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Research Article

The Characterization of Substructures of c-Anti Fuzzy Subgroups with Application in Genetics

Kalaichelvan Kalaiarasi,[1] **P. Sudha**,[1] **Nasreen Kausar**,[2] **Sajida Kousar**,[3] **Dragan Pamucar**,[4] and **Nasr Al Din Ide**[5]

[1]*Research Department of Mathematics, Cauvery College for Women (Affiliated to Bharathidasan University), Tiruchirappalli 620018, Tamil Nadu, India*
[2]*Department of Mathematics, Faculty of Arts and Science, Yildiz Technical University, Esenler, Istanbul 34210, Turkey*
[3]*Department of Mathematics & Statistics, Faculty of Basic and Applied Sciences, International Islamic University Islamabad, Islamabad, Pakistan*
[4]*Faculty of Organizational Sciences, University of Belgrade, Belgrade, Serbia*
[5]*Department of Mathematics, University of Aleppo, Aleppo, Syria*

Correspondence should be addressed to Nasr Al Din Ide; ide1112002@yahoo.ca

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Fuzzy and anti fuzzy normal subgroups are the current instrument for dealing with ambiguity in various decision-making challenges. This article discusses γ-anti fuzzy normal subgroups and γ-fuzzy normal subgroups. Set-theoretic properties of union and intersection are examined and it is observed that union and intersection of γ-anti fuzzy normal subgroups are γ-anti fuzzy normal subgroups. Employee selection impacts the input quality of employees and hence plays an important part in human resource management. The cost of a group is established in proportion to the fuzzy multisets of a fuzzy multigroup. It was a good idea to introduce anti-intuitionistic fuzzy sets and anti-intuitionistic fuzzy subgroups, as well as to demonstrate some of their algebraic features. Product of γ-anti fuzzy normal subgroups and γ-fuzzy normal subgroups is defined, the product's algebraic nature is analyzed, and the findings are supported by presenting γ-anti typical sections with blurring and γ-ordinary parts with the weirdness of well-defined and well-established groups of genetic codes.

1. Introduction

The algebraic theory contains various applications not only in theocratical and applied mathematics such as algebraic geometry, cryptography, game theory, and harmonic analysis but also in other scientific fields like physics, genetics, and engineering. The algebraic structures are conventionally defined on a nonempty set by employing binary operations. The simplest among them is groupoid, precisely a non-vain set endowed with a bipartite action. With the inclusions of different properties, the groupoid can be transformed into semigroup and monoid group. The group is a central algebraic structure and serves as a baseline for various algebraic structures such as ring, field, vector space, and module. Significate research has been carried out for deep analysis of algebraic properties and applications ranging from mathematics to DNA structure, cars moving on the road to aircraft flying in the air, and wristwatches to complicated computing systems. Uncertainty, imprecision, and ambiguity are the common factors associated with real-life decision-making problems or any kind of experimental data. The classical probability theory is sometimes not enough to handle all such cases. In 1965, Zadeh [1] laid the notation of blurry place as an ideal framework to incorporate the uncertainty fragment in logic. The set is formalized by defining a map called membership responsibility through a nonempty set to the interval $[0, 1]$, where the images of elements under this function are called degree or grade of membership. The concept was so inspiring that it grabbed the attention of researchers from every field of knowledge. Several new theories are established parallel to the classical ones by considering the fuzzy sets and logic.

Rosenfeld [2] sought to incorporate fuzzy ideas in group theory in 1970 and labeled the results as a fuzzy subgroup. Rosenfeld looked at the basic group theocratic aspects of the newly discovered algebra. Algebraists later studied the structural features of fuzzy subgroups. Anthony [3, 4] refined Rosenfeld's concept by enhancing the need for pictures of elements and their inverses. They are less expensive to design, cover a broader variety of operating situations, and are more easily adaptable to plain language concepts. Fuzzification is the process of transforming a crisp input value into a fuzzy value through the application of knowledge base information. Zadeh relates the ordinary sets and fuzzy sets using level sets. Level sets are seen in fuzzy groups, and it is demonstrated that a flexible selection of a subgroup G is a hazy subgroup if and only if all of the applied principles are constituents G (see [5, 6]). Liu [7] established flexible resilient parts and fuzzy ideals in 1982. Mukherjee et al. [8–10] suggested an association connecting fuzzy normal subgroups, fuzzy cosets, and group-theoretic analogs. Kumar et al. [11] solved the problems of sensitive ordinary groupings and flexible quotients. Tarnauceanu [12] also introduced the idea of curved typical counterparts for the group of limited groups. Choudhary et al. and Addis [13, 14] explored feature sustaining bridges and elementary presented in this research theorems. Malik et al. [15] and Mishref [16] developed fuzzy normal series to enhance the theory of group nilpotency and solubility.

