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Research Article

Robust Minimum Cost Consensus Model for Multicriteria Decision-Making under Uncertain Circumstances

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Recently, the consensus model of group decision-making in uncertain circumstances has received extensive attention. Existing models focus on either minimum cost (maintain the total budget) or maximum utility (improve satisfaction). Based on the minimum cost consensus model, a new multicriteria robust minimum cost consensus model with utility preference is proposed in this paper. Firstly, considering the inherent uncertainty of input data, the unit adjustment cost of experts is described under three robust scenarios. Subsequently, a cost consensus model that expresses the views of decision-makers in a variety of uncertain preference forms such as utility function and Gaussian distribution is proposed. Finally, through the application in emergency decision-making, the cost model and the utility model were compared and analyzed to verify the effectiveness and superiority of the proposed model.

1. Introduction

Group decision-making (GDM) is a procedure in which multiple people participate in the negotiation and make a final decision [1–4]. In GDM, through sufficient communication and multiple effective discussions, decision-makers (DMs) of different individuals explore solutions by gradually changing their views to achieve a consensus for or against a particular issue. However, due to the differences in behavioral preferences and background knowledge, DMs often hold different views on the same problem, resulting in conflicting opinions among groups [5, 6]. Hence, the consensus negotiation process requires a moderator with outstanding leadership and social skills. He/she needs to take various effective measures and persuade the DMs to gradually revise his/her views and reach a final agreement. The resource consumption or financial compensation in the process of reaching a consensus is collectively referred to as "consensus cost." Therefore, how to obtain consensus at the minimum cost has become one of the research hotspots.

The existing minimum cost consensus model (MCCM) research is mainly divided into two categories: the first is Ben-Arieh et al. [7, 8] and the other is Gong et al. [9, 10]. In 2007, Ben-Arieh and Easton [7] proposed a consensus on the minimum cost based on multicriteria decision-making under certain budget constraints. Afterward, they [8] further investigated the above consensus models with quadratic cost functions and solved the maximum number of experts with budget constraints. Besides, Gong et al. [9, 10] studied MCCM with the maximum total utility under the limited budget and introduced various utility/preference constraints scenarios. To further study the MCCM, Zhang et al. [11] constructed a new framework to reach minimum cost consensus by aggregation operators and proved that their model is a more general extension of Ben-Arieh et al. Considering the degree of consensus and cost in GDM, Zhang et al. [12] proposed a soft cost consensus model and explained its economic significance. Wu et al. [13] and Zhang et al. [14] applied the feedback mechanism in MCCM to social networks and explored scheme recommendation and DMs' trust measure. Li et al. [15] and Gong et al. [16]

obtained more reliable results by studying the consistency of DM consensus models. Liu et al.[17] focused on the issues where DMs address hesitant fuzzy preference relations to voice their evaluation information. In order to evaluate consensus-reaching processes in group decision-making more objectively, Álvaro Labella et al. [18] proposed a new comprehensive minimum consensus model considering the distance to global opinion and consensus degree. Focusing on large-scale group decision-making problems, Rodriguez et al. [19] constructed a comprehensive minimum cost consensus model (CMCC) with fuzzy preference relation modeling expert opinions and obtained reliable results. Considering multiple self-confidence levels, Zhang et al. [20] proposed an optimization-based consensus model with a self-confident comparative linguistic expression preference relation to minimizing the information loss between the decision-makers' self-confidence preference and the corresponding individual preference. Zhang et al. [21] combined group decision-making and game theory to build a bilevel optimization model with maximum return modifications and minimum cost feedback consensus mechanism. Zhang et al. [22] proposed an automatic mechanism to reach consensus in the form of heterogeneous preference and studied the minimum adjustment distance of the GDM process.

The above models either consider the negotiation cost or study the utility level of GDM, which well deal with the consensus decision-making problem. Lowering the unit adjustment cost of the expert is conducive to maintaining the total budget, and improving the utility level can enhance the satisfaction of each decision-maker. However, they all think about decision-making issues from only one perspective. Therefore, we combine the negotiation cost with DM's utility to study the consensus problem in GDM. Moreover, since the moderator has a certain leading role in the entire decision-making process and has a deeper understanding of specific issues, it gives him full authority to convince others to reach consensus [23, 24]. Therefore, we regard the moderator as an independent decision-making unit with different utility preferences. Besides, since most of the existing consensus models focus on the deterministic cost and describe less uncertainty, another point of our research is to consider the unit adjustment cost in an uncertain environment. Through a variety of preferences to express the opinions of decision-makers, Tan et al. [25] established a stochastic optimization cost consensus model considering the minimum budget and maximum utility. Gong et al. [26] combined the uncertainty theory to discuss uncertain chance-constrained minimum cost consensus models. Other literature studies on these works can be seen in [27–34], respectively.

The robust optimization (RO) method, first proposed by Soyster [35] in the 1970s, is an effective tool for dealing with optimization problems under uncertainty. Ben-Tal and Nemirovski [36, 37] and Bertsimas and Sim [38, 39] proposed a series of new robust counterpart models, which improved the theoretical framework of robust optimization and gradually formed a mainstream research field. The superiorities of robust optimization are as follows: (i) In the modeling process, RO takes account of uncertainty and describes variables in the form of a set. (ii) They have strong robustness even in the worst case, and the optimal solution has low sensitivity to perturbation of parameters. The key to the robust optimization method mainly lies in the choice of uncertainty set and robust counterpart theory. Unknown parameters are expressed in the form of an uncertainty set so that the model can still obtain robust results in the worst case. As an optimization method under uncertainty, RO makes up for the deficiency of stochastic optimization and fuzzy programming theories. Thus, the innovation of robust optimization theory is of great significance, and its application fields are extremely wide. Zhang et al. [40] studied vehicle routing problems with time windows using RO and proposed a new decision criterion. Ji et al. [41] built a mixed-integer robust programming model to solve the two-echelon inventory routing problem of perishable products. In combination with deep learning, Ning and You [42] proposed a data-driven robust optimization framework and applied it to the supply chain. Dai and Wang [43] introduced two sparse and robust portfolio mean-variance models using objective regularization. Considering the worst case, Han et al. [44] studied the robust minimum cost consensus model under four different forms of uncertain sets. Kaustuv et al. [45] addressed perturbation in the decision variable space and proposed a new robust consensus measure, which provides a reformulation mechanism for multiobjective optimization problems. Jeff Hong and Huang [46] studied a robust statistical model that uses simple data segmentation verification steps to learn the prediction set to achieve limited sample nonparametric statistical guarantees.

