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Research Article

Certain Concepts in Vague Graph with an Application

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Vague graphs (VGs), belonging to the FGs family, have good capabilities when faced with problems that cannot be expressed by FGs. When an element's membership is not clear, neutrality is a good option that can be well-supported by a VG. The previous definition limitations in irregular-FG have led us to offer new definitions in VGs. So, in this paper, we introduce the concepts of strongly edged irregular vague graphs (SEIVGs), strongly edged totally irregular vague graphs (SETIVGs), and perfectly regular vague graphs (PRVGs) with several examples. A comparative study between SEIVGs and SETIVGs is presented. Finally, an application of vague influence digraph has been presented.

1. Introduction

Graphs have long been used to describe objects and the relationships between them. Many of the issues and phenomena around us are associated with complexities and ambiguities that make it difficult to express certainty. These difficulties were alleviated by the introduction of fuzzy sets by Zade [1]. The FS focuses on the membership degree of an object in a particular set. The existence of a single degree for a true membership could not resolve the ambiguity on uncertain issues, so the need for a degree of membership was felt. Afterward, to overcome the existing ambiguities, Gau and Buehrer [2] introduced false membership degrees and defined a VS as the sum of degrees not greater than 1. The first definition of FGs was proposed by Kauffman [3] in 1993, from Zade's fuzzy relations [4, 5]. But Rosenfeld [6] introduced another elaborated definition, including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness, etc. Ramakrishna [7] introduced the concept of VGs and studied some of their properties. Balakrishnan et al. [8] presented k-neighborhood RGs. Akram et al. [9–11] defined the vague hypergraphs and Cayley vague graphs. Borzooei and Rashmanlou [12] investigated new concepts of regularity in VGs. Santhimaheswari et al. [13] discussed edge irregularity in FGs. Gani and Radha [14] introduced RFGs and TRFGs. Nandhini and Kamaraj [15] studied strongly irregular interval-valued fuzzy graphs. Gani and Latha [16] defined the concept of IFGs, NIFGs, and HIFGs in 2008. Radha and Kumaravel investigated the concept of edge degree, total edge degree, and edge regular fuzzy graphs and discussed the degree of an edge in some fuzzy graphs [17]. Kosari et al. [18] defined a vague graph structure with an application in medical sciences. Rashmanlou et al. [19] introduced some remarks on VGs. Rao et al. [20–22] studied dominating set, equitable dominating set, valid degree, isolated vertex, and some properties of VGs with the novel application. Kou et al. [23] investigated g-eccentric nodes and vague detour g-boundary nodes in VGs. Shi et al. [24, 25] introduced total dominating set, perfect dominating set, and global dominating set in product vague graphs. Banitalebi et al. [26] defined 2-domination in vague graphs. Ghorai et al. [27] presented regular product vague graphs and product vague line graphs.

VGs have a wide range of applications in the field of psychological sciences as well as in the identification of individuals based on oncological behaviors. With the help of VGs, the most efficient person in an organization can be identified according to the important factors that can be useful for an institution. Likewise, a VG is capable of focusing on determining the uncertainty, combined with inconsistent and indeterminate information, of any real-world problem, in which FGs may not lead to adequate results. Therefore, in this paper, the new concepts of PRVG, SEI, and SETIVGs are introduced. A comparative study between SEIVGs and SETIVGs has been conducted. Finally, an application of vague influence digraph has been presented.

2. Preliminaries

Definition 1 (see [2]). *A VS A is a pair (t_A, f_A) on set X where t_A and f_A are taken as real-valued functions which can be defined on $V \longrightarrow [0,1]$ so that $t_A(p) + f_A(p) \leq 1$, $\forall\, p \in X$.*

Definition 2 (see [7]). *A VG is defined as a pair $G = (A, B)$, where $A = (t_A, f_A)$ is a VS on V and $B = (t_B, f_B)$ is a VS on $E \subseteq V \times V$ so that for each $pq \in E$, $t_B(pq) \leq t_A(p) \wedge t_A(q)$, $f_B(pq) \geq f_B(p) \vee f_B(q)$.*

Definition 3 (see [12]). (*i*) *Let $G = (A, B)$ be a VG. The degree of a node p is $d_G(p) = (d_G^t(p), d_G^f(p))$, where $d_G^t(p) = \sum_{p \neq q} t_B(pq)$ and $d_G^f(p) = \sum_{p \neq q} f_B(pq)$, for $pq \in E$.*

(*ii*) The TD of a node p is defined as $td(p) = (td^t(p), td^f(p))$, where $td^t(p) = \sum t_B(pq) + t_A(p) = d_G^t(p) + t_A(p)$ and $td^f(p) = \sum f_B(pq) + f_A(p) = d_G^f(p) + f_A(p)$, $pq \in E$.

