Hindawi
Discrete Dynamics in Nature and Society
Volume 2022, Article ID 1551937, 14 pages
https://doi.org/10.1155/2022/1551937

Research Article

An Optimal Credit Scoring Model Based on the Maximum Default Identification Ability for Chinese Small Business

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Received 25 October 2021; Accepted 9 December 2021; Published 19 January 2022

Academic Editor: Juan L. G. Guirao

The reasonable credit scoring model must have strong default identification ability, which means the credit scoring can effectively distinguish between defaulting and nondefaulting customers. The premise to determine the credit score of small enterprises is to determine the weight of indicators. This paper studies 3,045 Chinese small business loans, and two novel weighting methods “Wilks' Lambda method” and “AUC value method” are proposed, The greater the weight they meet, the greater the ability of default identification. The five weighting methods of “Wilks' lambda method,” “AUC value method,” “G1 method,” “entropy method,” and “mean square variance method” are compared. An important contribution of the paper is to discover that Wilks' Lambda method is the most effective method for small business.

1. Introduction

The essence of credit is a borrowing and lending relationship which aims at pay back. Credit risk is default risk, that is, the possibilities that the borrower repays the principal and interest as scheduled. Credit risk evaluation is to reveal the nature of a debt default risk, which essentially estimates the customer's credit status and determine the order of loan customers.

A reasonable credit risk evaluation system must have strong default identification ability, which is able to effectively distinguish between defaulting customers and nondefaulting customers. One reason to determine the weight of the credit evaluation indicator is the key to determine the quality of the credit evaluation system. In most of weighting methods, the choice of an appropriate weighting method is the key for credit risk evaluation. If the choice of the weighting method is not appropriate, it will directly affect the evaluation result, which means the poor credit enterprises will be evaluated as good businesses and this will mislead the decision-making for financial institutions. The weight also can reflect the importance of the indicator; that is, we can determine the key indicators that play an important role in credit risk evaluation according to the weight.

This paper studies 3,045 small business loans of a commercial bank of China, and two novel weighting methods “Wilks' Lambda method” and “AUC value method” are proposed. The greater the weight they meet, the greater the ability of default identification. The five weighting methods of “Wilks' lambda method,” “AUC value method,” “G1 method,” “entropy method, and mean square variance method” are compared.. An important contribution of the paper is to discover that Wilks' Lambda method is the most effective method for small business. The weight results show that nonfinancial indicators such as “consumer price indicator” and “enterprise credit in 3 years” have the largest weight and play an important role in default prediction of small enterprises. The credit scoring model is constructed according to Wilks' Lambda method, which has the maximum default identification ability.

The rest of the paper is structured as follows: Section 2 is the review of the literature. Section 3 subsequently describes the model of weight indicator. Section 4 constructs the standard to choose the optimal weighting methods. Section 5 is the empirical study, and the final section concludes the study.

2. Review of the Literature

In the existing research, artificial intelligence methods such as neural network, SVM, and statistical methods such as logit regression are used to build the credit scoring model. Chai et al. established a credit scoring system by using both partial correlation analysis and probit regression [1]. Bai et al. used fuzzy rough-set theory and fuzzy C-means clustering to evaluate farmer credit level [2]. Tong et al. introduced mixture cure models to the area of credit scoring [3]. Harris used SVM assessment for the credit risk [4]. Tanoue et al. forecasted default with a multistage model [5]. Chi et al. established the credit risk rating system by logit regression [6]. Shi et al. developed an approach combining Pearson correlation analysis with F test significance discrimination for credit risk [7]. Shi et al. proposed a credit rating model that considers the impact of LGD [8]. Mizen and Tsoukas forecasted default ratings in an ordered probit model [9]. Danenas and Garsva [10] and Hilscher and Wilson [11] constructed the linear credit scoring equation. Hasumi and Hirata studied the Japanese credit scoring market using data on 2,000 small- and medium-sized enterprises and a small-business credit scoring (SBCS) model [12]. Min and Lee proposed a DEA model for credit scoring [13].

For all the credit scoring models, the key point is to determine the weight of indicators. The existing weighting methods can be divided into three categories: subjective weight, objective weight, and combined weight [14].

Subjective weight was decided by the experts according to their experience, knowledge, and personal preferences. For example, the method of analytical hierarchy process (AHP) was used to weight the evaluation indicator [15–17]. Vidal used the Delphi method to determine the subjective weight of evaluation indicators [18].

Objective weight was decided by the data which belong to objective information. The objective weight methods include entropy method [19], standard deviation method [20], variation coefficient method [21], and goal programming method [22, 23]. Chen et al. used an entropy weight method to weight industries when analyzing the systemic risk of different industries and thereby established a credit evaluation model [24].

The existing research based on the discrete degree of the data to determine the indicator weight did not take into account the default identification ability. In fact, for the credit risk evaluation, the standard "the bigger the default identification ability, the greater the indicator weight" should be satisfied.

Optimal weight also belongs to the objective weight, which can get the weight through the goal programming method. The special point is that we can get the optimal weight according to the evaluation results, which means the default customer has the lower evaluation scoring and the nondefault customer has the higher scoring. The disadvantage lies in that the optimal weight only ensures the evaluation result to be the most optimal, but the size of weight does not reflect the importance of the indicator, whereas the objective and subjective weights do.

Combined weight combines the subjective weight and objective weight, which can ensure that the result not only relies on subjective experience of experts but also reflects the objective information of data [25, 26]. Ono used the Propensity Score Match method to weight an indicator and established a credit scoring model for Japan's small businesses [27]. Huang used the second order of the least square method and the GMM-SYS method to weight an indicator, thereby examining the relationship between trade credit and bank credit [28].

In fact, the combined weight was not reasonable, because the combined weight was a combination of different weighting methods, especially combining the objective weight and subjective weight, which will combine a good method and a bad method and lead to the final result being not better. On the issue of credit risk evaluation, the combined weight may combine the weighting method with large default identification ability and less default identification ability and lead to the result lacking default identification ability.

In the research of credit risk evaluation, the weighting method often randomly chooses subjectively and there was not a standard. This study has done the following two tasks: one is calculating the indicator weight based on the default identification ability and the other is determining the optimal weighting method in the five different weighting methods according to the maximum default identification ability.

3. Weighting Methods

3.1. Standardization of Rating Indicator Data. The standardization of indicator data aims to transform the original indicator data into standardized values between 0 and 1, in order to eliminate the impact of indicators dimension. There are four types of indicators named positive indicator, negative indicator, interval indicator, and qualitative indicator. The standardization process is as follows.

