Research Article

A Stochastic User Equilibrium Formulation for the Cumulative Prospect Theory-Based Cross-Nested Logit

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The cumulative prospect theory provides a better description for route choice behavior of the travelers in an uncertain road network environment. In this study, we proposed a multiclass cumulative prospect value- (CPV-) based cross-nested logit (CNL) stochastic user equilibrium (SUE) model. For this model, an equivalent variational inequality (VI) model is provided, and the existence and equivalence of the model solutions are also proved. The method of successive averages (MSA), method of successive weighted averages (MSWA), and self-regulated averaging (SRA) method are designed and compared. In addition, the proposed multiclass CPV-based CNL SUE model is also compared with the multiclass utility value- (UV-) based CNL SUE model. The results show that the path flow assigned by the multiclass CPV-based CNL SUE model is more consistent with the actual situation. The impact of different model parameters on the cumulative prospect value (CPV) is investigated.

1. Introduction

Traffic assignment is a foundational problem in transportation research. Stochastic user equilibrium (SUE) model is an important method of traffic assignment. The route choice model is the key of the SUE model. The multinomial logit (MNL) model is typically used in the route choice model due to its simple mathematical structure. However, the MNL model has the property of independence of irrelevant alternatives (IIA) [1], that is, it does not consider the similarities between different paths. To this end, the cross-nested logit (CNL) model is developed to deal with this problem [2]. At present, the SUE model based on the CNL route choice has been studied. Bekhor et al. [3] proposed a general CNL-SUE model. On this basis, Bekhor et al. [4] proposed a path-based algorithm for solving the CNL-SUE model and tested the performance of the algorithm. Along the same line, Du et al. [5] developed a path-based traffic assignment algorithm to solve the CNL SUE model by using the Barzilai-Borwein (BB) step size. In addition, Hu and Zhou [6] proposed a multiuser multimode CNL SUE model.

The CNL model is usually modeled based on expected utility theory (EUT). However, under uncertain road network environment, route choice behavior of travelers usually violates the EUT [7]. EUT assumes that travelers are perfectly rational when choosing a path. In reality, the route choice behavior of travelers is affected by environmental factors and psychological state. Therefore, cumulative prospect theory (CPT) [8, 9] is further developed. The CPT has been widely used in the process of traffic assignment modeling. Lo et al. [10] and Shao et al. [11], respectively, consider stochastic demand and link capacity degradation to establish travel time reliability models. On this basis, Siu and Lo [12] proposed a methodology to model the doubly uncertain transportation network with stochastic link capacity degradation and stochastic demand. Connors and Sumalee [13] provided a general modeling framework to describe the route choice behavior of travelers under uncertain travel time. Xu et al. [14] introduced the endogenous reference point to the UE model based on cumulative prospect theory. Wang and Sun [15] encapsulated the stochastic perception error (SPE) of travelers into the UE model. Yang and Jiang

[16] extend the model in [15] to develop a CPT-based SUE model.

At present, there is no research on embedding the cumulative prospect theory into the CNL model framework in the SUE modeling process. Thus, the objective of this study is to propose a multiclass cumulative prospect value (CPV-) based CNL SUE model through using CPV replacing utility value (UV) as path performance. The equivalent variational inequality (VI) model and associated solution algorithm are provided. The Nguyen–Dupuis network is selected to compare the path flow assigned by the multiclass CPV-based CNL SUE model and the multiclass UV-based CNL SUE model. The computational efficiency of the three algorithms, i.e., the method of successive averages (MSA) [17], the method of successive weighted averages (MSWA) [18], and the self-regulated averages (SRA) [19], are also compared. The influence of model parameters on the CPV is analyzed.

The organization of this study is as follows. First, the multiclass CPV-based CNL SUE model is proposed. Then, the equivalent VI model and associated solution algorithms are provided. Afterwards, the numerical examples are investigated to demonstrate the performance of the proposed multiclass CPV-based CNL SUE model. Finally, conclusions of the study are provided.

