Research Article
Decision-Making Optimization of Risk-Seeking Retailer Managed Inventory Model in a Water Supply Chain

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Water retailer managed inventory is a classical and inevitable inventory management mode in present economic society. Stochastic models can more clearly explain demand uncertainty and are closely related to water supply chains. Risk preferences are widely valued in behavioral operation management. Related to the risk preferences in inventory management, the research on risk aversion is dominant, while risk-seeking is insufficient. Based on the model assumptions, the risk-seeking retailer’s optimal inventory model with stochastic demand in a water supply chain is studied. The risk-seeking retailer’s optimal inventory quantity, optimal inventory cost, supplier profit, retailer profit, and the profit of the entire water supply chain are derived. The validity of the equations is proved. The sensitivity analysis of the risk-seeking retailer’s optimal inventory decision-making is carried out. The risk level effects on the five dimensions, the retail price, wholesale price, unit shortage cost, unit inventory cost, and unit residual value, are displayed through numerical simulation. The optimal inventory quantity and optimal inventory cost of the risk-seeking retailer are obtained.

1. Introduction

1.1. Background. With the global population growth and economic development, water resources are becoming increasingly scarce. Water is a crucial business resource, but subsistent evidence suggests that the management of water resources is often poorly operated [1]. According to the United Nations World Water Development Report 2019, 45 percent of global GDP will be in the face of risk by 2050, if the natural environment and the unbearable pressures on worldwide water resources continue to degenerate at current rates. 40 percent of worldwide grain production is in the same situation. Moreover, the aggregate risk of displacement due to disasters has increased twofold since the 1970s. The exhaustion of water and other natural resources is progressively seen as the driving force behind the displacement of internal and international migration. Rogers et al. [2] considered water as an economic commodity and used prices to motivate efficiency, equity, and sustainability. In China, a series of regulations were issued to regulate water resources management. For example, “Urban Water Supply Regulations” (China State Council 2018) regulate the use of water for urban life, production, and other constructions. “Industrial Water Usage Quota for 18 Industries including Iron and Steel Industry” (Ministry of Water Resources China 2019) stipulates bound boundaries for water use in high-water-consumption industries. It is predicted that the “Industrial Water Usage Quota for 18 Industries including Iron and Steel Industry” will save 1 billion cubic meters of water annually after it is strictly implemented. Because water has commodity properties, research on water resources from the perspective of supply chain has been paid more and more attention.

In the traditional supply chain, retailers are at the end of the supply chain and directly face customers. On the one hand, the retailer keeps inventory and decides the inventory level to deal with demand uncertainty. On the other hand, the retailers also have to bear the shortage cost due to stockout. It is little motivated for upstream suppliers to reserve buffer stock to meet demand of end-users [3]. This classical inventory management mode is the retailer managed inventory (RMI) that is also known as customer
managed inventory (CMI). The retailer retains the ownership of the inventory and accordingly bears the inventory cost, management cost, and some risks and responsibilities [4]. The upstream enterprise and the retailer in the supply chain are two completely independent individuals who own and manage inventory independently. The two parties transact business without cooperative management of inventory. Meanwhile, both parties bear various costs and risks caused by uncertain market demand, respectively, as shown in Figure 1.

Uncertainty is an essential feature of market. As the internal and external environments change rapidly, the uncertainty increases significantly. Several international organizations made predictions about world GDP and trade for this year and next year (Table 1). Under the influence of uncertainty, there are large gaps between pessimistic and optimistic forecasts. For water supply chains, uncertainty also means risks. Retailers’ attitudes towards risks, namely, risk preferences, are unavoidable topics.

1.2. Aims. Owing to the background above, this paper focuses on the decision-making optimization of the risk-seeking retailer managed inventory mode under stochastic demand in a water supply chain, which consists of one supplier and one retailer. Based on model assumptions, the profit expressions of supplier, retailer, and the entire water supply chain are firstly provided. Then, sensitivity analyses of related factors are given through analytical equations combined with figures. The conclusions are drawn finally.

1.3. Literature Review. Based on the background discussed in the Introduction, our study involves three aspects of literature. The first stream is about the research on water supply chains, the second one is on the retailer managed inventory with stochastic demand, and the third one is on risk preferences in inventory management.