Definition 4 (see [28]). *Let G be a VG. The degree of an edge pq is defined as $d_G(pq) = (d_G^t(pq), d_G^f(pq))$, where $d_G^t(pq) = d_G^t(p) + d_G^t(q) - 2t_B(pq)$, $d_G^f(pq) = d_G^f(p) + d_G^f(q) - 2f_B(pq)$.*

Definition 5 (see [28]). *Let G be a VG. The TD of an edge pq is defined as $td_G(pq) = (td_G^t(pq), td_G^f(pq))$, where $td_G^t(pq) = d_G^t(p) + d_G^t(q) - t_B(pq)$, $td_G^f(pq) = d_G^f(p) + d_G^f(q) - f_B(pq)$. Note that the degree of an edge pq in the underlying edge is defined as $d_G(pq) = d_G(p) + d_G(q) - 2$.*

Definition 6 (see [19]). *A VG G is called SVG if $t_B(q_iq_j) = \min\{t_A(q_i), t_A(q_j)\}$ and $f_B(q_iq_j) = \max\{f_A(q_i), f_A(q_j)\}$, for each edge $q_iq_j \in E$. A VG G is called complete-VG if $t_B(q_iq_j) = \min\{t_A(q_i), t_A(q_j)\}$ and $f_B(q_iq_j) = \max\{f_A(q_i), f_A(q_j)\}$, $\forall\, q_iq_j \in E$.*

Definition 7 (see [12]). *A VG G is said to be: (i) (R_1, R_2)-regular if $d_G(q_i) = (R_1, R_2)$, for all $q_i \in V$ and also G is called to be RVG of degree (R_1, R_2). (ii) (R_1, R_2)-TR if $td_G(q_i) = (R_1, R_2)$, $\forall\, q_i \in V$. (iii) (R_1, R_2)-ERVG if $d_G(pq) = (R_1, R_2)$, $\forall\, pq \in E$. (iv) (R_1, R_2)- TERVG if $td_G(pq) = (R_1, R_2)$, $\forall\, pq \in E$.*

All the basic notations are shown in Table 1.

Table 1: : Some basic notations.

Notation	Meaning
FG	Fuzzy graph
VS	Vague set
VG	Vague graph
TD	Total degree
IVG	Irregular vague graph
HIFG	Highly irregular fuzzy graph
NIFG	Neighborly irregular fuzzy graph
SVG	Strong vague graph
CVG	Connected vague graph
RFG	Regular fuzzy graph
TRFG	Totally regular fuzzy graph
PR	Perfect regular
ER	Edge regular
TR	Totally regular
TER	Totally edge regular
CF	Constant function
AM	Adjacency matrix
UCG	Underlying crisp graph
ERVG	Edge regular vague graph
TIVG	Totally irregular vague graph
HIVG	Highly irregular vague graph
TERVG	Totally edge regular vague graph
PRVG,	Perfect regular vague graph
SEIVG	Strongly edge irregular vague graph
SETIVG	Strongly edge totally irregular vague graph
PERVG	Perfect edge regular vague graph
SIVG	Strongly irregular vague graph
NEIVG	Neighborly edge irregular vague graph

3. New Concepts of Regularity in Vague Graphs

In this section, PRVGs, SEIVGs, and SETIVGs are introduced and several properties of them are studied.

Definition 8. A PRVG is a VG that is both regular and TR.

Definition 9. A PERVG is a VG that is both ER and TER.

Lemma 1. *Let $G = (A, B)$ be a PRVG. Then, $t_A\colon V \longrightarrow [0,1]$, and $f_A\colon V \longrightarrow [0,1]$ are CFs.*

Proof. G is a PR, so G is both (R_1, R_2)-regular and (S_1, S_2) TR. Then, we have $td_G^t(q) = d_G^t(q) + t_A(q) = d_G^t(p) + t_A(p) = td_G^t(p)$, $td_G^f(q) = d_G^f(q) + f_A(q) = d_G^f(p) + f_A(p) = td_G^f(p)$, $\forall\, p, q \in V$. Since $d_G(q) = d_G(p) = (R_1, R_2)$ and $td_G(q) = td_G(p) = (S_1, S_2)$, we have $t_A(q) = t_A(p)$, $f_A(q) = f_A(p)$, hence, if G is PR, then (t_A, f_A) should be CF.