In addition, there are few literature studies on the minimum cost consensus problem in an uncertain environment. Existing studies have explored group consensus decision-making models based on optimized consensus rules either from the perspective of stochastic optimization or from fuzzy programming. As an improvement, based on the classical MCCM, three robust minimum cost consensus models with utility preferences are studied and a new optimization method for solving the uncertainty of unit adjustment cost is proposed. In this paper, RO is applied to cost consensus with utility preference problems according to the actual condition. By putting the unit adjustment cost of DMs into an uncertainty set and describing the utility preference of GDM with different functions, the optimal solution satisfying most constraints can be obtained in the worst case.

The main contributions of this paper include the following:

(i) Considering the consensus problem of multicriteria decision-making with minimum cost and maximum utility under an uncertain environment, we put forward the study to describe DMs' preferences in multiple uncertain preference forms.

(ii) Robust optimization is applied to MCCM. Aiming at the minimum cost consensus problem with utility preferences in the GDM, a novel method to solve the uncertainty of DM's unit adjustment cost is presented. This method is more general than the stochastic optimization approach and can illustrate the real scene more accurately.

(iii) Also, three uncertain scenarios are discussed. The unit adjustment cost fluctuated in the box uncertainty set, ellipsoidal uncertainty set, and budget uncertainty set, respectively, and the robust MCCM with utility preferences is studied.

The rest of this paper is organized as follows: Section 2 is the model description. Section 3 is a numerical experiment of emergency decision-making. Section 4 is the conclusion and the next step. The proofs of uncertain sets and robust counterparts are shown in the Appendix.

2. Model Description

Based on the group decision theory, this section considers the minimum cost consensus problem with utility preference and constructs a robust optimization model under three different uncertainty sets.

2.1. Basic MCCM. In the initial stage of the consensus process, the opinion deviation between decision-makers may be very large. By introducing the moderator feedback mechanism to modify the opinions, it helps the decision-makers adjust their preferences to reach a consensus. To improve consensus efficiency and minimize the preference adjustment or consensus cost, Dong et al. [47] first initiated the minimum adjustment consensus model (MACM) by introducing linguistic preferences and achieving the minimum adjustment of the original and adjusted preferences. Their model is as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m} d\left(o_i - o'\right) \\ \text{s.t.} \quad & \left|o' - o^c\right| \leq \alpha, \forall i = 1, \ldots, m, \end{aligned} \tag{1}$$

where $d(o_i - o')\,(i = 1, \ldots, m)$ is the distance between o_i and o'. Different aggregation functions can be used in model (1) to get different MACMs. OWA is a general one by Dong et al. [47].

On the other hand, Ben-Arieh et al. [7, 8] addressed a minimum cost consensus model, which is based on the following assumption: a moderator represents the interests of the group and individual DMs represent different interest groups. Based on the differences between the opinions, the moderator will incur specific costs to promote consensus. Therefore, their model becomes

$$\begin{aligned} & \min B = \sum_{i=1}^{m} c_i \left|o_i - o'\right| \\ & \text{s.t. } o' \in O, \end{aligned} \tag{2}$$

where m is the number of individual DMs, c_i is the unit adjustment cost paid by the moderator d' to the individual d_i, o_i is the opinion of individual d_i, and $|o_i - o'|$ denotes the opinion deviation between o' and o_i.

Based on the above model, Gong et al. [9, 10] constructed a minimum cost consensus model considering the preference utility of individual DMs. Instead of trying to minimize cost, moderators would encourage individuals to change their original opinions with a more relaxed budget. Zhang et al. [11] combined the advantages of model (1) and model (2) and proposed an optimization model to solve the basic MCCM problem. Zhang et al. [48] reviewed the origin and basic research paradigm of MCCM and classified MCCM into two decision scenarios: MCCM in classic group decision-making and complex group decision-making problems (such as social networks and large-scale). To solve the consensus decision problem in the Grains to Greens Program, Tan et al. [25] fully considered minimizing negotiation cost and maximizing utility value and then proposed an MCCM with utility preference. So, model (2) can be improved:

$$\begin{aligned} \min \quad & B - \Theta\lambda \\ \text{s.t.} \quad & \sum_{i=1}^{m} c_i \left|o_i - o'\right| = B \\ & \lambda \leq U_1(o_i), \lambda \leq U_2(o') \\ & o' \in [a_i, b_i], i \in M, o' \in O, \lambda \in [0, 1], \end{aligned} \tag{3}$$

where λ is the utility value of the GDM process, Θ is the utility adjustment coefficient, B can be viewed as an advance budget, $U_1(o_i)$ and $U_2(o')$ are the utility functions of d_i and d', respectively, and a_i and b_i are the limits of the individual DMs' opinion preference interval.

Theorem 1. *If o_i and o' represent individual opinion and consensus opinion, respectively, model (3) is equivalent to the following optimization model:*

$$\begin{aligned} \min \quad & B - \Theta\lambda \\ \text{s.t.} \quad & c^T d \leq B \\ & o_i - o' = \varepsilon_i, i = 1, \ldots, m \\ & \varepsilon_i \leq d_i \\ & -\varepsilon_i \leq d_i \\ & \lambda \leq U_1(o_i), \lambda \leq U_2(o') \\ & o' \in [a_i, b_i], i \in M, o' \in O, \lambda \in [0, 1]. \end{aligned} \tag{4}$$

Proof. Let $c = (c_1, \ldots, c_m)$ and $d = (d_1, \ldots, d_m)$, according to $|x| = \min\{x, -x\}$; let $|o_i - o'| = |\varepsilon_i| = d_i$; then, $\varepsilon_i \leq d_i$ and $-\varepsilon_i \leq d_i$ can be obtained. Thus, model (3) and model (4) are equivalent.

In most consensus negotiation processes, the unit adjustment cost paid by the moderator to the individual DMs is usually fixed. However, in a practical situation, the unit adjustment cost is affected by many uncertainties. As far as we know, the existing consensus models often use fuzzy numbers or intervals to express uncertainty. In this case, some of the constraints are not satisfied and the optimal solution would not be feasible. Therefore, it is necessary to consider how to reduce the impact of uncertainty on the model. As is known to all, robust optimization is a common method to cope with uncertain problems. Better results can be obtained by selecting an appropriate uncertainty set to accurately describe the uncertain parameters. This drives us to explore a robust optimization method to deal with group decision-making problems. □

2.2. Robust MCCM. In this section, from the inspiration of Tan [27], we comprehensively consider individuals and moderators' utility preferences and use the probability distribution of random variables to represent the opinions of DMs. This paper introduces the left and right membership functions to express the utility of DMs. Since Ji Lai and Hwang [49] proposed in 1994, it has been widely used in many fields. When reaching a consensus, the moderator must not only consider budget constraints but also try to satisfy the utility preferences of all DMs. Based on the previous research, we propose a robust minimum cost consensus model with utility preferences under the following two cases:

(i) Case 1: individuals' opinions belong to the utility preferences function decision-making type, and the moderator's opinions obey random distribution. Assuming that o' obeys the Gaussian distribution, o_i are expressed as the linear right membership function.

