

Research Article

Edge Colouring of Neutrosophic Graphs and Its Application in Detection of Phishing Website

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Graph colouring enjoys many practical as well as theoretical uses. Graph colouring is still a very active subject of research. This article introduces a new concept of the chromatic number of the neutrosophic graph (NG). There is a discussion of the difference between the fuzzy colouring of the fuzzy graph (FG) and the NG. Finally, a real-life application for detecting phishing websites has been demonstrated by NG colouring.

1. Introduction

The graph is one of the greatest ways to portray relationships between objects. The objects depict the vertices and the relationship between objects are considered as edges. The edges within FGs measure the degree via membership value (MV) within [0,1]. The FGs were presented in 1973 by Kaufmann [1], and it was developed in 1975 by Rosenfeld [2]. Samanta and Pal [3, 4] introduced many types of FGs and provided many applications of FG. Mahapatra et al. [5–9] presented many applications of the FG. Pal et al. Reference [10] have discussed about the modern trends in fuzzy graphs and their application.

The concept of a neutrosophic set was generalized as an intuitionistic set by Smarandache [11]. The neutrosophic set is characterized by the fact that true-membership value (TMV), indeterminacy-membership value (IMV), and falsity-membership value (FMV) are automatically independent of each other in a range of $]^-0, 1^+$ [. Some studies attempted to build a three-way fuzzy concept lattice using the neutrosophic set for the characterization of uncertainty based on its acceptation, rejection, and uncertain components, independently [12]. The fourth dimension data representation is defined as Turiyam set [13, 14]. There are four

possible 4 – tuples in the Turiyam Set: Truth membership value (*T*), Indeterminacy membership value (*I*), Falsity membership value (*F*), and Liberalization membership value (*L*). The dimensions of all of these quantities are independent and $0 \le T + I + F + L \le 4$. The idea of NG was introduced in 2016 by Broumi et al. [15] and it is also used in MATLAB code for energy and spectrum analysis [16, 17]. Akram and Shahzadi [18] defined a new definition of NG by the concept of neutrosophic set.

Graph colouring is one of the oldest research areas. Many problems like map colouring, traffic light problems etc., can be solved by graph colouring. Casselgren and Petrosyan [19] introduced edge colouring of a graph by the concept of the improper interval in 2021. The complexity of counting problems in edge-colouring in a graph has been solved by Cai and Govorov [20]. Umamaheswari and Mithra [21] has introduced the fractional chromatic number on edge-colouring of graph theory. Samanta et al. [22] introduced fuzzy colouring in FGs. Munoz et al. [23] introduced the colouring of an FG in 2005. Mahapatra et al. [24, 25] introduced the edge colouring of an FG and radio k colouring in FGs. For more details readers can read the articles [26–32].

Nowadays, searching for a suitable journal is difficult for a researcher. Many researchers targeted some lesser-known journals and had their papers published in them. So, searching for a suitable, reputed journal for an article is a critical task. Also, some of the journals are fake. They have used the exact name of a reputed journal. Therefore it is difficult to find a suitable journal that is not a forgery. By colouring an NG, we represent a website, which can easily indicate a suitable journal and the chance of a website being phishing.

1.1. Significance of the Work. Some of the important significances of this study are listed below:

- (i) In this article, we shall introduce a new concept of graph colouring with different merits.
- (ii) Sometimes, data is displayed with falsity and indeterminacy. Sometimes, data is not available, or it contradicts the facts. So, data may contain three values: true value, falsity, and indeterminacy. All three values—true value, falsity, and indeterminacy—are considered in NGs. In this article, we represent a website, which can easily indicate a suitable journal and the chance of a website being phishing.
- (iii) A small network has been considered here to verify the proposed method.
- (iv) Some essential properties of the edge colour have been demonstrated in the NG.

1.2. Framework of the Paper. This paper is organized in the following manner:

In Section 2: Some important definitions of the NG are provided.

In Section 3: The definition of the strength score of an edge of an NG is provided.

In Section 4: The procedure of edge-colouring of a NG is provided.