It is clear that this is not a sufficient condition, as any IVG with a CF (t_A, f_A) is neither TR nor PR. Therefore, we can classify PRVGs as the graphs that are regular with a CF (t_A, f_A). □

Theorem 1. *A VG G is PR if and only if it holds in the following conditions: (i) $\sum_{k \neq i} t_B(q_jq_k) = \sum_{k \neq j} t_B(q_jq_k)$, $\sum_{k \neq i} f_B(q_iq_k) = \sum_{k \neq j} f_B(q_jq_k)$, $\forall i, j \in \{1, \ldots, |q|\}$. (ii) $t_A(q_i) = t_A(q_j)$, $f_A(q_i) = f_A(q_j)$, $\forall i, j \in \{1, \ldots, |q|\}$.*

Proof. Suppose G is a PR. Thus, G is RVG. Hence, it satisfies (i). From Lemma 1 we conclude that (ii) is hold, too.

Conversely, we assume that G is a VG holds in both conditions (i) and (ii). Since (i) is the definition of regularity, G is (R_1, R_2)-regular. From (ii) we have $(t_A, f_A) = (S_1, S_2)$ is a constant function, $td_G^t(q) = R_1 + S_1$, $td_G^f(q) = R_2 + S_2$, and $\forall q \in V$. So, G is both regular and TR. Therefore, G is PR. □ □

Remark 1. Let G be a PRVG and let $(t_A, f_A) = (R, S)$, $\forall q \in V$. Then, the order of G is $O(G) = R|V|$.

Proposition 1. *Let G be a PRVG and let $d(q) = (R_1, R_2)$, $\forall q \in V$. Then the size of G is $S_t(G) = R_1|V|/2$ and $S_f(G) = R_2|V|/2$.*

Proof. Since G is PR, so $d_G^t(q) = R_1$, $d_G^f(q) = R_2$, and $\forall q \in V$. Hence, $\sum_{q\in V} d_G^t(q) = R_1|V|$, $\sum_{q\in V} d_G^f(q) = R_2|V|$. However, since $d_G^t(q) = \sum_{p\neq q} t_B(pq)$, $d_G^f(q) = \sum_{p\neq q} f_B(pq)$, we have $\sum_{q\in V} d_G^t(q) = \sum_{q\in V}\sum_{p\neq q} t_B(pq) = 2\sum_{pq\in E} t_B(pq) = 2S_t(G)$, $\sum_{q\in V} d_G^f(q) = \sum_{q\in V}\sum_{p\neq q} f_B(pq) = 2\sum_{pq\in E} f_B(pq) = 2S_f(G)$. Thus, we conclude that the size of a PRVG is $(R_1|V|/2, R_2|V|/2)$. □

Lemma 2. *Let G be a PERVG. Then, t_B and f_B are constant functions.*

Proof. Since G is PER, so G is both (R_1, R_2)-ER and (S_1, S_2)-TER. Hence we get $S_1 = td_G^t(pq) = d_G^t(pq) + t_B(pq) = R_1 + t_B(pq) = td_G^t(xy) = d_G^t(tu) + t_B(tu) = R_1 + t_B(tu)$, $S_2 = td_G^f(pq) = d_G^f(pq) + f_B(pq) = R_2 + f_B(pq) = td_G^f(tu) = d_G^f(tu) + f_B(tu) = k_2 + f_B(tu)$, hence, $t_B(pq) = t_B(tu)$ and $f_B(pq) = f_B(tu)$. Since pq and tu were arbitrary chosen edges, the proof is complete. □

Theorem 2. *A VG G is PER if and only if it holds in the following conditions:*

$$
\begin{aligned}
&(i) \sum_{z\neq p} t_B(pz) + \sum_{z\neq q} t_B(qz) - 2t_B(pq) = \sum_{z\neq t} t_B(tz) + \sum_{z\neq u} t_B(uz) - 2t_B(tu),\\
&\sum_{z\neq p} f_B(pz) + \sum_{z\neq q} f_B(qz) - 2f_B(pq) = \sum_{z\neq t} f_B(tz) + \sum_{z\neq u} f_B(uz) - 2f_B(tu),\ \forall pq, tu \in E,\\
&(ii)\ t_B(pq) = t_B(tu),\ f_B(pq) = f_B(tu),\ \forall pq, tu \in E.
\end{aligned}
\tag{1}
$$