(ii) Case 2: individuals' opinions obey random distribution, and the moderator's opinions obey the utility preferences function decision-making type. Assuming that o' is expressed as the linear left membership function, o_i obey the Gaussian distribution.

In order to construct a robust optimization model of the MCCM with utility preferences, the unit adjustment cost of uncertain parameters is placed in an uncertainty set. And, the corresponding robust counterpart of the cost constraint can be represented as

$$c^T d \leq B, \forall \left(c = \left[c^0\right] + \sum_{j=1}^{L} \zeta_j \left[c^j\right] = C^0 + C^j \zeta \colon \zeta \in Z \right). \tag{5}$$

Expression (5) is also called the factor model, which is widely used in the robust optimization community.

2.2.1. Box Uncertainty Set. If the utility preferences are case 1 and the uncertainty set is box, then

$$Z^{\text{Box}} = \left\{ \zeta \in \mathbb{R}^L \colon \|\zeta\|_\infty \leq 1 \right\}. \tag{6}$$

Then, the robust optimization model is given as

$$\begin{aligned}
\min \quad & B - \Theta\lambda \\
\text{s.t.} \quad & \left[c^0\right]^T d + \sum_{j=1}^{L} u_j \leq B \\
& -u_j \leq \left[c^j\right]^T d \leq u_j, j = 1, \ldots, L \\
& \left|o_i - o'\right|_1 = \varepsilon_i, i = 1, \ldots, m \\
& \varepsilon_i \leq d_i \\
& -\varepsilon_i \leq d_i \\
& \lambda \leq \frac{o_i - \min\{o_i\}}{\max\{o_i\} - \min\{o_i\}} \\
& o_i \in \left[a_i, b_i\right], i \in M \\
& o' \sim N\left(\mu, \sigma^2\right) \\
& \lambda \in [0, 1].
\end{aligned} \tag{7}$$

If the utility preferences are case 2 and the uncertainty set is box (6), then the robust optimization model is given as follows:

$$\begin{aligned}
\min \quad & B - \Theta\lambda \\
\text{s.t.} \quad & \left[c^0\right]^T d + \sum_{j=1}^{L} u_j \leq B \\
& -u_j \leq \left[c^j\right]^T d \leq u_j, j = 1, \ldots, L \\
& \left|o_i - o'\right|_1 = \varepsilon_i, i = 1, \ldots, m \\
& \varepsilon_i \leq d_i \\
& -\varepsilon_i \leq d_i \\
& \lambda \leq \frac{o'_u - o'}{o'_u - o'_l} \\
& o' \in \left[o'_l, o'_u\right] \\
& o_i \sim N\left(\mu_i, \sigma_i^2\right) \\
& \lambda \in [0, 1].
\end{aligned} \tag{8}$$

In the above models, the objective function is a linear combination of consensus negotiation cost B and utility value λ. In order to find the optimal solution, it is necessary to minimize the moderator's budget and maximize the utility value of DMs. Θ is the adjustment coefficient to ensure maximum utility.

2.2.2. Ellipsoidal Uncertainty Set. Consider case 1, if the uncertainty set is ellipsoid as follows:

$$Z^{\mathrm{Epd}}=\left\{\zeta\in\mathbb{R}^L:\ \|\zeta\|_2\le\Omega\right\}. \tag{9}$$

The corresponding robust optimization model can be constructed as

$$\begin{aligned}
\min\quad & B-\Theta\lambda\\
\text{s.t.}\quad & \left[c^0\right]^T d+\sum_{j=1}^{L}u_j\le B\\
& -u_j\le\left[c^j\right]^T d\le u_j,\ j=1,\ldots,,L\\
& \left|o_i-o'\right|_1=\varepsilon_i,\ i=1,\ldots,m\\
& \varepsilon_i\le d_i\\
& -\varepsilon_i\le d_i\\
& \lambda\le\frac{o_i-\min\{o_i\}}{\max\{o_i\}-\min\{o_i\}}\\
& o_i\in\left[a_i,b_i\right],i\in M\\
& o'\sim N\left(\mu,\sigma^2\right)\\
& \lambda\in[0,1].
\end{aligned} \tag{10}$$

Consider case 2, if the uncertainty set is ellipsoid (9). The corresponding robust optimization model is established as follows:

$$\begin{aligned}
\min\ & B-\Theta\lambda\\
\text{s.t.}\quad & \left[c^0\right]^T d+\sum_{j=1}^{L}u_j\le B\\
& -u_j\le\left[c^j\right]^T d\le u_j,\ j=1,\ldots,,L\\
& \left|o_i-o'\right|_1=\varepsilon_i,\ i=1,\ldots,m\\
& \varepsilon_i\le d_i\\
& -\varepsilon_i\le d_i\\
& \lambda\le\frac{o_u'-o'}{o_u'-o_l'}\\
& o'\in\left[o_l',o_u'\right]\\
& o_i\sim N\left(\mu_i,\sigma_i^2\right)\\
& \lambda\in[0,1].
\end{aligned} \tag{11}$$

2.2.3. Budgeted Uncertainty Set. While the cost consensus with utility preferences is case 1, the uncertainty set is of the following budgeted type [38]:

$$Z^{\mathrm{Bud}}=\left\{\zeta\in\mathbb{R}^L:\ \|\zeta\|_1\le1,\|\zeta\|_\infty\le\Gamma,1\le\Gamma\le L\right\}. \tag{12}$$

Correspondingly, we get the following optimization model:

$$\begin{aligned}
\min\quad & B-\Theta\lambda\\
\text{s.t.}\quad & \left[c^0\right]^T d+\sum_{j=1}^{L}u_j\le B\\
& -u_j\le\left[c^j\right]^T d\le u_j,\ j=1,\ldots,,L\\
& \left|o_i-o'\right|_1=\varepsilon_i,\ i=1,\ldots,m\\
& \varepsilon_i\le d_i\\
& -\varepsilon_i\le d_i\\
& \lambda\le\frac{o_i-\min\{o_i\}}{\max\{o_i\}-\min\{o_i\}}\\
& o_i\in\left[a_i,b_i\right],i\in M\\
& o'\sim N\left(\mu,\sigma^2\right)\\
& \lambda\in[0,1].
\end{aligned} \tag{13}$$