2. Study Model

2.1. CPV-Based CNL Route Choice Model. It is well-known that traditional CNL route choice model is developed based on EUT. However, EUT usually exhibits inconsistency with traveler's route choice behavior in an uncertain traffic environment. Therefore, we use CPV instead of the utility value as path performance in the CNL framework to establish a more realistic route choice model. The steps of the CPV-based CNL route choice modeling are as follows:

First, the bureau of public roads (BPR) function is used to calculate the link travel time, namely,

$$T_a(x_a, C_a) = t_a^0 \left[1 + \alpha \left(\frac{x_a}{C_a} \right)^\beta \right], \quad \forall a, \quad (1)$$

where T_a is the travel time on link a , t_a^0 and C_a are the free-flow travel time and actual capacity on link a , respectively, and α and β are deterministic parameters.

In reality, some events (e.g., natural disasters and traffic accidents) will cause link capacity to decrease, so that the link capacity becomes a random variable. Assume that the link capacity C_a is uniformly distributed in the interval $[\zeta_a \bar{c}_a, \bar{c}_a]$ [10]. Since C_a is a random variable, T_{ki}^w is also a random variable. According to Lo et al. [10], the mean and variance of path travel time are given as follows:

$$E(T_{ki}^w) = \sum_a \left\{ \delta_{a,k}^w \left[t_a^0 + \alpha t_a^0 x_a^\beta \frac{1 - \zeta_a^{1-\beta}}{\bar{c}_a^\beta (1 - \zeta_a)(1 - \beta)} \right] \right\}, \quad \forall w, k, i, \quad (2)$$

$$\text{var}(T_{ki}^w) = \sum_a \left\{ \delta_{a,k}^w \alpha^2 (t_a^0)^2 x_a^{2\beta} \left[\frac{1 - \zeta_a^{1-2\beta}}{\bar{c}_a^{2\beta} (1 - \zeta_a)(1 - 2\beta)} - \left(\frac{1 - \zeta_a^{1-\beta}}{\bar{c}_a^\beta (1 - \zeta_a)(1 - \beta)} \right)^2 \right] \right\}, \quad \forall w, k, i, \quad (3)$$

where $\bar{c}_a$ is the design capacity of link a , $\zeta_a \bar{c}_a$ is the worst-degraded capacity, ζ_a is a fraction of the design capacity, and $\delta_{a,k}^w$ represents the relationship between path k and link a in O-D pair w . Note that $\delta_{a,k}^w = 1$ if link a is on path k ; $\delta_{a,k}^w = 0$, otherwise.

Second, the endogenous reference point proposed by Xu et al. [14] is adopted here. In other words, the minimum travel time budget is defined as the reference point. The expression of the reference point is as follows:

$$P\{T_{ki}^w \leq \mathcal{B}_{ki}^w = E(T_{ki}^w) + \xi_i \sigma_{ki}^w\} = \rho_i, \quad \forall w, k, i, \quad (4)$$

$$\mathcal{B}_{ki}^w = E(T_{ki}^w) + \sigma_{ki}^w \Phi^{-1}(\rho_i), \quad \forall w, k, i, \quad (5)$$

$$u_{0i}^w = \min_k \{\mathcal{B}_{ki}^w\}, \quad \forall w, k, i, \quad (6)$$

where $\mathcal{B}_{ki}^w$ is the travel time budget of user class i on path k between O-D pairs w , $E(T_{ki}^w)$ and σ_{ki}^w are the mean and standard deviation of T_{ki}^w , respectively, ρ_i is the on-time arrival probability of user class i , Φ^{-1} is the inverse function of the standard normal cumulative distribution function, u_{0i}^w is the reference point of user class i between O-D pairs w , and ξ_i is a parameter related to on-time arrival probability of user class i .

Third, the CPV of the path is calculated by the value function, and the probability weight function is as follows:

$$g(T_{ki}^w) = \begin{cases} (u_{0i}^w - T_{ki}^w)^\delta, & T_{ki}^w \leq u_{0i}^w, \\ -\lambda (T_{ki}^w - u_{0i}^w)^\eta, & T_{ki}^w > u_{0i}^w, \end{cases} \quad \forall w, k, i, \quad (7)$$

$$\omega(p) = \exp[-(-\ln p)^\gamma], \quad (8)$$

$$v_{ki}^w = \int_{\underline{u}_{ki}^w}^{\underline{u}_{0i}^w} \frac{d\omega^+ [F(T_{ki}^w)]}{dT_{ki}^w} g(T_{ki}^w) dT_{ki}^w + \int_{\underline{u}_{0i}^w}^{\underline{u}_{ki}^w} -\frac{d\omega^- [1 - F(T_{ki}^w)]}{dT_{ki}^w} g(T_{ki}^w) dT_{ki}^w, \quad \forall w, k, i, \quad (9)$$