Proof. Let G be a PER, then, the proof of parts (i) and (ii) is obvious according to definitions of PER and ERVG. Conversely, assume that G is a VG that satisfies both conditions (i) and (ii), i.e., G is (R_1, R_2)-ER and has a constant $t_B = S_1$ and $f_B = S_2$. Because of (i) is the definition of ER, (ii) shows that $td_G^t(pq) = R_1 + S_1 = td_G^t(tu)$, $td_G^f(pq) = R_2 + S_2 = td_G^f(tu)$, and $for\, pq, tu \in E$. □

Remark 2. Let G be a PERVG and $t_B(pq) = S_1$, $f_B(pq) = S_2$, and $for\, pq \in E$. Then, the size of $G$is $S(G) = (S_1|E|, S_2|E|)$.

Lemma 3. *If G is regular and $(t_B, f_B) = (c, d)$ is a CF, then, G is PER.*

Proof. Suppose G is an RVG and $(t_B, f_B) = (S_1, S_2)$ be a CF. Then, $(d_G^t(q), d_G^f(q)) = (R_1, R_2)$, $\forall q \in V$ where (R_1, R_2) is a multiple of (S_1, S_2). So, the degree of an arbitrary edge of G is

$$
d_G^t(pq) = d_G^t(p) + d_G^t(q) - 2t_B(pq) = 2R_1 - 2S_1 = 2(R_1 - S_1),
\tag{2}
$$

Hence, G is ER. Since the TD of an edge in a VG is $td_G^t(pq) = d_G^t(pq) + t_B(pq)$, $td_G^f(pq) = d_G^f(pq) + f_B(pq)$, we have that $td_G^t(pq) = 2(R_1 - S_1) + S_1 = 2R_1 - S_1$, $td_G^f(pq) = 2(R_2 - S_2) + S_2 = 2R_2 - S_2$, for all edges in G. Thus, G is TER and therefore PER.

Since PRVGs are regular, so, PRVGs with a CF (t_B, f_B) are necessarily PER. However, it is not the case in general that TRVGs with a constant function (t_B, f_B) are PER; let us just consider the following example. □

Example 1. Let G be a VG as Figure 1.

Let $(t_B, f_B) = (0.1, 0.3)$ be a CF. Then, the degree of a and b is $(0.2, 0.6)$ while the degree of c and d is $(0.3, 0.9)$, yet the TD of each node is precisely $(0.4, 0.9)$. Hence, this counterexample has a constant (t_B, f_B) and is TR but is not RVG. Now consider the edges ac and cd. We have that $d(a, c) = (0.3, 0.9) \neq (0.4, 1.2) = d(c, d)$. Thus, it is not ER. Since (t_B, f_B) is constant, clearly this counterexample cannot be TER as it is not ER.

Theorem 3. *If G is PR and complete, then G is PER.*

Proof. Since G is PR, $t_A(p) = t_A(q)$, $f_A(p) = f_A(q)$, and $\forall p, q \in V$. Also G is complete, so $t_B(pq) = t_A(p) \wedge t_A(q)$, $f_B(pq) = f_A(p) \vee f_A(q)$, and $\forall pq \in E$. Combined, these two facts yield that (t_B, f_B) is a constant function. Thus, we may apply Lemma 3 and complete the proof. □

Definition 8. The AM $A(G)$ of a VG G is an $n \times n$ matrix defined as $A(G) = [a_{ij}]$, where $a_{ij} = (t_B(q_i q_j), f_B(q_i q_j))$. We note that $t_B(q_i q_j)$ shows the strength of the relationship between q_i and q_j, and $f_B(q_i q_j)$ describes the strength of the non-relationship among q_i and q_j.

Remark 3. Let G be a PERVG with $t_B = S_1$ and $f_B = S_2$ and let G^* be the UCG of G. Then, $A(G) = SA(G^*)$.

Given this simple remark, we can immediately enumerate several spectral properties of PERVGs.

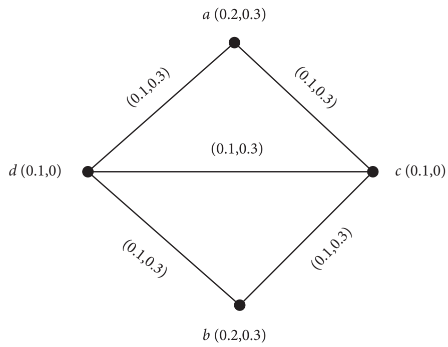

Figure 1: Vague graph G.