In Subsection 4.1: An algorithm for edge-colouring of an NG is provided. Also, this definition has been described in detail with examples. Some important theorems about the edge-colouring of NGs are also given.

In Section 4: An application of the suitable journal and chance of a Phishing Website in a neutrosophic graph with detailed analysis is provided.

In Section 5: The entire conclusion is given.

Authors contributions.

The author's contributions to graph colouring and FG colouring have been shown in Table 1.

List of abbreviations.

All the abbreviations are shown in Table 2.

Basic notations.

All the basic notations are shown in Table 3.

2. Preliminaries

Akram and Sitara [41] defined NG as follows.

Definition 1. A NG $\xi = (V, \sigma, \mu)$ is a non-empty set V together with a pair of functions $\sigma = (\sigma_T, \sigma_I, \sigma_F)$: $V \longrightarrow [0, 1]^3$ and $\mu = (\mu_T, \mu_I, \mu_F)$: $V \times V \longrightarrow [0, 1]^3$ such that for all $a, b \in V$

$$\mu_{T}(a,b) \leq \min\{\sigma_{T}(a), \sigma_{T}(b)\},$$

$$\mu_{I}(a,b) \leq \min\{\sigma_{I}(a), \sigma_{I}(b)\},$$
(1)

$$\mu_{F}(a,b) \leq \min\{\sigma_{F}(a), \sigma_{F}(b)\},$$

where, $\mu_T(a, b), \mu_I(a, b), \mu_F(a, b)$ represent the TMV, IMV and FMV of the edge (a, b) in ξ , respectively. $\sigma_T(a), \sigma_I(a), \sigma_F(a)$ represent the TMV, IMV and FMV of the vertex *a* in ξ , respectively.

Strength of an edge was introduced by Akram [42].

Definition 2. $\xi = (V, \sigma, \mu)$ be a NG then the strength of an edge (a, b) is denoted by $(O_{T(a,b)}, O_{I(a,b)}, O_{F(a,b)})$ and defined as

$$O_{T(a,b)} = \frac{\mu_T(a,b)}{\min\{\sigma_T(a), \sigma_T(b)\}},$$

$$O_{I(a,b)} = \frac{\mu_I(a,b)}{\min\{\sigma_I(a), \sigma_I(b)\}},$$

$$O_{F(a,b)} = \frac{\mu_F(a,b)}{\min\{\sigma_F(a), \sigma_F(b)\}},$$
(2)

where, $\mu_T(a, b)$, $\mu_I(a, b)$, $\mu_F(a, b)$ represent the TMV, IMV and FMV of the edge (a, b) in ξ , respectively. $\sigma_T(a)$, $\sigma_I(a)$, $\sigma_F(a)$ represent the TMV, IMV and FMV of the vertex *a* in ξ , respectively.

Samanta et al. [22] introduced the fuzzy colour as follows.

Definition 3. $X = x_1, x_2, ..., x_n$ is a set of basic colours. Then fuzzy set (X, f) is called the set of fuzzy colours where $f: X \longrightarrow [0, 1]$ and fuzzy colour is denoted by $(x_i, f(x_i))$, where x_i is the basic colour and $f(x_i)$ is the membership value of the basic colour x_i .

3. Strength Score of an Edge of NG

There is some limitation in the Definition 1. If $\min\{\sigma_T(a), \sigma_T(b)\} = 0$ or $\min\{\sigma_I(a), \sigma_I(b)\} = 0$ or $\min\{\sigma_F(a), \sigma_F(b)\} = 0$ then the strength values can not be calculated. Throughout this study, we assume that if $\min\{\sigma_T(a), \sigma_T(b)\} = 0$ then $O_T(a, b) = 0$; if $\min\{\sigma_I(a), \sigma_I(b)\} = 0$ then $O_I(a, b) = 0$; if $\min\{\sigma_F(a), \sigma_F(b)\} = 0$ then. $O_F(a, b) = 0$

First, we introduced the strength score of an edge of an NG. Table 4.