Let x_{ij} be the standardized value of the j^{th} customer in the i^{th} indicator, v_{ij} be the original value of the j^{th} customer in the i^{th} indicator, n be the number of customers, q_1 be lower boundary of the optimal region, and q_2 be the upper boundary. The positive indicator, negative indicator, and the interval indicator can be expressed as follows:

$$x_{ij} = \frac{v_{ij} - \max_{1\le i\le n}\left(v_{ij}\right)}{\max_{1\le i\le n}\left(v_{ij}\right) - \max_{1\le i\le n}\left(v_{ij}\right)}, \tag{1}$$

$$x_{ij} = \frac{\max_{1\le i\le n}\left(v_{ij}\right) - v_{ij}}{\max_{1\le i\le n}\left(v_{ij}\right) - \max_{1\le i\le n}\left(v_{ij}\right)}, \tag{2}$$

$$x_{ij} = \begin{cases} 1 - \dfrac{q_1 - v_{ij}}{\max\left(q_1 - \min_{1\le i\le n}\left(v_{ij}\right), \max_{1\le i\le n}\left(v_{ij}\right) - q_2\right)}, & v_{ij} < q_1\,(3-a), \\ 1 - \dfrac{v_{ij} - q_2}{\max q_1 - \min_{1\le i\le n}\left(v_{ij}\right), \max_{1\le i\le n}\left(v_{ij}\right) - q_2}, & v_{ij} > q_2\,(3-b), \\ 1, & q_1 \le v_{ij} \le q_2\,(3-c). \end{cases} \tag{3}$$

The standardization of qualitative indicators is through expert interview, survey, etc. It is given in Table 1.

3.2. Subjective Weighting Method Based on the G1 Method. The subjective weight of evaluation indicator can be obtained based on the experts' experience. The G1 method reflects the importance of indicators by the order that experts gave. If the order is given, the relative importance of any two adjacent indicators can be obtained, and this is the parameter used to calculate the weight. The steps of calculating the weight is as follows:

Step 1: determine the importance order of indicators by experts. The most important indicator is in the first place, and the least important indicator is in the last place.

Step 2: determine the value of the ratio r_i between two adjacent indices x_{i-1} and x_i, and the values of the ratio are shown in Table 2.

Step 3: calculate the weight of the last indicator ω_n^1. The superscript "1" denotes the first weighting method. The formula is

$$w_n^1 = \left(1 + \sum_{k=2}^{n}\prod_{i=k}^{n} r_i\right)^{-1}. \tag{4}$$

Step 4: calculate the weight of the other indicator ω_i^1. On the basis of formula (4), the other indicators' weight is calculated

$$w_{i-1}^1 = r_i w_i^1, \quad i = n, n-1, \ldots, 2. \tag{5}$$

Through formulas (4) and (5), we can get the weight of every indicator. The results satisfied that the higher the ranking, the more important the indicator and the larger the weight is.

3.3. Objective Weighting Method Based on Information Content. There are entropy weight method, the mean square deviation method, and other methods that can measure the information content. The more discrete the data of the indicator, the more information the indicator reflects, and the greater the weight, so we can ensure that the more the information content of the important indicators, the greater the weight.

3.3.1. Entropy Weight Method. Let x_{ij} be the standardized value of the j^{th} customer in the i^{th} indicator, $\overline{x}_i$ be the average of the i^{th} indicator, s_i be the standard deviation of the i^{th} indicator, n be the number of customers, m be the number of indicators, e_i be the entropy of the i^{th} indicator, ω_i^2 be the weight of the i^{th} indicator, and the superscript "2" be the second weighting method. The formula is as follows:

$$e_i = -\frac{1}{\ln(m)} \times \sum_{j=1}^{m}\left(\frac{x_{ij}}{\sum_{j=1}^{m} x_{ij}} \times \ln\left(\frac{x_{ij}}{\sum_{j=1}^{m} x_{ij}}\right)\right), \tag{6}$$

$$\omega_i^2 = \frac{(1-e_i)}{\left(n - \sum_{i=1}^{n} e_i\right)}. \tag{7}$$

The value of entropy e_i denotes the information content, $(1-e_i)$ denotes the difference coefficient, and the larger the difference coefficient, the larger the information content of the i^{th} indicator, so the larger the weight.

3.3.2. Mean Square Deviation Method. Let x_{ij} be the standardized value of the j^{th} customer in the i^{th} indicator, n be the number of customers, m be the number of indicators, s_i be the mean square deviation of the i^{th} indicator, ω_i^3 be the weight of the i^{th} indicator, and the superscript "3" denote the third weighting method. The formula is as follows:

$$s_i = \sqrt{\frac{\sum_{j=1}^{m}\left(x_{ij} - (1/m)\sum_{j=1}^{m} x_{ij}\right)^2}{m}}, \tag{8}$$

$$\omega_i^3 = \frac{s_i}{\sum_{i=1}^{n} s_i}. \tag{9}$$

The value of mean square deviation s_i reflects the discrete data; the more discrete the data, the more the information content of the indicator and the larger the weight.

3.4. Objective Weighting Method Based on Default Identification Ability. A reasonable credit risk evaluation system must have strong default identification ability, which can effectively distinguish between default customers and nondefault customers. Determining the weight of the credit evaluation index reasonably is the key to determining the quality of the credit evaluation system. If the choice of weighting method is not appropriate, it will directly affect the evaluation result, which means the poor credit enterprises will be evaluated as good businesses, and this will mislead the decision-making for financial institutions.

Table 1: The standard used for standardization of qualitative indicators.

(1) Layer	(2) Indicator	(3) Standardized description	(4)Standardized data
Basic situation of legal person	Education background	(1) Bachelor's degree or above	1.00
		(2) Associate degree	0.90
		(3) High school or technical secondary school degree	0.70
		(4) Junior middle school and primary school education	0.40
		(5) Others or lack of data	0.00
...	...	...	...
Business reputation	Enterprise tax record	(1) Tax history more than 3 years and no tax arrears records	1
		(2) Tax history less than 3 years and no tax arrears records	0.75
		(3) One-time tax arrears record and paid in full later	0.5
		(4) No tax record	0.25
	...	...	...

Note: the table shows the scores of qualitative indicators determined by experts, ranging from 0 to 1. It provides a basis for subsequent quantitative weight calculation.

Table 2: Value of ratio r_i.