While the cost consensus with utility preferences is case 2, the uncertainty set is budgeted. We obtain the following optimization model:

$$\begin{aligned} &\min B - \Theta\lambda \\ &\text{s.t.} \quad \left[c^0\right]^T d + \sum_{j=1}^{L} u_j \le B \\ &\quad - u_j \le \left[c^j\right]^T d \le u_j, j = 1, \ldots, L \\ &\quad \left|o_i - o'\right|_1 = \varepsilon_i, i = 1, \ldots, m \\ &\quad \varepsilon_i \le d_i \\ &\quad - \varepsilon_i \le d_i \\ &\quad \lambda \le \frac{o_u' - o'}{o_u' - o_l'} \\ &\quad o' \in \left[o_l', o_u'\right] \\ &\quad o_i \sim N\left(\mu_i, \sigma_i^2\right) \\ &\quad \lambda \in [0, 1]. \end{aligned} \tag{14}$$

3. Numerical Experiments

In this section, we demonstrate the robust MCCM with utility preferences in solving an emergency decision-making problem. All the optimization models are solved using Anaconda 1.9.12 (Python 3.7). All experiments are conducted on Mac, which has 8 GB of RAM and a 2.3 GHz Intel Core i5.

The flood disaster of the Huaihe River Basin in China is a hot topic with a long history and high participation of the masses. There are many dams with different locations and functions in the Huaihe River Basin. When extreme weather occurs, such as plum rains, some dams need to be selected and opened to release water to protect other areas. For example, on July 20, 2020, due to continuous heavy rainfall, Wangjiaba in the Huaihe River Basin of Anhui Province opened a sluice to release water to protect the lives and property of residents in Henan, Jiangsu, and Anhui. More than 30,000 acres of arable land were flooded and 170,000 people in 131 villages were evacuated. Generally speaking, the occurrence of extreme weather is sudden and accidental, which requires local authorities to make decisions in a short time. This is a kind of emergency decision-making problem. However, the choice of disaster area not only depends on geographical location and utility preferences, but the local government should also consider giving reasonable compensation (planting subsidies, housing allowances, etc.) to the victims.

To narrow the differences of opinion and achieve consensus, local governments need to pay some negotiation "cost" (manpower, expenses, and so on). The local government hopes to control the "cost" within its fiscal budget based on reaching a consensus as much as possible. Furthermore, victims' financial condition, land cultivation, and behavioral habits can also lead to different preferences. In the process of consensus negotiation, the preferences of local governments are affected by macroarrangements and appropriation budgets. Therefore, based on flood subsidy, the local government and victims have formed an uncertain group consensus system. Here, we regard the local government as the moderator and victims as individual DMs, thus forming the DMs in GDM. On this basis, the preferences of local governments and victims are represented by utility function and probability distribution, and the group consensus model in different situations is constructed. The goal is to ensure satisfaction with the majority of DMs and to control the budget.

3.1. Numerical Examples. Assume that the consensus negotiation of flood disaster problems involves local government d' and 4 victims d_1, d_2, d_3, and d_4. The unit cost of each victim is $c_1 = 7$, $c_2 = 8$, $c_3 = 2$, and $c_4 = 11$. Since $c = [c^0] + \sum_{j=1}^{L} \zeta_j [c^j]$, suppose initial adjustment cost $c = c^0 = (7, 8, 2, 11)$. The negotiated cost budget of the local government is B. Based on case 1, we assume o' obeys the Gaussian distribution with parameter (10, 2). The victims' opinions are illustrated by the linear right partial membership functions in the intervals $o_1 = [9, 14], o_2 = [11, 17], o_3 = [8, 13]$, and $o_4 = [7, 15]$. We use Monte Carlo simulation, optimization package cvxpy1.1 (https://www.cvxpy.org), and Genetic Algorithm (Geatpy 2.6.0) to obtain the optimal solution of models (7), (10), and (13). In the process of test, we take the utility adjustment coefficient Θ changing from 2^0 to 2^{10}, respectively.

From the results of Tables 1–3, we get some conclusions of models (7), (10), and (13). When the utility adjustment coefficient Θ is not adjusted ($\Theta = 1$), the objective function of the model satisfies both maximum utility and minimum cost. Naturally, every individual's opinions are approximated to the lower threshold of their range. The utility values λ and consensus costs are relatively low under the individual DMs' right membership functions. As Θ gradually increases, the minimum cost is sacrificed to satisfy the victims' satisfaction. The opinion of individual DMs is close to the upper limit, the utility value is high, and the cost is also high. When Θ approaches about 2^4, the utility value λ reaches a relatively superior level and it is a perfect utility at $\Theta = 2^{10}$.

Similarly, based on case 2, we assume that o' is expressed as a linear left membership function on the interval $[10, 16]$, and the victims' opinions obey the Gaussian distribution: $o_1 \sim N(11, 3), o_2 \sim N(12, 1), o_3 \sim N(8, 2)$, and $o_4 \sim N(13, 3)$. By the same experimental method, we get the optimal solution of models (8), (11), and (14).

Tables 4–6 show the results of models (8), (11), and (14). When $\Theta = 1$, the moderator's opinion o' is tending to the right limits of its range. Since the moderator is the left partial decision-making type, the value of λ is low. As Θ gradually increases, the moderator's opinion approaches the lower boundary and the utility value is relatively high. For local

Table 1: Result of model (7).

Θ	o_1	o_2	o_3	o_4	o'	λ	B
1	9.669	11.363 2	8.414 5	7.714 2		0.074 5	21.830 7
2	9.682 2	12.047 2	8.381 5	7.608 5		0.341 4	23.203 4
4	10.914 3	13.331 7	9.000 1	8.353 2		0.586 9	26.119 5
8	10.672 7	13.858 4	10.878	11.935		0.646 2	34.237 4
16	11.781 9	13.049 5	11.326 5	12.577		0.733 8	33.851 7
32	12.282 1	14.402 6	10.672 5	11.420 3		0.854 6	41.548 4
64	11.237 6	14.897 7	12.422 3	12.614 2		0.878	45.243 3
128	11.896 5	15.303 4	12.077 8	13.0001		0.911 6	48.262 5
256	12.651	15.997	12.438 1	14.491 2		0.924	51.705 6
512	13.946	16.026 3	12.596	14.434 7		0.930 5	52.662 8
1024	13.105 6	16.350 7	12.077 8	14.871		0.948 4	58.329 9

Table 2: Result of model (10).