Regarding the first stream, the water footprint theory proposed by professor Hoekstra [5] provided an effective accounting and evaluation tool for the research of water security strategy, which has become one of the hot spots in the field of water resources management. Based on this theory, water was regarded as one of the evaluation indexes of a certain agricultural or industrial supply chain [6–9], instead of being regarded as a complete supply chain. There are relatively few studies on water resources as a complete supply chain. Garcia-Caceres et al. [10] considered drinking water as a supply chain. A decision support system is presented to explore an optimal plan in order to promote the efficiency and sustainability in a drinking water treatment chain. Du et al. [11] studied the efficiency of water supply chain members under two-divided contracts of pricing and wholesale price considering different competition intensities and rainfall utilization performance degrees. In the literature of Chen and Wang [12], the operational strategy and policy for internal incentives and subsidy in a water-saving service supply chain under the scenario with maximum social welfare are discussed. Chen and Chen [13] developed, analyzed, and compared four decision models of game theory in two cases of considering or without considering backlogging for the interbasin water transfer supply chain. Loss of water transport in the condition of joint pricing and stock management is considered. By reading relevant literature, we summarize and condense the following noteworthy characteristics of water supply chains:

1. Water supply chain is characterized by high risk and multiple risks. It is obviously affected not only by climate [14] but also by human behavior [15].
2. Water resources can form a supply chain independently [16] or a compound supply chain with other supply chains [17, 18].
3. Because of the special form of water, it is transported in one direction through prebuilt pipes and is very dependent on the infrastructure.
4. Water supply chain is characterized by regionalization and this characteristic is also obvious for risks in water supply chains [19, 20].

Inventory level is one of the important indicators reflecting economic benefits of enterprises. Many scholars have proved that RMI is an inevitable inventory management mode, and, in some cases, it has certain advantages. For example, Li et al. [21] through comparative studies verified that the operation efficiency of supply chains under the VMI mode is not necessarily superior to the RMI mode in any case, and the contracts that are applicable to the RMI mode are not necessarily equally applicable to the VMI mode. Anand et al. [22] proved that, in reality, retailers may hold and manage inventory in equilibrium as strategic inventory and suppliers are unable to prevent this. Hong and Park [23] compared the policies between RMI and VMI and found that the total cost of supply chain under VMI is sometimes higher than that in the traditional RMI mode. Li et al. [24] developed a scattered supply chain for two products with the retailer as Stackelberg leader. By comparing three models, the authors arrived at the conclusion that it is optimal to offer retail-sponsored gift cards in such a supply chain. Therefore, the research on the RMI mode has important practical significance.

Because of uncertainty, the assumption of stochastic demand is more practical and representative for reality than that of deterministic demand [25]. The stochastic model can more clearly explain the demand uncertainty and is closer to the present research and practice [26]. Many researches are conducted based on the stochastic demand hypothesis. A strict policy for carbon emissions considering partial backorders is explored by Ghosh et al. [27] in order to determine the optimal strategy, including order quantity, reordering point, and shipment number in a two-stage supply chain with stochastic demand. Chan et al. [28] proposed three models of synchronized cycles under stochastic customer demand and found that the total expected cost in the stochastic synchronous cycles model is better than that under the other stochastic policies. The inventory and routing decisions of supplier are studied by Onggo et al. [29] in order to minimize the costs for inventory, transportation, food-waste, and stockout of perishable products under customers’
stochastic demand. Chavarro et al. [30] generated and utilized 240 stochastic cases to evaluate deterministic and stochastic solution methods in the case that customer demand is considered as a normal density function. The key information for assignment decision considering customers’ hierarchy is identified by Fleischmann et al. [31]. A robust and nearly optimal decentralized assignment approach is developed to fulfill the hierarchical random demand. More relevant studies can be found in the recent literature [32–34].

“Rational man” or “economic man” was assumed in the classical decision theory and caused large differences between theoretical research results and real situations. The preference and expected utility theory proposed by Von Neumann and Morgenstern [35] is the formal origin of the risk preference theory. In the inventory management area, Zanakis et al. [36] analyzed the differences between theoretical research and management practice in detail, which gradually attracted the attention of academic circles [37–39]. Related to the risk preferences in inventory management, the research on risk aversion is dominant [40–43]. For our research topic, we should also deeply explore recent literature in terms of the impact of risk preference on water supply chain. Du et al. [44] considered risk preference of water supplier and water distributor in order to study water pricing strategies in two competitive water supply chains. However, they assumed that the water supplier and water distributor were risk-neutral and acted rationally. This is the limitation of their research. Chen et al. [17] established a synthetic model conception for a shale gas-water supply chain considering system dynamics and two-stage stochastic risk-averse programming, which generated a weight factor to measure the decision-makers’ averse attitude degree towards risk. Li et al. [45], based on risk interval, proposed a stochastic optimization model aiming at many uncertainties in agricultural water resources. The application of CVaR and risk-aversion measures was considered. Fu et al. [46] contributed to the methods of water allocations by considering risk preferences of decision-makers. However, they only mentioned the level of risk-aversion. To sum up, with regard to risk preferences in water supply chain, research on risk-aversion prevails, while risk-seeking type is barely involved. The Conditional Value at Risk (CVaR)

Table 1: The recently updated forecasts for world Gross Domestic Product (GDP) and trade.