An intuitionistic blurry set [17] is an induction of an unclear set endowed with two functions from a nonempty adjust to the interval $[0,1]$ known as membership and non-integration function. The idea is not identical to the occurrence and non-occurrence of an event in classical probability theory as in this case the quantity of the grades of integration and non-integration could be real in any number between 0 and 1. In 2017, Al-Husban et al. [18] described a complex intuitionistic fuzzy normal subgroup. As defined by Rosenfeld, a blurry subdivision of a category G is a blurry subdivision if the degree of membership of the product of two elements is greater than or equal to the minimum of their individual degrees. The replacement of minimum by maximum defines a new type of fuzzy-subgroups called anti fuzzy subgroups [19]. Onasanya [20] investigated existing fuzzy group-theoretic properties for anti fuzzy subgroups. α-anti blurry subdivisions and α-blurry subdivisions are depicted by Sharma [21, 22]. Shuaib et al. [23, 24] manifested some characterizations and properties of o-fuzzy subgroups.

Most recently, several generalizations of fuzzy sets and subgroups [25–31] are developed not only for the sake of new algebraic structures but also to utilize them for wide range of applications [32–34]. Advancements in fuzzy sets are introduced mainly in search of a better and more efficient tool to deal with uncertainties more accurately and effectively. Ultimately algebraic structures are also upgraded but conducting analysis in the generalized fuzzy environment. In this article, the authors aim to study group theocratic concepts in an advanced manner by employing the impulse of γ-anti blurry sets. Normal subgroups and their analytic behavior are examined; also group homomorphisms are used to define fuzzy extension rules.

2. Preliminaries

Definition 1 (see [14]). A fuzzy set constructed from either a nonempty set A is a technique η: $A \longrightarrow [0,1]$.

Definition 2 (see [14]). A grouping $(L,\star)$ is a semiset L minus a discrete activity L that meets the following properties:

(i) *Closure*. for all $a,b \in L$, the element $a\star b$ is a uniquely defined element of L

(ii) *Associativity*. We have $a\star(b\star c) = (a\star b)\star c$ for every $a,b,c \in L$

(iii) *Identity*. for any $a \in L$, there exists an identity element e such that $e\star a = a$ and $a\star e = a$

(iv) *Inverse*. There exists an inverse element $a^{-1} \in L$ for each $a \in L$ such that $a\star a^{-1} = e$ and $a^{-1}\star a = e$

Definition 3 (see [19]). Assume M is a fuzzy subset (FSS) of a group L. Then, M is a fuzzy subgroup (FSG) if $M(a^{-1}b) \geq \min\{A(a), A(b)\}$ for all $a,b \in L$ where $A(a), A(b)$ are fuzzy membership functions.

Definition 4 (see [21]). Assume M is a FSS of a group L. Then, M is an AFSG (anti fuzzy subgroup) if $M(a^{-1}b) \leq \max\{A(a), A(b)\}$, for all $a,b \in L$ where $A(a), A(b)$ are fuzzy membership functions.

Definition 5 (see [32]). A function t^*: $[0,1]\times[0,1] \longrightarrow [0,1]$ is said to be a t-conorm on $[0,1]$ if and only if t^* satisfies the following properties for all $u,v,w,s \in [0,1]$:

(i) $t^*(u,v) = t^*(v,u)$

(ii) $t^*(u,t^*(v,w)) = t^*(t^*(u,v),w)$

(iii) $t^*(u,0) = t^*(0,u) = u, t^*(1,1) = 1$

(iv) If $u \leq w$ and $v \leq s$ then $t^*(u,v) \leq t^*(w,s)$

Definition 6 (see [32]). Let S_p: $[0,1]\times[0,1] \longrightarrow [0,1]$ be the algebraic sum t-conorm on $[0,1]$, then it is defined by $S_p(a,b) = a + b - ab, \quad 0 \leq a \leq 1,\ 0 \leq b \leq 1.$

Tuning is the most laborious and tedious part of building a fuzzy system. It often involves adjusting existing fuzzy sets and fuzzy rules. With appropriate examples, the composition of the fuzzy relations is described in two ways: max-min composition and max-product composition. This study also introduces the composition features of fuzzy relations.