(1) r_i value	(2) comparison of two adjacent indicators
1.0	Indicator x_{i-1} has the same importance as x_i
1.2	Indicator x_{i-1} has slightly more importance than x_i
1.4	Indicator x_{i-1} has obviously more importance than x_i
1.6	Indicator x_{i-1} has strongly more importance than x_i
1.8	Indicator x_{i-1} has extremely more importance than x_i
1.1, 1.3, 1.5, 1.7	Corresponding to the above intermediate cases of two adjacent judgments

Note: the table shows the ratio of the importance of the two indicators, according to the experts.

Therefore, this paper puts forward the idea of assigning weights to indicators according to the standard of default identification ability. Indicators with stronger capability of distinguishing default state should be given greater weight. We construct the statistics related to the default state, such as the F-statistics and Wilks' Lambda χ^2-statistics, which can identify the default identification capability. We can also identify the default identification capability through the judgment of default, like the ROC curve and gene coefficient.

3.4.1. Objective Weighting Method Based on Wilks' Lambda Method. The steps to calculate the weight based on Wilks' Lambda method:

Step 1: evaluate the sum of squares within group SS_{wi} for the i^{th} indicator.

According to the customer's actual default state, the i^{th} indicator is divided into two groups, the default group (denoted as 1) and the nondefault group (denoted as 0). Let m be the number of customers, m_0 be the number of nondefault customers, m_1 be the number of default customers, $x_{ij}^{(0)}$ be the standardized value of the j^{th} nondefault customer in the i^{th} indicator, $\overline{x}_i^{(0)}$ be the average of nondefault customers in the i^{th} indicator, $x_{ij}^{(1)}$ be the standardized value of the j^{th} default customer in the i^{th} indicator, $\overline{x}_i^{(1)}$ be the average of default customers in the i^{th} indicator, $\overline{x}_i$ be the average of the i^{th} indicator, and n be the number of customers. The sum of squares within group SS_{wi} for the i^{th} indicator is

$$SS_{\mathrm{wi}} = \sum_{j=1}^{m_0} \left(x_{ij}^{(0)} - \overline{x}_i^{(0)} \right)^2 + \sum_{j=1}^{m_1} \left(x_{ij}^{(1)} - \overline{x}_i^{(1)} \right)^2. \quad (10)$$

Equation (10) refers to the sum of nondefault customer values deviating from the average value and the default customer values deviating from their mean value for the i^{th} indicator. The smaller the sum of squares within group SS_{wi}, the less the value differences between default and nondefault customers.

Step 2: evaluate the sum of squares between groups SS_{bi} for the i^{th} indicator.

The sum of squares between groups SS_{bi} for the i^{th} indicator is

$$SS_{\mathrm{bi}} = m_0 \left(\overline{x}_i^{(0)} - \overline{x}_i \right)^2 + m_1 \left(\overline{x}_i^{(1)} - \overline{x}_i \right)^2, \quad (11)$$

Equation (11) refers to the default and the nondefault customer average values deviating from the mean of all customers for the i^{th} indicator. The larger of the sum of squares between groups SS_{bi}, the larger the value differences between default and nondefault customers.

Step 3: evaluate the eigenvalue γ_i for the i^{th} indicator.

Take the maximum value of discriminant criterion named eigenvalue γ_i in discriminant analysis into the indicator weighting. That is,

$$\gamma_i = \frac{SS_{\mathrm{bi}}}{SS_{\mathrm{wi}}}. \quad (12)$$

Step 4: evaluate Wilks' Lambda value Λ_i for the i^{th} indicator:

$$\Lambda_i = \frac{1}{1+\gamma_i}. \quad (13)$$

Step 5: evaluate the statistics χ_i^2 for the i^{th} indicator.

Let m be the number of customers and G be the number of groups; in this study, there are two groups named default group and nondefault group, so $G=2$, and let J be the number of variables, because we calculate the statistics of one indicator each time, so $J=1$. The formula of χ_i^2 statistics is

$$\chi_i^2 = -\left(m - \frac{J+G}{2} - 1\right)\ln\,\Lambda_i, \quad (14)$$

The meaning of equations (12) to (14): for the i^{th} indicator, the smaller the sum of squares within group SS_{wi}, the less the value differences between default and nondefault groups and the larger the sum of squares between groups SS_{bi}, the larger the value differences between default and nondefault groups. Thus, the eigenvalue γ_i is larger and Wilks' Lambda value Λ_i is also larger which means the stronger the indicator ability to distinguish the default situation.

Step 6: evaluate the weight ω_i^4 of the i^{th} indicator.

Normalization processing is used for the value of χ_i^2 statistic which is calculated by formula (14), the weight of the i^{th} indicator ω_i^4 is obtained, and the superscript "4" denotes the fourth weighting method named Wilks' Lambda method. The formula is as follows:

$$\omega_i^4 = \frac{\chi_i^2}{\sum_{i=1}^{m}\chi_i^2}, \quad (15)$$

Meaning of formula (15): the larger the χ_i^2 statistic, the stronger the indicator ability to distinguish the default situation and the larger the weight of the i^{th} indicator.

The meaning of the weighting method based on Wilks' Lambda method: for the i^{th} indicator, the smaller the sum of squares within group SS_{wi}, the less the value differences between default and nondefault groups and the larger the sum of squares between groupw SS_{bi}, the larger the value differences between default and nondefault groups and the larger the χ_i^2 statistic, which means the stronger the indicator ability to distinguish the default situation. Also, the weight of the i^{th} indicator is larger, and the method makes the weight reflect the ability of identifying default state, which makes up the disadvantage that the existing indicator system had nothing to do with the ability to identify the default situation.

3.4.2. Objective Weighting Method Based on ROC Curve. Calculating the AUC value reflects the default identification ability through the ROC curve. When the AUC value is greater, the indicator can distinguish the default customers from nondefault customers and the default identification accuracy is higher. This means the indicator has stronger default identification ability, so the indicator weights should be greater.

The steps to calculate the weight based on the ROC curve method:

Step 1: building the logistic regression equation

Let $P(y=1)$ denote the default probability of the j^{th} customer; z_j denote the Latent variables; x_{ij} denote the standardization score of the i^{th} indicator and the j^{th} customer; n denote the number of customers; m denote the number of indices; α denote the constant; β_i denote the regression coefficient of the i^{th} indicator; and ε denote the random error term. The logistic regression model is

$$P(y=1) = \frac{1}{1+e^{-z_j}}, \quad (16)$$

$$z_j = \alpha + \sum_{i=1}^{m}\beta_i x_{ij} + \varepsilon, \quad (17)$$

The regression coefficient β and its standard error SE_β can be obtained by using maximum likelihood estimation in equation (16), and this process can be realized by SPSS software.