Theorem 4. *Let G be a PERVG and G^* be its UCG. If λ is an eigenvalue of G^*, then, $S\lambda$ is an eigenvalue of G.*

Proof. This follows directly from the fact that $AX = \lambda X$ implies $SAX = S\lambda X$. □

Definition 9. The energy of VG G is defined as

$$E(G) = \left(E\left(t_B\left(q_l q_m\right), E\left(f_B\left(q_l q_m\right)\right)\right) = \left(\sum_{l=1}^{n} |\lambda_l|, \sum_{l=1}^{n} |\mu_l|\right). \quad (3)$$

Theorem 5. *Let G be a PERVG with UCG G^*. If the energy of G^* is $E(G)$, then, $E(G) = SE(G^*)$.*

Proof. We know that $E(G^*) = \sum_{i=1}^{n} |\lambda_i|$ that the λ_i shows the eigenvalues of G^*. From Theorem 4 the energy of G is calculated as $E(G) = (\sum_{i=1}^{n} |S\lambda_i|, \sum_{i=1}^{n} |S\mu_i|) = (S\sum_{i=1}^{n} |\lambda_i|, S\sum_{i=1}^{n} |\mu_i|) = \mathrm{SE}(G^*)$. □

Definition 10. Let G be a VG. Then,

(*i*) G is called to be an IVG if there is a node that is neighbor to nodes with different degrees.

(*ii*) G is called to be a TIVG if there is a node that is neighbor to nodes with different TDs.

(*iii*) G is called to be a SIVG if every couple of nodes have a different degree.

(*iv*) G is called to be a HIVG if every node in G, which is neighbor to the nodes having different degrees.

Definition 11. Let G be a CVG. Then

(i) G is called a NEIVG if its neighbor edges have different degrees.

(ii) G is called a NETIVG if its neighbor edges have different TDs.

(iii) G is called a SEIVG if each edge has a different degree (or) no two edges have the same degree.

(iv) G is called a SETIVG if each edge has a different TD or no two edges have similar TDs.

Example 2. Let G be a VG that is a cycle with length five. With a simple calculation in Figure 2 we have

$$d_G(u) = (0.5, 1.3),\ d_G(v) = (0.3, 1.1),\ d_G(w) = (0.4, 1.3),$$
$$d_G(x) = (0.3, 1.6),\ d_G(y) = (0.5, 1.7). \quad (4)$$

Degrees of the edges are calculated as follows:

$$d_G(uv) = (0.6, 1.4),\ d_G(vw) = (0.3, 1.2),\ d_G(wx) = (0.3, 1.5),$$
$$d_G(xy) = (0.6, 1.5),\ d_G(yu) = (0.2, 1.4). \quad (5)$$

Obviously, each edge has a different degree. So, G is SEIVG. TDs of the edges are as follows:

$$\begin{aligned}
td^t_G(uv) &= d^t_G(uv) + t_B(uv) = 0.6 + 0.1 = 0.7,\\
td^f_G(uv) &= d^f_G(uv) + f_B(uv) = 1.4 + 0.5 = 1.9,\\
td^t_G(vw) &= d^t_G(vw) + t_B(vw) = 0.3 + 0.2 = 0.5,\\
td^f_G(vw) &= d^f_G(vw) + f_B(vw) = 1.2 + 0.6 = 1.8,\\
td^t_G(wx) &= d^t_G(wx) + t_B(wx) = 0.3 + 0.2 = 0.5,\\
td^f_G(wx) &= d^f_G(wx) + f_B(wx) = 1.5 + 0.7 = 2.2,\\
td^t_G(xy) &= d^t_G(xy) + t_B(xy) = 0.6 + 0.1 = 0.7,\\
td^f_G(xy) &= d^f_G(xy) + f_B(xy) = 1.5 + 0.9 = 2.4,\\
td^t_G(yu) &= d^t_G(yu) + t_B(yu) = 0.2 + 0.4 = 0.6,\\
td^f_G(yu) &= d^f_G(yu) + f_B(yu) = 1.4 + 0.8 = 2.2.
\end{aligned} \quad (6)$$

Clearly, each edge in G has a different TD. Thus, G is SETIVG. Hence, G is both SEIVG and SETIVG.