3. γ-Anti FSS (γ AFSS) and Their Attributes

This section deals with the definition of γ-anti fuzzy subset and some results based on this definition.

Definition 7 (see [3]). Let M be a nonempty set and H be an FSS of $M$and $\gamma \in [0,1]$. Then, the FSS is called the γ-anti FSS (γ AFSS) of M if

$$H_\gamma(a) = S_p\{H(a), 1-\gamma\}, \quad \text{for all } a \in M. \tag{1}$$

Remark 1

(i)

$$\begin{aligned} H_1(a) &= S_p\{H(a), 1-1\} \\ &= H(a) - H(a)\cdot(0) \\ &= H(a). \end{aligned} \tag{2}$$

(ii)

$$\begin{aligned} H_0(a) &= S_p\{H(a), 1-0\} \\ &= S_p\{H(a), 1\} \\ &= H(a) + 1 - H(a)\cdot 1 \\ &= 1. \end{aligned} \tag{3}$$

Theorem 1. *Let H and K be two arbitrary FSS of M. Then,*

(i) $(H\cup K)_\gamma = H_\gamma \cup K_\gamma$
(ii) $(H\cap K)_\gamma = H_\gamma \cap K_\gamma$

Proof. Consider

(i)

$$\begin{aligned} (H\cup K)_\gamma(a) &= S_p\{(H\cup K)(a), 1-\gamma\} \\ &= S_p\{\max\{H(a),K(a)\}, 1-\gamma\} \\ &= \max\{S_p\{H(a),1-\gamma\}, S_p\{K(a),1-\gamma\}\} \\ &= \max\{H_\gamma(a), K_\gamma(a)\} \\ &= H_\gamma(a)\cup K_\gamma(a), \quad \text{for all } a\in M \\ &= \left(H_\gamma \cup K_\gamma\right)(a). \end{aligned} \tag{4}$$

Hence, we have

$$(H\cup K)_\gamma(a) = \left(H_\gamma\cup K_\gamma\right)(a). \tag{5}$$

(ii)

$$\begin{aligned} (H\cap K)_\gamma(a) &= S_p\{(H\cap K)(a), 1-\gamma\} \\ &= S_p\{\min\{H(a),K(a)\}, 1-\gamma\} \\ &= \min\{S_p\{H(a),1-\gamma\}, S_p\{K(a),1-\gamma\}\} \\ &= \min\{H_\gamma(a), K_\gamma(a)\} \\ &= H_\gamma(a)\cap K_\gamma(a), \quad \text{for all } a\in M \\ &= \left(H_\gamma \cap K_\gamma\right)(a). \end{aligned} \tag{6}$$

Hence, we have

$$(H\cap K)_\gamma(a) = \left(H_\gamma\cap K_\gamma\right)(a). \tag{7}$$

□

Definition 8. Suppose $f\colon M \longrightarrow N$ be a category M to column N component. If H and K are flexible sections (FSS) of M and N, alternately, then $f(H)$ and $f^{-1}(K)$ are now the portrait of soft set H and the inverse image of fuzzy rules K, respectively, defined as

$$f(H)(b) = \begin{cases} \operatorname{Sup}\left\{H(a)\colon a\in f^{-1}(b)\right\}; & \text{iff}^{-1}(b) \neq \phi, \\ 1; & \text{iff}^{-1}(b) = \phi, \end{cases} \tag{8}$$

for every $b \in K$ and $f^{-1}(K)(a) = K(f(a))$, for every $a \in H$.

Theorem 2. *Let $f\colon M \longrightarrow N$ be a mapping and HandK be two FSS of M and N, respectively, then*

(a) $f^{-1}(K_\gamma) = (f^{-1}(K))_\gamma$
(b) $f(H_\gamma) = (f(H))_\gamma$

Proof.

(a)

$$\begin{aligned} f^{-1}\left(K_\gamma\right)(a) &= K_\gamma(f(a)) \\ &= \max\{K(f(a)), 1-\gamma\} \\ &= \max\left\{f^{-1}(K)(a), 1-\gamma\right\} \\ &= \left(f^{-1}(K)\right)_\gamma(a). \end{aligned} \tag{9}$$

Hence, we have

$$f^{-1}\left(K_\gamma\right)(a) = \left(f^{-1}(K)\right)_\gamma(a), \quad \text{for all } a\in H. \tag{10}$$