where δ and η are the degrees of diminishing sensitivity, $0 < \delta, \eta < 1$, λ is the loss-aversion coefficient, $\lambda \geq 1$, and γ is the degree of curvature of the probability weight function, $0 < \gamma < 1$. In addition, $\omega(p)$ and p are the perceived probability and actual probability of the event, respectively. Note that $\omega(p)$ increases monotonously with respect to p , and it has nothing to do with the value of γ . $F(T_{ki}^w)$ is the distribution function of T_{ki}^w , and ω^+ and ω^- are the gain area

and loss area of probability weighting function, respectively. In equation (9), $\underline{u}_{ki}^w$ and $\underline{u}_{0i}^w$ are the lower bound and upper bound of the travel time of user class i on path k between O-D pairs w , respectively. The lower bound is free flow time, and the upper bound is $\underline{u}_{ki}^w = E[(T_{ki}^w)] + 3\sqrt{\text{var}(T_{ki}^w)}$.

Finally, we apply the CPV instead of the utility value as the path performance in the CNL framework. The probability of user class i choosing path k between O-D pairs w is

$$P_{ki}^w = \frac{\exp[\theta_i v_{ki}^w + \ln \sum_{m \in M} \alpha_{mk}^w (\sum_l \alpha_{ml}^w \exp(\theta_l v_{li}^w))^{\mu_m^w - 1}]}{\sum_{j \in K_w} \exp[\theta_i v_{ji}^w + \ln \sum_{m \in M} \alpha_{mj}^w (\sum_l \alpha_{ml}^w \exp(\theta_l v_{li}^w))^{\mu_m^w - 1}]}, \quad \forall w, k, i, \quad (10)$$

where k, j , and l are the path indicators, v_{li}^w is the CPV of user class i on path l between O-D pairs w , θ_i is the dispersion parameter of user class i , μ_m^w is the nesting coefficient, $0 < \mu_m^w \leq 1$, and e_{mk}^w is the inclusion coefficient. According to Prashker and Bekhor [20], $e_{mk}^w = (L_m/L_k)^\tau \delta_{mk}^w$ and $\mu_m^w = 1 - (1/G_m^w) \sum_k e_{mk}^w$, where L_m is the length of link m , L_k is the length of path k , G_m^w is the number of paths including link m between O-D pairs w , and τ is a parameter that characterizes the travelers' perception of the similarities between different paths, $\tau = 1$.

It is possible to decompose equation (10) as marginal probability and conditional probability, that is,

$$P_{ki}^w = \sum_m P_i^w(m) P_i^w(k|m), \quad \forall w, k, i. \quad (11)$$

The marginal probability is described as

$$P_i^w(m) = \frac{(\sum_k (e_{mk}^w \exp(\theta_i v_{ki}^w))^{1/\mu_m^w})^{\mu_m^w}}{\sum_m (\sum_k (e_{mk}^w \exp(\theta_i v_{ki}^w))^{1/\mu_m^w})^{\mu_m^w}}, \quad \forall w, m, i, \quad (12)$$

and the conditional probability is described as

$$P_i^w(k|m) = \frac{(e_{mk}^w \exp(\theta_i v_{ki}^w))^{1/\mu_m^w}}{\sum_l (e_{ml}^w \exp(\theta_i v_{li}^w))^{1/\mu_m^w}}, \quad \forall w, k, m, i. \quad (13)$$

2.2. Multiclass CPV-Based CNL SUE Model. According to the SUE principle, the network equilibrium state satisfies the following condition:

$$\sum_m f_{mki}^w = q_i^w P_{ki}^w, \quad \forall w, k, i. \quad (14)$$

The regular constraints must hold, that is,

$$q_i^w = \sum_m \sum_k f_{mki}^w, \quad \forall w, i, \quad (15)$$

$$x_{ai} = \sum_w \sum_k \sum_m f_{mki}^w \delta_{ak}^w, \quad \forall a, i, \quad (16)$$

$$f_{mki}^w \geq 0, \quad \forall w, k, m, i, \quad (17)$$

where q_i^w is the travel demand of user class i between O-D pairs w , f_{mki}^w is the flow of user class i on path k of nest m between O-D pairs w , and x_{ai} is the flow of user class i on link a .

