm{d}V_2(t)\}}{\mathrm{d}t} + 2\beta E\left\{V_2^{((\alpha+1)/2)}(t)\right\}. \quad (72)$$

Therefore, network (53) is finite-time output strictly passive in the sense of expectation under controller (55). □

4.3. Finite-Time Synchronization

Theorem 8. *Assume that a continuous, positive-definite function $\widehat{V}(t)$ satisfies the following inequality:*

$$\varphi_2\left(E\|e(t)\|_2\right) \leq \widehat{V}(t), \quad (73)$$

where φ_2: $[0, +\infty) \longrightarrow [0, +\infty)$ is continuous and strictly monotonically increasing function and $\varphi_2(s)$ is positive for $s > 0$ with $\varphi_2(0) = 0$. If network (51) is finite-time passive (finite-time input strictly passive, finite-time output strictly passive) in the sense of expectation with respect to $\widehat{V}(t)$, then SCCN (53) is finite-time synchronized in the sense of expectation under controller (55).

Here we omit the proof of the theorem. The readers can refer to the proof of Theorem 4.

5. Numerical Examples

Example 1. The following SCCNs are discussed:

$$\mathrm{d}z_i(t) = \left[f(z_i(t)) + 0.7\sum_{j=1}^{6} G_{ij}\Gamma z_j(t) + u_i(t) + v_i(t)\right]\mathrm{d}t + h(z_i(t))\mathrm{d}\omega(t), \quad i = 1, 2, \ldots, 6, \quad (74)$$

where $f_s(\varsigma) = (1/4)(|\varsigma + 1| - |\varsigma - 1|), s = 1, 2, 3, \Gamma = \mathrm{diag}(0.15, 0.05, 0.25)$, $h(z_i(t)) = \mathrm{diag}(0.2, 0.4, 0.2)$,

$$G = \begin{pmatrix} -0.4 & 0.1 & 0 & 0.1 & 0.2 & 0 \\ 0.1 & -0.4 & 0 & 0.2 & 0.1 & 0 \\ 0 & 0 & -0.5 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.2 & -0.6 & 0 & 0.1 \\ 0.2 & 0.1 & 0.1 & 0 & -0.5 & 0.1 \\ 0 & 0 & 0.2 & 0.1 & 0.1 & -0.4 \end{pmatrix}. \quad (75)$$

Obviously, $H(1)$ holds under the condition that $P = I_3, \lambda = 0.8$, and $\Delta = \mathrm{diag}(0.15, 0.14, 0.12)$. Choose $A_2 = \mathrm{diag}(0.6, 0.8, 0.9)$ and

$$A_1 = \begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.4 & 0.3 & 0.3 \\ 0.6 & 0.1 & 0.4 \end{pmatrix}. \quad (76)$$

Take $Q = \mathrm{diag}(0.2I_3, 0.6I_3, 0.7I_3, 0.5I_3, 0.3I_3, 0.4I_3)$. According to Theorem 1, the SCCNs (74) can realize finite-time passivity in the sense of expectation under controller (17). Then, we can easily find the parameters $\gamma_1 = 0.0163$ and $\gamma_2 = 0.0300$ satisfying the condition of Theorems 2 and 3. The simulation results are shown in Figures 1 and 2.

By Theorem 4, network (74) under finite-time output strictly passive can achieve finite-time synchronization. Figure 3 shows the simulation results.

Example 2. The following SCCNs with time-varying delay are discussed:

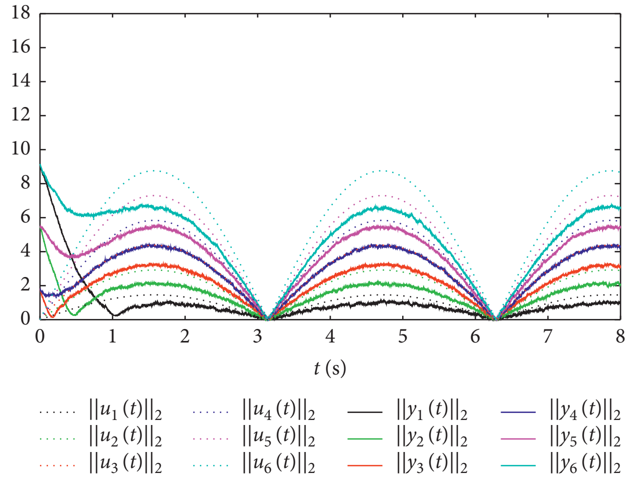

FIGURE 1: The norms of the input and output vectors $\|u_i(t)\|_2, \|y_i(t)\|_2$, $i = 1, 2, \ldots, 6$.