(b)

$$\begin{aligned} f\left(H_\gamma\right)(b) &= Sup\left\{H_\gamma(a)\colon f(a) = b\right\} \\ &= \operatorname{Sup}\{\max\{H(a),1-\gamma\}\colon f(a)=b\} \\ &= \max\{\operatorname{Sup}\{H(a)\colon f(a)=b\}, 1-\gamma\} \\ &= \max\{f(H)(b), 1-\gamma\} \\ &= (f(H))_\gamma(b). \end{aligned} \tag{11}$$

Hence, we have

$$f\left(H_\gamma\right)(b) = (f(H))_\gamma(b), \quad \text{for all } b\in K. \tag{12}$$

Thus, we have proved that the union and intersection of two γ-FSS are also a γ-FSS. □

4. Mathematical Dominion of γ-Anti FSGs (γ AFSGs)

In this section, we have discussed the γ-anti fuzzy subgroups (γ-AFSGs) and some results based on γ-AFSG.

Definition 9. Let L be a group and M be a FSS of L and $\gamma \in [0,1]$. Then, M is called γ-APFSG of L if

(i) $M_\gamma(ab) \le \max\{M_\gamma(a), M_\gamma(b)\}$, for all $a, b \in L$
(ii) $M_\gamma(a^{-1}) = M_\gamma(a)$

Theorem 3. *Let $M\colon L \longrightarrow [0,1]$ be a μ-APFSG of a group L, then*

(i) $M_\gamma(a) \ge M_\gamma(e)$, $\forall a \in L$ and $e \in L$
(ii) $M_\gamma(ab^{-1}) = M_\gamma(e)$

Proof. (i)

$$\begin{aligned} M_\gamma(e) &= M_\gamma\left(aa^{-1}\right) \\ &\le \max\left\{M_\gamma(a), M_\gamma\left(a^{-1}\right)\right\} \\ &= \max\left\{M_\gamma(a), M_\gamma(a)\right\} \\ &= M_\gamma(a) \end{aligned} \tag{13}$$

Hence, $M_\gamma(e) \le M_\gamma(a)$.

This implies that $M_\gamma(a) \ge M_\gamma(e)$.

(ii)

$$\begin{aligned} M_\gamma(a) &= M_\gamma\left(ab^{-1}b\right) \\ &\le \max\left\{M_\gamma\left(ab^{-1}\right), M_\gamma(b)\right\} \\ &= \max\left\{M_\gamma(e), M_\gamma(b)\right\} \\ &= M_\gamma(b) \end{aligned} \tag{14}$$

Hence, $M_\gamma(a) \le M_\gamma(b)$

Similarly, $M_\gamma(b) \le M_\gamma(a)$.

This implies that $M_\gamma(a) = M_\gamma(b)$. □

Theorem 4. *Every APFSG of a group L is a γ-APFSG of a group L.*

Proof. Let M be an APFSG of a category L and let a and b be two elements in L. Since M is an APFSG of a group L, then we have

$$M\left(a^{-1}b\right) \le \max\{A(a), A(b)\}, \quad \text{for all } a, b \in L. \tag{15}$$

To prove

(i) $M_\gamma(ab) \le \max\{M_\gamma(a), M_\gamma(b)\}$, for all $a, b \in L$
(ii) $M_\gamma(a^{-1}) = M_\gamma(a)$, for all $a \in L$

(i) Consider

$$\begin{aligned} M_\gamma(ab) &= S_p\{M(ab), 1-\gamma\} \\ &\le S_p\{\max\{M(a), M(b)\}, 1-\gamma\} \\ &= \max\left\{S_p\{M(a), 1-\gamma\},\right. \\ &\quad \left.\cdot S_p\{M(b), 1-\gamma\}\right\} \\ &= \max\left\{M_\gamma(a), M_\gamma(b)\right\} \end{aligned}$$

Hence, $M_\gamma(ab) \le \max\left\{M_\gamma(a), M_\gamma(b)\right\}$, for all $a, b \in L$. (16)

(ii) Consider

$$\begin{aligned} M_\gamma\left(a^{-1}\right) &= S_p\left\{M\left(a^{-1}\right), 1-\gamma\right\} \\ &= S_p\{M(a), 1-\gamma\} \\ &= M_\gamma(a). \end{aligned} \tag{17}$$

Therefore, M is μ-APFSG of L. □

Note 1. The converse of Theorem 4 might not be true.

