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Research Article

Two-Player Location-Price Game in a Spoke Market with Linear Transportation Cost

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This paper investigates the location game of two players in a spoke market with linear transportation cost. A spoke market model has been proposed, which is inspired by the Hotelling model and develops two-player games in price competition. Using two-stage (position and price) patterns and the backward guidance method, the existence of price and location equilibrium results for the position games is proved.

1. Introduction

In 1928, John von Neumann proved the basic principle of game theory [1]. Nowadays, game theory is not only a new field of modern mathematics but also an important subject of operational research. The game theory mainly studies the interaction between the mathematical theory and the incentive structure for studying the competitive phenomena [2]. It is one of the standard analysis tools for economics and is widely applied in finance, securities, international relations, computer science, political science, and many other fields [3–8].

As an important research object in the field of industrial organization and supply chain management, the location problem attracts attention more and more. In 1929, the game theory was applied to the positional problem by Hotelling and the classic Hotelling model was constructed [9]. In this model, it is assumed that consumers are uniformly distributed in a linear street, and there are two companies of the same size which determine their locations to maximize the profits. In the subsequent decades, various position problems developed from the classical model were considered, and many results were obtained. The result of d'Aspremont et al. [10] shows that the price equilibrium solution is ubiquitous for the modified Hotelling model and that the seller tends toward the difference of maximization. The Cournot competition with uneven distribution of consumers in a linear city model was studied in [11], and a necessary condition of agglomeration equilibrium was obtained. The author in [12] claimed that if there is no pure strategy equilibrium, the Hotelling model exhibits a mixed-strategy equilibrium. The Hotelling spatial competition model was extended by the author in [13] from three aspects: shape of the demand curve, the number of firms, and type of space. In [14], the Hotelling model for duopolistic competition with a class of utility functions was examined. In the meantime, when the curvature of the utility functions is high enough, the existence of an equilibrium was proven. The relationship between the equilibrium location of the Hotelling model and the consumer density was analyzed by the authors in [15], and it was pointed out that the higher the consumer density, the closer the equilibrium position. In [16], the author investigated the existence of equilibrium states in the Hotelling model in the case of n players and analyzed the effect of the number of companies on the equilibrium results of the Hotelling game. The Hotelling duopoly model with brand loyalty and network effects was considered in [17]. Also, the results show that when the transportation cost is a linear function, there is a pure strategy price equilibrium. The authors in [18] found that the principal consideration affecting the equilibrium locations is production technology when the impact of labor inputs and production technology on spatial competition was considered in the Hotelling model. Under general conditions, the result in [19] shows that there is a pure strategy price-location Nash equilibrium in the Hotelling duopoly model based on the cost-of-location function. Based on the developed duopoly game by the Hotelling model, the competition between online retailers and brick-and-mortar retailers was investigated by the authors in [20]. Hotelling introduced the bounded linear region of basic location space into location game for the first time [9]. Inspired by the hot ring model, there has been a lot of literature on the location game of linear location space. In [10], a modified Hotelling instance was proposed by using nonlinear transportation cost instead of linear transportation cost, and the ineffectiveness of the principle of minimum differentiation was proven. By correcting some assumptions of the Hotelling model, spatial duopoly competition was discussed by the authors in [21], and the equilibrium position of bounded sections was found to be the same as the social optimum position of n enterprises. In [22], the existence of the Nash equilibrium of locations and prices in the learning markets was verified, and the impact of the freight rates and the magnitude of changes in marginal costs on one or two companies was also examined. In addition, the comparison of Cournot competition with Bertrand competition was made in the game of location [23], when the position space is a linear limit area.

In fact, the market usually includes a variety of complex traffic networks. In order to accurately reflect the actual market, complex places such as spokes and circles are considered by many researchers. Based on the quadratic transportation cost function, the author in [24] considered the location space as a circular road and proved the existence and uniqueness of a unique price equilibrium in multiplayer location game. Furthermore, as for the circle market, the authors in [25] considered the problems of nonexistence and existence of an equilibrium for a location-price game. In [26], the authors explored a linear and circular model with spatial Cournot competition and examined the dependence between demand density and location equilibrium. For multiple participants in a circular market, the authors in [27] claimed that the unique equilibrium position is equidistantly distributed. By using a spoke model, the nonlocalised spatial competition was considered by the authors in [28], and the influence of the number of enterprises on the equilibrium price was also analyzed. In addition, an explicit partial game complete set of equilibrium positions was induced by the author in [29] by assuming that crossing finite roads and transport costs proportional to the distance square root. In the spoke model, the location choices and spatial price discrimination were considered by the author in [30].