Step 2: the prediction of the default probability

Taking the data of customers into formulas (16) and (17), the default probability $P(y=1)$ can be predicted.

Step 3: the classification of model identification results

From the calculated default probability $P(y=1)$ with the real default state of customers, if the default probability is $P(y=1)\geq 0.5$, the customers are discriminated default; else $P(y=1)<0.5$, the customers are not default.

The classification result by comparing predicted and real default state is shown in Table 3.

Step 4: the construction of the ROC curve

According to the classification results in Table 3, the two variables are defined, which are the horizontal and vertical coordinates of the ROC curve.

$$\text{TPR} = \frac{\text{TP}}{(\text{TP}+\text{FN})}. \quad (18)$$

$$\text{FPR} = \frac{\text{FP}}{(\text{FP}+\text{TN})}, \quad (19)$$

Vertical coordinate: also known as the true positive rate (TPR), it is the ratio of predict the correct default sample TP accounted for the total sample (TP + FN), with the formula expressed as

Horizontal coordinate: also known as the false positive rate (FPR), it is the ratio of wrongly predicted sample TP that nondefault customers are predicted default, accounted for the total sample (TP + FN), with the formula expressed as

Table 3: Classified result of the predicted model.

Actual default state	Predicted default state 1 (default)	0 (nondefault)
1 (default)	The number of actual default customers judged to be correct true positive (TP)	The number of actual default customers judged to be wrong false negative (FN)
0 (nondefault)	The number of actual nondefault customers judged to be wrong false positive (FP)	The number of actual nondefault customers judged to be correct true negative (TN)

Step 5: the calculation of AUC value

Computing the area under the ROC curve, the value is AUC which belongs to 0-1. The greater the AUC value of the indicator, the stronger the ability of default identification of the indicator.. If AUC = 1, it means the predicted results are entirely consistent with actual state and this is the most ideal situation.

Step 6: evaluate the weight ω_i^5 of the i^{th} indicator.

Normalization processing is used for the value of AUC, the weight of the i^{th} indicator ω_i^5 is obtained, and the superscript "5" denotes the fifth weighting method named ROC curve.

$$\omega_i^5 = \frac{\mathrm{AUC}_i}{\sum_{i=1}^m \mathrm{AUC}_i}. \tag{20}$$

The meaning of the weighting method based on ROC curve: for the i^{th} indicator, the ROC curve can be constructed according to the number of default customers judged correctly TP accounted for the proportion of all default customers (TP + TN) and the number of nondefault customers judged correctly TN accounted for the proportion of all nondefault customers (FN + TN). The larger the area under the ROC curve, the stronger the default identification ability is and the larger the weight of the indicator is, which makes the weight reflect the ability of identifying default state and makes up the disadvantage that the existing indicator system had nothing to do with the ability to identify the default situation.

4. Selection of the Optimal Weighting Model

How to confirm the optimal weighting method for credit risk evaluation among many weighting methods? The standard to select the optimal weighting method is that the credit score has default identification ability. In other words, the credit scores of nondefaulting customers are relatively high, and the credit scores of defaulting customers are relatively low.

(1) Calculate the credit evaluation score z_j

We get the weight of indicator ω^t in Section 2, where the superscript "t" denotes the t^{th} weighting method. For the weight and the standard value of indicators, we can get the customers' credit score by the linear weighted method.

Let z_j denote the credit score of the j^{th} customer; x_{ij} denote the standardization score of the i^{th} indicator and the j^{th} customer; n denote the number of customers; and m denote the number of indices; so the evaluation function is

$$z_j = \sum_{i=1}^m w_i^t x_{ij}, \tag{21}$$

Formula (21) considers that there is a linear relationship between the indicators and credit score. Some nonlinear evaluation model has the similar result when choosing the weighting method. For example, Logit model, Tobit model, Probit model, etc. are nonlinear models and their ranking of small business credit scores is the same as formula (21). Therefore, this paper chooses the linear model to find the optimal weighting method. Wilks' Lambda weighting method is still the best, and the evaluation result is still the best under the condition of the nonlinear evaluation model.

(2) Determine positive and negative ideal points

The positive ideal point means the best evaluation result is hypothetical; in the credit risk evaluation, it means the nondefault customers all have the best value and the default customers all have the worst value. Conversely, the negative ideal point means the worst evaluation result; the nondefault customers have the worst evaluation result and the default customers have the best evaluation results.

For the linear weighted evaluation, the sum of the indicators' weight is always equal to one and the customers' data belong to the interval zero to one by standard processing, so the credit score belongs to the interval zero to one. The evaluation score vector $\mathbf{Z}$ for n customers meets

$$\mathbf{Z} = \left\{ z_1^{(0)}, \ldots, z_0^{(0)}, z_1^{(1)}, \ldots, z_{n1}^{(1)}, \ldots \right\}. \tag{22}$$

As shown above, n_0 denotes the number of nondefault customers; n_1 denotes the number of default customers; and superscript "(0)" denotes the nondefault customers and "(1)" denotes default customers.

So the positive ideal point $\mathbf{Z}^+$ and the negative ideal point $\mathbf{Z}^-$ satisfied

$$Z^{+} = \left\{z_j^{+}\right\} = \{1, \ldots, 1, 0, \ldots, 0\},$$
$$Z^{-} = \left\{z_j^{-}\right\} = \{0, \ldots, 0, 1, \ldots, 1\}, \tag{23}$$

$\mathbf{Z}^{+}$ and $\mathbf{Z}^{-}$ have the same structure as $\mathbf{Z}$.

(3) Calculated the Euclidean distance

Let D^{+} denote the distance between credit score and positive ideal point, D^{-} denote the distance between credit score and negative ideal point, z_j denote the credit score of the j^{th} customer, z_j^{+} denote the positive ideal value of the j^{th} customer, z_j^{-} denote the negative ideal value of the j^{th} customer, and n denote the number of customers. Then,

$$D^{+} = \sqrt{\sum_{j=1}^{n}\left(z_j - z_j^{+}\right)^2}, \tag{24}$$

$$D^{-} = \sqrt{\sum_{j=1}^{n}\left(z_j - z_j^{-}\right)^2}, \tag{25}$$

Formula (24) represents the close relationship between the evaluation value of customers and the positive ideal value. Formula (25) represents the close relationship between the evaluation value of customers and the negative ideal value.