Theorem 5. *Union of two γ-APFSGs of a group L is also γ-APFSG of L.*

Proof. Authorize H and K be two γ-APFSGs of a category L. Let us assume that $a, b \in L$.

$$\begin{aligned} (H \cup K)_\gamma(ab) &= \left(H_\gamma \cup K_\gamma\right)(ab) \\ &= \max\left\{H_\gamma(ab), K_\gamma(ab)\right\} \\ &\le \max\left\{\max\left\{H_\gamma(a), H_\gamma(b)\right\},\right. \\ &\quad \left.\cdot \max\left\{K_\gamma(a), K_\gamma(b)\right\}\right\} \\ &= \max\left\{\max\left\{H_\gamma(a), K_\gamma(a)\right\},\right. \\ &\quad \left.\cdot \max\left\{H_\gamma(b), K_\gamma(b)\right\}\right\} \\ &= \max\left\{(H \cup K)_\gamma(a),\right. \\ &\quad \left.(H \cup K)_\gamma(b)\right\} \end{aligned} \tag{18}$$

Hence, $(H \cup K)_\gamma(ab) \le \max\left\{(H \cup K)_\gamma(a), \cdot (H \cup K)_\gamma(b)\right\}$.

Consider

$$\begin{aligned} (H \cup K)_\gamma\left(a^{-1}\right) &= \left(H_\gamma \cup K_\gamma\right)\left(a^{-1}\right) \\ &= \max\left\{H_\gamma\left(a^{-1}\right), K_\gamma\left(a^{-1}\right)\right\} \\ &= \max\left\{H_\gamma(a), K_\gamma(a)\right\} \\ &= \left(H_\gamma \cup K_\gamma\right)(a) \\ &= (H \cup K)_\gamma(a). \end{aligned} \tag{19}$$

Therefore, $H \cup K$ is γ-APFSG of L. □

Example 1. Let $L = Z$, the set of integers be the group under the binary operation "+."

Let us assume the two FSS H and K of Z as

$$H(a) = \begin{cases} 0.3, & \text{if } a \in 3Z, \\ 0.7, & \text{otherwise.} \end{cases} K(a) = \begin{cases} 0.2, & \text{if } a \in 2Z, \\ 0.5, & \text{otherwise.} \end{cases} \quad (20)$$

Let us take $\gamma = 1$. Obviously, H and K are 1-APFSG of Z.

Now, $(H \cup K)(a) = \max\{H(a), K(a)\}$.

To prove: association of two γ-APFSG of Z is not a γ-APFSG of Z.

Therefore, $(H \cup K)(a) = \begin{cases} 0.3, & \text{if } a \in 3Z, \\ 0.2, & \text{if } a \in 2Z - 3Z, \\ 0.7, & \text{otherwise.} \end{cases}$ Let $a = 9$ and $b = 2$. Then,

$$\begin{aligned} (H \cup K)(9) &= 0.3 \\ (H \cup K)(2) &= 0.2 \\ (H \cup K)(9-2) &= (H \cup K)(7) \\ \max\{(H \cup K)(a), (H \cup K)(b)\} &= \max\{0.3, 0.2\} \\ &= 0.3. \end{aligned} \quad (21)$$

Therefore, $(H \cup K)(a-b) \rangle \max\{(H \cup K)(a), (H \cup K)(b)\}$. Hence, $H \cup K$ is not a 1-APFSG of Z. Therefore, union of two 1-APFSG of Z is not a 1-APFSG of Z.

Definition 10. Let H and K be two γ-APFSGs of groups L_1 and L_2, respectively. Then, product of μ-APFSGs of H and K is defined as

$$H_\mu \times K_\mu = \max\{H_\mu(a), K_\mu(b)\}, \quad \text{for all } a \in L_1, b \in L_2. \quad (22)$$

Theorem 6. *Let H and K be two γ-APFSGs of groups L_1 and L_2, respectively. Then $H_\mu \times K_\mu$ is μ-APFSG of $L_1 \times L_2$.*

Proof. Let $a_1, a_2 \in L_1$ and $b_1, b_2 \in L_2$, then $(a_1, b_1), (a_2, b_2) \in L_1 \times L_2$,