3. Equivalent VI Model and Solution Algorithm

3.1. Equivalent VI Model. The solution of the multiclass CPV-based CNL SUE model is equivalent to seeking a feasible path flow, so that the following variational inequality (VI) model holds

$$\sum_w \sum_m \sum_k \sum_i \left[\frac{1 - \mu_m^w}{\theta_i} \ln \left(\sum_k f_{mki}^{w*} \right) + \frac{\mu_m^w}{\theta_i} \ln f_{mki}^{w*} - \frac{1}{\theta_i} \ln (e_{mk}^w) - v_{ki}^w \right] \times (f_{mki}^w - f_{mki}^{w*}) \geq 0. \quad (18)$$

3.2. Equivalence and Existence

(1) Equivalence

According to the VI theorem, equation (18) can be written as the following equivalent complementary relaxation condition:

$$\begin{aligned} \left(\frac{1 - \mu_m^w}{\theta_i} \ln \left(\sum_k f_{mki}^{w*} \right) + \frac{\mu_m^w}{\theta_i} \ln f_{mki}^{w*} - \frac{1}{\theta_i} \ln (e_{mk}^w) - v_{ki}^w \right) \times f_{mki}^{w*} &= 0, \\ \left(\frac{1 - \mu_m^w}{\theta_i} \ln \left(\sum_k f_{mki}^{w*} \right) + \frac{\mu_m^w}{\theta_i} \ln f_{mki}^{w*} - \frac{1}{\theta_i} \ln (e_{mk}^w) - v_{ki}^w \right) &\geq 0. \end{aligned} \quad (19)$$

When $f_{mki}^{w*} > 0$, we have

$$\frac{1 - \mu_m^w}{\theta_i} \ln \left(\sum_k f_{mki}^{w*} \right) + \frac{\mu_m^w}{\theta_i} \ln f_{mki}^{w*} - \frac{1}{\theta_i} \ln (e_{mk}^w) - v_{ki}^w = 0. \quad (20)$$

After a series of transformations, the marginal probability is obtained as follows:

$$P_i^w(m) = \frac{\sum_k f_{mki}^{w*}}{\sum_m \sum_k f_{mki}^{w*}} = \frac{\left\{ \sum_k (e_{mk}^w)^{(1/\mu_m^w)} \exp[(\theta_i v_{ki}^w)/\mu_m^w] \right\}^{\mu_m^w}}{\sum_m \left\{ \sum_k (e_{mk}^w)^{(1/\mu_m^w)} \exp[(\theta_i v_{ki}^w)/\mu_m^w] \right\}^{\mu_m^w}}. \quad (21)$$

And then, the conditional probability is also obtained as follows:

$$P_i^w(k|m) = \frac{f_{mki}^{w*}}{\sum_k f_{mki}^{w*}} = \frac{(e_{mk}^w)^{(1/\mu_m^w)} \exp[(\theta_i v_{ki}^w)/\mu_m^w]}{\sum_k (e_{mk}^w)^{(1/\mu_m^w)} \exp[(\theta_i v_{ki}^w)/\mu_m^w]}. \quad (22)$$

Obviously, equations (21) and (22) correspond to equations (12) and (13). Thus, the solution of the variational inequality model in equation (18) is equivalent to the solution of the multiclass CPV-based CNL SUE model.

(2) Existence

Let $F(f_{mki}^w) = ((1 - \mu_m^w)/\theta_i) \ln (\sum_k f_{mki}^w) + (\mu_m^w/\theta_i) \ln f_{mki}^w - (1/\theta_i) \ln (e_{mk}^w) - v_{ki}^w$. The distribution function of link travel time is assumed to be a continuous function of the link flow. According to equations

(4)–(6), the reference point is a continuous function of the path flow. According to equations (7)–(9), the CPV of the path is a continuous function of the path flow. Thus, $F(f_{mki}^w)$ is also a continuous function of the path flow. In addition, according to equations (15)–(17), the set of feasible path flow is a compact convex set. Based on the VI theorem, the solution of equation (18) exists.