In this paper, strongly motivated by the above discussion, we developed a location game in the spoke market, where two players make price competition in the market. The main problem is how to choose the optimal point on the spokes for each player as its location such that its profit is maximized.

2. Descriptions of the Spoke Model

In terms of geometry, the market is made up of N spokes converging at one common point, where these spokes are $OA_1, OA_2, \ldots, OA_N$. Each of them has a fixed length, normalized to $l_j = 1$, $j = 1, 2, \ldots, N$. For example, a spoke model with $N = 7$ is shown in Figure 1. Then, the total length of the market is N. Consumers are evenly distributed on each spoke with a constant density, normalized to $\rho = 1$. Therefore, N represents the total number of customers.

A customer on a spoke i is denoted by $x_i \in [0, 1]$. For example, when $x_i = 0$ the customer is located at the center O of the market. while customers at $x_i = 1$ are located at extreme point A_i of the spoke OA_i.

Suppose that each player prices the products. For any customer, the products from one of the players are sold for the same price. Also, the transportation costs are paid by the customer. Let p_k be the mill price of the products of player k, where $k = 1, 2$. Furthermore, $T(d)$ is a linear transportation cost function, which is described as follows:

$$T(d) = cd, \tag{1}$$

where d is the distance from the player to the customer and c is a positive number representing the transportation cost constant. From (1), it is seen that the transportation cost $T(d)$ increases linearly with transportation distance d.

The net utility of a customer at x_i buying products of player k is expressed as follows:

$$u(x_i, p_k, y_k) = u - p_k - T(d(x_i, y_k)), \tag{2}$$

where u represents the utility directly obtained by customers buying homogeneous products, which is large enough to make sure that $u(x_i, p_k, y_k)$ is positive. $d(x_i, y_k)$ is the distance from customer position x_i to the location y_k of firm k. If the customer located at x_i and the firm j located at y_k are on the same spoke OA_i, the distance $d(x_i, y_k) = |x_i - y_k|$. If they are on different spokes, the distance $d(x_i, y_k) = x_i + y_k$. Thus, this distance $d(x_i, y_k)$ can be expressed as

$$d(x_i, y_k) = \begin{cases} |x_i - y_k|, & x_i \in OA_i, y_k \in OA_i, \\ x_i + y_k, & x_i \in OA_i, y_k \notin OA_i. \end{cases} \tag{3}$$

3. Main Results

It is natural that the customer located at x_i will buy the products of firm 1 if $u(x_i, p_1, y_1) > u(x_i, p_2, y_2)$. Otherwise, the customer will choose the products from firm 2. The set of all customers buying the products of firm k is called an attraction domain for this firm. In order to obtain the number of customers in the attraction domain, we should find the marginal customers whose net utilities for firms 1 and 2 are indifferent. So, the location x_i of the mage customer satisfies

$$p_1 + cd(x_i, y_1) = p_2 + cd(x_i, y_2). \tag{4}$$

Lemma 1. *Suppose that firms 1 and 2 are both on spoke 1 and $y_1 \le y_2$; then, the number of customers M_k in attraction domains for firm k is as follows:*

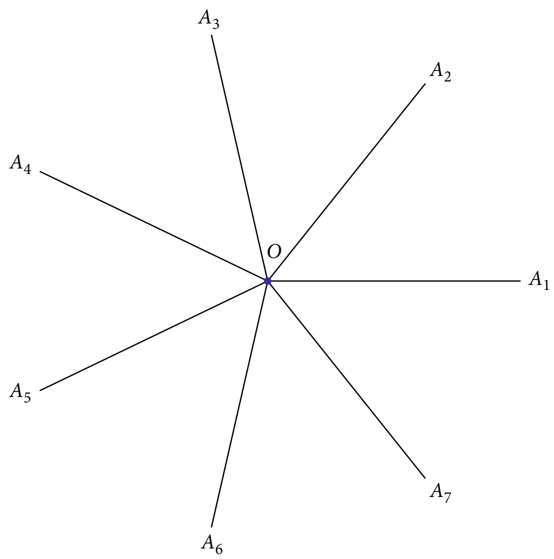

Figure 1: A spoke model with $N = 7$.

