The Mathematical Modeling Approach for the Wastewater Treatment Process in Saudi Arabia during COVID-19 Pandemic

Abdullah Ahmadini, Ahmed Msmali, Zico Mutum, and Yashpal Singh Raghav

Department of Mathematics, Faculty of Science Jazan University, Jizan, Saudi Arabia

Correspondence should be addressed to Zico Mutum; mmeetei@jazanu.edu.sa

Received 15 January 2022; Revised 3 August 2022; Accepted 20 September 2022; Published 7 October 2022

Academic Editor: Anibal Coronel

Copyright © 2022 Abdullah Ahmadini et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The novel coronavirus disease (COVID-19) pandemic has had devastating effects on healthcare systems and the global economy. Moreover, coronavirus has been found in human feces, sewage, and in wastewater treatment plants. In this paper, we highlight the transmission behavior, occurrence, and persistence of the virus in sewage and wastewater treatment plants. Our approach follows the process of identifying a coronavirus hotspot through existing wastewater plants in major cities of Saudi Arabia. The mathematical distributions, including the log-normal distribution, Gaussian model, and susceptible-exposed-infected-recovered (SEIR) model, are adopted to predict the coronavirus load in wastewater plants. We highlight not only the potential virus removal techniques from wastewater treatment plants but also methods of tracing SARS-CoV-2 in humans through wastewater treatment plants. The results indicate that our modeling approach may facilitate the analysis of SARS-CoV-2 loads in wastewater for early prediction of the epidemic outbreak and provide significant implications to the public health system.

1. Introduction

The contamination of wastewater with viruses causing various diseases is an emerging threat that is increasing owing to the effects of climate change [1]. With the outbreak of the COVID-19 pandemic, contamination of water with viruses has become an issue of concern. The coronavirus can be transmitted from wastewater to workers in wastewater treatment plants. However, a hot climate may reduce the persistence of the virus in sewage systems; thus, the spread of the virus in such climates may not be significant [2].

Globally, researchers are testing wastewater for the novel coronavirus to estimate the total number of infections in a community, given that most people will not be tested. Coronavirus can be detected in feces within three days of infection, which is significantly earlier than the time when people begin to show severe symptoms and seek medical care [3]. Therefore, wastewater testing helps assess the spread of coronavirus [4]. Coronavirus has been correlated with viral RNA, which can be detected using real-time reverse transcription-quantitative polymerase chain reaction (RT-qPCR). The occurrence of the virus in wastewater has been found to positively correlate (correlation coefficient $r > 0.9$) with local hospital admissions [5].

On March 2, 2020, the Ministry of Health in Saudi Arabia announced the first case of COVID-19. By April 9, 2020, the number of COVID-19-confirmed cases in the country had increased to 2932. This number increased rapidly in June and July 2020. Recently, however, the number of newly confirmed cases is gradually reducing. Detection of SARS-CoV-2 in wastewater provides the capability of wastewater-based epidemiology for COVID-19 surveillance [6]. The wastewater approach for estimating the spread of coronavirus has scarcely been used in Saudi Arabia. A few studies [2, 7, 8] have been conducted on wastewater surveillance to track COVID-19 in Saudi Arabia. In this study, we investigate some models that will aid our understanding of the occurrence and persistence of coronavirus in wastewaters. Using a mathematical distribution, including normal distribution, log-normal distribution, and the susceptible-exposed-infected-recovered (SEIR) epidemiology model, we suggest a possible model that can be used to identify coronavirus hotspots by analyzing wastewaters.
In this paper, Section 2 highlights wastewater treatment plants in Saudi Arabia and their models. Section 3 focuses on the techniques used for virus removal from wastewater treatment plants, and Section 4 provides the material and methods used for tracing the coronavirus in sewage and untreated wastewater plants, and Section 5 provides the discussion and results of our work. Finally, Section 6 presents the conclusions of our study.

2. Wastewater and Models

Saudi Arabia has witnessed one of the fastest socioeconomic developments over the last few decades with increased urbanization. Consequently, water demand for all purposes has tremendously increased in a country with a hot climate and limited natural water resources. Saudi Arabia is the world’s third-largest per capita consumer of water with no lakes and rivers [9]. This gap between water demand and supply is filled by groundwater resources, water desalination, and the economical use of treated wastewater. In Saudi Arabia, wastewater treatment plants are, therefore, essential to bridge this gap. Treated wastewater is considered an important part of water resources and aims to achieve 100% use for nonpotable water demand by 2025 [10]. The Ministry of Water and Electricity aims to provide entire sewage collection and treated wastewater services to every city with a population above five thousand by 2025 [11].

At present, there are thirty-three wastewater treatment plants in Saudi Arabia with a total capacity of 748 million cubic meters per year. Fifteen more are under construction. Water reuse in Saudi Arabia is growing in both urban and rural areas. Treated wastewater is mainly used for landscaping, irrigation, and in industries such as refining. In Riyadh, the capital city of Saudi Arabia, nearly 50% of treated wastewater (about 120 million cubic meters) has been successfully used in commercial and industrial enterprises [12]. The major wastewater treatment plants serving various cities are listed in Table 1. The table shows the insufficiency of the wastewater management system in Saudi Arabia.

The most commonly used secondary treatment mechanism for wastewater in Saudi Arabia is the conventional activated sludge system, as shown in Table 2. Other common treatment techniques are filtration and chlorination; in addition, a few wastewater treatment plants use reverse osmosis. A major challenge of wastewater treatment plants (WWTPs) in Saudi Arabia is the lack of proper connectivity between the sewage system and existing plants.