(4) Calculate the neartude C_t

As shown above, D^{+} denotes the distance between credit score and positive ideal point and D^{-} denotes the distance between credit score and negative ideal point, so the formula of neartude based on the t^{th} weighting method is

$$C_t = \frac{D^{-}}{D^{-} + D^{+}}, \tag{26}$$

The value of neartude C_t satisfied $0 \le C_t \le 1$. If the condition $z_j = z_j^{+}$, the evaluation value is equal to the positive ideal value which means the default customers' credit score is the worse value 0 and the nondefault customers' credit score is the best value 1, so the neartude $C_t = 1$. similarly, if $z_j = z_j^{-}$, the evaluation value id equal to the negative ideal value which means the default customers' credit score is the best value 1 and the nondefault customers' credit score is the worst value 0, so the neartude $C_t = 0$.

The larger the value of neartude C_t, the closer the final credit score is to the positive ideal value 1 and the farther away the score is from the negative ideal value 0; this means the larger the value of neartude, the more distinguished the evaluation result of the default and nondefault customers.

(5) Select the optimal weighting method

According to the analysis of formula (26), we know that the larger the neartude C_t, the more distinguished the evaluation result of the default and nondefault customers. This means the credit score has greater default identification ability, because the evaluation score is a function of weight and the corresponding weighting method is optimal.

In short, the greater the neartude value, the better the weighting method.

The meaning of the selection optimal weighting method: through the distance of nondefault customers' scores to positive ideal point and default customers' scores to negative ideal point, we construct the neartude which reflected the default identification ability; the greater the neartude value, the easier for the weighting method to distinguish between the default and nondefault customers, so we can select the optimal method among different weighting methods; by this way, it overcomes the existing research's disadvantages that nondefault and default customers' scores had a large number of overlaps; this can also avoid the deficiency of random selection of weighting methods without considering the purpose of evaluation.

5. Empirical Study

5.1. Credit Risk Evaluation Indicator System and the Indicator Data

5.1.1. Credit Risk Evaluation Indicator System. We get the credit risk evaluation indicator system by the logistic regression model which includes sixteen indicators. Because the establishment of the indicator system is not the main content of this paper, our research is how to choose an optimal weighting method for credit risk evaluation. So, we just use the indicator system directly. The indicator system including sixteen indicators is shown in Table 4. The explanation of 16 indicators is in [6]. So, there is no further explanation in this paper.

The indicator system is shown in column (a) in Table 4.

5.1.2. Data Obtained. There are two types of data in this paper. First is using the data of 3045 small enterprise loans in 28 cities of a regional commercial bank in China in recent 20 years, in which there are 2995 nondefault small enterprises and 50 default small enterprises. Second, we asked 43 experts from one regional commercial bank's head office to rank the indicators based on their significance.

The data can be obtained by 16 indicators in the order of $X_1, X_2, \ldots, X_{16}$ annotation, showed in column a in Table 4. Table 4 is constituted by 2 parts: the first part is the original data showed in columns 1–3045, recorded for matrix (v_{ij}); the second part is the standardization data showed in columns 3046–6090, recorded for matrix (x_{ij}). The process of standardization is shown in (3).

The ranking data of 43 experts on the indicators are shown in rows 1–16 in Table 5. We will convert the rankings to values in Section 5.2.

Table 4: The original data and standardized data of 16 indicators of small enterprises.

(a) indicator	The 3045 original data v_{ij}			The 3045 standardized data x_{ij}					
				50 default customers			2995 nondefault customers		
	(1) customer 1	. . .	(3045) customer 3045	(3046) customer 1	. . .	(3095) customer 50	(3096) customer 51	. . .	(6090) customer 3045
X_1 net cash flow ratio from current liabilities operating activities	−0.054	. . .	0.136	0.472	. . .	0.461	0.000	. . .	0.496
X_2 super-quick ratio	0.23	. . .	0.28	0.041		0.037	0	. . .	0.050
X_3 total outstanding loans to total assets ratio	0	. . .	0	1		1	0	. . .	1
X_4 net cash flow from operating activities (yuan)	−2206060	. . .	45459230	0.487		0.496	0.859	. . .	0.734
X_5 working capital allocation ratio	−0.22	. . .	0.09	0		1	0	. . .	0.061
X_6 retained earnings growth rate	0.888	. . .	0.888	0.519	. . .	0.501	0.503	. . .	0.513
X_7 consumer price indicator	101.4	. . .	105.4	1	. . .	0.951	1	. . .	0.976
X_8 controlled income of each urban resident (yuan)	9101	. . .	26921	0.182		0.452	0.182	. . .	0.719
X_9 Engel coefficient	39.400	. . .	36.200	0.651	. . .	0.838	0.651	. . .	0.790
X_{10} working time in relevant industry	0	. . .	10 years	0	. . .	0.400	1.000	. . .	1.000
X_{11} account opening status	General settlement account	. . .	General settlement account	0.5		0	0	. . .	0.5
X_{12} product sales range	Export	. . .	Domestic sales	1		0	0	. . .	0.5
X_{13} dwelling condition	Home ownership	. . .	Home ownership	1		1	0	. . .	1
X_{14} working time holding the position	None	. . .	4	0	. . .	0.400	1.000	. . .	0.400
X_{15} enterprise credit in 3 years	Have default record and unclear	. . .	Have credit record and no default	0	. . .	0	0	. . .	1.000
X_{16} score of pledged collateral	Land use right of industrial land	. . .	Other corporate guarantees	0.669	. . .	0.649	0.100	. . .	0.570
Default or nondefault	1	. . .	0	1	. . .	1	0	. . .	0

Table 5: The ranking of the experts for 16 indicators and numerical transformation.

(a) No.	(b) Experts	Importance ranking of indicators a_{ij}			Converting the rankings to values b_{ij}		
		(1) X_1 net cash flow ratio from current liabilities operating activities	. . .	(16) X_{16} score of pledged collateral	(17) X_1 net cash flow ratio from current liabilities operating activities	. . .	(32) X_{16} score of pledged collateral
1	WHL	4	. . .	3	0.931	. . .	0.846
2	JYK	1	. . .	11	1.000	. . .	0.813
3	WP	12	. . .	1	0.632	. . .	0.776
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
42	XJL	9	. . .	3	0.776	. . .	0.931
43	LF	5	. . .	7	0.905	. . .	0.734

5.1.3. Standardization of Indices Data. For the data matrix (v_{ij}) in rows 1–16 and columns 1–3045 in Table 4, each data set v_{ij} represents the original data of the i^{th} indicator for the j^{th} customer. Among them, we can find a maximum and a minimum value from 3045 data in each row that were $\max(v_{ij})$ and $\min(v_{ij})$ and needed in formulas (1)–(3).