$$\begin{aligned} & H_\mu \times K_\mu\left((a_1, b_1)\left(a_2^{-1}, b_2^{-1}\right)\right) \\ & = H_\mu \times K_\mu\left(a_1 a_2^{-1}, b_1 b_2^{-1}\right) \\ & = \max\left\{H_\mu\left(a_1 a_2^{-1}\right), K_\mu\left(b_1 b_2^{-1}\right)\right\} \\ & \leq \max\left\{\max\left\{H_\mu(a_1), H_\mu\left(a_2^{-1}\right)\right\}, \max\left\{K_\mu(b_1), K_\mu\left(b_2^{-1}\right)\right\}\right\} \\ & \leq \max\left\{\max\left\{H_\mu(a_1), H_\mu(a_2)\right\}, \max\left\{K_\mu(b_1), K_\mu(b_2)\right\}\right\} \\ & = \max\left\{\max\left\{H_\mu(a_1), K_\mu(b_1)\right\}, \max\left\{H_\mu(a_2), K_\mu(b_2)\right\}\right\} \\ & = \max\left\{H_\mu \times K_\mu(a_1, b_1), H_\mu \times K_\mu(a_2, b_2)\right\}. \end{aligned} \quad (23)$$

Hence, $H_\mu \times K_\mu((a_1, b_1)(a_2^{-1}, b_2^{-1})) \leq \max\{H_\mu \times K_\mu(a_1, b_1), H_\mu \times K_\mu(a_2, b_2)\}$. With appropriate examples, the composition of the fuzzy relations is described in two ways: max-min composition and max-product composition. This study also introduces the composition features of fuzzy relations. □

Definition 11. Let H be a γ-APFSG of a group L and $\gamma \in [0,1]$. For any $a \in H$, the γ-AFLCS of H in L is represented by $aH_\gamma(g) = S_p\{H(m^{-1}), \gamma\}$, for all $a, g \in L$. The γ-AFRCS is defined as

$$(Ha)(g) = S_p\left\{H\left(ga^{-1}\right), \gamma\right\}, \quad \text{for all } a, g \in L. \quad (24)$$

Definition 12. Let H be a γ-APFSG of a group L and $\gamma \in [0,1]$. Then, H is said to be μ-APFNSG of L if and only if $aH = Ha$, for all $a \in H$.

Theorem 7. *Every APFNSG of a category L is a γ-APFNSG of L.*

Proof. Let H be an APFNSG of a category L. Then, for any $a \in H$,

$$\begin{aligned} aH &= \text{Ha} \\ \Rightarrow (aH)(g) &= (\text{Ha})(g) \text{ for all } g \in L. \end{aligned} \quad (25)$$

Hence, we have

$$\begin{aligned} H\left(a^{-1}g\right) &= H\left(ga^{-1}\right) \\ \Rightarrow S_p\left\{H\left(a^{-1}g\right), \gamma\right\} &= S_p\left\{H\left(ga^{-1}\right), \gamma\right\} \\ \Rightarrow aH_\gamma(g) &= H_\gamma a(g) \\ \Rightarrow aH_\gamma &= H_\gamma a, \quad \text{for all } a \in L. \end{aligned} \quad (26)$$

Hence, H is a γ-APFNSG of L. □

Remark 2. The contrary of the accompanying hypothesis is not really true.

Example 2. Let $L = D_3 = \langle u, v \colon u^3 = v^2 = e, vu = u^2 v \rangle$ be the dihedral group. Let us define the FSG of D_3 as

$$H(a) = \begin{cases} 0.6, & \text{if } a \in \langle v \rangle, \\ 0.4, & \text{otherwise,} \end{cases} \quad (27)$$

To prove: H is not a APFNSG of L. Let us take $\gamma = 0$, then we have

$$\begin{aligned} \left(aH_\gamma\right)(g) &= S_p\left\{H\left(a^{-1}g\right), 1-\gamma\right\} \\ &= S_p\left\{H\left(a^{-1}g\right), 1\right\} \\ &= H\left(a^{-1}g\right) + 1 - H\left(a^{-1}g\right) \\ &= 1 \\ \left(H_\gamma a\right)(g) &= S_p\left\{H\left(ga^{-1}\right), 1-\gamma\right\} \\ &= S_p\left\{H\left(ga^{-1}\right), 1\right\} \\ &= H\left(ga^{-1}\right) + 1 - H\left(ga^{-1}\right) \\ &= 1 \\ \Rightarrow aH_\gamma &= H_\gamma a. \end{aligned} \quad (28)$$

Therefore, H is a o-APFNSG of L.

Now,

$$
\begin{aligned}
H\left(u^2(uv)\right) &= H\left(u^3 v\right) \\
&= H(ev) = H(v) = 0.6, \\
H\left((uv)u^2\right) &= H(u(uv)u) = H(u(vu)u) = H\left(u\left(u^2 v\right)u\right) \\
&= H\left(\left(u^3 v\right)u\right) = H((ev)u) = H(vu) \\
&= 0.4 \\
\Rightarrow H\left(u^2(uv)\right) &\neq H\left((uv)u^2\right).
\end{aligned}
\tag{29}
$$

$\Rightarrow H$ is not γ-APFNSG of L.