The knowledge and understanding of biological and chemical wastewater treatment include both advanced and conventional techniques. Advanced techniques include the application of mathematics, statistics, physics, chemistry, and bioprocess engineering. An overview of the sequence of wastewater treatment processes adopted by most of the existing wastewater treatment plants is shown in Figure 1.

The wastewater treatment process mainly consists of a sump pump (SP), septic tank (ST), anaerobic filter (UAF), and constructed wetlands (HFCW). The purpose of the sump pump is to remove the coarse solids in the wastewater and pump the water into a two-chamber septic tank. Wastewater flows by gravity through an anaerobic filter and then through the constructed wetlands. Finally, the level tank (LT) controls the water level in the constructed wetland. Wetlands can improve wastewater quality by filtering before being released to open water.

The most widely used mathematical model for full-scale wastewater treatment plants is the activated sludge model no.1 (ASM1) [13]. The ASM1 model describes the process for organic and nitrogen removal from wastewater and validates its optimal performance using statistical models. An advanced mechanical description of the biological processes is provided in the updated version of the ASM model. The activated sludge model is one of the most important treatment processes for various wastewater, and

### Table 1: Major wastewater treatment plants in Saudi Arabia (source: King Abdullah University of Science and Technology).

<table>
<thead>
<tr>
<th>Region</th>
<th>Existing flow capacity (1000 m³/day)</th>
<th>Under construction</th>
<th>Planned flow capacity (1000 m³/day)</th>
<th>Future capacity (by the year 2035)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al Baha</td>
<td>56</td>
<td>81</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Al Jouf</td>
<td>69</td>
<td>96</td>
<td>2</td>
<td>38</td>
</tr>
<tr>
<td>Assir</td>
<td>294</td>
<td>419</td>
<td>4</td>
<td>82.5</td>
</tr>
<tr>
<td>Eastern province</td>
<td>648</td>
<td>905</td>
<td>13</td>
<td>527.3</td>
</tr>
<tr>
<td>Hail</td>
<td>93</td>
<td>130</td>
<td>1</td>
<td>19.2</td>
</tr>
<tr>
<td>Jizan</td>
<td>203</td>
<td>301</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>Madinah</td>
<td>275</td>
<td>390</td>
<td>4</td>
<td>351</td>
</tr>
<tr>
<td>Makkah</td>
<td>1100</td>
<td>1532</td>
<td>15</td>
<td>888</td>
</tr>
<tr>
<td>Najran</td>
<td>78</td>
<td>107</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Northern borders</td>
<td>47</td>
<td>71</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>Qaseem</td>
<td>185</td>
<td>258</td>
<td>5</td>
<td>131.5</td>
</tr>
<tr>
<td>Riyadh</td>
<td>1066</td>
<td>1513</td>
<td>10</td>
<td>993.5</td>
</tr>
<tr>
<td>Tabouk</td>
<td>120</td>
<td>169</td>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>Total</td>
<td>4234</td>
<td>5972</td>
<td>3135</td>
<td>2188.3</td>
</tr>
</tbody>
</table>
approximately 90% of municipal wastewater treatment plants use it in their treatment process [14, 15]. Mathematical modeling of the wastewater treatment process plays an important role in the management of an effective treatment technique for existing plants.

In this section, we briefly illustrate some of the mathematical models used to study the quality of treated wastewater. The first-order kinetic mathematical models are commonly used to compare contaminant removal efficiency and mass reduction in wastewater treatment plants. It is a linear model used to predict pollutant removal in wastewater treatment systems [16]. The model equation is as follows:

$$\frac{dC}{dt} = -kC,$$  \hspace{1cm} (1)

where $C_{out} = C_{in}e^{-k(HRT)}$.

$C_{out}$ is the concentration of the pollutant at the system inlet (mg/L), and HRT is the duration of hydraulic retention time of the system expressed in days. The monitoring data are adjusted to generate equation (1), and a similar process for this treatment is mentioned in [17]. The first-order kinetic model equation provides the parameter $k$, representing the rate of removal of pollutants or contaminants from wastewater.

The second-order kinetic mathematical model [18] was used to predict the substrate removal from the wastewater treatment system as follows:

$$\frac{dC}{dt} = -kX \left( \frac{C_{out}}{C_{in}} \right)^2,$$ \hspace{1cm} (2)

Assume $C = C_{out}$, and $t = HRT$.

Integrating the above equation (2), we get
The above equation (3) is linearized, and we obtain
\[
\frac{C_{\text{out}}}{C_{\text{in}}} = \frac{1}{1 + kXt/C_{\text{in}}},
\]
where \( C_{\text{in}}/kX \) is the model constant, and \( X \) is the biomass concentration in the reactor. The coefficient \( b \) is close to one and generally reflected the impossibility of attaining a zero of chemical oxygen demand (COD).

So, the above equation (4) can be written as follows:
\[
\frac{C_{\text{out}}}{C_{\text{in}}} = \frac{tb + C_{\text{in}}}{kX}.
\]

Data adjustment can be performed for the wastewater treatment system for each step by nonlinear least squares. The statistical significance of each parameter model is measured by \( p \) value, and \( R^2 \) is estimated to measure the goodness of fit to validate the model.

Multiple linear regression is another commonly used model to study the relationship between the dependent variable and independent variables and is also known as a predictor [19]. The different water quality parameters measured were used as the independent variables, and parameters for the water quality at the outlet were considered as the dependent variables. The equation for this model is given as \( Y = \alpha X_1 + \beta X_2 + \gamma X_3 \), where \( Y \) is the independent variable and \( X \) is the dependent variable. Analyzing the multiple regression equations provides useful information for different water quality outflows.