3.3. Solution Algorithm. The SUE model is usually solved by the method of the successive averages (MSA) algorithm. However, the predetermined iterative step length causes the convergence speed of the MSA algorithm to be very slow. Therefore, we apply the MSA algorithm, MSWA algorithm, and SRA algorithm to solve the equivalent VI model and compare the computational efficiency of these three algorithms. The calculation steps of these three algorithms are as follows:

Step 1. Initialization.

Determine the feasible path set K_w . Calculate the reference point and the CPV based on zero flow. According to equation (14), the initial path flow pattern is obtained by assigning the O-D demand to the link. Set iteration counter $n = 1$.

Step 2. Update the CPV of the path.

Recalculate the mean and variance of the path travel time based on equations (2) and (3). For each O-D pair, redetermine the reference point according to equations (4)–(6). Update the CPV of each path according to equation (9).

Step 3. Direction finding.

Based on the updated CPV of the path, reassign the O-D demand to the link to obtain the auxiliary path flow $\mathbf{g}^{(n)}$, and then, iterative direction $\mathbf{g}^{(n)} - \mathbf{f}^{(n)}$ is also obtained.

Step 4. Update path flow.

Set $\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} + \chi^{(n)} (\mathbf{g}^{(n)} - \mathbf{f}^{(n)})$. For the MSA algorithm, $\chi^{(n)} = (1/n)$. For the MSWA algorithm, $\chi^{(n)} = (n^d / (1^d + 2^d + 3^d + \dots + n^d))$, $d \geq 0$. For the SRA algorithm, $\chi^{(n)} = (1/\mathcal{H}^n)$, where

$$\mathcal{H}^n = \begin{cases} \mathcal{H}^{(n-1)} + \Gamma, \Gamma > 1, & \text{if } \|\mathbf{f}^{(n)} - \mathbf{g}^{(n)}\| \geq \|\mathbf{f}^{(n-1)} - \mathbf{g}^{(n-1)}\|, \\ \mathcal{H}^{(n-1)} + L, L < 1, & \text{if } \|\mathbf{f}^{(n)} - \mathbf{g}^{(n)}\| < \|\mathbf{f}^{(n-1)} - \mathbf{g}^{(n-1)}\|, \end{cases} \quad (23)$$

Step 5. Convergence test

If $\|\mathbf{f}^{(n)} - \mathbf{g}^{(n)}\| \leq \varepsilon$, then stop. If not, set $n = n + 1$ and go to Step 2.

4. Numerical Examples

In this section, we first compare the computational efficiency of the MSA algorithm, MSWA algorithm, and SRA algorithm. Then, we compare the path flow assigned by the multiclass CPV-based CNL SUE model and the multiclass UV-based CNL SUE model. Finally, we analyze the influence of different model parameters on the CPV of path.

4.1. Preliminary. The Nguyen–Dupuis network is selected to test the performance of the proposed multiclass CPV-based CNL SUE model. The Nguyen–Dupuis network includes 13 nodes and 19 links, as shown in Figure 1. For each O-D pair, the travel demand is $q_{1-2} = 660$, $q_{1-3} = 495$, $q_{4-2} = 412.5$, and $q_{4-3} = 495$, respectively.

The free-flow time, design capacity, and the worst-degraded coefficient are given in Table 1, and the path composition and path length are given in Table 2. It should be noted that the values in Tables 1 and 2 are adopted from studies by Xu et al. [14] and Jiang and Xie [21], respectively.

4.2. Comparison of Computational Efficiency among MSA, MSWA, and SRA. The three different algorithms (i.e., MSA, MSWA, and SRA) are implemented on the Nguyen–Dupuis network to compare their computational efficiency. According to Liu et al. [18], we set $\Gamma = 1.9$, $\mathcal{L} = 0.1$, and $d = 2$. The computational efficiency of MSA, MSWA, and SRA is given in Table 3.

In Table 3, it can be seen that the MSA algorithm may provide a faster convergence speed than the SRA algorithm and the MSWA algorithm at the initial stage. However, when the number of iterations gradually increases, the SRA achieves a preset accuracy with a fastest convergence speed in all algorithms. This is because the step size of the SRA algorithm will be adjusted according to the information provided in the two iterations. In contrast, the step size of the MSA algorithm is predetermined, and it is difficult to give an optimal weight parameter of the MSWA algorithm in advance. Therefore, the computational efficiency of the SRA algorithm outperforms the MSA algorithm and the MSWA algorithm.