Example 3. SEIVGs need not be SETIVGs. From Figure 3 we have

$$\begin{aligned}
&d_G(u) = (0.5, 1),\ d_G(v) = (0.4, 1.3),\ d_G(w) = (0.3, 1.1),\\
&d_G(uv) = (0.3, 1.1),\ d_G(vw) = (0.5, 1),\ d_G(wu) = (0.4, 1.3),\\
&td_G(uv) = (0.6, 1.7),\ td_G(vw) = (0.6, 1.7),\ td_G(wu) = (0.6, 1.7).
\end{aligned} \quad (7)$$

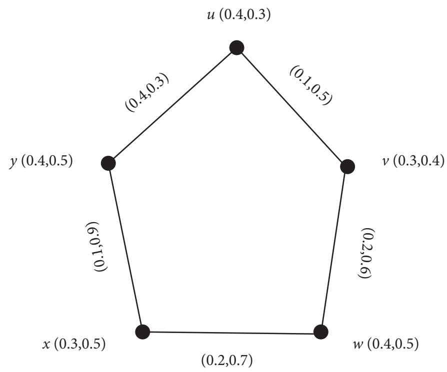

Figure 2: Vague graph G.

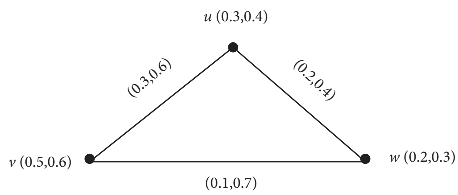

Figure 3: Vague graph G.

We note that G is SEIVG because each edge has a different degree. Furthermore, G is not SETIVG, since all the edges have same TDs.

Hence, SEIVG need not be SETIVG.

Theorem 6. *Let G be a CVG and (t_B, f_B) is a CF. If G is SEIVG, then, G is SETIVG.*

Proof. Let us assume that (t_B, f_B) is a CF. Let $(t_B(pq), f_B(pq)) = (S_1, S_2)$, and $\forall\, pq \in E$, where (S_1, S_2) is constant. Let pq and tu be arbitrary edges in E. Assume G is SEIVG. Then, $d_G(pq) \neq d_G(tu)$, where pq and tu are any pair of edges in E. So, $d_G(pq) \neq d_G(tu)$, $d_G^t(pq) + S_1 \neq d_G^t(tu) + S_1$, $d_G^f(pq) + S_2 \neq d_G^f + S_2(tu)$, $d_G^t(pq) + t_B(pq) \neq d_G^t(tu) + t_B(tu)$, $d_G^f(pq) + f_B(pq) \neq d_G^f(tu) + f_B(tu)$, $td_G^t(pq) \neq td_G^t(tu)$, and $td_G^f(pq) \neq td_G^f(tu)$, where pq and tu are any pair of edges in E.

Hence, G is SETIVG. □

Theorem 7. *Let G be a CVG and (t_B, f_B) is a CF. If G is SETIVG, then, G is SEIVG.*

Proof. Let us assume that (t_B, f_B) is a CF. Let $(t_B(pq), f_B(pq)) = (S_1, S_2)$, $\forall\, pq \in E$, where (S_1, S_2) is constant. Let pq and tu be arbitrary edges in E. Let us assume G is SETIVG. Then, $td_G^t(pq) \neq td_G^f(tu)$, $td_G^f(pq) \neq td_G^f(tu)$, where pq and tu are any pair of edges in E. $td_G^t(pq) \neq td_G^f(tu)$, $td_G^f(pq) \neq td_G^f(tu)$. Hence, $d_G^t(pq) + t_B(pq) \neq d_G^t(tu) + t_B(tu)$ and $d_G^f(pq) + f_B(pq) \neq d_G^f(tu) + f_B(tu)$. Therefore, $d_G^t(pq) + S_1 \neq d_G^t(tu) + S_1$, $d_G^f(pq) + S_2 \neq d_G^f + S_2(tu)$, where pq and tu are any pair of edges in E. So, G is SEIVG. □

Remark 4. Let G be a CVG. If G is both SEIVG and SETIVG, then, (t_B, f_B) need not be a CF.

Example 4. In Example 2, *G is both SEIVG and SETIVG. But (t_B, f_B) is not CF.*

Theorem 8. *Suppose that G is a CVG. If G is SEIVG, then, G is NEIVG.*

Proof. Let G be a CVG. Let us assume that G is SEIVG. Then, each edge in G has a different degree. Hence, neighbor edges have different degrees. So, G is NEIVG. □

Theorem 9. *Let G be a CVG. If G is SETIVG, then, G is NETIVG.*

Proof. Let us assume that G is a CVG that is SETIVG, then, each edge in G has a different TD. So, neighbor edges have different degrees. Thus, G is NETIVG. □

Remark 5. The converse of Theorems 3.26 and 3.27 are not established.