3. COVID-19 and Wastewater

The occurrence of coronavirus in wastewater has shown the risk of persistent and fatal viral pandemics. Wastewater-based epidemiology (WBE) is a promising tool for assessing COVID-19 [20]. At present, several applications of WBE exist, including the detection of illicit drugs, surveillance of pathogenic enteric viruses, and assessment of potential industrial chemical exposures [21]. Some viruses released from the feces of infected humans are able to survive for days to months in aquatic environments [22]. For instance, SARS-CoV-1 can survive for more than 17 days at 4°C and three days at 20°C in hospital sewage, municipal sewage, and chlorine-free tap water [23]. Therefore, the occurrence of coronaviruses in wastewater is of significant concern. It has been reported that sewage contributes to SARS-CoV-2 transmission in urban regions. Later, it reinforced the potential for secondary transmission of the disease through wastewater, especially in regions that have unsatisfactory health and sanitation infrastructure [24].

Wastewater is one of the tools to help with the surveillance of COVID-19, and its testing could be used as an early warning sign if the virus returns [25]. Several articles [3, 26, 27] have been published confirming the detection of SARS-CoV-2 in sewage. It is necessary to implement proper disinfection of sewage to avoid infection with virus. Figure 2 illustrates the possible aspects of coronavirus contamination in treated wastewater.

3.1. Ultraviolet (UV) and Chlorine Treatment. One of the most commonly used wastewater disinfection processes includes the use of ultraviolet (UV) and chlorine. The disinfection process by using chemical compounds containing chlorine is useful for the efficient inactivation of SARS-CoV-2 in wastewater treatment plants [28]. They observed that no COVID-19 was detected in the effluent after disinfection. The disinfection efficiency with chlorine is recommended by applying a dose higher than 6.5 mg/L and a contact time of at least 1.5 hours. It is more efficient to use UV radiation for disinfection of wastewater in hospitals that care for patients with COVID-19 due to the low amount of by-products [29]. According to the WHO, the presence of residual chlorine of 0.5 mg/L is standard and should be guaranteed in all water systems.

3.2. Ozone Treatment. Ozone is widely used for the disinfection of many viruses in wastewater. According to the International Ozone Association (IOA), research and testing have not been conducted on coronavirus. Therefore, a concrete conclusion cannot be made related to the disinfection of SARS-CoV-2 coronavirus by using ozone [30]. Wastewater treatment using ozone could sufficiently disinfect viruses and provide a solid barrier to prevent them from entering the environment with the effluent, and ozone seems to be more effective than chlorine. There is little information about the SARS-CoV-2 removal by using ozone disinfection processes, although the coronavirus is expected to be more sensitive to disinfection than other viruses. However, in [31], the authors reported that ozone could be one of the disinfectant methods to effectively inactivate SARS-CoV-2. Some published results also show that ozone can effectively disinfect some viruses including SARS-CoV-2 in aerosols or fomites [32].

The disinfection efficiency of the virus in wastewater depends on the ozone concentration and contact time. In [33], these values are calculated for specified log-inactivation levels of a number of viruses in wastewater at pH 7.6 and 16°C, with ranging within a specific value. Ozone is an efficient disinfection process as compared with conventional sewage treatment and able to reduce viral concentration by one to two log10 [34].

3.3. Sand Filtration. Sand filtration is one of the most widely used wastewater treatment methods for wastewater treatment plants. In Saudi Arabia, most of the tertiary wastewater treatment plants are using sand filtration after the secondary stage of treatment for improving the quality of water [35]. Sand filtration is frequently used for the removal of dissolved particles and usually has less than 1 log 10 virus removal. However, in [36], the authors reported on the molecular mechanism for the unprecedented high virus removal using a practical sand filter. They developed functionalized sand filters using water extract of Moringa oleifera seeds. They
then tested the efficiency of the obtained sand using the MS2 bacteriophage virus, achieving a 7 log₁₀ virus removal. The virus removal activity of functionalized sand filter is due to protein binding. Some reported that the use of zero-valent iron sand filtration can remove virus contamination from wastewater. The zero-valent iron sand filters are efficiently able to remove some specific viruses such as MS2 and AIV (<1–2 log), but the removal efficiency varies among different viral species [37].

3.4. Membranes for Virus Removal. Water filtration membranes are categorized into four main types based on their pore size; they are as follows: reverse osmosis (RO), nanofiltration, ultrafiltration, and microfiltration. Figure 3 shows the pore size and capabilities of these membranes for filtering particles, and dissolved ions including salts, bacteria, and pathogens.

Membranes such as reverse osmosis, nanofiltration, and ultrafiltration should be suitable for removing SARS-CoV-2 (considering the size is 100 μm) from wastewater [39]. In [40], the authors highlighted the use of polymeric and ceramic membranes for virus removal from water. It was reported that the virus removal efficiencies were highly variable since the range reported was 0.2–7.

The reverse osmosis and high-pressure nanofiltration membranes were popularly used for the treatment of water and desalination. The treatment process through these membrane systems was based on the solute transport by diffusion through the nonporous active layer of the membrane [41]. In this section, we outline some recent literature on water filtration membranes used for the removal of viruses including SARS-CoV-2 from wastewater treatment plants.