Definition 4. Let, $\xi = (V, \sigma, \mu)$ be a NG and the strength of an edge (a, b) is $(O_{T(a,b)}, O_{I(a,b)}, O_{F(a,b)})$ then the strength score an edge (a, b) is denoted by $S_{(a,b)}$ and defined by $S_{(a,b)} = 2 + O_{T(a,b)} - O_{I(a,b)} - O_{F(a,b)}/3$.

Year	Authors	Contributions	
1973	Meyer [33]	Equitable colouring	
1979	Leighton [34]	Colouring algorithm for large scheduling problems	
2004	Malafiejski [35]	Sum coloring of graphs	
2006	Furmańczyk [36]	Equitable colouring of graph products	
2010	Malaguti et al. [37]	A survey on vertex colouring problems	
2013	Galinier et al. [38]	Recent advances in graph vertex colouring	
2015	Lewis [39]	A guide to graph colouring: Algorithms and applications	
2016	Samanta et al. [22]	Colouring of FGs	
2018	Zhou et al. [40]	Improving probability learning based local search for graph coloring	
2020	Mahapatra et al. [25]	Applications of edge colouring of FGs	
2021	Mahapatra et al. [8]	Colouring of COVID-19 affected region based on fuzzy directed graphs	
This paper	Mahapatra et al.	Neutrosophic edge-colouring	

TABLE 2: List of abbreviations.

Abbreviations	Meaning
FG	Fuzzy graph
NG	Neutosophic graph
MV	Membership value
TMV	True membership value
IMV	Indeterminacy membership value
FMV	Falsity membership value

TABLE 3: Some basic notations.

Notation	Meaning	
G	Crisp graph	
ξ	NG	
V	Set of vertices	
Ε	Set of edges	
$\sigma_T(x), \sigma_I(x), \sigma_F(x)$	The TMV, IMV and FMV of the vertex x of ξ	
$\mu_T(x, y), \mu_I(x, y), \mu_F(x, y)$	The TMV, IMV and FMV of the edge xy of ξ	
$(O_{T(a,b)}, O_{I(a,b)}, O_{F(a,b)})$	The TMV, IMV and FMV of strength of an edge (a, b)	
$S_{(\mu,\nu)}$	Strength score an edge (u, v)	
X	Chromatic number of the NG	
Xs	Strong chromatic number of the NG	

TABLE 4: Vertex membership value the NG of Figure 1.

σ	1	2	3	4	5	6
σ_T	0.6	0.9	0.2	0.5	0.5	0.8
σ_I	0.2	0.1	0.1	0.5	0.6	0.5
σ_F	0.1	0.1	0.5	0.3	0.8	0.3

Let, $\xi = (V, \sigma, \mu)$ be a NG then an edge (a, b) is said to Table 5be strong if the strength score is grater than or equal to 0.5 ie, edge (a, b) is said to strong if $S_{(a,b)} \ge 0.5$ otherwise Table 6this edge is weak.

Example 1. In Figure 1, a NG has been considered. TMV, IMV and FMV of all vertices are considered as in Table 4 and TMV, IMV and FMV of all edges are considered as in Table 5. Now, strength score of all edges of the NG of Figure 1 has been shown in Table 6. Here, the edges (1, 5), (2, 4), (2, 5), (5, 6) are strong and other all edges of this NG are weak.

TABLE 5: Edge membership value of the NG of Figure 1.

μ_F
0.4
0.6
0.2
0.5
0.4

Theorem 1. Let, $\xi = (V, \sigma, \mu)$ be a NG and $S_{(a,b)}$ be the strength score of an edge (a, b) then $0 \le S_{(a,b)} \le 1$.

Proof. $\xi = (V, \sigma, \mu)$ be a NG and the strength of an edge (a, b) is $(O_{T(a,b)}, O_{I(a,b)}, O_{F(a,b)})$. So, the value of the strength $0 \le (O_{T(a,b)} \le 1, 0 \le O_{I(a,b)} \le 1, 0 \le O_{F(a,b)} \le 1)$. Also, the strength score of an sedge (a, b) is $S_{(a,b)} = 2 + O_{T(a,b)} - O_{I(a,b)} - O_{F(a,b)}/3$. If the value of strength is (0, 1, 1) then the value of $S_{(a,b)}$ is 0 and the value of $S_{(a,b)}$ is 1 when the value of strength is (1, 0, 0). So, the value of strength score of an edge (a, b) is $0 \le S_{(a,b)} \le 1$. □

TABLE 6: Strength score of all edges of the NG of Figure 1.