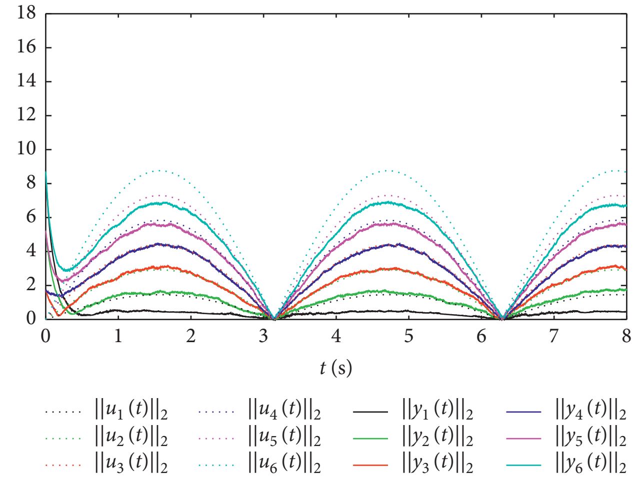

FIGURE 4: The norms of the input and output vectors $\|u_i(t)\|_2, \|y_i(t)\|_2$, $i = 1, 2, \ldots, 6$.

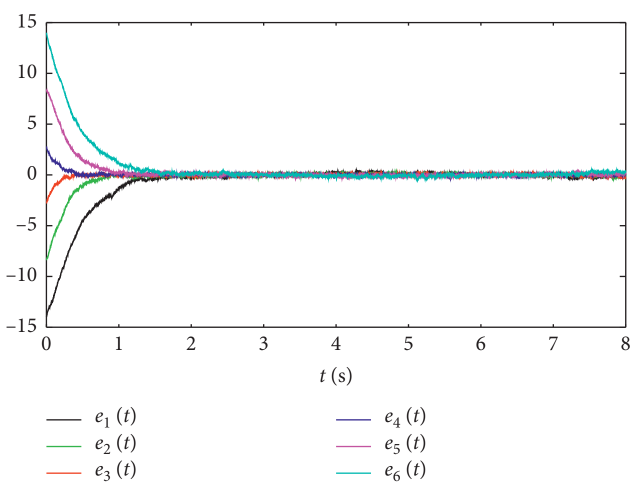

FIGURE 2: The error vectors $e_i(t)$, $i = 1, 2, \ldots, 6$.

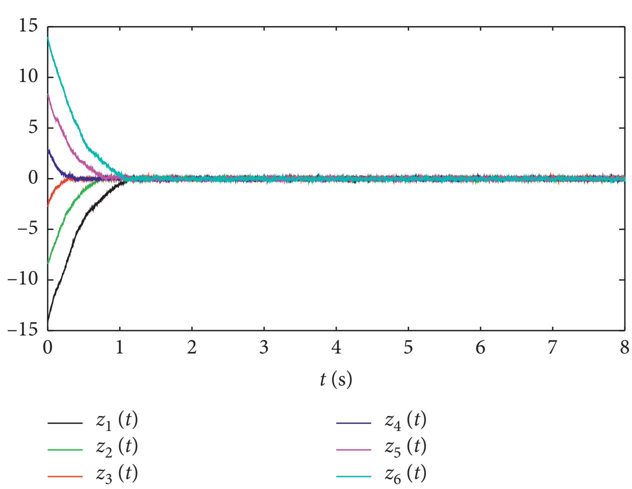

FIGURE 3: The state vectors $z_i(t)$, $i = 1, 2, \ldots, 6$.