3.4.1. Reverse Osmosis. Recently, in [42], the authors highlighted the treatment of waterborne pathogens by reverse osmosis. The reverse osmosis (RO) treatment method is considered as the last grade purification/treatment level. Any impurities with a molecular weight greater than 200 are not able to pass through the RO system. Similarly, the larger the ionic charge of the contaminants, the more likely to be rejected by the RO membrane [43]. However, RO can be used in combination with other complementary processes such as pretreatment and post-treatment of particular matters to produce high-quality recycled water [44]. In [45], the authors reported that the membrane bioreactor (MBR)/reverse osmosis (OR) systems were showing virus removal efficiencies (range of 2.3–2.9 log₁₀). They stated that reverse osmosis or combined system (MBR/RO) was a more advanced wastewater treatment system with higher virus removal efficiency (>5 log₁₀) for MS2 coliphages, whose virus size is smaller than SARS-CoV-2.

Szczuka et al. [46] reported that using forward-reverse osmosis for wastewater treatment had achieved higher than 98% rejection of organic contaminants, achieved at least 6.7 log₁₀ removal of bacteriophage MS2 spiked into wastewater, and achieved at least 5.4 log₁₀ removal of native E. coli in gray water and sewage. In [47], the authors used sand filters and reverse osmosis system for removing noroviruses of size 38–40 nm in diameter [48], from wastewater.

3.4.2. Ultrafiltration. Ultrafiltration is often considered as one of the best pretreatment steps prior to reverse osmosis for the removal of human pathogens including bacteria, protozoa, and viruses from wastewater. Recently, in [49], the authors reviewed the use of ultrafiltration membranes for removing bacteria and viruses from wastewater. They reported that the use of ultrafiltration produced better water quality compared to conventional treatment due to the very fine pore structure. Using ultrafiltration for virus removal does not require preconditioning of water samples, so it is widely used for water quality [50].

Ultrafiltration is a suitable membrane technique for removing coronaviruses with a mean particle diameter of
120 nm and envelope diameter of 80 nm [51]. The adsorption property of solids in wastewater may facilitate the removal of coronavirus. It is also used for the detection of viruses in wastewater. Ultrafiltration devices, such as the Centricon Plus-70, Amicon Ultra-15, and automatic Concentrating Pipette, have been successfully used to detect SARS-CoV-2 from wastewater [52].

### 3.4.3. Nanomaterials

Nanotechnology and its application for water and wastewater treatment have emerged as a promising area and fast-developing field. It has been successfully used for the removal of viruses, bacteria, protozoans, and other contaminants from wastewater. The membranes with silver nanoparticles (AgNPs) have shown efficiency against MS2 bacteriophage virus and considered the best water treatment processes [53]. It is also reported that silver nanoparticles used in a nanocomposite filter system can be reached up to 100 percent removal of several different viruses. Electrospun nanofiber membranes (ENMs) are popularly used to treat pathogen contamination in wastewater. It has high flux and high rejection rate compared to conventional membranes, and offers cost-effective, lightweight, and lower energy efficiency. ENMs have high porosity with approximately 80% while conventional membranes have 5–35% porosity [54]. Nanofibers containing tetraethoxysilane and ammonium thiomolybdate mixed with polyacrylonitrile have enhanced more than 90% of virus removal efficiency. The use of nanoparticles alone or as a part of composite membranes could be more efficient in removing SARS-CoV-2 from wastewater [53].

### 3.4.4. Microfiltration

Microfiltration membranes have average pore size, and the virus removal from the wastewater treatment plant is lower compared to other membranes. In [55], the authors reported that the virus size (63 nm) was smaller than the microfiltration membrane size (0.2 μm), so it could pass through the membranes when the permeate flux was high or the membranes were clean. Moreover, in [56], they investigated that the presize of microfiltration was larger than 100 nm, which was more suitable for the removal of protozoa and bacteria rather than virus.

## 4. Material and Methods

We used the log-normal distribution, Gaussian model, and SEIR model to predict the infection of SARS-CoV-2 through wastewater. The theoretical assumptions and data considered in this study were collected and obtained from the website of the Ministry of Health (MOH), Saudi Arabia (https://covid19.moh.gov.sa/).

### 4.1. Normal Distribution

The normal distribution and its properties provide an important role in the analytical investigation of the collection of coronavirus traces in untreated wastewater from wastewater treatment plants. Data of coronavirus traces collected from N wastewater treatment plants are assumed to follow a normal distribution. The rate of change in viral load at untreated wastewater plant with respect to time is given as follows:

$$\frac{dX_i(t)}{dt} = \nu_i X_i(t) - \omega_i t X_i(t), \quad (6)$$

where \(\nu_i\) represents the change in mean and deviation due to daily new viral load; \(\omega_i\) represents the change in deviation of dataset related to the daily average viral load/concentration in untreated wastewater from i wastewater treatment plant. Let \(X_i(t)\) be the viral load/concentration of coronavirus trance in untreated wastewater from \(i\) wastewater treatment plant, where \(i = 1, 2, 3, \ldots, N\).
during the time interval $t_0 \leq t \leq T$. Let $\mu$ be the mean of viral load found in untreated wastewater from the wastewater treatment plant and $\sigma$ be the related standard deviation. Then, equation (6) can be written as follows:

$$\frac{dX_i(t)}{dt} = \frac{\mu_i - X_i(t)}{\sigma_i^2}$$  \hspace{1cm} (7)

The exact solution of equation (7) is given as follows:

$$X_i(t) = \frac{1}{\sigma_i \sqrt{2\pi}}X_i(0)\exp\left\{-\frac{(t-\mu_i)^2}{2\sigma_i^2}\right\}.$$  \hspace{1cm} (8)

We can plot the set $\{X_i(t): i = 1, 2, 3, \ldots, N\}$ to forecast the possible viral load in untreated wastewater plants.

