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Research Article

Synchronous Behavior of Coupled Oscillatory Network with Different Interconnection Patterns

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This paper concerns with the synchronous behavior of coupled oscillatory networks with positive and negative interconnections. Furthermore, the coupling strength is related to the frequencies of oscillators, and we study the dynamical behavior of power networks with weight coefficient between oscillators. Most importantly, time delay is inevitable in the propagation power through the transmission line. Thus, the impact of time delay on synchronous state of oscillatory network is discussed. Simulation results show that the impact of positive interconnection is not obvious on the synchronous state of power network. Nevertheless, for negative coupling, large coupling strength between oscillators decreases the synchronous performance. In addition, we find that when the weight coefficient is even, the synchronous ability of network increases with the increase in weight coefficient whatever the coupling strategy and topological structure be. However, when weight parameter is odd, the entire network with negative coupling cannot achieve synchronization. Finally, our results show that time delay has a critical value $\tau = 1$, and the evolutions of order parameters are different under different values of time delay.

1. Introduction

Nonlinear dynamics has been developed to study general dynamical systems with nonlinearity and has become an important mathematical tool in power system dynamics [1, 2]. Power systems are very complicated dynamical system, and their dynamical behavior is of importance for a secure operation of power network. Moreover, power networks play crucial roles in any country's aspects and are an essential part of the infrastructure of our modern society [3, 4]. On the other hand, the second-order Kuramoto-like oscillator model has been established as a standard model describing the collective phenomenon, which is of great interest in power systems [5, 6]. Synchronization is considered the most ubiquitous collective behavior appearing in natural systems [7, 8]. Additionally, the topology of the network and the distribution of power generation and consumption play important roles in synchronization. In recent years, many researchers have investigated synchronous stability based on the Kuramoto-like model [9–12]. For instance, researchers presented the synchronized condition in terms of complex network topology and parameters. Reference [9] investigated the stability of decentralized power networks against dynamical perturbations. Synchronization of coupled units has been investigated in several networks. In most studies, homogeneous coupling patterns between oscillators are assumed. Thus, various interactions coexist within the same system in practice. An important example is that of systems with positive and negative interactions [7]. For example, cooperation and competition relationship coexist in ecological networks. Moreover, more complex coupling formats may increase the dynamical behavior of the coupled network. In addition to this, in many real-world networks, there is a weight related to lines and nodes [13–15]. In this way, the synchronous state is dependent on competition between the phase differences and the natural frequencies of oscillators. It is generally believed that the coupling types of the oscillatory network are impacted by the native frequencies of the oscillators [16]. This implies that the coupling strength plays a crucial role on the synchronization in oscillatory networks [17–21]. So far, many previous works mainly considered the influence of the topology structure, instead of the oscillator itself, on the synchronization of oscillatory networks. Most importantly, due to the distance between generators and consumers, time delay is inevitable in the propagation of power through the transmission line [22–24]. Other researchers have investigated the synchronization of coupled oscillators. For example, the notion of a master stability function can be used to determine the stability of synchronous solutions of identical oscillators with linear coupling [25, 26]. Moreover, by assuming that all oscillators have an identical natural frequency, authors studied the dynamics behavior of oscillators [8, 15, 22, 27–30]. In this work, we carry out simulations on synchronization in populations of coupled Kuramoto oscillators with different natural frequency, with the aim of developing results to analyze the influence of coupling patterns and time delay in synchronous state of power network.

This paper is organized as follows. First, a coupled oscillatory network model with positive and negative connection is presented. Moreover, the condition of synchronous state stability is analyzed in Section 2. Then, in Section 3, the effect of the coupling strategies on the synchronizability of elementary networks is investigated. In Section 4, the impact of time delay on synchronous behavior is studied. Finally, the conclusions of this work are drawn in Section 5.