Theorem 8. *Let H be γ-APFNSG of a group L. Then, $H_\gamma(b^{-1}ab) = H_\gamma(a)$ or $H_\gamma(ab) = H_\gamma(ba)$ for all $a, b \in L$.*

Proof. Let H be a γ-APFNSG of a group L.

$$
\begin{aligned}
&\Rightarrow aH_\gamma && = H_\gamma a, \quad \text{for all } a \in L \\
&\Rightarrow \left(aH_\gamma\right)\left(b^{-1}\right) && = \left(H_\gamma a\right)\left(b^{-1}\right),\ b^{-1} \in L \\
&\Rightarrow S_p\left\{H\left(a^{-1}b^{-1}\right), 1-\gamma\right\} && = S_p\left\{H\left(b^{-1}a^{-1}\right), 1-\gamma\right\}. \\
&\Rightarrow H_\gamma(ba) - 1 && = H_\gamma(ab) - 1 \\
&\Rightarrow H_\gamma(ba) && = H_\gamma(ab);
\end{aligned}
\tag{30}
$$

[As His a γ − AFSG of L so $H_\gamma(g^{-1}) =$ $H_\gamma(g)$ for all $g \in L$]. □

5. Application

A conventional genetic sequence is made of three base pairs, and the organization among those DNA bases or RNA is precise. Mathematically, these genetic codes can be interpreted and analyzed by defining appropriate algebraic structures. The two main types of nucleic acids are DNA and RNA which are long chains of repeating nucleotides. In RNA, the base thymine (A, C, G, T in DNA) replaces with uracil (U). RNA is formed by the transcription process of DNA and is mostly involved in protein synthesis. This entire process commits to encoding the codons (triplets). These triplets are called standard genetic code SGC [35]. It is a domain transformation of crisp into fuzzy inputs that are used to establish the degree of truth for each rule premise. The mathematical explanation of gene mutation is provided by group automorphism, making it simple to identify the mutation. Different mathematical models suggested binary interpretation of the GC of the DNA bases. These binary representations suggested that there must exist some partial order on the codon set. The partial order on GC is defined by using chemical Base classes (codon and heterocyclic) and protonated values. Numerous algebraic structures for genetic code (GC) have been presented to investigate the consequence of the significant link between the mutational process and the coding apparatus on protein-coding regions [36]. Mathematically, a GC is identical to a cube enclosed in 3D space, as a result of steady phylogenetic analyses of DNA protein-coding regions. Sanchez and Barreto proposed that GC may be characterized as such linear average of treatment in order elliptic groups and suggested that it can be quite logical to extend it to the whole genome defined on the GC, where population's GA in the alike steer to the similar canonical decomposition into p-groups [37]. The four-base Boolean lattice is constructed by considering that bases with the same hydrogen bond number in the DNA molecule and with separate chemical types must be supportive elements in the lattice. By using this complementary behavior of DNA bases, Sanchez et al. [38–40] define two dual Boolean codon lattices of GC. The boolean lattice of the GC is presumed to be the direct product of three copies of the four-base two dual Boolean lattices. Here, we will construct γ-anti fuzzy subgroups and γ-fuzzy subgroups for the DNA base. We proceed in the following manner. Let $L = \{A, C, G, U\}$ define a binary operation L as follows:

$$
\begin{array}{ccccc}
\cdot & A & C & G & U \\
A & A & C & G & U \\
C & C & A & U & G \\
G & G & U & A & C \\
U & U & G & C & A
\end{array}.
\tag{31}
$$

Define blurring subset μ of G as

$$
\mu(x) = \begin{cases} 1, & \text{if } x = A, \\ 0.5, & \text{if } x = C \text{ or } x = G, \\ 0.3, & \text{if } x = U. \end{cases}
\tag{32}
$$