(a) $M_1 = (1/2)(p_2 - p_1 + y_1 + y_2) + N - 1$ *and* $M_2 = 1 - (1/2)(p_2 - p_1 + y_1 + y_2)$ *if* $y_1 - y_2 \le p_1 - p_2 < y_2 - y_1$.

(b) $M_1 = N$ *and* $M_2 = 0$ *if* $p_1 - p_2 < y_1 - y_2$.

(c) $M_1 = 0$ *and* $M_2 = N$ *if* $p_1 - p_2 > y_2 - y_1$.

(d) $M_1 = (1/2)(N - 1 + y_1)$ *and* $M_2 = (1/2)(N + 1 - y_1)$ *if* $p_1 - p_2 = y_2 - y_1$.

Lemma 2. *Suppose that firm 1 is on spoke 1, firm 2 is on spoke 2, and* $y_1 \le y_2$*; then, the number of customers* M_k *in attraction domains for firm* k *is as follows:*

(a) $M_1 = (1/2)(p_2 - p_1 + y_1 + y_2) + N - 1$ *and* $M_2 = 1 - (1/2)(p_2 - p_1 + y_1 + y_2)$ *if* $y_1 - y_2 \le p_1 - p_2 < y_2 - y_1$.

(b) $M_1 = N$ *and* $M_2 = 0$ *if* $p_1 - p_2 \le -y_1 - y_2$.

(c) $M_1 = 0$ *and* $M_2 = N$ *if* $p_1 - p_2 \ge y_1 + y_2$.

(d) $M_1 = (N/2)$ *and* $M_2 = (N/2)$ *if* $p_1 - p_2 = y_2 - y_1$.

The structure of the game played by the two firms is as follows:

(1) Location stage: each firm determines its location $y_k \in [0, 1]$ on its spoke simultaneously.

(2) Price stage: each firm chooses the price strategy $p_k(y_1, y_2)$ based on the locations y_1 and y_2.

In the following, the backward induction method will be employed to solve the game. In the second stage, for given positions on the spokes, firms simultaneously determine their prices to ensure maximum profits in current location. In the following, we only discuss the case of the two firms in the same spoke. For the other case, where they are in different spokes, the discussion is very similar. Obviously, their profits π_i can be computed by

$$\pi_i(p_1, p_2, y_1, y_2) = p_i M_i(p_1, p_2, y_1, y_2), \quad (5)$$

where $i = 1, 2$. For example, the relationship between π_1 and p_1 can be seen in Figure 2. To obtain a price equilibrium, we need solve the following equations derived from the first-order conditions of the profits in p_1 and p_2:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial p_1} = 0, \quad \text{or } \dfrac{\partial \pi_1}{\partial p_1} \text{ does not exist,} \\[2ex] \dfrac{\partial \pi_2}{\partial p_2} = 0, \quad \text{or } \dfrac{\partial \pi_2}{\partial p_2} \text{ does not exist.} \end{cases} \quad (6)$$

According to Lemma 1 and by solving equation (6), we have the price equilibrium:

$$\begin{aligned} p_1^* &= \frac{1}{3}(y_1 + y_2) + \frac{4}{3}N - \frac{2}{3}, \\ p_2^* &= -\frac{1}{3}(y_1 + y_2) + \frac{2}{3}N + \frac{2}{3}. \end{aligned} \quad (7)$$

In the first stage, firms simultaneously determine their location based on optimal price strategy (7). Noting that $y_1 \le y_2$, the location equilibrium is

$$\begin{aligned} y_1^* &= 0, \\ y_2^* &= 1. \end{aligned} \quad (8)$$

Summarizing the above discussion, we obtain the main result.

Proposition 1. *In a spoke market with transportation cost function (1) and net utility (2), if the two players develop the location game with price competition, the equilibrium location is that* $y_1^* = 0$ *and* $y_2^* = 1$*, which means that one player is at the center point of the market and another player is at the extreme point.*

Remark 1. In the proposition, the equilibrium location reflects the principle of maximum differentiation, which makes the two players avoid vicious price war in the same place.