2. Mathematical Model and Linear Stability

The Kuramoto-like model is considered as the paradigm model to describe power grids [28]. The model can be expressed as follows:

$$\begin{cases} \dot{\theta}_i = \omega_i, \\ \dot{\omega}_i = -\alpha_i\omega_i + P_i + K\sum_{j=1}^{N} A_{ij}\sin\left(\theta_j - \theta_i\right), i = 1, 2, \ldots, N, \end{cases} \quad (1)$$

where θ_i and $\dot{\theta}_i = \omega_i$ are the instantaneous phase and frequency of the $i-th$ oscillator, respectively. This model is designed to represent distribution power network. It describes the collective dynamics of N subgrids with an effective damping coefficient α rotating at grid frequency $\Omega = 2\pi \times 50$ HZ in the normal operation state. K is the line capacity of the power transmission between subgrid i and j. P_i denotes the power that is supplied $(P_i > 0)$ or consumed $(P_i < 0)$ at node i, and α_i is the damping constant that we take equally for all oscillators. We focus on two types of interconnection that coexist in the oscillatory network. Hence, the coupling matrix can be defined as following. If the connection between oscillators is positive, then the element is $A_{ij} = 1$, while the connection is negative, the element is $A_{ij} = -1$ and $A_{ij} = 0$ denotes there are no connections between oscillators. In this way, the coupling matrix can be described as $A = A^+ + A^-$; thus, equation (1) is rewritten as follows:

$$\begin{cases} \dot{\theta}_i = \omega_i, \\ \dot{\omega}_i = -\alpha_i\omega_i + P_i + K\sum_{j=1}^{N}\left[A_{ij}^{+}\sin\left(\theta_j - \theta_i\right) + A_{ij}^{-}\sin\left(\theta_i - \theta_j\right)\right], i = 1, 2, \ldots, N. \end{cases} \quad (2)$$

This model is more realistic since we mostly observe the interconnections of power networks with two types between elements.

In this paper, we suppose the power distribution with the form as follows:

$$P_i = (-1)^{i+1}P + \lambda_i, \quad i = 1, 2, \ldots, N-1, \quad (3)$$

where $\lambda_i \in [0, 1]$ is the adjustable parameter and $\sum_{i=1}^{N} P_i = 0$.

In order to keep power balance in real power network, the synchronous state of the network is given by

$$\begin{cases} \dot{\omega}_i = 0, \\ \dot{\omega}_i = \omega_s, i = 1, 2, \ldots, N, \end{cases} \quad (4)$$

where $r(t) = 1/N\sum_{j}^{N} e^{i\phi_j(t)}$ is the synchronous frequency of the coupled network.

Define the variable $\triangle_{ji} = \theta_j - \theta_i$.

So, equation (2) can be rewritten as follows:

$$-\alpha w_s + P_i + K\sum_{i=1}^{N}\left[A_{ij}^{+}\sin\left(\triangle_{ji}\right) - A_{ij}^{-}\sin\left(\triangle_{ji}\right)\right] = 0. \quad (5)$$

Based on this condition and the summation of all the equations of the algebraic system in (2), we can find the relations:

$$\begin{cases} \omega_s = 0 & N \text{ is odd} \\ \omega_s = \dfrac{\sum_{i=1}^{N} P_i}{\alpha N} = \dfrac{\overline{P}}{\alpha}N \text{ is even} \end{cases}, \quad (6)$$

where $\overline{P}$ represents the average value of all P_i.

In order to quantity the synchronization behavior, we introduce the order parameter $r(t)e^{i\Psi(t)} = 1/N\sum_{j=1}^{N} e^{i\phi_j(t)}$ which corresponds to the coherence of the collective motion of the oscillators [27] where $\Psi(t)$ indicates the average phase of oscillators and the range of order parameter is $0 \leq r(t) \leq 1$. In addition, the value of $\Psi(t)$ represents the average phase of the ensemble of oscillators.