4.2. Log-Normal Model. The log-normal distribution is an important statistical distribution to model many natural phenomena. The small percentage changes in the log-normal model correspond to numerous growth processes, and they are represented in log scales. If the effect of these changes is not significant, then the distribution will be closer to normal. We adopted the log-normal distribution to describe the spread of COVID-19 virus through wastewater treatment plants.

Suppose the random change $R$ is log-normal distribution, then the normal distribution $Z = \ln(R)$ follows the dynamics of the spread of COVID-19 virus. For instance, if $Z = \exp(R)$ has the behavior of the spread of the virus, then the property of $R = \exp(Z)$ will be the log-normal distribution. In this case, the data collected from the wastewater treatment plants are assumed to follow log-normal distribution; then, the mathematical equation is given as follows:

$$X_i(t) = \frac{1}{\sigma_i \sqrt{2\pi}}X_i(0)\exp\left\{-\frac{(\ln(t) - \mu_i)^2}{2\sigma_i^2}\right\}.$$  \hspace{1cm} (9)

Comparing between the collected data and equation (8), we can determine the parameters $\mu_i$ and $\sigma_i^2$ as follows:

$$\mu_i = \frac{\langle(\theta_i + \theta_2)\langle\theta_i - \theta_2\rangle/2 - \ln[X_i(\theta_i)/X_i(\theta_2)]\rangle \theta_2 - \theta_1\langle\ln[X_i(\theta_2)/X_i(\theta_1)]\rangle}{\langle\theta_i - \theta_2\rangle/2 - \ln[X_i(\theta_2)/X_i(\theta_1)]\rangle \langle\theta_2 - \theta_1\rangle/\ln[X_i(\theta_2)/X_i(\theta_1)]\rangle \langle\theta_1 - \theta_2\rangle/\ln[X_i(\theta_1)/X_i(\theta_2)]\rangle}$$  \hspace{1cm} (10)

and variance

$$\sigma_i^2 = \frac{\langle(\theta_i - \mu_i)^2 \rangle - \langle(\theta_i - \mu_i)^2 \rangle}{2\ln[X_i(\theta_i)/X_i(\theta_1)]\rangle \langle(\theta_i - \mu_i)^2 \rangle}{\ln[X_i(\theta_i)/X_i(\theta_1)]\rangle \langle\theta_i - \mu_i\rangle/\ln[X_i(\theta_1)/X_i(\theta_2)]\rangle}$$  \hspace{1cm} (11)

Similarly, with the log-normal distribution equation (9) we have the parameters $\mu_i$ and $\sigma_i^2$ for the $i$ wastewater treatment plant as follows:

$$\mu_i = \frac{\langle(\ln\theta_i + \ln\theta_2)\langle\ln\theta_i - \ln\theta_2\rangle/2 - \ln[X_i(\ln\theta_i)/X_i(\ln\theta_2)]\rangle \langle\ln\theta_2 - \ln\theta_1\rangle/\ln[X_i(\ln\theta_2)/X_i(\ln\theta_1)]\rangle \langle\ln\theta_1 - \ln\theta_2\rangle/\ln[X_i(\ln\theta_1)/X_i(\ln\theta_2)]\rangle}{\langle\theta_i - \ln\theta_2\rangle/2 - \ln[X_i(\ln\theta_2)/X_i(\ln\theta_1)]\rangle \langle\ln\theta_2 - \ln\theta_1\rangle/\ln[X_i(\ln\theta_2)/X_i(\ln\theta_1)]\rangle \langle\theta_2 - \ln\theta_1\rangle/\ln[X_i(\ln\theta_2)/X_i(\ln\theta_1)]\rangle}$$  \hspace{1cm} (12)

and variance

$$\sigma_i^2 = \frac{\langle(\ln\theta_i - \mu_i)^2 \rangle - \langle(\ln\theta_i - \mu_i)^2 \rangle}{2\ln[X_i(\ln\theta_i)/X_i(\ln\theta_1)]\rangle \langle(\ln\theta_i - \mu_i)^2 \rangle}{\ln[X_i(\ln\theta_i)/X_i(\ln\theta_1)]\rangle \langle\ln\theta_i - \mu_i\rangle/\ln[X_i(\ln\theta_1)/X_i(\ln\theta_2)]\rangle}$$  \hspace{1cm} (13)

where $\{\theta_1, \theta_2, \theta_3, \theta_4\}$ represents the number of days within the time interval $t_0 \leq t \leq T$. We considered the collection of viral loads from untreated wastewater treatment plants for these consecutive days.

Figures 4(a) and 4(b) show the normal equation (8) and the log-normal equation (9) for different viral loads. Considering that the collection from untreated wastewater plants was performed in five major cities (Madinah, Jeddah, Jazan, Riyadh, and Makkah), it is possible to observe high coronavirus traces owing to the high number of infected persons. Therefore, we chose different mean and standard deviation distributions, as shown in Figures 5 and 6. It is assumed that an adequate collection of data was performed. The data collected from the wastewater treatment plants in the major cities were plotted with respect to time. For instance, we assume that the data collected from each wastewater treatment plant from the respective major cities have a daily mean $\mu = 0.2, \mu_2 = 0.3, \mu_3 = 0.4, \mu_4 = 0.5, \mu_5 = 2$ and standard deviation $\sigma_1 = 0.1, \sigma_2 = 0.2, \sigma_3 = 0.28, \sigma_4 = 0.3, \sigma_5 = 1$ to be represented by the normal distribution.