To illustrate the dynamic behaviors of two players in the market and verify the validity of the results, we design the computer simulation algorithm in the following. At the beginning, each player randomly chooses its position y_i in a spoke market. Then, the two players start the first round of position game. Player 1 calculates its optimal price and profit at the current position by formulas (5) and (7). Player 1 tries to move a small step so that its new position increases its profit. Then, player 2 moves a small step in the same manner as player 1. In this way, the two players adjust their positions in each round and move alternately in the spoke market until their profits no longer increase.

Therefore, the algorithm for simulating dynamic behaviors in the spoke market can be given as follows:

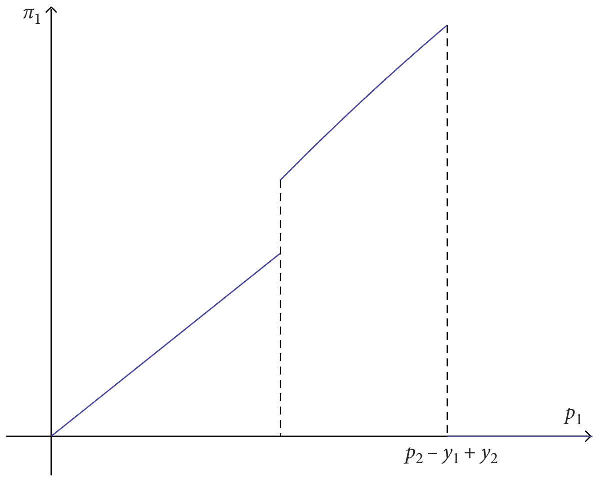

Figure 2: The relationship between π_1 and p_1.

(i) Step 1: initialize the move step size h with a small number. Initialize the positions y_1 and y_2 of the two players with two random numbers in $[0, 1]$.

(ii) Step 2: compute p_1 and p_2 by formula (7).

(iii) Step 3: compute $X_1 = \pi_1(p_1, p_2, y_1, y_2)$, $X_2 = \pi_1(p_1, p_2, y_1 + h, y_2)$ and $X_3 = \pi_1(p_1, p_2, y_1 - h, y_2)$ by formula (5).

(iv) Step 4: if $X_1 < X_2$, assign the value $y_1 + h$ to y_1. If $X_1 < X_3$, assign the value $y_1 - h$ to y_1.

(v) Step 5: compute $Y_1 = \pi_1(p_1, p_2, y_1, y_2)$, $Y_2 = \pi_1(p_1, p_2, y_1, y_2 + h)$ and $Y_3 = \pi_1(p_1, p_2, y_1, y_2 - h)$ by formula (5).

(vi) Step 6: if $Y_1 < Y_2$, assign the value $y_2 + h$ to y_2 and return to Step 2. If $Y_1 < Y_3$, assign the value $y_2 - h$ to y_2 and return to Step 2.

4. Conclusion

The location game with price competition in a spoke market is established for two players, where the transportation cost is linear. Employing a two-stage approach, the location equilibrium of the location game is proved for the considered market. The obtained result shows that one player should be at the center point of the market while another one should be at the extreme point when the location game is in equilibrium. This paper considers the geometry of the market as spokes. In fact, the geometry of the market could be very complex in the real world. Therefore, we will mainly consider the location games on complex grids in future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] J. von Neumann, "Zur theorie der gesellschaftsspiele," *Mathematische Annalen*, vol. 100, no. 1, pp. 295–320, 1928.

[2] M. Valizadeh and A. Gohari, "Playing games with bounded entropy," *Games and Economic Behavior*, vol. 115, pp. 363–380, 2019.

[3] S. Lasaulce and H. Tembine, *Game Theory and Learning for Wireless Networks. Fundamentals and Applications*, Academic Press, Cambridge, MA, USA, 2011.

[4] P. De Giovanni and G. Zaccour, "A two-period game of a closed-loop supply chain," *European Journal of Operational Research*, vol. 232, no. 1, pp. 22–40, 2014.

[5] S. Goyal, H. Heidari, and M. Kearns, "Competitive contagion in networks," *Games and Economic Behavior*, vol. 113, pp. 58–79, 2019.

[6] E. Talamàs, "Price dispersion in stationary networked markets," *Games and Economic Behavior*, vol. 115, pp. 247–264, 2019.

[7] W.-C. Guo and F.-C. Lai, "Spatial cournot competition in two intersecting circular markets," *The Annals of Regional Science*, vol. 64, no. 1, pp. 37–56, 2020.