4.3. Comparative Analysis of Path Flows Based on CPT and EUT. To demonstrate the performance of the proposed multiclass CPV-based CNL SUE model, the path flow assigned by the proposed model and the multiclass UV-based CNL SUE model are shown in Figure 2. Travelers are divided into four categories based on their familiarity with the conditions of the road network. The dispersion parameters and nesting coefficients of these four user classes are set to $\theta = \mu = 0.25$, $\theta = \mu = 0.5$, $\theta = \mu = 0.75$, and $\theta = \mu = 1$, respectively. In addition, the parameters in the BPR function are $\alpha = 0.15$ and $\beta = 4$, the on-time arrival probability is defined as $\rho = 0.8$, and the iteration accuracy is set to $\varepsilon = 0.0001$. According to the parameter calibration results of Kahneman and Tversky [22] and Prelec [23], the parameters in the value function are set to $\delta = \eta = 0.88$ and

FIGURE 1: Nguyen–Dupuis network.

TABLE 1: Link characteristics.

No.	Link	t_a^0	$\bar{c}_a$	ζ_a
1	1-5	7	300	0.8
2	1-12	9	200	0.8
3	4-5	9	200	0.7
4	4-9	12	200	0.8
5	5-6	3	350	0.6
6	5-9	9	400	0.6
7	6-7	5	500	0.7
8	6-10	13	250	0.8
9	7-8	5	250	0.7
10	7-11	9	300	0.7
11	8-2	9	500	0.7
12	9-10	10	550	0.6
13	9-13	9	200	0.8
14	10-11	6	400	0.7
15	11-2	9	300	0.7
16	11-3	8	300	0.6
17	12-6	7	200	0.8
18	12-8	14	300	0.7
19	13-3	11	200	0.7

$\lambda = 2.25$, and the parameter in the probability weight function is considered as $\gamma = 0.74$.

In Figure 2, it can be seen that the path flow assigned by the proposed multiclass CPV-based CNL SUE model and the multiclass UV-based CNL SUE model is significantly different. This is because the proposed model assumes that travelers are bounded rationally when choosing a path, while the multiclass UV-based CNL SUE model is based on an assumption that travelers are perfectly rational when choosing a path.

It can also be seen from the path flow assigned by the multiclass CPV-based CNL SUE model that some travelers choose a path with a high mean travel time but low travel time standard deviation. For example, when $\theta = 0.25$, the

TABLE 2: Path composition and length.

O-D	Path	Node sequence	Path length
1-2	1	1-12-8-2	32
	2	1-5-6-7-8-2	29
	3	1-5-6-7-11-2	33
	4	1-5-6-10-11-2	38
	5	1-5-9-10-11-2	41
	6	1-12-6-7-8-2	35
	7	1-12-6-7-11-2	39
	8	1-12-6-10-11-2	44
4-2	15	4-9-10-11-2	37
	16	4-5-6-7-8-2	31
	17	4-5-6-7-11-2	35
	18	4-5-6-10-11-2	40
	19	4-5-9-10-11-2	43
1-3	9	1-5-9-13-3	36
	10	1-5-6-7-11-3	32
	11	1-5-6-10-11-3	37
	12	1-5-9-10-11-3	40
	13	1-12-6-7-11-3	38
	14	1-12-6-10-11-3	43
4-3	20	4-9-13-3	32
	21	4-9-10-11-3	36
	22	4-5-9-13-3	38
	23	4-5-6-7-11-3	34
	24	4-5-6-10-11-3	39
	25	4-5-9-10-11-3	42

TABLE 3: Computational efficiency of MSA, MSWA, and SRA.

Number of iterations n	Convergence criterion $\epsilon^k = \mathbf{f}^k - \mathbf{g}^k$		
	MSA	MSWA	SRA
100	75.94	491.00	257.91
200	24.31	348.15	22.24
400	12.13	19.18	4.62
800	6.06	2.39	0.36
1600	3.03	0.30	0.01

mean travel time on path 5 is greater than that of path 6, and the travel time standard deviation is smaller than that of path 6. More flow is assigned on path 5 than that of path 6. In contrast, the path flow assigned by the multiclass UV-based CNL SUE model indicates that all travelers choose the path with a low mean travel time, regardless of the travel time standard deviation. Similarly, the path flow under other dispersion parameters and nesting coefficients shows a similar situation, so we will not repeat them here. Furthermore, some empirical studies [24, 25] have shown that travelers are bounded rationally when choosing a path. The path flow assigned by the proposed multiclass CPV-based CNL SUE model is aligned with the actual situation. Therefore, it can be concluded from the above analysis that the proposed multiclass CPV-based CNL SUE model can deal with perfectly rational issue.