4.3. Gaussian Model. The Gaussian model was used to predict the time evolution of the COVID-19 pandemic in Germany [57], Italy [58], and China [59]. The best justification for the Gaussian distribution for detecting the dynamic behavior of SARS-CoV-2 is given by the central limit theorem. For instance, when several independent random variables are added, their normalized sum satisfies the Gaussian distribution, even if the original variables are not normally distributed. Numerical simulations of earlier pandemics indicate that the time evolution of pandemic waves is usually shown by an exponential rise until a maximum is reached and then rapidly decreases. We adopted a simple Gaussian distribution to estimate the dynamic nature of coronavirus infection in wastewater. Let
$X_t$ be the number of viral loads in an untreated wastewater plant per day, and its time evolution is given by the following Gaussian function:

$$X_t = X_t(0) \exp \left\{ \frac{(t-T)^2}{w^2} \right\},$$  \quad (14)

as shown in Figure 5(b), where $X_t(0)$ denotes the maximum value at a certain time $T$, and $w$ is the width of the Gaussian distribution. By monitoring the new viral loads in wastewater and using equation (14), we can derive the relative change in COVID-19 infection per day as follows:

$$P(t) = \frac{dX_t(t)/dt}{X_t(t)} = \frac{d(lnX_t(t))/dt}{X_t(t)} = \frac{2(T-t)}{w^2},$$ \quad (15)

where $P(t)$ denotes the Gaussian distribution of the viral load of coronavirus traces in untreated wastewater. The monitored data are often reported in terms of the doubling time ($\tau$) and the effective reproduction factors. The corresponding exponential function at any time is given as follows:

$$X_t(t) \propto e^{t\ln2/\tau} = 2^{t/\tau}.$$ \quad (16)
Figure 6: Continued.
Using equations (16) in (15), we obtain the relative changes in the daily rate as follows:

\[ P(t) = \frac{d \ln X_i(t)}{dt} = \frac{\ln 2}{\tau} \]  

Equating the two results in equations (15) and (17), we obtain the time-dependent Gaussian doubling time as follows:

\[ \tau(t) = \frac{A}{T - t}, \quad A = (\ln \sqrt{2})w^2 = 0.35w^2. \]  

Using equations (16) in (15), we obtain the relative changes in the daily rate as follows:

\[ P(t) = \frac{d \ln X_i(t)}{dt} = \frac{\ln 2}{\tau} \]  

Equating the two results in equations (15) and (17), we obtain the time-dependent Gaussian doubling time as follows:

\[ \tau(t) = \frac{A}{T - t}, \quad A = (\ln \sqrt{2})w^2 = 0.35w^2. \]  

The daily number of newly infected cases can also be defined using the cumulative case rate. Using equation (15) and the peak time \( t - T \), the corresponding cumulative number of cases at time \( t \) in terms of the error function can be written as

\[ C(t) = \int_{-\infty}^{t} c(s)ds = \frac{C_{\text{tot}}}{2} \left[ 1 + \text{erf} \left( \frac{t - T}{\sqrt{\sigma}} \right) \right], \]  

where \( C_{\text{tot}} = \sqrt{\pi \sigma} c_{\text{max}}w \) denotes the total number of infected cases. Similar values for cumulative death and infection cases relevant to the first wave of the COVID-19 pandemic were obtained in [60]. Using equation (19), we can write the respective relative changes as follows:

\[ P(t) = \frac{C'(t)}{C(t)}. \]  

Figure 6: SEIR model with (a) \( \beta = 0.0095, \gamma = 0.004, \) and \( \sigma = 0.01; \) (b) \( \beta = 0.0095, \gamma = 0.03, \) and \( \sigma = 0.1; \) (c) \( \beta = 0.01, \gamma = 0.004, \) and \( \sigma = 0.0025; \) (d) \( \beta = 0.09, \gamma = 0.01, \) and \( \sigma = 0.09; \) and (e) \( \beta = 0.00348, \gamma = 0.03, \) and \( \sigma = 0.1. \)
Using equation (14) and equating the results with the corresponding exponential function lead to the time-dependent cumulative Gaussian doubling time as follows:

\[
G(t) = \frac{C(t) \ln 2}{c(t)} = \chi \exp \left( \frac{t - T}{w} \right) \left[ 1 + e^{f \left( \frac{t - T}{w} \right)} \right],
\]

(21)

where \( \chi = \sqrt{\pi} \ln (\sqrt{2}) \approx 0.614 \). Figure 5(a) shows the Gaussian instantaneous time and cumulative doubling time as a function of time.

The Gaussian model can predict the time (months) evolution of the number of infections, where \( X_i(0) \) denotes the maximum number of infections. The best-fit parameters have a peak time of \( T = 4.4 \) months, where \( t = 0 \) in \( X_i(t) \) corresponds to March 2, 2020 (the first day of COVID-19 infection in Saudi Arabia). The maximum infection cases were observed during June and July 2020, approximately 4.4 months after the first day of infection in Saudi Arabia.

4.4. Susceptible-Exposed-Infected-Recovered (SEIR) Model

The simulation of the well-known SEIR model was performed to represent the dynamic behavior of the spreading process of the COVID-19 pandemic through wastewater. In epidemiology, the SEIR model is a compartmental pandemic model widely used to characterize the outbreak of COVID-19. This model, the spread of infection depends on the number of susceptible populations and the number of infected populations. The incubation period is considered when individuals have been infected but do not show symptoms. Because the coronavirus disease has a long incubation period, it is reasonable to model the pandemic with another compartment that is exposed to humans, but not virus spreaders [61].