Edge	O_T	O_I	O_F	Strength score
(1,6)	0.833	1.0	0.667	0.389
(1, 5)	0.8	0.5	0.625	0.558
(1,2)	0.833	1.0	1.0	0.278
(2,4)	1.0	1.0	0.667	0.444
(2,5)	0.8	0.5	0.75	0.517
(3,4)	0.5	1.0	0.667	0.278
(4,5)	1.0	0.8	0.75	0.483
(4,6)	0.8	0.8	0.667	0.444
(5,6)	1.0	0.8	0.625	0.525
(2, 3)	1.0	1.0	0.8	0.4



FIGURE 1: A Neutrosophic graph.

Theorem 2. Let, $\xi = (V, \sigma, \mu)$ be a complete NG then the value of strength of an edge (a, b) is (1, 1, 1) and the strength score of the edge is $S_{(a,b)} = 1/3$.

Proof. $\xi = (V, \sigma, \mu)$ be a complete NG then the edge membership value of the edge (a, b) is

$$\mu_T(a,b) = \min\{\sigma_T(a), \sigma_T(b)\},$$

$$\mu_I(a,b) = \min\{\sigma_I(a), \sigma_I(b)\},$$

$$\mu_F(a,b) = \min\{\sigma_F(a), \sigma_F(b)\}.$$
(3)

Then the strength of the edge (a, b) is

$$O_{T(a,b)} = \frac{\mu_T(a,b)}{\min\{\sigma_T(a), \sigma_T(b)\}} = 1,$$

$$O_{I(a,b)} = \frac{\mu_I(a,b)}{\min\{\sigma_I(a), \sigma_I(b)\}} = 1,$$
(4)

$$O_{F(a,b)} = \frac{\mu_F(a,b)}{\min\{\sigma_F(a), \sigma_F(b)\}} = 1.$$

So, the value of strength of an edge (a, b) is (1, 1, 1) and the strength score of the edge (a, b) is $S_{(a,b)} = 2 + 1 - 1 - 1/3 = 1/3$.

Note 1. Let, $\xi = (V, \sigma, \mu)$ be a complete NG then all edges are weak. The strength score of all edge are $S_{(a,b)} = 1/3 < 0.5$. So, all edges of the graph ξ are weak.

4. Edge Colouring of NG

In neutrosophic edge colouring, if two edges are adjacent, the vertices have fuzzy colours with different basic colours. Otherwise, their fuzzy colours may be the same as the same basic colour. Here, we used fuzzy colours to colour the NG.

Let $\xi = (V, \sigma, \mu)$ be neutrosophic connected graph and $C = (c_1, c_2, \dots, c_k)$ be a set of basic colours. Suppose, any edge (a, b) gets a fuzzy colour $(c_i, f(c_i))$, where c_i is the basic colour of the edge (a, b) and $f(c_i)$ is the depth of the basic colour c_i . Also, the depth of the colour is equal to the strength score of the edge (a, b) i.e. $f(c_i) = S_{(a,b)}$.

4.1. Algorithm to Colour the Edges of an FG

Input: A NG $\xi = (V, \sigma, \mu), |V| = n.$

Output: Complete edge coloured NG.

Step 1: Calculate the strength score of all edges. Also, vertices are to be labelled as 1, 2, ..., n.

Step 2: First of all, the vertex "1" is focused on colouring all its incident edges in such a way that no two incident edges are of the same colours. Here, the depths of the colours are equal to the strength scores of the corresponding edges.

Step 3: Proceed to direct neighbours of "1" except the previous one, which is already focused, and label them 11, 12, ..., 1m if there are *m* such neighbours. Start with the vertex 11 and repeat Step 2 and so on for 12, 13, ..., 1m.