Θ	o_1	o_2	o_3	o_4	o'	λ	B
1	9.675 8	11.278 1	8.034 7	7.167 3		0.064 5	19.099 2
2	10.419 5	12.792 5	9.785 1	7.229 7		0.383 1	22.240 3
4	10.051 8	12.711 4	10.665 2	8.858 8		0.585 8	25.385 5
8	11.675 8	14.590 4	9.845 1	8.172 3		0.621 5	31.597 2
16	11.767 6	14.201 3	9.548 8	10.856 2		0.621 9	36.659
32	11.274 6	15.401 2	10.815 7	11.931 7		0.657 7	42.530 3
64	12.407 3	14.705 6	11.910 4	11.667		0.697 9	44.202 2
128	13.045 5	15.946 4	12.034 9	12.477 8		0.915 8	50.121 4
256	13.403 4	16.438 1	12.066 1	13.813 8		0.938 5	51.930 3
512	13.703 9	16.512 5	12.466 7	13.967 2		0.951 2	52.355 3
1024	13.869 4	16.964 3	12.724 6	14.201 9		0.985 3	55.060 6

Table 3: Result of model (13).

Θ	o_1	o_2	o_3	o_4	o'	λ	B
1	9.313 5	11.687 3	8.586 5	7.691 2		0.030 1	20.913 6
2	11.956 7	11.784 5	10.635 4	10.001 2		0.150 9	38.663 5
8	11.368 9	13.350 6	9.530 2	8.621 6		0.270 9	36.191 2
16	12.914 7	14.966 4	10.243 9	9.138 8		0.644	33.349 5
32	12.747 8	13.266 5	10.474 4	11.940 2		0.728 1	38.808 1
64	12.319 7	13.565 3	12.809 1	13.323 9		0.666 8	47.502 2
128	13.005	15.267 1	12.380 2	14.454 6		0.892 3	51.702
256	13.350 3	15.088 2	12.246 4	14.136 2		0.929 7	53.839 9
512	13.452 7	16.443 3	12.287 1	14.304 5		0.948 1	57.048 2
1024	13.375 7	16.871 5	12.599 7	14.719 3		0.965	57.642

Table 4: Result of model (8).

Θ	o_1	o_2	o_3	o_4	o'	λ	B
1					15.812 2	0.074 2	24.828 4
2					14.892 3	0.11	26.995 6
4					14.129 4	0.240 9	28.242 2
8					14.297 8	0.377 9	30.545 8
16					12.203	0.712 8	39.212 7
32					11.867 4	0.825 6	41.294 8
64					11.273 4	0.852 7	44.909 6
128					11.457 1	0.928 2	50.343
256					11.008 3	0.927 9	51.543 9
512					10.709 3	0.958 4	56.904 6
1024					10.029 3	0.992 5	58.995 5

Table 5: Result of model (11).

Θ	o_1	o_2	o_3	o_4	o'	λ	B
1					15.997 7	0.121 8	19.614 5
2					15.056 2	0.261 5	24.868 4
4					14.083 2	0.476 4	27.876 3
8					14.291 3	0.572 1	29.905 1
16					13.634 9	0.802 8	31.313 5
32					13.412 8	0.837 1	40.883 6
64					12.846 9	0.862 1	41.787 5
128					12.867 5	0.861 8	51.668 2
256					11.440 4	0.920 8	52.971 6
512					10.561 5	0.945 7	53.615 8
1024					10.535 6	0.997 2	54.681 1

Table 6: Result of model (14).

Θ	o_1	o_2	o_3	o_4	o'	λ	B
1					15.824 2	0.043	20.036 2
2					14.400 3	0.271 5	20.748 9
4					14.685 8	0.407 5	26.172 3
8					13.982	0.426 9	34.246 3
16					11.697 3	0.674	44.267 2
32					15.242	0.756 9	49.256 6
64					11.324	0.787 2	46.305 8
128					11.213	0.895 6	49.954 5
256					11.498 4	0.937 4	52.256 6
512					11.058 1	0.950 6	54.472 3
1024					10.270 7	0.956 9	55.374 6

governments, although the minimum consensus cost when $\Theta > 1$ is higher than the minimum cost when $\Theta = 1$, in actual situations, the moderator tries to find a balance between cost and utility.

Generally, as the utility adjustment coefficient Θ increases, the consensus cost, and utility are significantly enhanced. While the priority Θ reaches about 2^7, the variation of the minimum consensus cost B and utility value λ is tiny. Since the negotiated budget has reached a high level, even if the coefficient Θ increases, the consensus cost will not increase significantly.

3.2. Influence of Uncertainty Parameters. In Section 4.1, we analyzed the variation of minimum cost and maximum utility under different priority Θ values. The key to determining the robustness of the consensus model lies in the parameter ζ. It is important to note whether the bias of ζ will affect the total cost and utility of consensus negotiation. In models (7) and (8), since the parameter of the box set is 1, it forms an interval set. However, parameters such as the radius of an ellipsoid in models (10) and (11) are unknown. Therefore, it is necessary to study the relationship between the change of some uncertain parameters and the minimum cost and maximum utility.

Figures 1 and 2 show the change of consensus cost B and utility λ under different unknown parameters in case 1 and case 2, respectively. The upper left illustrates the variation of the consensus cost with Ω in the ellipsoidal uncertainty set. The larger Ω is, the larger B is. However, unlike the situation in Section 4.1, the change in utility is not always increasing. At first, λ grows with an increase in Ω and then becomes stable after reaching a certain threshold. The utility curve in case 1 is balanced around $\Omega = 5$, while λ in case 2 tends to stabilize at $\Omega = 7$. In the lower left, consider the variation of B, when the uncertainty set is budgeted. Both B and λ are increasing at first with the growth of π and show a gradually increasing trend. Finally, when $\pi = 5$, both curves reach the maximum to varying degrees and remain smooth. Generally speaking, as the radius of the ellipsoid becomes larger, the consensus cost increases significantly. The increase in ellipsoid set radius means that the perturbation of unit adjustment cost increases, making it more difficult to reach a consensus. In models (10) and (11), the utility curve shows a stable situation after climbing first. Not only because the utility value λ has an upper limit but also because no amount of consensus cost in flood disasters is enough to compensate for the mental damage of the victims. Also, the local government's budget is limited and it is impossible to increase the cost of consensus negotiation endlessly. It can be found that the consensus cost of models (10) and (11) is lower than that of other models and the utility value is higher than of other models. So, they are considered the most robust.