In the SEIR model, we assume that the time is long term with no vital dynamics, and the population \( N \) size is constant. In this model, individuals are classified into four groups or compartments according to their infectious status. We classified the infection types as either with or without symptoms. The compartments of the model are as follows:

(i) Susceptible (\( S \)): a number of humans that are not infected by the virus but may catch the disease.
(ii) Exposed (\( E \)): a number of humans that are already infected but cannot spread the virus.
(iii) Infected (\( I \)): a number of infected humans and transmission of the virus to susceptible individuals.
(iv) Recovered (\( R \)): a number of humans that are recovered from infection.

A simple model of SEIR is given as follows:

\[
\frac{d}{dt} S(t) = -\beta S(t) I(t),
\]

(22)

\[
\frac{d}{dt} E(t) = \beta S(t) I(t) - \alpha E(t),
\]

(23)

\[
\frac{d}{dt} I(t) = \sigma E(t) - \gamma I(t),
\]

(24)

\[
\frac{d}{dt} R(t) = \gamma I(t).
\]

(25)

The above system of equations (22)–(25) can be solved using any numerical method, and a simulation can be presented to predict the behavior of SARS-CoV-2. The above equation can be written as follows:

\[
\frac{S(t_{n+1}) - S(t_n)}{\Delta t} = -\beta S(t_n) I(t_n),
\]

\[
\frac{E(t_{n+1}) - E(t_n)}{\Delta t} = \beta S(t_n) I(t_n) - \alpha E(t_n),
\]

\[
\frac{I(t_{n+1}) - I(t_n)}{\Delta t} = \sigma E(t_n) - \gamma I(t_n),
\]

\[
\frac{R(t_{n+1}) - R(t_n)}{\Delta t} = \gamma I(t_n),
\]

(26)

where \( \Delta t \) represents a very small step size in the time domain. The parameters \( \beta, \sigma, \) and \( \gamma \) defined in the above equations represent the probability of infection, incubation rate, and average rate of people recovered at time \( t \) (in days), respectively. The model parameters are fitted values from the official website of the Ministry of Health [62]. We assumed the average incubation period to be five days.

The number of possible viral loads in the wastewater plants can be predicted, as shown in Figures 6(a)–6(e). From the model diagram, we can see that the population of infectious individuals increases at the beginning of the outbreak and progressively decreases over time. The main feature of this model is the incorporation of the importance of coronavirus contamination in untreated wastewater by infected people.

4.5. SEIR Model Linked to Wastewater

The SARS-CoV-2 RNA is possible to detect in human feces a few days to a week before the onset of symptoms. Both symptomatic and asymptomatic humans shed coronavirus through their feces, and studies have shown the presence of SARS-CoV-2 in wastewater samples. Since then, there have been numerous modeling approaches of using wastewater-based epidemiology for early detection of SARS-CoV-2. However, wastewater-based surveillance (WBS) is linked to the COVID-19 epidemic status and the stage of the outbreak is still in its infancy.

The SEIR model is widely used for predicting the outbreak of COVID-19 epidemic in many countries [63]. In [64], the authors used the SEIR model to study the estimation of the number of infections on the base of the mass rate of virus RNA in wastewater treatment plants. The model was used to predict the infected individuals with new cases in
a specific region through two wastewater treatment plants. They stated that the model overcomes some of the limitations associated with individual testing campaigns and provides useful information to the policymakers. The authors in [65] adopted an epidemiological SEIR model to link quantified SARS-CoV-2 levels in wastewater estimating infected individuals within the sewershed, and to support policy decisions. They also described the concentration of SARS-CoV-2 in wastewater due to fecal shedding from the infected individuals. The model was fitted to wastewater-based surveillance data and clinical reports from six wastewater treatment plants located in three Canadian cities and estimated important epidemiological matrices (prevalence, effective reproduction number, and forecasted incidence).

The SEIR stochastic model and an extended Kalman filter were used to reconstruct epidemic dynamics from SARS-CoV-2 viral load in wastewater [66]. The authors stated that their approach integrates noisy wastewater data and daily case numbers into a dynamic epidemiological model and enhances the surveillance of wastewater for early epidemic prediction. In [67], a stochastic wastewater-based SEIR model was used to show the importance of viral load in wastewater for the early detection and prediction of an outbreak in a community. The SEIR model based on viral load data could reliably predict the number of infections in the future. In [68], a SEIR model was used to represent scenarios of wastewater screening for SARS-CoV-2 at a college campus of 12,500 students during the time of low viral presence and transmission, and low exposure to the surrounding communities. The model predicted that wastewater surveillance could maintain an overall number of infections similar to the traditional surveillance testing and reduce the number of false-positive cases.

5. Results and Discussion

The possible COVID-19 hotspots in different wastewater treatment plants of five major cities in Saudi Arabia can be predicted using log-normal mathematical models and a Gaussian model. The results are shown in Figures 4(a)–4(b) and 5(a)–5(b), respectively. Moreover, further prediction of the next hotspot was considered using the susceptible-exposed-infected-recovered (SEIR) mathematical model. The results for predicting possible viral loads in wastewater treatment plants are shown in Figures 6(a)–6(e).