Example 5. Suppose G is a VG that is a path with four nodes.

From Figure 4, $d_G(u) = (0.1, 0.5)$, $d_G(v) = (0.2, 1)$, $d_G(w) = (0.2, 1)$, $d_G(x) = (0.1, 0.5)$, $d_G(uv) = (0.1, 0.5)$, $d_G(vw) = (0.2, 1)$, and $d_G(wx) = (0.1, 0.5)$. Here, $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence, G is NEIVG. But G is not SEIVG, since $d_G(uv) = d_G(wx)$. Also, $td_G(uv) = (0.2, 1)$, $td_G(vw) = (0.3, 1.5)$, and $td_G(wx) = (0.2, 1)$. Note that $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence, G is NETIVG. But G is not SETIVG, since $td_G(uv) = td_G(wx)$.

Theorem 10. *Let G be a CVG on and (t_B, f_B) is a constant function. If G is SEIVG, then, G is an IVG.*

Proof. Let G be a CVG, (t_B, f_B) be a CF, and $(t_B(pq), f_B(pq)) = (S_1, S_2)$, $\forall\, pq \in E$, where (S_1, S_2) is constant. Suppose that G is SEIVG. Then, every pair of edges has different degrees. Let pq and qt be adjacent edges in G having distinct degrees. Then, $d_G^t(pq) \neq d_G^t(qt)$ and $d_G^f(pq) \neq d_G^f(qt)$. So, $d_G^t(p) + d_G^t(q) - 2t_B(pq) \neq d_G^t(q) + d_G^t(t) - 2t_B(qt)$ and $d_G^f(p) + d_G^f(q) - 2f_B(pq) \neq d_G^f(q) + d_G^f(t) - 2f_B(qt)$. Hence, $d_G^t(p) + d_G^t(q) - 2S_1 \neq d_G^t(q) + d_G^t(t) - 2S_1$ and $d_G^f(p) + d_G^f(q) - 2S_2 \neq d_G^f(q) + d_G^f(t) - 2S_2$. Therefore, $d_G^t(p) + d_G^t(q) \neq d_G^t(q) + d_G^t(t)$ and $d_G^f(p) + d_G^f(q) \neq d_G^f(q) + d_G^f(t)$, that we get $d_G(p) \neq d_G(t)$. So, $\exists$ a node q, which is a neighbor to nodes u and t and has different degrees. Hence, G is an IVG. □

4. Application of Vague Influence Digraph to Find the Most Effective Person in a Hospital

The issue of treatment has been one of the most important issues for any country, because any society that benefits from healthy people will naturally make significant progress. But, there are very important issues that can play a significant role in the treatment and improvement of patients, one of which is the amount of labor and also the ability of staff in a hospital

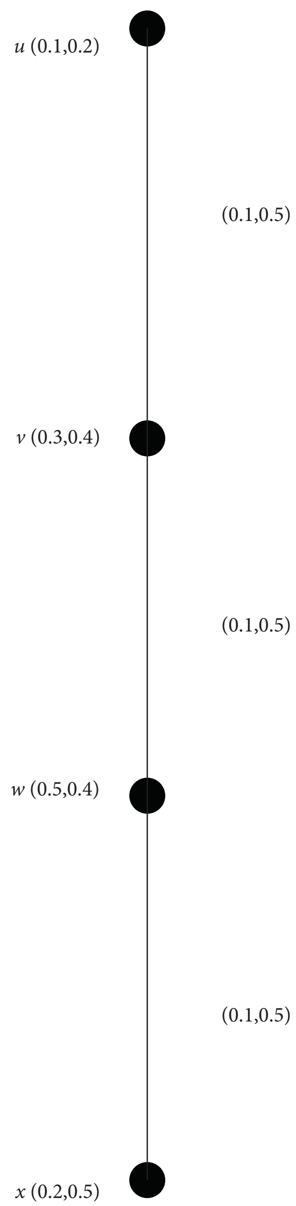

Figure 4: Vague graph G.

Table 2: Name of employees in a hospital and their services.

Name	Services
Ali	Head of the hospital
Hassan	Pharmacy manager
Jafar	Head of the laboratory
Mohammad	Financial affairs manager
Ehsan	Head of nurses
Nima	Head of security
Staff(St)	Staff

or treatment center, because the more capable and faster the workforce is, the more patients will be accepted and also the treatment process of patients will be done more quickly.