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<tr>
<td>Forecast for WTO Trade (April 2020)</td>
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<tr>
<td>Optimistic scenario</td>
<td>−2.5</td>
<td>7.4</td>
<td>−12.9</td>
<td>21.3</td>
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<tr>
<td>Pessimistic scenario</td>
<td>−8.8</td>
<td>5.9</td>
<td>−31.9</td>
<td>24.0</td>
<td>3.6</td>
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<td>World Economic Prospects of the IMF (April 2020)</td>
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<tr>
<td>Optimistic scenario</td>
<td>−3.0</td>
<td>5.8</td>
<td>−11.0</td>
<td>8.4</td>
<td>3.6</td>
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<tr>
<td>Pessimistic scenario</td>
<td>−5.2</td>
<td>4.2</td>
<td>−13.4</td>
<td>5.3</td>
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<td>Economic Prospects of the OECD (June 2020)</td>
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<tr>
<td>Single-hit scenario</td>
<td>−6.0</td>
<td>5.2</td>
<td>−9.5</td>
<td>6.0</td>
<td>1.6</td>
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<tr>
<td>Double-hit scenario</td>
<td>−7.6</td>
<td>2.8</td>
<td>−11.4</td>
<td>2.5</td>
<td>1.5</td>
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<td>Memo terms</td>
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<tr>
<td>IMF GDP set at market exchange rate</td>
<td>−4.2</td>
<td>5.4</td>
<td>−11.0</td>
<td>8.4</td>
<td>2.6</td>
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<tr>
<td>World Bank GDP set at purchasing power parity</td>
<td>−4.1</td>
<td>4.3</td>
<td>−13.4</td>
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</tr>
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proposed by Rockafellar and Uryasev [47] is a common method to evaluate the degree of risks. Poormoai and Atan [48] used CVaR to combine risk-averse and risk-seeking attitudes of decision-makers. The parameters of optimal policy were obtained with maximization of the weighted utility function. Xu et al. [49] introduced CVaR into a news-vendor model and came to the conclusion that the risk-averse attitude of news-vendor facing opportunity loss may more likely cause excessive orders compared to risk-seeking attitude. Kouvelis and Li [50] used VaR constraints to study integrated risk management including risk-seeking framework in a news-vendor setting using profit maximization.

Nevertheless, the research on water supply chains from the perspective of inventory mode and risk-seeking preference is rarely carried out in the present literature and practices. This article happens to be a useful supplement to this field.

2. Methods

2.1. Model Assumptions. This paper studies the inventory decision-making of a two-echelon water supply chain under the RMI mode with stochastic demand. The basic assumptions are as follows:

(1) The water supply chain is composed of one supplier and one retailer, and only the retailer is used as the main decision-making subject to study the effect of risk-seeking preference on its inventory decision. The supplier’s output and inventory decisions are not controlled. Therefore, it is assumed that the supplier’s ending profit is only the difference between sales revenue and production cost without supplier’s inventory cost.

(2) The cost of water transportation is not considered.

(3) The retailer has only one order opportunity during the sales cycle and cannot replenish water.

(4) The wholesale price of water is determined by the supplier. The retail price of water is determined by the retailer without considering the functional relationship between price and demand.

(5) It is assumed that the market is open and there is no upper limit to the market demand. Let demand $x \in [0, +\infty)$ be a continuous random variable. Its probability density function is $f(x)$ and its cumulative distribution function is $F(x)$. $F(x)$ is a strictly monotonically increasing continuous function, and its inverse distribution exists, denoted as $F^{-1}(x)$. Since a specific functional form is required for sensitivity analysis, suppose that $F(x) = 1 - e^{-x}$, so $F^{-1}(x) = -\ln(1 - x)$.

2.2. Problem Description. Based on the above assumptions, the RMI mode with stochastic demand in the water supply chain is described as follows.

In a sales cycle, the retailer orders $q$ unit water from the supplier at wholesale price $w$ and sells it to consumers at retail price $p$. Due to the uncertainty of market demand $x$, the best situation is that the retailer has an accurate understanding of market demand, the order quantity is exactly equal to the quantity demanded, and there is neither inventory cost nor shortage cost. But, in most cases, the deviation between the retailer’s forecast of market demand and the actual market demand may lead to inventory costs (unit inventory cost is $h$) due to too many orders. At this time, at the end of the sales cycle, the retailer must deal with the remaining water at the unit residual value $r$, which is the preferential price; meanwhile, there may be shortage costs due to insufficient inventory (unit shortage cost is $b$).