Step 4: Repeat Step 3 until all edges of the graph have been coloured.

4.2. Chromatic Number of NGs. The minimum number of basic colours needed to colour an NG is called a chromatic number of an NG. The chromatic number of an NG is denoted by χ . Suppose minimum *n* number of basic colours are used to colour an NG, then the chromatic number $\chi = n$.

4.3. Strong Chromatic Number of NGs. The strong chromatic number will be an improved concept of the chromatic number. Let $\xi = (V, \sigma, \mu)$ be an NG. Then, the strong chromatic number is the minimum number of basic colours used to colour an NG, whose maximum depth is greater than or equal to 0.5. These colours are called strong colours. The strong chromatic number is denoted by χ_s .

Example 2. In Figure 1, considered a NG. TMV, IMV and FMV of all vertices are considered as in Table 4 and TMV, IMV and FMV of all edges are considered as in Table 5. Now, the strength score of all edges of the NG of Figure 1 has been shown in Table 6. Here, 4 basic colours Red, Green, Blue, Yellow are used to colour this NG of Figure 1. Also, the edge-colouring NG with colour depth has been shown in Figure 2 and the edge-colouring NG has been shown in Figure 3. Here, four basic colours are used, so the chromatic number is $\chi = 4$. Also, each basic colour's maximum depth is greater than 0.5. So, the strong chromatic number of this NG is $\chi_s = 4$.



FIGURE 2: A colouring NG with depth of colour.



FIGURE 4: A network.

Theorem 3. Let, $\xi = (V, \sigma, \mu)$ be a complete NG then the chromatic number of ξ is $\chi = |V| - 1$ and strong chromatic number is $\chi_s = 0$.

Proof. $\xi = (V, \sigma, \mu)$ be a complete NG. So, the degree of all vertices are same and the degree of all vertices a are |V| - 1. Therefore, the minimum number of basic colours needed to coloured this complete NG is |V| - 1. So, $\chi = |V| - 1$. Since, ξ be a complete NG so, all edges are weak and the strength score of all edge are 1/3 < 0.5. So, the depth all basic colours are 1/3. So, the strong chromatic number is zero i.e. $\chi_s = 0$.

Theorem 4. Let, $\xi = (V, \sigma, \mu)$ be a line NG and |V| > 2 then the chromatic number is $\chi = 2$.

Proof. $\xi = (V, \sigma, \mu)$ be a line neutrosophic graph. So, the maximum degree of any vertex is 2. Therefore, the minimum number of basic colours needed to coloured this line NG is 2. So, $\chi = 2$. Therefore, the chromatic number of a line NG is $\chi = 2$.



FIGURE 5: A network of 10 journals and 3 researchers.

Theorem 5. Let, $\xi = (V, \sigma, \mu)$ be a star NG then the chromatic number is $\chi = |V| - 1$.

Proof. $\xi = (V, \sigma, \mu)$ be a star NG. So, degree of the centre vertex is |V| - 1. Therefore, the minimum number of basic colours needed to coloured this line NG is |V| - 1. So, $\chi = |V| - 1$. Therefore, the chromatic number of a star NG is $\chi = |V| - 1$.

5. Application in Detection of Phishing Website

Different types of problems can be solved by graph colouring. One of them is the application to indent the phishing website. Nowadays, searching for a suitable journal is a difficult task for a researcher. Many re-searchers targeted some non-reputed journals and published their papers in these non-reputed journals. So, searching for a suitable, reputed journal for an article is a very important task. Also, some of the journals are fake. They have used the same name as a reputed journal. As a result, finding a suitable journal that is not a forgery is extremely difficult. Through the colouration of an NG, we represent a website, which can easily indicate a suitable journal and the chance of a phishing website.

Suppose a neutral website can be designed for searching for a suitable journal for an article. Every journal information has been updated there, and every researcher can register on this website with their own information. Now, every piece of information contains some falsity, and some information is not identified. So, this type of website can be designed by the NG.

Let us assume a small website where n number of researchers are registered for searching a journal, and mnumber of the journals are registered. The researcher and journal are considered a vertex of a network, and an edge exists if they have precise matching. So, we get a network in Figure 4.