2.1. Linear Stability Analysis. In the following, we intend to study the linear stability of the synchronous state. Firstly, we linearize (1) around a steady state $(\theta_i^*, 0)$ Thus, one can obtain the linearized dynamics using the matrix differential equation:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I_N \\ L & -\alpha \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} = J \begin{bmatrix} \theta \\ \omega \end{bmatrix}, \quad (7)$$

where I_N denotes unit matrix and L is a Laplacian matrix denoting the structure of the network, which is given by

$$L_{ij} = \begin{cases} -K\cos\left(\theta_j^* - \theta_i^*\right), & i \neq j, \\ -\sum_{l \neq i}^{n} L_{il}, & i \neq j. \end{cases} \quad (8)$$

Since L is Laplacian matrix, one can diagonalize by substituting $J = QLQ^{-1}$, where the elements of matrix Q are the eigenvectors of L and J is the diagonalized matrix composed by u_j.

In the following, we can rewrite (7) using $Y_1 = Q^{-1}X_1$ and $Y_2 = Q^{-1}X_2$.

$$\begin{bmatrix} \dot{Y}_{1j} \\ \dot{Y}_{2j} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ J & -\alpha \end{bmatrix} \begin{bmatrix} Y_{ij} \\ Y_{2j} \end{bmatrix}. \quad (9)$$

It is well known that if the largest nonzero Lyapunov exponent is negative, then the stationary solution is linearly stable and is unstable otherwise. Hence, the synchronous state stability is determined by the following eigenvalues:

$$\lambda_{\pm,j} = \frac{-\lambda_A \pm \sqrt{\lambda_A^2 - 4u_j}}{2}, \quad \text{for } j = 1, 2, \ldots, n. \quad (10)$$

So, the synchronous state of the system is stable if and only if the real parts of the eigenvalues are negative, that is,

$$\max \Re\left(\lambda_{\pm,j}\right) < 0. \quad (11)$$

That is, the stability of the oscillatory power network is governed by the eigenvalues of Laplacian matrix. As we know, coupling strategies correspond to the Laplacian matrix.

In the next step, we will analyze the effect of coupling schemes on synchronizability of oscillatory network.

3. Effects of Coupling Strategy on Synchronous State

3.1. The Chain Oscillatory Network. In this subsection, firstly, we give an elementary model consisting of three elements, a generator and two consumers under different coupling strategies, described as in Figure 1.

According to Figure 1, we give the dynamical model of the chain network with three nodes.

$$\begin{cases} \ddot{\theta}_1 = -\alpha\dot{\theta}_1 + P_1 + K\left[A_{ij}^{-}\sin\left(\theta_2 - \theta_1\right) + A_{ij}^{+}\sin\left(\theta_3 - \theta_1\right)\right], \\ \ddot{\theta}_2 = -\alpha\dot{\theta}_2 - P_2 + K\left[A_{ij}^{-}\sin\left(\theta_2 - \theta_1\right)\right], \\ \ddot{\theta}_3 = -\alpha\dot{\theta}_3 - P_3 + K\left[A_{ij}^{+}\sin\left(\theta_3 - \theta_1\right)\right. \end{cases} \quad (12)$$

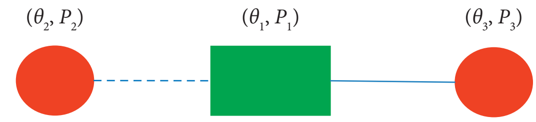

Figure 1: The chain network model. A generator (square) injects a power, and two loads (circles) consume power, each consuming a power in arbitrary units where dash line is negative coupling and solid line is positive coupling.