2.3. Optimal Decision-Making of Risk-Seeking Retailer’s Managed Inventory in the Water Supply Chain. Based on the above problem description, during the sales cycle, when the retailer’s order quantity is $q$ and the market demand is $x$, the retailer’s profit function in the water supply chain is as follows:

$$
\pi_r(q, x) = p \min[\{q, x\}] - wq - h(q - x)^+ - b(x - q)^+ + r(q - x)^+,
\tag{1}
$$

where $X^* = \max[0, X]$. The first term at the right end of equation (1) represents the retailer’s sales revenue and therein $\min[\{q, x\}]$ represents the actual sales volume of the retailer, that is, the smaller one between the quantity ordered and the quantity demanded; the second item represents its order cost; the third item represents its inventory holding cost; the fourth item represents its shortage cost when demand exceeds supply; and the last item represents its income from selling surplus water when supply exceeds demand.

Since demand $x$ is a continuous random variable, the profit described by equation (1) is also a continuous random variable. Set $Z = \pi_r(q, x)$; the distribution function of $Z$ is the continuous function $G(z) = P[Z \leq z]$. The probability density function of $Z$ is the continuous function $g(z)$, and $z$ represents the given profit level. The certain risk level $\beta \in (0, 1]$ is given, and the conditional value at risk CVaRT$_\beta(Z)$ of the profit function for the risk-seeking retailer can be expressed as

$$
\text{CVaRT}_\beta(Z) = E(Z|Z \geq z_\beta) = \frac{1}{1 - \beta} \int_{z_\beta}^{\infty} zg(z)dz.
\tag{2}
$$

That is, the conditional value at risk CVaRT$_\beta(Z)$ of the risk-seeking retailer is used to represent the expected profit of the part, which is higher than the risk value $z_\beta$ on the profit function. At this point, the retailer does not control the part below the value at risk but only makes decisions on the part above the value at risk. This is the essential difference between a risk-seeking retailer and a risk-averse retailer.

In the case of retailer managed inventory under stochastic demand, the objective function of the risk-seeking retailer is

$$
\max \text{CVaRT}_\beta(Z).
\tag{3}
$$

We have defined the distribution function of random profit $Z$ as a continuous function $G(z) = P[Z \leq z]$, where $z$...
is the given profit level. Thus, in the case of the risk-seeking retailer managed inventory under stochastic demand, \( G(z) \) is as follows.

1. When \( x \leq q \),
\[
P\{px - wq - h_r (q - x) + r_r (q - x) \leq z\} = P\left\{x \leq \frac{(w + h_r - r_r)q + z}{p + h_r - r_r}\right\}.
\]  
(4)

Let \( q_1 = (w + h_r - r_r)q + z/p + h_r - r_r \) be set, denoted as
\[
G_1(z) = F(q_1).
\]  
(5)

2. When \( x > q \),
\[
P\{pq - wq - b(x - q) \leq z\} = P\left\{x \geq \frac{(p - w + b)q - z}{b}\right\}.
\]  
(6)

Let \( q_2 = (p - w + b)q - z/b \) be set, denoted as
\[
G_2(z) = 1 - F(q_2).
\]  
(7)

Because of the target profit level \( z \leq (p - w)q \), \( q_1 \leq q \) and \( q_2 \geq q \).

Now we present the relationship between the stochastic demand and profit in the case of retailer managed inventory under stochastic demand [51], as shown in Figure 2, where (a) represents the target profit level \( z \leq \pi_A \) and (b) represents the target profit level \( z \in (\pi_A, \pi_B] \) in the water supply chain.

From Figure 2, we can visually see the relationship between the retailer's profit and the stochastic demand with the fixed order quantity \( q \) in the water supply chain. We assume that the market is open and that there is no upper limit to the market demand \( x \). Let \( x \in (0, +\infty) \) be set; therefore, the retailer's profit function is in one continuous decreasing state in the interval \((q, +\infty)\). Point A denotes the retailer's profit when the market demand \( x = 0 \). The retailer holds the largest inventory at this point and its corresponding profit is \( \pi_A = (-w - h_r + r_r)q \). Point B represents the retailer’s profit when the market demand is exactly equal to the order quantity, which is also the maximum profit \( \pi_B = (p - w)q \) under the fixed order quantity. In order to further analyze the optimal inventory strategy of the risk-seeking retailer, we discuss it in the following two cases:

1. When \( z \leq \pi_A \) and \( z = G_2^{-1}(\beta) \), combining equation (7), the following can be obtained:
\[
z_\beta = (p - w + b)q - bF^{-1}(1 - \beta).
\]  
(8)

Meanwhile, from \( z \leq \pi_A \) and equation (8), the following can be obtained:
\[
q \leq \frac{bF^{-1}(1 - \beta)}{p + b + h_r - r_r}.
\]  
(9)

2. When \( z \in (\pi_A, \pi_B] \) and \( G_1(z_\beta) + G_2(z_\beta) = \beta \), combining equations (5) and (7), the following can be obtained:
\[
1 + F(q_1) - F(q_2) = \beta.
\]  
(10)

We get \( q > bF^{-1}(1 - \beta)/p + b + h_r - r_r \).