[8] L. F. Maia, W. Viana, and F. Trinta, "Transposition of location-based games: using procedural content generation to deploy balanced game maps to multiple locations," *Pervasive and Mobile Computing*, vol. 70, Article ID 101302, 2021.

[9] H. Hotelling, "Stability in competition," *The Economic Journal*, vol. 39, no. 153, pp. 41–57, 1929.

[10] C. d'Aspremont, J. J. Gabszewicz, and J.-F. Thisse, "Computation of multi-facility location nash equilibria on a network under quantity competition," *Econometrica*, vol. 47, no. 4, pp. 1145–1150, 1979.

[11] B. Gupta, D. Pal, and J. Sarkar, "Spatial cournot competition and agglomeration in a model of location choice," *Regional Science and Urban Economics*, vol. 27, no. 3, pp. 261–282, 1997.

[12] E. Gal-or, "Hotelling's spatial competition as a model of sales," *Economics Letters*, vol. 9, no. 1, pp. 1–6, 1982.

[13] D. Graitson, "Spatial competition a la Hotelling: a selective survey," *The Journal of Industrial Economics*, vol. 31, no. 1/2, pp. 11–25, 1982.

[14] N. Economides, "Minimal and maximal product differentiation in Hotelling's duopoly," *Economics Letters*, vol. 21, no. 1, pp. 67–71, 1986.

[15] S. P. Anderson, J. K. Goeree, and R. Ramer, "Location, location, location," *Journal of Economic Theory*, vol. 77, no. 1, pp. 102–127, 1997.

[16] S. Brenner, "Hotelling games with three, four, and more players," *Journal of Regional Science*, vol. 45, no. 4, pp. 851–864, 2005.

[17] L. Lambertini and R. Orsini, "On Hotelling's 'stability in competition' with network externalities and switching costs," *Papers in Regional Science*, vol. 92, no. 4, pp. 873–883, 2013.

[18] W.-C. Guo, F.-C. Lai, and D.-Z. Zeng, "A Hotelling model with production," *Mathematical Social Sciences*, vol. 73, pp. 40–49, 2015.
[19] J. Hinloopen and S. Martin, "Costly location in Hotelling duopoly," *Research in Economics*, vol. 71, no. 1, pp. 118–128, 2017.
[20] J. Chen and B. Chen, "When should the offline retailer implement price matching?" *European Journal of Operational Research*, vol. 277, no. 3, pp. 996–1009, 2019.
[21] A. P. Lerner and H. W. Singer, "Some notes on duopoly and spatial competition," *Journal of Political Economy*, vol. 45, no. 2, pp. 145–186, 1937.
[22] A. Smithies, "Optimum location in spatial competition," *Journal of Political Economy*, vol. 49, no. 3, pp. 423–439, 1941.
[23] J. H. Hamilton, J.-F. Thisse, and A. Weskamp, "Spatial discrimination: Bertrand vs. Cournot in a model of location choice," *Regional Science and Urban Economics*, vol. 19, no. 1, pp. 87–102, 1989.
[24] M. Peitz, "The circular road revisited: uniqueness and supermodularity," *Research in Economics*, vol. 53, no. 4, pp. 405–420, 1999.
[25] M. A. de Frutos, H. Hamoudi, and X. Jarque, "Equilibrium existence in the circle model with linear quadratic transport cost," *Regional Science and Urban Economics*, vol. 29, no. 5, pp. 605–615, 1999.
[26] W.-C. Guo and F.-C. Lai, "Spatial Cournot competition in a linear-circular market," *The Annals of Regional Science*, vol. 54, no. 3, pp. 819–834, 2015.
[27] B. Gupta, F.-C. Lai, D. Pal, J. Sarkar, and C.-M. Yu, "Where to locate in a circular city?" *International Journal of Industrial Organization*, vol. 22, no. 6, pp. 759–782, 2004.
[28] Y. Chen and M. H. Riordan, "Price and variety in the spokes model," *The Economic Journal*, vol. 117, no. 522, pp. 897–921, 2007.
[29] R. M. Braid, "The locations of firms on intersecting roadways," *The Annals of Regional Science*, vol. 50, no. 3, pp. 791–808, 2013.
[30] C. Reggiani, "Spatial price discrimination in the spokes model," *Journal of Economics and Management Strategy*, vol. 23, no. 3, pp. 628–649, 2014.