The original data can be standardized, and the standardization data are shown in columns 3046–6090 in Table 4.

There is a need to point out that there was one interval-type indicator in the 16 indices, which is the consumer price indicator. The best range of consumer price indicator is [101, 105]. Taking the original data v_{ij} into formula (3), the standardized data x_{ij} can be obtained.

According to the standardized method of qualitative indices in Section 3.1, the indices value are changed into [0, 1] range.

5.2. Calculation of Five Types of Indicator Weights

5.2.1. Subjective Weight Based on the G1 Method. The importance order of indicators is determined by experts. The most important indicator "X_{10} working time in relevant industry" is in the first row in Table 6, and the least important indicator "X_8 controlled income of each urban resident (yuan)" is in the last row in Table 6.

The value of the ratio r_i between two adjacent indices x_{i-1} and x_i was determined according to the rules in Table 2 by experts, and the results of the ratio are shown in column 2 in Table 6.

The weight ω_{16}^1 of the least important indicator "X_8 controlled income of each urban resident" is determined as follows: put the data r_i in rows 2–16 and column 2 in Table 6 into formula (4); the subjective weight ω_{16}^1 is $w_{16}^1 = (1 + \sum_{k=2}^{16} \prod_{i=k}^{16} r_i) - 1 = [1 + (1 \times 1.8 \times 1.4 \times \ldots \times 1.1 \times 1) + \ldots + (1.1 \times 1) + 1]^{-1} = 0.007$.

The result is shown in column 3 and row 16 in Table 6.

On the basis of ω_{16}^1, the weight of other indicators was calculated according to formula (5). For example, the indicator in row 15 in Table 6 "X_7 consumer price indicator" shows $\omega_{15}^1 = r_{16} \times \omega_{16}^1 = 1 \times 0.007 = 0.007$. And so on, the weight of other indicators can be reverse-calculated; the results are shown in Table 6.

5.2.2. Objective Weight Based on the Entropy Weight Method. Taking the indicator "X_1 net cash flow ratio from current liabilities operating activities," for example, and putting the standardized data in row 2 and columns 3046–3090 in Table 4 into formula (6), we get the entropy value of indicator X_1: $e_1 = 0.996$; the result is shown in column 2 in Table 7.

Similarly, we can calculate the entropy of other indicators; the results are shown in column 2 in Table 7.

Putting the entropy values in column 2 in Table 7 into formula (7), the weights of indicators were obtained which are shown in column 3 in Table 7.

5.2.3. Objective Weight Based on the Mean Square Deviation Method. Taking the indicator "X_1 net cash flow ratio from current liabilities operating activities," for example, and putting the standardized data in row 2 and columns 3046–3090 in Table 4 into formula (8), we get the mean square deviation value of indicator X_1: $s_1 = 0.104$; the result is shown in column 4 in Table 7.

Similarly, we can calculate the mean square deviation value of other indicators; the results are shown in column 4 in Table 7.

Putting the mean square deviation values in column 4 in Table 7 into formula (9), the weights of indicators were obtained which are shown in column 5 in Table 7.

The first two weighting methods are based on information content, and the latter two weighting methods are based on default identification ability.

5.2.4. Objective Weight Based on Wilks' Lambda Method. Taking the indicator "X_1 net cash flow ratio from current liabilities operating activities," for example, and putting the standardized data in row 2 and columns 3046–3090 in Table 4 into formulas (10)–(14), we get the χ_1^2 statistics value of indicator X_1: $\chi_1^2 = 12.712$; the result is shown in column 6 in Table 7.

Similarly, we can calculate the χ^2 statistics value of other indicators; the results are shown in column 6 in Table 7.

Putting the χ^2 statistics values in column 6 in Table 7 into formula (15), the weights of indicators were obatined which are shown in column 7 in Table 7.

5.2.5. Objective Weight Based on the ROC Curve Method. Taking the indicator "X_1 net cash flow ratio from current liabilities operating activities," for example, and putting the standardized data in row 2 and columns 3046–3090 in Table 4 into formulas (16) and (17), we get the logistic regression model of indicator X_1 and then we can calculate the default probability $P_j(y = 1)$ of the j^{th} customer. Compare the calculated default probability $P(y = 1)$ with the size of 0.5 $P(y = 1) < 0.5$, means that customers are defaulted, and vice versa.

According to the classification result in Table 3, we can obtain the ROC curve based on formulas (18) and (19). Computing the area under the ROC curve, and the value is $\text{AUC}_1 = 0.608$, and the result is shown in column 8 in Table 7. Similarly, we can calculate the AUC value of other indicators; the results are shown in column 8 in Table 7. Putting the AUC values in column 8 in Table 7 into formula (20), the weights of indicators were obtained which are shown in column 9 in Table 7.

In order to show the difference among the 5 types of weighting methods, the weights of indicators are drawn in Figure 1.

5.3. Selection of the Optimal Weighting Method. In the five types of weighting methods, through calculating the neartude to select an optimal weighting method, we can get a series of indicator weights.

Taking the G1 weighting method for example and putting the G1 weight ω_i^1 in column 3 in Table 6 into formula (21), we can obtain the credit score of every customer; the evaluation results are represented by vectors:

$$Z^1 = (0.348, 0.709, \ldots, 0.559). \tag{27}$$

Putting the results Z^1, the positive ideal point $Z^+ = \{z_j^+\} = \{1, \ldots, 1, 0, \ldots, 0\}$, and the number of customers $m = 3045$ into formula (24), the distance between credit score and positive ideal point is obtained as $D^+ = 21.841$. Similarly, we can get the distance between evaluation result Z^1 and the negative ideal point $Z^- = \{z_j^-\} = \{0, \ldots, 0, 1, \ldots, 1\}$ satisfying $D^- = 35.221$.

Putting the two distances into formula (26), the neartude of the G1 weighting method was calculated:

Table 6: Weights based on the G1 method.