**Proposition 1.** In the water supply chain under RMI mode with stochastic demand, the risk-seeking retailer takes CVaR-profit maximization as its decision-making objective and its optimal inventory quantity \( Q^* \) is as follows:

1. When \( q \leq bF^{-1}(1 - \beta)/p + b + h_r - r_r \),
\[
Q^* = \begin{cases} 
F^{-1}\left(1 - \beta\right) \frac{p - w + b}{p + h_r + b - r_r} - x, & q^* > x \\
0, & q^* \leq x.
\end{cases}
\]  
(11)

2. When \( q > bF^{-1}(1 - \beta)/p + b + h_r - r_r \) and \( f(q) > b(w + h_r - r_r)^2 f(q_1) + (p + h_r - r_r)(p - w + b)^2 f(q_2) /b(p + h_r - r_r)(p + h_r + b - r_r), \)
\[
Q^* = \begin{cases} 
(p + b + h_r - r_r)^{-1} \left( F\left(q^*\right) - \frac{(p - w + b)(1 - \beta)}{p + h_r + b - r_r} \right) + bF^{-1}\left(1 - \beta\right) \frac{w + h_r - r_r}{p + h_r + b - r_r}, & q^* > x \\
0, & q^* \leq x.
\end{cases}
\]  
(12)
Proof. Based on the groundwork mentioned above, the solution process is divided into the two following situations:

(1) When \( q \leq bF^{-1}(1 - \beta)/p + h_r - r_r, z_\beta = (p - w + b)q - bF^{-1}(1 - \beta) \). By observing Figure 2, when \( z = z_\beta \), the corresponding demand is \( F^{-1}(1 - \beta) \). According to the definition formula (2) of the Conditional Value at Risk-seeking CVaRT \( \beta (Z) \), we can draw that the retailer does not make decisions on the worst profit in the demand interval \([F^{-1}(1 - \beta), +\infty)\) (the profit possibility of this part is \( \beta \)) but only controls the demand interval \([0, F^{-1}(1 - \beta)]\) corresponding to the stochastic profit \( z \geq z_\beta \). In this case, formula (2) is equivalent to

\[
CVaRT_\beta (Z) = \frac{1}{1 - \beta} \left( \int_q^0 ([p + h_r - r_r)t - (w + h_r - r_r)t]dF(t) + \int_q^{F^{-1}(1-\beta)} [(p - w + b)q - bt]dF(t) \right). 
\] (13)

Therefore, the maximum exists. The optimal order quantity can be obtained by setting the first-order condition equal to 0:

\[
q^* = F^{-1}\left((1 - \beta) - \frac{p - w + b}{p + h_r + b - r_r}\right). 
\] (16)

(2) When \( q > bF^{-1}(1 - \beta)/p + h_r - r_r, \) \( z_\beta \) satisfies \( 1 + F(q_1) - F(q_2) = \beta \). Similarly, by observing Figure 2, we can obtain that the retailer only controls the demand segment \([q_1, q_2]\) corresponding to the stochastic profit \( z \geq z_\beta \). At this point, equation (2) is equivalent to

\[
\frac{\partial CVaRT_\beta (Z)}{\partial q} = \frac{1}{1 - \beta} \left( -(p + h_r + b - r_r)F(q) + (p - w + b)(1 - \beta) \right). 
\] (14)

We have

\[
\frac{\partial^2 CVaRT_\beta (Z)}{\partial q^2} = \frac{1}{1 - \beta} \left( -(p + h_r + b - r_r)f(q) \right) < 0. 
\] (15)
Solving the first partial derivative of the above equation with respect to order quantity \( q \), we can get

\[
\frac{\partial \text{CVaRT}_\beta(Z)}{\partial q} = \frac{1}{1-\beta} \left( -(p + h_r + b - r_r)F(q) + (w + h_r - r_r)F(q_1) + (p - w + b)F(q_2) \right) .
\]

(18)

where

\[
\frac{\partial^2 \text{CVaRT}_\beta(Z)}{\partial q^2} = \frac{1}{1-\beta} \left( -(p + h_r + b - r_r)f(q) + \frac{(w + h_r - r_r)^2}{p + h_r - r_r} f(q_1) + \frac{(p - w + b)^2}{b} f(q_2) \right) .
\]

(19)

The sign symbol of the above equation cannot be judged by the existing conditions, so we use the inverse method to solve it. Assuming that when \( q > bF^{-1}(1-\beta)/p + b + h_r - r_r \), the maximum exists,

\[
\frac{\partial^2 \text{CVaRT}_\beta(Z)}{\partial q^2} < 0 ,
\]

(20)

\[
q^* = \left( p + h_r - r_r \right) F^{-1} \left( F(q^*) - \left( (p - w + b)(1-\beta)/(p + h_r + b - r_r) \right) \right) + bF^{-1} \left( F(q^*) + \left( (w + h_r - r_r)(1-\beta)/(p + h_r + b - r_r) \right) \right) \frac{b}{p + b + h_r - r_r} .
\]