4.4. Sensitivity Analysis. To analyze the influence of different model parameters on the CPV of the path, we select all paths

in the O-D pair (4, 2). It should be noted that the colors in Figure 3 represent CPVs of paths.

In Figure 3, it can first be seen that the CPV hardly changes with the parameter μ . This means that the parameter μ has no significant influence on the CPV. In addition, it can also be seen that the parameters ρ , ζ_a , and δ have a great impact on the CPV, while the parameter θ has a small impact on the CPV.

Second, as the increase of the parameters θ , ρ , δ , and γ , the CPV of the path gradually increases. Specifically, when the parameter θ becomes larger, travelers know more road network information. At this time, the travelers' route choice tends to be deterministic, which will lead to a slight increase in the CPV of the path. When the parameter ρ increases, the reference point becomes larger. This means that travelers increase their psychological expectations of the travel time standard. At this time, travelers' benefits increase, and the CPV of the path naturally increases. When the parameter δ becomes larger, the degree of diminishing sensitivity in the profit area decreases. When the path travel time is lower than the reference point, travelers may obtain greater benefits, so the CPV of the path increases. When the parameter γ increases, travelers pay more attention to large-probability events. This means that the weight of high-probability events will increase. It is a high-probability event that travelers obtain benefits. So the CPV of the path increases.

Third, as the parameters λ and η increase, the CPV of the path decrease steadily. Specifically, when the parameter λ becomes larger, the travelers pay more attention to the loss. The proportion of the loss part in the CPV will increase, so the CPV of the path decreases. When the parameter η

(a)

(b)

(c)

(d)

(e)

(f)

FIGURE 2: Continued.

FIGURE 2: The path flow under different parameters. (a) $\theta = 0.25$. (b) $\theta = 0.5$. (c) $\theta = 0.75$. (d) $\theta = 1$. (e) $\mu = 0.25$. (f) $\mu = 0.5$. (g) $\mu = 0.75$. (h) $\mu = 1$.

FIGURE 3: Continued.

FIGURE 3: The variation of the CPV with model parameters. (a) Sensitivity analysis of μ and θ . (b) Sensitivity analysis of μ and ρ . (c) Sensitivity analysis of μ and ζ_a . (d) Sensitivity analysis of μ and λ . (e) Sensitivity analysis of μ and δ . (f) Sensitivity analysis of μ and η . (g) Sensitivity analysis of μ and γ .

increases, the degree of diminishing sensitivity in the loss area decreases. When the path travel time is greater than the reference point, travelers may suffer more losses, so the CPV of the path decreases.

Finally, as the parameter ζ_a increases, the CPV of the path initially increases and then decreases. When the parameter $\zeta_a = 0.2$, the link capacity is very low, so the CPV of the path is small. As the parameter ζ_a increases, the reliability of the link travel time increases. At this time, travelers' psychological expectations for travel time standards decrease, and travelers' losses will increase, so the CPV of the path will naturally decrease.

5. Conclusion

This study proposes a multiclass CPV-based CNL SUE model by using CPV instead of UV as path performance in the CNL framework. An equivalent variational inequality model is provided. Three solution algorithms (i.e., MSA, MSA, and SRA) are designed and compared. The results show that the SRA algorithm has the highest computational efficiency. In addition, we compare the path flow assigned by the proposed multiclass CPV-based CNL SUE model and the multiclass UV-based CNL SUE model. The results indicate that the path flow assigned by the proposed model is more consistent with

the actual route choice behavior of travelers. The influence of parameters on the CPV of the path is also investigated.

The model parameters have an important influence on the process of traffic assignment modeling. We will combine survey and experimental data to investigate calibration of model parameters in the future. In addition, travel demand exhibits elastic characteristics under the saturated traffic condition. We will extend fixed demand to elastic demand in future research.

Data Availability

The data used to support the findings of this study are included within the article and are cited at relevant places within the text as references.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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