Now the membership values of corresponding vertices of the journal website may depend on the following issues.

- (1) Date of initiating the journal website
- (2) Number of visitors of this website
- (3) The link of this website is the same as the indexing providing the link
- (4) Subject
- (5) Subject classification



- (6) Abstracting and indexing
- (7) Etc

And the membership values of vertices to the corresponding researcher depend on the following parameters.

- (1) Subject
- (2) Subject classification
- (3) Require quality of journal (Q_1, Q_2, Q_3) etc.
- (4) Number of publications
- (5) Quality of publication
- (6) Etc

Now, the TMV, IMV, FMV of every vertex are calculated from the number of true parameters, number of indeterminacy parameters, and number of falsity parameters. The membership value of each edge depends on the matching between several true parameters, the number of indeterminacy parameters, number of falsity parameters.

Suppose, in particular, 10 journals and 3 researchers are registered, and this network has been shown in Figure 5. When a researcher R_1 logs in this website, then the researcher gets a subgraph of Figure 5. In Figure 6, an NG is considered, and this is the subgraph of Figure 5. Also, TMV, IMV and FMV of all vertices are considered as in Table 7 and TMV, IMV and FMV of all edges are considered as in Table 8. Now, the strength score of all edges of the NG of Figure 2 has been shown in Table 9. Here, 4 basic colours Red, Green, Blue, Yellow are used to colour this NG of Figure 2. Also, the edge-colouring NG with colour depth has been shown in Figure 8. Here, 4 basic colours are used, so the chromatic number is $\chi = 4$. So, the strong chromatic number of this NG is $\chi_s = 3$.

Now, the chromatic number indicates the number of journals matched with this researcher, and the strong chromatic number indicates the number of highly matched journals. Also, the depth of the colour or strength score indicates how much matches between a journal and this researcher. Here, the strength score of the journal J_4 is 0.33, which is very low compared to other journals. So, the website where the journal was published may be phishing.

Now, the chromatic number indicates the number of journals is matching this researcher and strong chromatic number indicates number of journals are highly matching with this researcher. Also, the depth of the colour or strength score indicates how much matches between a journal and

TABLE 7: Vertex membership value the NG of.

σ	R_1	J_1	J_2	J ₃	J_4
σ_T	0.9	0.9	0.4	0.9	0.7
σ_I	0.1	0.1	0.8	0.1	0.5
σ_F	0.2	0.3	0.8	0.01	0.1

TABLE 8: Edge membership value of the NG of Figure 6.

μ	(R_1, J_1)	(R_1, J_2)	(R_1, J_3)	(R_1, J_4)
μ_T	0.9	0.4	0.9	0.6
μ_I	0.05	0.01	0.01	0.01
μ_F	0.1	0.1	0.1	0.01

TABLE 9: Strength score of all edges of the NG of Figure 6.

Edge	O_T	O_I	O_F	Strength score
(R_1, J_1)	1.0	0.5	0.33	0.723
(R_1, J_2)	1.0	1.0	1.0	0.33
(R_1, J_3)	1.0	0.1	0.5	0.8
(R_1, J_4)	0.857	0.1	0.5	0.752







FIGURE 8: Colouring of NG of researcher R_1 .

this researcher. Here, the strength score of the journal J_4 is 0.39, which is very low compared to other journals. So, the journal may be a phishing website of a journal.

6. Conclusion

In this study, a concept of edge-colouring has been introduced. A related term, "chromatic number", is also defined in a different way. A new concept of neutrosophic graph colouring was developed in this study. The addition of a chromatic number of neutrosophic graphs was defined, and some theorems have been demonstrated. To measure the strength of an edge, the strength score has been introduced, and related properties are examined. A small application has been shown to identify phishing websites. Here, we have assumed the impact factor, indexing and citations to check the quality of a journal. But there may exist some journals which are good but not indexed. There are some other parameters to be included. In the near future, more parameters with uncertainty may be included to identify the phishing website using the extended model of neutrosophic fuzzy graphs.

Data Availability

There are no data associated with this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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