(21)

In conclusion, we have the following:

1. When \( q \leq bF^{-1}(1-\beta)/p + b + h_r - r_r \), the optimal order quantity \( q^* \) of the risk-seeking retailer in the water supply chain is

\[
q^* = F^{-1} \left( (1-\beta) \frac{p - w + b}{p + h_r + b - r_r} \right) .
\]

(22)

2. When \( q > bF^{-1}(1-\beta)/p + b + h_r - r_r \) and \( f(q) > b(w + h_r - r_r)^2 f(q_1) + (p + h_r - r_r)(p - w + b)^2 f(q_2)/b(p + h_r - r_r)(p + h_r + b - r_r) \), the optimal order quantity \( q^* \) of the risk-seeking retailer in the water supply chain is

\[
q^* = \left( p + h_r - r_r \right) F^{-1} \left( F(q^*) - \left( (p - w + b)(1-\beta)/(p + h_r + b - r_r) \right) \right) + bF^{-1} \left( F(q^*) + \left( (w + h_r - r_r)(1-\beta)/(p + h_r + b - r_r) \right) \right) \frac{b}{p + b + h_r - r_r} .
\]

(23)
Thus, equations (11) and (12) of the optimal inventory quantity of the retailer in the water supply chain hold. So, we can directly get Proposition 2–Proposition 5.

**Proposition 2.** In the case of the risk-seeking retailer managed inventory under stochastic demand scenario in the water supply chain, the optimal inventory cost $C^*$ of the risk-seeking retailer aiming at maximizing CVaR-profit is given by

$$C^* = h_r (q^* - x)^+.$$ (24)

**Proposition 3.** In the case of the risk-seeking retailer managed inventory under stochastic demand scenario in the water supply chain, the supplier’s profit can be expressed by

$$\pi_m = (w - c) q^*. \quad (25)$$

**Proposition 4.** In the case of the risk-seeking retailer managed inventory under stochastic demand scenario in the water supply chain, the retailer’s profit can be expressed by

$$\pi_r = (p - w + b) q^* - b x - (p + h_r + b - r_r) (q^* - x)^+. \quad (26)$$

**Proposition 5.** In the case of the risk-seeking retailer managed inventory under stochastic demand scenario in the water supply chain, the profit of the entire supply chain is given by

$$\pi_c = (p + b - c) q^* - b x - (p + h_r + b - r_r) (q^* - x)^+. \quad (27)$$

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### 3. Results and Discussion

Then, the sensitivity of the risk-seeking retailer’s optimal inventory strategy is analyzed under RMI mode with stochastic demand in the water supply chain. The effects of relevant factor changes on the retailer’s inventory strategy are discussed.

Since Proposition 1 gives the optimal inventory quantity under two conditions, as shown in equations (11) and (9), in this section, we only carry out sensitivity analysis on the optimal inventory strategy when $x < q^* \leq b F^{-1} (1 - \beta)/p + b + h_r - r_r$, as shown in Corollaries 1 and 2.

**Corollary 1.** In the case of the risk-seeking retailer managed inventory under stochastic demand scenario in the water supply chain, when $x < q^* \leq b F^{-1} (1 - \beta)/p + b + h_r - r_r$, the optimal inventory quantity $Q^*$ of the risk-seeking retailer increases monotonously with respect to the retail price $p$, unit shortage cost $b$, and unit residual value $r_r$, while it decreases monotonously with respect to the wholesale price $w$ and unit inventory cost $h_r$.

*Proof.* Solving the partial derivatives of the optimal inventory quantity $Q^*$ with respect to the retail price $p$, wholesale price $w$, unit shortage cost $b$, unit inventory cost $h_r$ and unit residual value $r_r$ in equation (11), we can obtain

$$\frac{\partial Q^*}{\partial p} = \frac{1}{f\left[F^{-1}(1 - \beta)\left(p - w + b/p + h_r + b - r_r\right)\right]} (1 - \beta) \frac{h_r + w - r_r}{(p + h_r + b - r_r)^2} \geq 0,$$

$$\frac{\partial Q^*}{\partial w} = \frac{1}{f\left[F^{-1}(1 - \beta)\left(p - w + b/p + h_r + b - r_r\right)\right]} (1 - \beta) \frac{1}{p + h_r + b - r_r} \leq 0,$$

$$\frac{\partial Q^*}{\partial b} = \frac{1}{f\left[F^{-1}(1 - \beta)\left(p - w + b/p + h_r + b - r_r\right)\right]} (1 - \beta) \frac{h_r + w - r_r}{(p + h_r + b - r_r)^2} \geq 0,$$

$$\frac{\partial Q^*}{\partial h_r} = \frac{1}{f\left[F^{-1}(1 - \beta)\left(p - w + b/p + h_r + b - r_r\right)\right]} (1 - \beta) \frac{w - p - b}{(p + h_r + b - r_r)^2} \leq 0,$$

$$\frac{\partial Q^*}{\partial r_r} = \frac{1}{f\left[F^{-1}(1 - \beta)\left(p - w + b/p + h_r + b - r_r\right)\right]} (1 - \beta) \frac{p - b - w}{(p + h_r + b - r_r)^2} \geq 0.$$ (28)