3.3. Comparison with the Other Models. In this section, we compare the robust MCCM with utility preferences with a series of consensus models that only consider the minimum

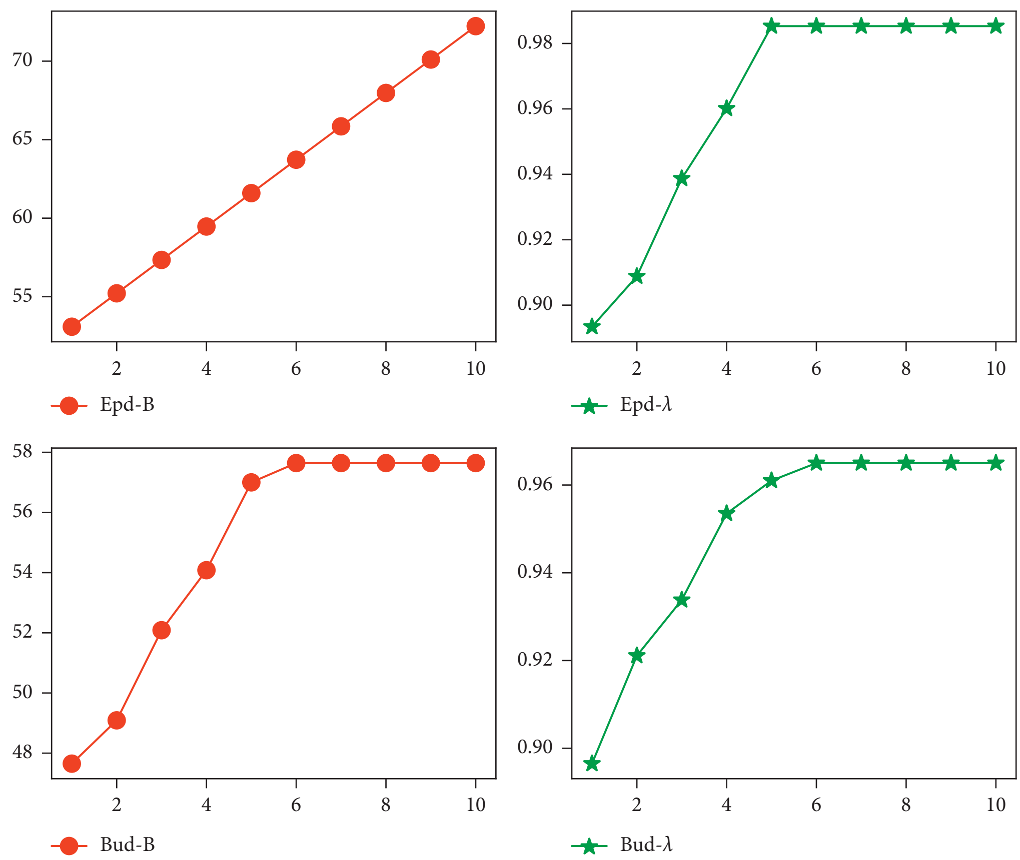

FIGURE 1: Fluctuation of cost B and utility λ under different uncertain parameters in case 1.

cost or maximize the DMs' utility and comprehensively reflect the advantages of our models. Only considering the maximum utility of MCCM, we need to fix the unit adjustment cost, so suppose that the moderator needs to pay the unit cost as the average of 7 in Section 3.1. Inspired by Gong et al. [9], the utility value of GDM is calculated.

Undoubtedly, the basic goal of the GDM model is to achieve a high consensus level as much as possible. Every decision-maker wants to get enough attention, and their views are highly valued. It can be seen from Table 7 that although the traditional MCCM has the lowest total cost of consensus negotiation, it ignores the value of decision-makers. When the objective function only considers the consensus model of maximum utility, although the utility value reaches a relatively good level in both cases, it is difficult to reflect the cost consumption in the negotiation process and it is not convenient for the local government to make a financial budget in time. Therefore, the minimum cost consensus model that comprehensively considers multicriteria decision-making is of practical significance.

To further test whether it makes sense to use robust optimization methods to solve the MCCM with utility preference, we use the same data in Section 3.1 to consider the utility preferences of the two cases to compare and analyze with the MCCM with utility preferences.

Table 8 shows the results from different models in two different cases. We found that the consensus cost and utility of robust models are significantly higher than of the original MCCM. It means that the moderator is too optimistic about the final consensus result in the original MCCM. Perhaps, these are unaccounted for uncertainties in the consensus decision-making process. This will increase the total cost and utility of the consensus to some extent. Under the box uncertainty set, the total cost increased the most (11.98%) and the utility improved the least (0.085 3). Under the ellipsoidal uncertainty set, the total cost increased the least (5.7%), while the utility improved the most (0.122 2). The solution of the budget uncertainty model is at the middle level. Here, we introduce a conservative coefficient ϕ to compare models more intuitively. When $\phi = 1$, it shows that the DMs are quite conservative. While $\phi = 0$, it indicates that the DMs have quite a few risk preferences and their decisions are often too optimistic. In Table 8, the conservative coefficients of the three robust models are 0.119 8, 0.057, and 0.106 6. The larger conservative coefficient illustrates that reaching consensus under the robust model is more difficult and the moderator will have to compensate for the uncertainty. Therefore, when selecting the uncertainty set, not only the influence of parameter fluctuations must be considered, but also the overall cost should be small and the utility must be as large as possible. So, the performance of the ellipsoidal uncertainty model is better than of other models.