(1) Indicator	(2) r_i value	(3) Weight ω_i^1
X_{10} working time in relevant industry	—	0.212
X_{16} score of pledged collateral	1	0.212
X_{15} enterprise credit in 3 years	1.8	0.118
X_{12} product sales range	1.4	0.084
X_1 net cash flow ratio from current liabilities operating activities	1.2	0.070
X_{14} working time holding the position	1	0.070
X_{13} dwelling condition	1.2	0.058
X_{11} account opening status	1.4	0.042
X_4 net cash flow from operating activities (yuan)	1.2	0.035
X_3 total outstanding loans to total assets ratio	1.2	0.029
X_2 super-quick ratio	1.4	0.021
X_5 working capital allocation ratio	1.4	0.015
X_6 retained earnings growth rate	1.2	0.012
X_9 Engel coefficient	1.6	0.008
X_7 consumer price indicator	1.1	0.007
X_8 controlled income of each urban resident (yuan)	1	0.007

Note: r_i values were determined according to the rules in Table 2 by experts.

Table 7: The 4 types of objective weighting results.

(1) Indicator	Entropy weight method		Mean square deviation method		Wilks' Lambda method		ROC curve	
	(2) Entropy value e_i	(3) Weight ω_i^2	(4) Mean square deviation s_i	(5) Weight ω_i^3	(6) χ^2 statistics value	(7) Weight ω_i^4	(8) AUC value	(9) Weight ω_i^5
X_1 net cash flow ratio from current liabilities operating activities	0.996	0.010	0.104	0.030	12.712	0.017	0.608	0.056
X_2 super-quick ratio	0.953	0.134	0.170	0.049	18.357	0.025	0.783	0.073
X_3 total outstanding loans to total assets ratio	0.998	0.005	0.114	0.033	17.268	0.024	0.514	0.048
X_4 net cash flow from operating activities (yuan)	0.998	0.007	0.085	0.025	25.302	0.035	0.578	0.054
X_5 working capital allocation ratio	0.963	0.103	0.178	0.051	4.155	0.006	0.599	0.056
X_6 retained earnings growth rate	0.993	0.019	0.119	0.034	5.041	0.007	0.566	0.052
X_7 consumer price indicator	0.981	0.054	0.044	0.013	172.99	0.237	0.713	0.066
X_8 controlled income of each urban resident (yuan)	0.974	0.074	0.131	0.038	107.26	0.147	0.573	0.053
X_9 Engel coefficient	0.967	0.092	0.070	0.020	136.39	0.187	0.702	0.065
X_{10} working time in relevant industry	0.944	0.157	0.344	0.099	41.982	0.057	0.652	0.060
X_{11} account opening status	0.931	0.194	0.304	0.087	4.076	0.006	0.711	0.066
X_{12} product sales range	0.980	0.056	0.287	0.082	33.274	0.046	0.765	0.071
X_{13} dwelling condition	1.000	0.001	0.479	0.137	19.604	0.027	0.657	0.061
X_{14} working time holding the position	0.995	0.014	0.373	0.107	27.918	0.038	0.835	0.077
X_{15} enterprise credit in 3 years	0.999	0.002	0.351	0.101	79.264	0.108	0.823	0.076
X_{16} score of pledged collateral	0.972	0.080	0.330	0.095	25.060	0.034	0.708	0.066

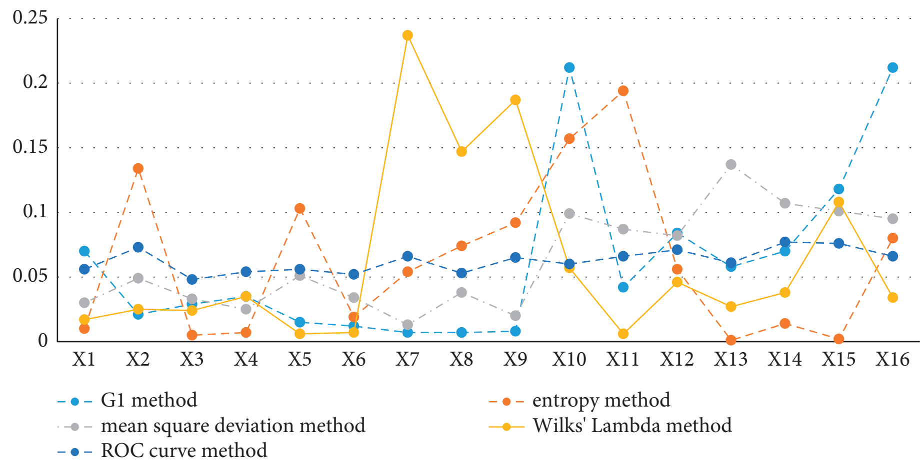

Figure 1: The weight of five types of weighting methods.

$$C_1 = \frac{35.221}{(35.221 + 21.841)} = 0.617. \tag{28}$$

Similarly, we can calculate the neartude of other four weighting methods, as follows: entropy weighting method: $C_2 = 0.486$. Mean square deviation weighting method: $C_3 = 0.576$. Wilks' Lambda weighting method: $C_4 = 0.703$. ROC curve weighting method: $C_5 = 0.580$.

The neartude of five types of weighting methods is shown in Figure 2.

From the abovementioned part, we know the greater the neartude C, the more distinguish the evaluation result of the default and nondefault customers and the better the corresponding weighting method.

In the five types of weighting methods, the neartude of Wilks' Lambda weighting method is highest at $C_4 = 0.703$ and so this weighting method is more suitable for credit risk evaluation.

5.4. Analysis of the Wilks' Lambda Weight of Credit Evaluation Indicators. The optimal weighting result based on Wilks' Lambda method is shown in Table 8. Adding the weight of the financial indicators in rows 1–6 in Table 8, the sum is 0.113, and adding the weight of nonfinancial indicators in rows 7–16 in Table 8, the sum is 0.887. So, we can get the conclusion that the nonfinancial indicators are more important than financial indicators in the area of credit risk evaluation for small business.

Adding the weights of the macroenvironment indicators in rows 7–9 in Table 8, the sum is 0.571. This shows that the macroeconomic factors are especially important in the credit risk evaluation of small business.

For small businesses, this result is obvious. Small businesses are more vulnerable to changes in external macroconditions because of their high risk, small amount, etc.

5.5. Credit Scoring Model. We get the optimal weighting method in Section 5.4. For the weight and the standard value of indicators, we can get the customers' credit score by the linear weighted method.