To sum up, the following conclusions are valid:
\[
\frac{\partial Q^*}{\partial p} \geq 0,
\frac{\partial Q^*}{\partial w} \leq 0,
\frac{\partial Q^*}{\partial b} \geq 0,
\frac{\partial Q^*}{\partial h_r} \leq 0,
\frac{\partial Q^*}{\partial r} \geq 0.
\]

(29)

In other words, Corollary 1 is valid.

Corollary 1 shows that the risk level has no effect on the monotonicity of the optimal inventory quantity in the five dimensions of the retail price, wholesale price, unit shortage cost, unit inventory cost, and unit residual value in the water supply chain. But, from Figure 3–7, we can see that, under different risk levels, the retail price, wholesale price, unit shortage cost, unit inventory cost, and unit residual value have different effect strengths on the optimal inventory quantity. When the retail price, unit shortage cost, and unit residual value increase, the risk-seeking retailer’s optimal inventory quantity also increases, as shown in Figures 3–5 and Figures 8–10. On the contrary, when the wholesale price and unit inventory cost increase, the optimal inventory quantity decreases, as shown in Figures 6, 7, 11, and 12.

**Corollary 2.** In the water supply chain, when \( x < q^* \leq bF^{-1}(1-\beta)/p + b + h_r - r_r \), the optimal inventory cost \( C^* \) of the risk-seeking retailer under RMI mode with stochastic demand increases monotonously with respect to the retail price \( p \), unit shortage cost \( b \), and unit residual value \( r_r \), while it decreases monotonously with respect to the wholesale price \( w \). When \( 0 < h_r \leq Q^*/B, \) \( r_r \) increases monotonously with respect to the unit inventory cost \( h_r \); when \( h_r > Q^*/B \), \( C^* \) decreases monotonously with respect to the unit inventory cost \( h_r \).

**Proof.** Because the proof process is similar to that of Corollary 1, we do not repeat it here and only give a brief analysis and figures. When \( x < q^* \leq bF^{-1}(1-\beta)/p + b + h_r - r_r \), solving the partial derivatives of the optimal inventory cost \( C^* \) with respect to the retail price \( p \), wholesale price \( w \), unit shortage price \( b \), unit residual value \( r_r \), and unit inventory cost \( h_r \) in equation (24), we can obtain

\[
\frac{\partial C^*}{\partial h_r} = Q^* - h_r - \frac{(1-\beta)(p-w+b)}{(p+h_r+b-r_r)^2F^{-1}(1-\beta)(p-w+b/p + h_r + b - r_r))}.
\]

(30)

Let \( B = (1-\beta)(p-w+b)/(p+h_r+b-r_r) \) be set; \( B > 0 \), where \( \frac{\partial C^*}{\partial h_r} = Q^* - h_rB \).

Thus, \( B > 0 \), where \( \frac{\partial C^*}{\partial h_r} = Q^* - h_rB \).

Because it is not easy to judge the sign symbol on the right side of the equation, we discuss it in the two following cases:

1. When \( 0 < h_r \leq Q^*/B \), it is easy to get \( \frac{\partial C^*}{\partial h_r} \geq 0 \)
2. When \( h_r > Q^*/B \), it is easy to get \( \frac{\partial C^*}{\partial h_r} < 0 \)

That is, Corollary 2 holds.