$$\mathrm{d}z_i(t) = \left[f(z_i(t)) + 0.8\sum_{j=1}^{6} G_{ij}\Gamma z_j(t-\tau(t)) + u_i(t) + v_i(t) \right]\mathrm{d}t + h(z_i(t))\mathrm{d}\omega(t), \quad i = 1, 2, \ldots, 6, \tag{77}$$

where $f_s(\varsigma) = (1/4)(|\varsigma + 1| - |\varsigma - 1|)$, $s = 1, 2, 3$, $\Gamma = \mathrm{diag}(0.15, 0.15, 0.15)$, $h(z_i(t)) = \mathrm{diag}(0.1, 0.2, 0.2)$. Taking $\tau(t) = 0.5 - 0.5e^{-t}$, we can get $d = 0.5$. The matrix G is chosen as

$$G = \begin{pmatrix} -0.8 & 0.1 & 0.2 & 0.1 & 0.2 & 0.2 \\ 0.1 & -0.5 & 0 & 0.2 & 0.1 & 0.1 \\ 0.2 & 0 & -0.7 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.2 & -0.6 & 0 & 0.1 \\ 0.2 & 0.1 & 0.1 & 0 & -0.5 & 0.1 \\ 0.2 & 0.1 & 0.2 & 0.1 & 0.1 & -0.7 \end{pmatrix}. \tag{78}$$

Obviously, $H(1)$ holds under the condition that $P = I_3, \lambda = 0.7$, and $\Delta = \mathrm{diag}(0.18, 0.16, 0.15)$. Choose $B_2 = \mathrm{diag}(0.6, 0.8, 0.9)$ and

$$B_1 = \begin{pmatrix} 0.6 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}. \tag{79}$$

Take $Q = \mathrm{diag}(0.5I_3, 0.6I_3, 0.7I_3, 0.5I_3, 0.7I_3, 0.8I_3)$, $M = I_6 \otimes \mathrm{diag}(0.1, 0.2, 0.4)$. According to Theorem 5, the SCCNs (77) can realize finite-time passivity under controller (55). Then, we can easily find the parameters $\gamma_3 = 0.0132$ and $\gamma_4 = 0.0369$ satisfying the condition of Theorems 6 and 7. The simulation results are shown in Figures 4 and 5.

By Theorem 8, network (77) under finite-time output strictly passive can achieve finite-time synchronization. Figure 6 shows the simulation results.

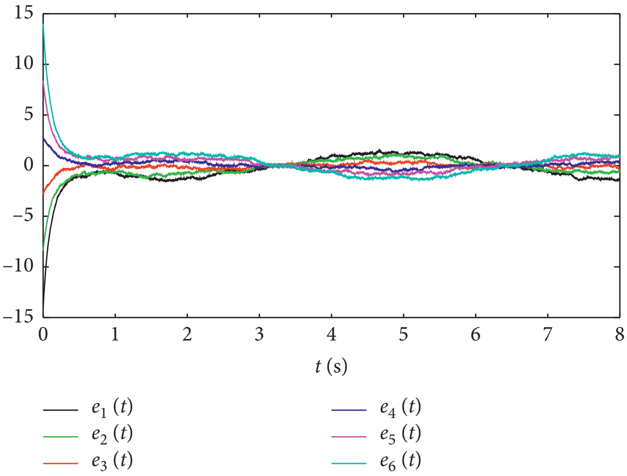

FIGURE 5: The error vectors $e_i(t)$, $i = 1, 2, \ldots, 6$.

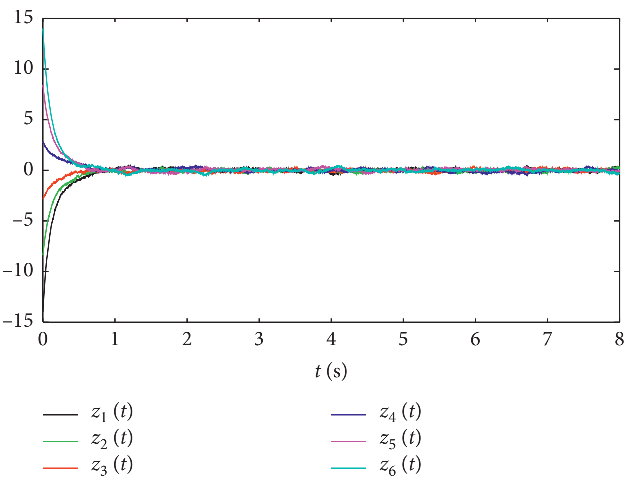

FIGURE 6: The state vectors $z_i(t)$, $i = 1, 2, \ldots, 6$.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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