Let z_j denote the credit score of the j^{th} customer; the credit score is given as

$$\begin{aligned} z_j &= 0.017X_1 + 0.025X_2 + 0.024X_3 + 0.035X_4 + 0.006X_5 \\ &\quad + 0.007X_6 + 0.237X_7 + 0.147X_8 + 0.187X_9 + 0.057X_{10} \\ &\quad + 0.006X_{11} + 0.046X_{12} + 0.027X_{13} + 0.038X_{14} \\ &\quad + 0.108X_{15} + 0.034X_{16} \\ &= 0.017. \end{aligned} \tag{29}$$

That is, 0.017 net cash flow ratio from current liabilities operating activities +0.025 super-quick ratio +0.024 total outstanding loans to total assets ratio +0.035 net cash flow from operating activities +0.006 working capital allocation ratio +0.007 retained earnings growth rate +0.237 consumer price indicator +0.147 controlled income of each urban resident +0.187 Engel coefficient +0.057 working time in relevant industry +0.006 account opening status +0.046 product sales range +0.027 dwelling condition +0.038 working time holding the position +0.108 enterprise credit in 3 years +0.034 score of pledged collateral.

6. Conclusions

Among most of weighting methods, the choice of an appropriate weighting method is the key for credit risk evaluation. If the choice of weighting method is not appropriate, it will directly affect the evaluation result, which means the poor credit enterprises will be evaluated as good businesses and this will mislead the decision-making for financial institutions. The weight can also reflect the importance of the indicator, that is, we can determine the key indicators that play an important role in credit risk evaluation according to the weight.

The reasonable credit risk evaluation system must have strong default identification ability, which means the evaluation results can be effectively distinguished between defaulting customers and nondefaulting customers. The existing research always used the combined weight which

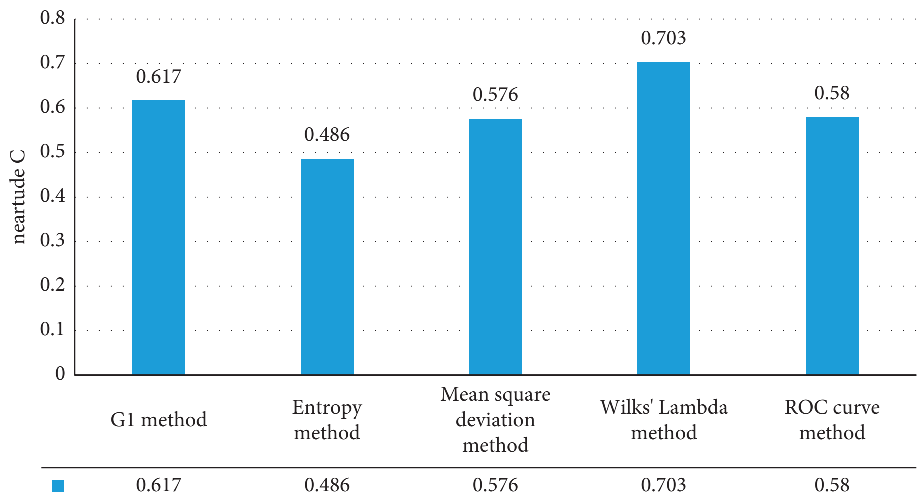

FIGURE 2: The neartude of five types of weighting methods.

TABLE 8: Optimal weighting result of Wilks' Lambda method.

Indicator	Wilks' Lambda weight	Sum of weight
X_1 net cash flow ratio from current liabilities operating activities	0.017	The weight of financial indicators is **0.113**
X_2 super-quick ratio	0.025	
X_3 total outstanding loans to total assets ratio	0.024	
X_4 net cash flow from operating activities (yuan)	0.035	
X_5 working capital allocation ratio	0.006	
X_6 retained earnings growth rate	0.007	
X_7 consumer price indicator	**0.237**	The weight of nonfinancial indicators is **0.887**
X_8 controlled income of each urban resident (yuan)	0.147	
X_9 Engel coefficient	0.187	
X_{10} working time in relevant industry	0.057	
X_{11} account opening status	0.006	
X_{12} product sales range	0.046	
X_{13} dwelling condition	0.027	
X_{14} working time holding the position	0.038	
X_{15} enterprise credit in 3 years	0.108	
X_{16} score of pledged collateral	0.034	

combines the subjective weight and objective weight. In fact, the combined weight was not reasonable, because the combined weight will combine a good method and a bad method and leads to the final result being not better.

This paper proposed the method for selecting an optimal weighting method suitable for credit risk evaluation. The contribution in theory was through the distance of nondefault customers' scores to positive ideal point and default customers' scores to negative ideal point, we construct the neartude which reflected the default identification ability; the greater the neartude value, the more distinguished the weighting method of the default and nondefault customers, and so we can select the optimal method among different weighting methods; by this way, it overcomes the existing research's disadvantages that nondefault and default customers' scores have a large number of overlaps; this can also avoid the deficiency of random selection of weighting methods without considering the purpose of evaluation.

This paper proposed two novel weighting methods "Wilks' Lambda method" and "AUC value method" according to the ability of default identification of the indicators. For comparison, three kinds of traditional subjective and objective weighting methods are listed. The subjective weight of evaluation indicator can be obtained by the G1 method, which reflects the experts' experience. The objective weights of evaluation indicator can be obtained by the entropy weight method and the mean square deviation method, which can measure the information content.

This paper also proposed how to confirm the optimal weighting method among those five weighting methods. The standard to select the optimal weighting method is the evaluation result with default identification ability which means the credit score can confirm the largest difference between default customers' scores and nondefault customers' scores.

The empirical study used loan data from 3,045 small business loans from a Chinese commercial bank and also used survey data from 43 experts from one regional commercial bank's head office. An important contribution of the paper is to discover that Wilks' Lambda method is the most effective method for small business and nonfinancial indicators such as "consumer price indicator" and "enterprise credit in 3 years" play an important role in prediction of small business default.

Our study opens up some potential future research avenues. First, increasing the amount of data or some other database in the empirical research could make the results more convincing. Second, a future study could develop more weighting methods based on default identification capability. These methods show the importance of explanatory indicators and give more reasonable evaluation results. Third, in terms of the weight assignment method based on information amount, the method is further improved, such as the method of topological entropy in [29, 30].

Data Availability

The data used to support the findings of this study have not been made available because of a contract with the commercial bank that supports the research on the confidentiality and nondisclosure of the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Youth Project of National Natural Science Foundation of China (No. 71901055), the Key Projects of National Natural Science Foundation of China (No. 71731003), and the General Projects of National Natural Science Foundation of China (No. 71873103).

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