Then, $\mu(ab) \geq \min\{\mu(a), \mu(b)\}$ and $\mu(a^{-1}) = \mu(a)$ for all $a, b \in G$ imply μ is a fuzzy subgroup of L. Let the fuzzy set η of L be defined by

$$
\eta(x) = \begin{cases} 0.9, & \text{if } x = A, \\ 0.7, & \text{if } x = C \text{ or } x = G, \\ 0.8, & \text{if } x = U. \end{cases}
\tag{33}
$$

then η is γ-APFSG of L as $\eta_\gamma(ab) \leq \max\left\{\eta_\gamma(a), \eta_\gamma(b)\right\}$ and $\eta_\gamma(a^{-1}) = \eta_\gamma(a)$ for all $a, b \in L$. Consider

$$
\begin{aligned}
\eta_\gamma(AC) &= S_p\{\eta(AC), 1-\gamma\} \\
&\leq S_p\{\max\{\eta(A), \eta(C)\}, 1\} \\
&= \max\left\{S_p\{\eta(A), 1\}, S_p\{\eta(C), 1\}\right\} \\
&= \max\left\{S_p\{0.9, 1\}, S_p\{0.7, 1\}\right\} \\
&= \max\{0.9 + 1 - 0.9, 0.7 + 1 - 0.7\} \\
&= \max\{1, 1\} \\
\eta_\gamma(AC) &= 1, \\
\max\left\{\eta_\gamma(A), \eta_\gamma(C)\right\} &= \max\left\{S_p\{\eta(A), 1\}, S_p\{\eta(C), 1\}\right\} \\
&= \max\left\{S_p\{0.9, 1\}, S_p\{0.7, 1\}\right\} \\
&= \max\{1, 1\} \\
&= 1.
\end{aligned}
\tag{34}
$$

This implies that

$$\eta_\gamma(AC) \leq \max\{\eta_\gamma(A), \eta_\gamma(C)\}. \quad (35)$$

The sixty-four codon system is

$$L^3 = \{AAA, AAC, AAG, AAU, ACA, ACC, ACG, ACU, AGA, AGC, AGG, AGU, AUA, AUC, AUG, AUU, CAA, CAC, CAG, CAU, CCA, CCC, CCG, CCU, CGA, CGC, CGG, CGU, CUA, CUC, CUG, CUU, GAA, GAC, GAG, GAU, GCA, GCC, GCG, GCU, GGA, GGC, GGG, GGU, GUA, GUC, GUG, GUU, UAA, UAC, UAG, UAU, UCA, UCC, UCG, UCU, UGA, UGC, UGG, UGU, UUA, UUC, UUG, UUU\}. \quad (36)$$

Using Definition 10, we compute γ-APFSG of the sixty-four codon system as follows:

$$\eta(xyz) = \begin{cases} 0.9, & \text{if } x, y \text{ or } z = A, \\ 0.7, & \text{if } x, y \text{ and } z = C \text{ or } x, y \text{ and } z = G, \\ 0.8, & \text{if } x, y \text{ or } z = U. \end{cases} \quad (37)$$

The groups L and L^3 are Abelian groups so the γ-APFSG is γ-AFNSG.

6. Conclusion

The goal of this study is to study anti fuzzified normative segments (γ-AFNSG and γ-FNSG). As per mathematical logic, the convergence of any assembly of subdivisions of a category G is also the division of G. However, the union of subgroups generally does not obey this rule. However, the union of any two γ-APFNSG is also a γ-APFNSG. The product of two γ-APFNSG of two different groups can be determined by taking the cross product of the groups under consideration with a componentwise binary operation. The cosets are also introduced which can be further used to define quotient structure in anti fuzzy subgroups. The work provides essential information about normal subgroups in γ-fuzzy and anti fuzzy subgroups. Nilpotent and soluble groups are governed by defining normal series, precisely, the chains of normal subgroups. In the future, nilpotent and soluble γ-fuzzy and anti fuzzy subgroups can be defined with the help of the results presented in this article. γ-AFNSG and γ-FNSG are constructed for the dihedral groups. The group automorphisms provide the mathematical description of gene mutation [41]. The dihedral group is the best example of finite non-Abelian groups generated by reflections and rotations of a regular polygon and plays an important role in group theory, geometry, and chemistry. Group automorphisms are observed as the algebra behind gene mutation here γ-AFNSG and γ-FNSG are established for the DNA base and codon system. It is a domain transformation of crisp into fuzzy inputs to determine the truth's degree for each rule premise. The group automorphism provides the mathematical description of gene mutation, so it will be easy to identify the mutation. In future, the automorphism for γ-AFNSG and γ-FNSG can be established by considering the extension principle of fuzzy sets and group homomorphism to analyze its impact to identify gene mutations.

Data Availability

The work is a contribution to theoretical fuzzy algebra so no data are required for the validity of the results obtained.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors' Contributions

This publication was developed in consultation with all the contributors.

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