We can assume that the viral load threshold is when the viral load in a wastewater treatment plant is equal to 1. From Figure 4(a), it can be concluded that the regions that are at maximum risk from COVID-19 infection are from untreated wastewater plants with sharply peaked curves. The black color curve shown in Figure 4(a) indicates a high number of COVID-19 infection cases on the first day, while the last region with a red color curve has a continuous number of moderately infected persons. However, as shown in Figure 4(b), the data collected from the wastewater plants were assumed to follow a log-normal distribution. It is presumed that the data were collected from wastewater treatment plants from five major cities in Saudi Arabia with daily averages and standard deviations in the case of the Gaussian model. It can be observed that wastewater treatment plants with black and green curves are considered as COVID-19 hotspot regions. The remaining curves were below the viral load threshold. In particular, we can conclude that these two regions, Makkah (average = 0.2 and standard deviation = 0.1) and Riyadh (average = 0.3 and standard deviation = 0.4), have a higher number of infected persons with SARS-CoV-2. The prediction of hotspots using a log-normal model is consistent with the real data of COVID-19 infection cases observed in the major provinces of Saudi Arabia. In March 2020, the province of Makkah had the highest number of COVID-19 infection cases (30.7%), followed by Riyadh (23%) [69]. In general, we can compare all the collected data from the wastewater treatment plants weekly or monthly, and each region can define a threshold. A COVID-19 hotspot region can be identified from a wastewater treatment plant with a viral load above the threshold value.

The Gaussian model can also be used to predict the possible peak time of infection using data collected from wastewater treatment plants. From Figure 5(b), the maximum probability of infection cases occurs during the time $T = 4.4$ (months), which is approximately four and a half months since the first day of the COVID-19 infection case in Saudi Arabia. The cumulative Gaussian model predicts the possible viral load in wastewater treatment plants over time. According to the model, the cumulative cases of infected people are represented by the red curve. This signifies that with a moderate rate of viral load in a wastewater treatment plant, there is a possibility of the occurrence of COVID-19 over a long period.

Using the epidemiological mathematical model SEIR and simulations can predict the next COVID-19 hotspot region. The model also shows a decrease in the susceptible population as people become exposed and infected and then recovered. The coronavirus load per day in untreated wastewater was predicted for five different regions, and Figures 6(a)–6(e) show the numerical simulations of the model with infection, incubation, and recovery rates for different regions in Saudi Arabia. From the simulation process, the SEIR model with infection type had a higher prediction of the hotspot. Figure 6(d) represents the hotspot because we assume that the threshold value for the SARS-CoV-2 virus load in an untreated wastewater treatment plant is 1. We can see that the viral load in untreated wastewater is higher and declines progressively over time.

All mathematical model approaches mentioned above may be promising methodologies for predicting possible COVID-19 hotspots. However, our results indicate that traveling restrictions, social distancing, vaccination, medical intervention, and security are important measures to flatten the curve.

6. Conclusions

In this study, we sought to identify COVID-19 hotspots in wastewater collected from wastewater treatment plants in major cities in Saudi Arabia. SARS-CoV-2 is spread not only
through the air but also through untreated wastewater, reaching wastewater treatment plants and the aquatic environment. Therefore, significant attention is required to examine wastewater treatment plants, especially in Saudi Arabia, where only a part of the wastewater is treated. We highlighted the recently developed wastewater treatment plants based on virus models that can be used to differentiate virus removal efficiencies. This includes some advanced techniques, such as treatment using ozone, UV, ultrafiltration, and nanofiltration membrane-based systems. It is a combination of different techniques in primary, secondary, and tertiary treatments that would facilitate the virus removal treatment process in wastewater treatment plants.

We used mathematical modeling, including the Gaussian model, the log-normal model, and the SEIR model, to understand the possible coronavirus load in untreated wastewater from wastewater treatment plants. The COVID-19 hotspot was predicted using these models, where the collected data were plotted as a function of time. The hotspots were identified when the viral load of the SARS-CoV-2 trace in wastewater was higher than the threshold value. Using the log-normal model to predict the region with the maximum infection of COVID-19 through wastewater provides a promising mathematical model. This model approach predicts two provinces (Riyadh and Makkah) in Saudi Arabia as COVID-19 hotspots, which is close to the real data of infection. The Gaussian model aids to estimate the time of the possible maximum outbreak of the pandemic. It was predicted that the peak outbreak would be 4.4 months since the first day of the outbreak in Saudi Arabia, which is consistent with the real data because June and July 2020 recorded the maximum number of infections. Describing the possible viral load in a wastewater treatment plant using the SEIR model has more advantages in the long-term prediction of the nature of the COVID-19 virus.

In conclusion, our models follow the process of predicting COVID-19 hotspots in wastewater. Wastewater treatment plants could be one of the solutions to limit the viral spread into the environment and a possible future detection, along with accessing the Saudi Health Ministry control measures and decisions on infection.

Our future research directions will focus on the extension of the mathematical models using stochastic and fractional derivative approaches that are more suitable to study the viral load of mutated SARS-CoV-2 in wastewater. The detection of SARS-CoV-2 in wastewater seems to appear earlier than the clinical testing, and it is an effective approach for epidemic monitoring and early warning system. Therefore, our further studies will also focus on developing effective mathematical models presenting the correlation between the viral load in wastewater and infected people for the long-term monitoring of the dynamic behavior of the epidemic.

Data Availability

All the data used in the manuscript are available online at the website of the Ministry of Health, Saudi Arabia (https://covid19.moh.gov.sa/).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors would like to thank the Scientific Research Ethics Committee, Jazan University, Ministry of Higher Education, Saudi Arabia, for financial support of this research with Grant no. W4-059.

References