The minimum consensus cost of model (13) is greater than of model (11) but less than of model (8) in case 2. The utility value of model (14) is smaller than of model (8) but greater than of model (11). The major difference with case 1

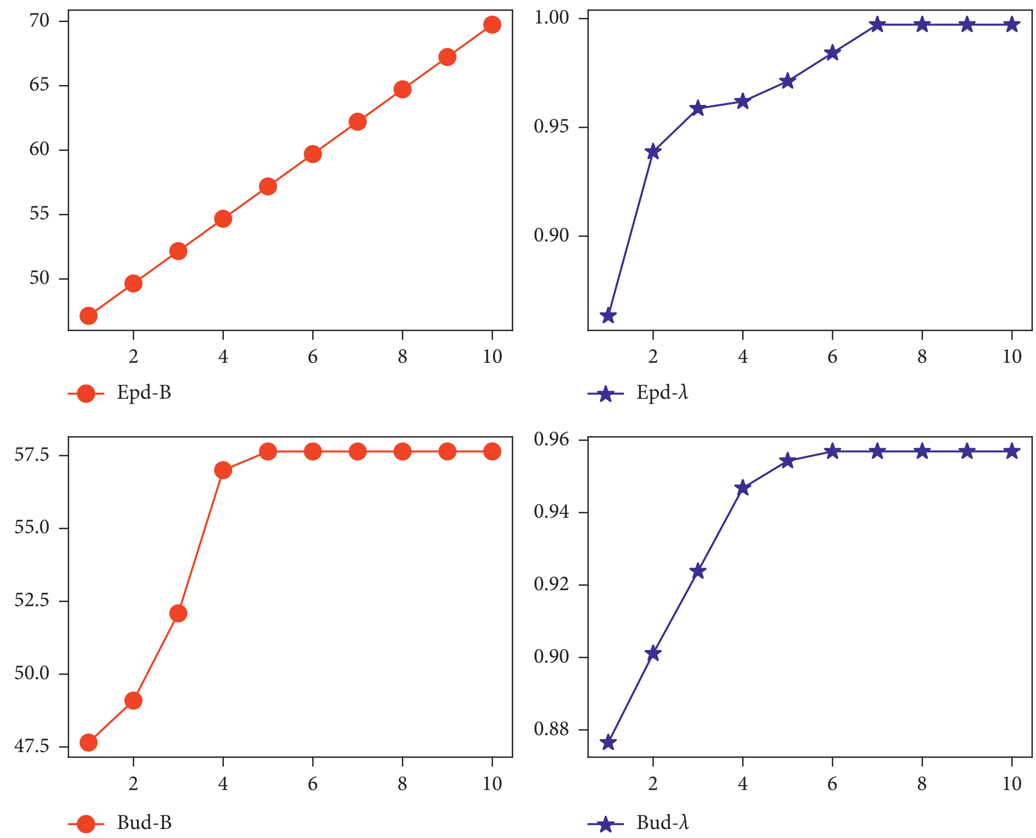

Figure 2: Fluctuation of cost B and utility λ under different uncertain parameters in case 2.

Table 7: Comparison with consensus models considering only one objective function.

Case	Object	MCCM	Box	Epd	Bud	MU
Case 1	B	50.316 8	58.329 9	55.060 6	57.642	—
	λ	—	0.948 4	0.985 3	0.965	0.934 2
Case 2	B	52.412 4	58.995 5	54.681 1	55.374 6	—
	λ	—	0.879	0.992 5	0.997 2	0.912 4

Table 8: Minimum cost consensus models with utility preferences and their conservative coefficient.

Case	Object	MU-MCCM	Box-MCCM	Epd-MCCM	Bud-MCCM
Case 1	B	52.091 2	58.329 9	55.060 6	57.642
	λ	0.863 1	0.948 4	0.985 3	0.965
	ϕ	0.000	0.119 8	0.057	0.106 6
Case 2	B	50.312 6	58.995 5	54.681 1	55.374 6
	λ	0.809	0.992 5	0.997 2	0.956 9
	ϕ	0.000	0.172 6	0.086 86	0.100 66

is that the ellipsoidal uncertainty model has a smaller cost and greater utility, but its conservative coefficient is larger than case 1.

In the following, we provide a summary of MCCM studies with different key elements (consensus measure, unit adjustment cost, DM's utility, and robustness) to more clearly show the superiority of our proposal (see Table 9). Our proposal considers both the DM's utility and the robustness of unit adjustment cost, which is more specific and thoughtful than other models.

Table 9: MCCM studies with different designs of key elements.

MCCM categories	Literature	Consensus measure	Unit adjustment cost	Utility	Robustness
MCCM with limited cost	[7, 8]	Hard	Heterogeneous	Not used	Not used
MCCM with limited cost	[9, 10]	Hard	Heterogeneous	Used	Not used
MCCM with limited cost	[25]	Soft	Heterogeneous	Used	Not used
MCCM with asymmetric cost	[27]	Hard	Heterogeneous	Used	Not used
Robust MCCM	[44]	Hard	Heterogeneous	Not used	Used
Our MCCM		Hard	Heterogeneous	Used	Used

4. Conclusion

In the process of GDM, uncertainty in the decision-making environment, unit adjustment cost of the moderator, and the preferences of DMs often lead to a total cost exceeding budget and poor utility. This paper uses robust optimization theory to solve group decision-making problems with uncertain consensus cost and utility preferences. The implementation is to transform the unit adjustment cost perturbation into an uncertainty set. Meanwhile, we consider utility functions and Gaussian distributions to simulate the utility preferences of decision-makers. Through numerical experiments, we obtain the robust model results in two cases. Our numerical experiments illustrate that the consensus cost gradually increases as the utility priority coefficient becomes greater. With the growth of uncertain parameters, the minimum consensus cost and maximum utility enhance to varying degrees. Among the three robust models, the ellipsoidal model shows the highest performance. Compared with the original MCCM, although the optimal solution of the robust MCCM is relatively conservative, it is crucial to find a balance between utility and cost in a short time and reach a consensus. We summarize our main contributions and findings as follows:

(1) We introduce a consensus model that considers both cost minimization and utility maximization. It is an extension of the existing consensus model.

(2) The robust MCCM with utility preference under two cases and three uncertain sets is considered. The three types of uncertainty sets are box set, ellipsoidal set, and budgeted uncertainty set. These MCCMs can transform into corresponding robust counterparts for solving.

(3) The Monte Carlo simulation method is used to obtain the optimal solution of robust MCCMS, which is more consistent with the actual decision.

Future research can consider two-stage decision-making [50–52], expert preferences with sentiment analysis [53, 54], and the multiobjective problem of opinion bias between individuals and moderators.