Corollary 2 shows that the risk level has no effect on the monotonicity of the optimal inventory cost in the five dimensions of the retail price, wholesale price, unit shortage cost, unit inventory cost, and unit residual value. But, from Figure 13–17, we can see that, under different risk levels, the retail price, wholesale price, unit shortage cost, unit inventory cost, and unit residual value have different effect strengths on the optimal inventory cost. When the retail price, unit shortage cost, and unit residual value increase, the risk-seeking retailer’s optimal inventory cost also increases, as shown in Figures 13–15 and Figure 18–20. On the contrary, when the wholesale price increases, the optimal inventory cost decreases, as shown in Figure 16 and Figure 21. When \( 0 < h_r \leq Q^*/B \), the optimal inventory cost increases with the increase of the unit inventory cost; when \( h_r > Q^*/B \), the optimal inventory cost decreases with the increase of the unit inventory cost, as shown in Figures 17 and 22.
Figure 3: Effect of risk level $\beta$ and retail price $p$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 4: Effect of risk level $\beta$ and unit shortage cost $b$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 5: Effect of risk level $\beta$ and unit residual value $r_r$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 6: Effect of risk level $\beta$ and wholesale price $w$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 7: Effect of risk level $\beta$ and unit inventory cost $h_r$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 8: Effect of retail price $p$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.
Figure 9: Effect of unit shortage cost $b$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 10: Effect of unit residual value $r_r$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 11: Effect of wholesale price $w$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.
Figure 12: Effect of unit inventory cost $h_r$ on the optimal inventory quantity $Q^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 13: Effect of risk level $\beta$ and retail price $p$ on the optimal inventory cost $C^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 14: Effect of risk level $\beta$ and unit shortage cost $b$ on the optimal inventory cost $C^*$ under RMI mode with stochastic demand in the water supply chain.
Figure 15: Effect of risk level $\beta$ and unit residual value $r_r$ on the optimal inventory cost $C^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 16: Effect of risk level $\beta$ and wholesale price $w$ on the optimal inventory cost $C^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 17: Effect of risk level $\beta$ and unit inventory cost $h_r$ on the optimal inventory cost $C^*$ under RMI mode with stochastic demand in the water supply chain.
Figure 18: Effect of retail price $p$ on the optimal inventory cost $C^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 19: Effect of unit shortage cost $b$ on the optimal inventory cost $C^*$ under RMI mode with stochastic demand in the water supply chain.

Figure 20: Effect of unit residual value $r_r$ on the optimal inventory cost $C^*$ under RMI mode with stochastic demand in the water supply chain.
4. Conclusions

Water supply chain is characterized by high risk and multiple risks. The decision-maker’s attitude to risk is one of the important factors that affect the decision result. Uncertain demand is the most fundamental uncertainty in supply chains. So, stochastic demand is incorporated into our research hypothesis. Based on the model assumptions, we study the optimal decisions of the risk-seeking retailer under RMI mode considering stochastic demand in a water supply chain. Firstly, we discussed the relationship between stochastic demand and profit. Next, we proved the expressions of the risk-seeking retailer’s optimal inventory quantity, and then the expression of the risk-seeking retailer’s optimal inventory cost, the profit expressions of the supplier, retailer, and the entire supply chain are derived. Finally, through sensitivity analysis and numerical simulation, we explored the impact of risk level on the optimal inventory decisions. We get the following conclusions:

(1) In the water supply chain, the risk-seeking level has no effect on the monotonicity of the optimal inventory quantity and optimal inventory cost in the five dimensions of the retail price, wholesale price, unit shortage cost, unit inventory cost, and unit residual value. But, under different risk levels, the five dimensions generate different effect strengths on the optimal inventory strategy.

(2) With regard to the optimal inventory quantity and optimal inventory cost of the risk-seeking retailer, their monotonicities are positively associated with the retail price, unit shortage cost, and unit residual value, while they are negatively correlated with the wholesale price.

(3) The unit inventory cost has different effects on the optimal inventory quantity and optimal inventory cost. It is negatively correlated with the optimal inventory quantity. Meanwhile, there is a certain

Figure 21: Effect of wholesale price \( w \) on the optimal inventory cost \( C^* \) RMI mode with stochastic demand under in the water supply chain.

Figure 22: Effect of unit inventory cost \( h_r \) on the optimal inventory cost \( C^* \) under RMI mode with stochastic demand in the water supply chain.
threshold for unit inventory cost, which makes the monotonicity of the risk-seeking retailer’s optimal inventory cost change before and after this threshold. We can explain this counterintuitive phenomenon. Before the unit inventory cost is less than the certain threshold, the increase in the risk-seeking retailer’s inventory quantity can still increase the expected profit. At this time, the risk-seeking retailer still increases the inventory quantity, and the inventory cost increases accordingly. When the unit inventory cost exceeds the threshold, the increase in the risk-seeking retailer’s inventory quantity leads to a decrease in the expected profit. At this time, the risk-seeking retailer reduces the inventory quantity, and the corresponding inventory cost also decreases.

(4) From the expressions of the supplier’s optimal profit and the profit of the entire supply chain, we can obtain that the supplier’s optimal profit and the profit of the entire supply chain are positively associated with the retailer’s order quantity. Therefore, the risk-seeking level of the retailer also affects the supplier’s profit and supply chain’s profit. It is important for the supplier to pay attention to the retailer’s risk preference.

(5) We considered only demand as a random variable in our research. There are also price uncertainty and cost uncertainty in practice. It makes sense to study the association of multiple random variables. Some scholars have made attempt methods to describe the relationships between various random variables [52].

Water is regarded as a crucial business resource. This correlational research is carried out from the perspective of supply chain operations and focuses on the optimal inventory strategy of the risk-seeking retailer under RMI mode considering stochastic demand, which makes the research more practical.

Data Availability

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation, to any qualified researcher.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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