In this section, we want to identify the most effective person in a hospital. To do this, we consider the vertices of the vague influence graph as the heads of each ward of the hospital, and the edges of the graph as the degree of interaction and influence each has on other. For this hospital, the set of staff is $F = \{\text{Ali, Hassan, Jafar, Mohammad, Ehsan, Nima, St}\}$.

Table 3: The level of staff capability.

	Ali	Hassan	Jafar	Mohammad	Ehsan	Nima	St
t_A	0.9	0.9	0.8	0.7	0.6	0.5	0.4
f_A	0.1	0.1	0.2	0.2	0.3	0.3	0.4

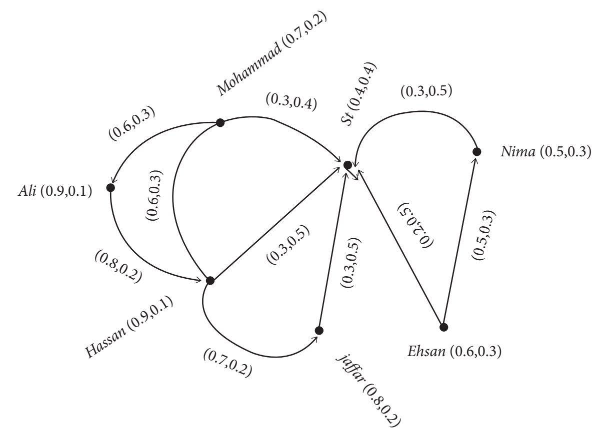

Figure 5: Vague influence digraph.

(a) Hassan has been working with Mohammad for 10 years and values his views on business issues.

(b) Ali has been the head of the hospital for a long time, and not only Hassan but also Mohammad are very satisfied with Ali's performance.

(c) For a hospital, the protection and maintenance of medical equipment and facilities is very important. Nima is a suitable person for this job.

(d) Jafar and Nima have a long history of conflict.

(e) Jafar has a very important role in the laboratory of the hospital.

Given the above, we consider a vague influence graph. The vertices represent each of the hospital staff members. Note that each staff member has the desired ability as well as shortcomings in the performance of their duties. Therefore, we use a vague set to express the weight of the vertices. The true membership indicates the efficiency of the employee, while the false membership shows the lack of management and shortcomings of each staff member. But the edges describe the level of relationships and friendships among employees. If these relationships were stronger, patients would be treated faster. Therefore, the edges can be considered as a vague set so that the true membership indicates a friendly relationship between both employees, and the false membership shows the degree of conflict and difference between the two officials. Names of employees and levels of staff capability are shown in Tables 2 and 3. The adjacency matrix corresponding to Figure 5 is shown in Table 4.Figure 5 shows

Table 4: Adjacency matrix corresponding to Figure 5.

	Ali	Hassan	Jaffar	Mohammad	Ehsan	Nima	St
Ali	(0, 0)	(0.8, 0.2)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Hassan	(0, 0)	(0, 0)	(0.7, 0.2)	(0, 0)	(0, 0)	(0, 0)	(0.3, 0.5)
Jafar	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0.3, 0.5)
Mohammad	(0.6, 0.3)	(0.6, 0.3)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0.3, 0.4)
Ehsan	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0.5, 0.3)	(0.2, 0.5)
Nima	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0.3, 0.5)
St	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)

that Ali has 90% of the power needed to do the hospital work as the head but does not have the 10% knowledge needed to be the boss. The directional edge of Mohammad-Hassan shows that there is 60% interaction and friendship between these two employees, and unfortunately, they have 30% disagreements and conflicts. Clearly, Mohammad has dominion over both Ali and Hassan, and his dominance over both is 60%. It is clear that, Mohammad is the most influential employee of the hospital because he controls both the head of the hospital and the manager of the pharmacy, who have 90% of the power in the hospital.

5. Conclusion

VGs have a wide range of applications in the field of psychological sciences as well as in the identification of individuals based on oncological behaviors. With the help of VGs, the most efficient person in an organization can be identified according to the important factors that can be useful for an institution. Today, VGs play an important role in social networks and allow users to find the most effective person in a group or organization. Hence, in this paper, certain notions of SEIVG and SETIVG are introduced. A comparative study between SEIVGs and SETIVGs has been conducted. Likewise, the concept of PRVG that is both regular and TR is given. Finally, an application of vague influence digraph has been introduced. In our future work, we will introduce the connectivity index, Winer index, and Randic index in VGs and investigate some of their properties.

Data Availability

No data were used for this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest with this study.

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