Appendix

Uncertainty Set and Robust Counterpart

In group decision-making problems, the uncertainty of input data can lead to inaccurate decision-making by DMs. Robust optimization is a method developed from the robust control theory for the shortcomings of traditional optimization methods, which effectively enhances the robustness of the model [36]. The general structure of uncertain LP problem is a collection:

$$\left\{\min_{x}\left\{A^{T}x + d\colon\, c^{T}x \le B\right\}\right\}_{(A,d,c,B)\in Z}, \tag{A.1}$$

where Z is an uncertain set with data varying. Assuming that $c = [c^0] + \sum_{j=1}^{L} \zeta_j [c^j]$, c is the true value, $[c^0]$ is the nominal value of the parameter, $[c^j]$ is the disturbance variable, and ζ_l is the uncertainty factor, which can take any value in the set Z. Then, we will study several uncertainty sets and their corresponding robust counterparts.

$$c^{T}x = \left[c^{0}\right]^{T}x + \max_{\zeta\in Z}\sum_{j=1}^{L}\zeta_j\left[c^{j}\right]^{T}x \le B. \tag{A.2}$$

Box Uncertainty Set

Proposition A.1. *Consider the case of interval uncertainty, where Z in (5) is a box,* $Z^{Box} = \left\{\zeta \in \mathbb{R}^{L}\colon\, \|\zeta\|_{\infty} \le 1\right\}$. *Inequality (A.3) is equivalent to constraint (A.2):*

$$\left[c^{0}\right]^{T}x + \sum_{j=1}^{L}\left|\left[c^{j}\right]^{T}x\right| \le B. \tag{A.3}$$

Proof. From the form of the box uncertainty set, constraint (A.2) can be transformed into

$$\left[c^{0}\right]^{T}x + \max_{\|\zeta\|_{\infty}\le 1}\sum_{j=1}^{L}\zeta_j\left[c^{j}\right]^{T}x \le B. \tag{A.4}$$

Then, we get

$$\max_{|\zeta_l|\le 1}\sum_{j=1}^{L}\zeta_j\left[c^{j}\right]^{T}x \le B - \left[c^{0}\right]^{T}x. \tag{A.5}$$

Maximizing the left side of the inequality above, let $\|\zeta_l\| = 1$, and we get explicit convex constraint (A.3), which can also represented as a series of linear inequalities:

$$\begin{cases} -u_j \le \left[c^{j}\right]^{T}x \le u_j,\ j = 1, \ldots, , L, \\ \left[c^{0}\right]^{T}x + \sum_{j=1}^{L}u_j \le B. \end{cases} \tag{A.6}$$

In summary, Proposition A.1 can be proved. □

Ellipsoidal Uncertainty Set

Proposition A.2. *Consider the case of ellipsoidal uncertainty where Z in (5) is a ellipsoid, $Z^E = \{\zeta \in \mathbb{R}^L\colon \|\zeta\|_\infty \le 1\}$. Inequality (A.7) is equivalent to constraint (A.2):*

$$\left[c^0\right]^T x + \Omega\sqrt{\sum_{j=1}^{L}\left(\left[c^j\right]^T x\right)^2} \le B. \quad \text{(A.7)}$$

Proof. According to the ellipsoidal uncertainty set, constraint (A.2) can be reformed as

$$\left[c^0\right]^T x + \max_{\|\zeta\|_\infty \le 1} \sum_{j=1}^{L} \zeta_j \left[c^j\right]^T x \le B. \quad \text{(A.8)}$$

Then, we get

$$\max_{\|\zeta\|_2 \le \Omega} \sum_{j=1}^{L} \zeta_j \left[c^j\right]^T x \le B - \left[c^0\right]^T x. \quad \text{(A.9)}$$

Maximizing the left side of the inequality above, let $\|\zeta\|_2 = \Omega$, where Ω is the radius of ball, and we can get explicit constraint form (A.7), which can also be represented as a set of linear inequalities:

$$\begin{cases} -u_j \le \left[c^j\right]^T x \le u_j, j = 1, \ldots, , L, \\ \left[c^0\right]^T x + \max\limits_{\|\zeta\|_\infty \le 1} \sum\limits_{j=1}^{L} \zeta_j \left[c^j\right]^T x \le B. \end{cases} \quad \text{(A.10)}$$

In summary, Proposition A.2 can be obtained. □

Budgeted Uncertainty Set

Proposition A.3. *When Z is the intersection of concentric coaxial box and ellipsoid, specifically called "uncertainty budgeted,"*

$$Z^{\text{Bud}} = \left\{\zeta \in \mathbb{R}^L\colon 0 \le \zeta_i \le 1, \sum_i \zeta_i \le \Gamma, 1 \le \Gamma \le L\right\}, \quad \text{(A.11)}$$

inequality (A.12) is equivalent to constraint (A.2):

$$\begin{aligned} &\left[c^0\right]^T x + \sum_{j=1}^{L} \left|z_j\right| + \pi\left\|\omega_j\right\|_\infty + \le B, \\ &z_j + \omega_j = -\left[c^j\right]^T x. \end{aligned} \quad \text{(A.12)}$$

Proof. Inspired by Ben-Tal [55], the representation of budgeted uncertainty set (12) becomes

$$Z = \left\{\zeta \in \mathbb{R}^L\colon P_1\zeta + p_1 \in K^1, P_2\zeta + p_2 \in K^2\right\}, \quad \text{(A.13)}$$

where we have the following:

(1) $P_1\zeta = [\zeta; 0]$, $p_1 = [0_L \times 1; 1]$, $K^1 = \{[z;t] \in \mathbb{R}^L \times \mathbb{R}\colon \|z\|_\infty \le t\}$, and $K^1_* = \{[z;t] \in \mathbb{R}^L \times \mathbb{R}\colon \|z\|_1 \le t\}$

(2) $P_2\zeta = [\zeta; 0]$, $p_1 = [0_L \times 1; \pi]$, $K^2 = K^1_* = \{[z;t] \in \mathbb{R}^L \times \mathbb{R}\colon \|z\|_1 \le t\}$, and $K^2_* = K^1$

Letting $y_1 = [z; \tau_1]$ and $y_2 = [\omega; \tau_2]$, whence 1-dimensional τ and L-dimensional z and ω, then the following set of constraints can be obtained:

$$\begin{aligned} &\tau_1 + \pi\tau_2 + \left[c^0\right]^T x \le B, \\ &\left(z + \omega_j\right) = -\left[c^j\right]^T x, j = 1, \ldots, L, \\ &\|z\|_1 \le \tau_1, \\ &\|\omega\|_\infty = \tau_2. \end{aligned} \quad \text{(A.14)}$$

Then, we get

$$\begin{aligned} &\sum_{j=1}^{L} \left\|z_j\right\| + \pi \max_j \left|\omega_j\right| + \left[c^0\right]^T x \le B, \\ &z_j + \omega_j = -\left[c^j\right]^T x. \end{aligned} \quad \text{(A.15)}$$

Maximizing the left side of the inequality above, we can also get a system of constraint linear inequalities (A.12). □

Data Availability

All data are randomly generated by Monte Carlo simulation, which can be copied, but may not get the same result.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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