As shown in (12), we notice that the capacities of transmission lines are considered the same values. However, in real situations, the transmission lines have different capacities and this ability is related to the frequency of nodes. Therefore, in the present paper, we focus our attention on oscillatory power network with different capacities. Thus, the chain dynamical network model translates to

$$\begin{cases} \ddot{\theta}_1 = -\alpha\dot{\theta}_1 + P_1 + P_1^c\left[A_{ij}^{-}\sin\left(\theta_2 - \theta_1\right) + A_{ij}^{+}\sin\left(\theta_3 - \theta_1\right)\right], \\ \ddot{\theta}_2 = -\alpha\dot{\theta}_2 - P_2 + P_2^c\left[A_{ij}^{-}\sin\left(\theta_2 - \theta_1\right)\right], \\ \ddot{\theta}_3 = -\alpha\dot{\theta}_3 - P_3 + P_3^c\left[A_{ij}^{+}\sin\left(\theta_3 - \theta_1\right),\right. \end{cases} \quad (13)$$

where weight coefficient c corresponds to the capacities of transmission lines. One can adjust the values of parameters to meet the power balance.

In what follows, we will study the influence of the weight coefficient c on synchronous behavior of oscillatory network with positive and negative couplings. Without loss of generality, we assume that $P_1 = 2$ and $P_2 = -1, P_3 = -1$. The average frequency value of $\omega_s = 0$ and the initial value is $(\theta_0, \omega_0) = (0, 0)$. The adjacent matrix is described by

$$A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}. \quad (14)$$

Figures (2) and (3) describe the influence of the coupling parameter $c$on synchronizability for oscillatory network with positive and negative coupling patterns. Figure 2(a) illustrates the order parameter evolutions of the oscillatory network with weight coefficient $c = 0$.

In Figures 2 and3, the order parameter is depicted as a function of coupling strength. We take the weight coefficients as odd in Figure 3(a) and they are even in Figure 3(b). In Figure 3(a), we can find that there is no consistency in the evolution of order parameters. Furthermore, the entire network achieves desynchronization of phases. Nevertheless, Figure 3(b) exhibits that the network remains asynchronous up to some critical value, at which its order parameter exhibits a jump to higher values. With the increase of the coupling strength, the order parameter is close to one. These interesting phenomena are explained using the oscillator's native frequency. Based on the proposed oscillatory network model, it is noticed that the weight parameter is odd and the native frequencies are negative, and the coupling between the generators and the consumer oscillators is repulsive. However, when the weight coefficient is odd and the native

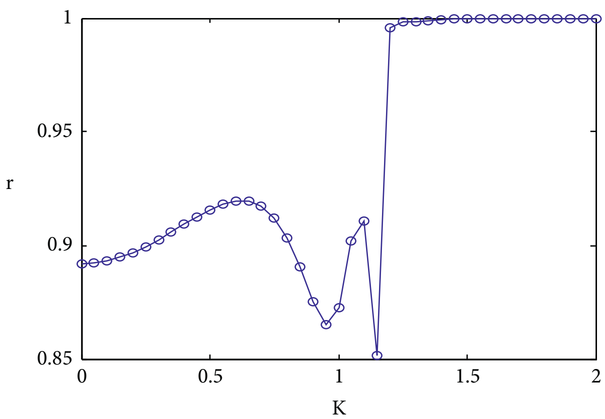

Figure 2: The evolution of order parameter of network with weight parameter $c = 0$.

frequencies are positive, the couplings between oscillators are attracted towards each other. This implies that the attracted and the repulsive couplings coexist in the power network with odd weight coefficient. On the other hand, if the weight coefficient is even, the coupling are always attracted, which denotes that there exists only attracted coupling in the whole network. Then, the oscillators gradually approach each other from the opposite direction. In this situation, the fluctuation of the order parameter is large. Then, while increasing the coupling strength, more and more oscillators are locked and all the oscillators slow down. Finally, when the coupling strength is large enough to lock all the oscillators, the frequency synchronization is achieved. That is, the order parameter grows larger with the increasing of the coupling strength.

3.2. The Ring Oscillatory Network. In order to investigate the effect of coupling patterns of oscillatory power network, we will study the case of a ring network, represented in Figure 4. Based on the topological structure, the dynamics of this kind of network is described by the following equation [29]:

$$\begin{cases} \dot{\theta}_i = \omega_i, \\ \dot{\omega}_i = -\alpha_i\omega_i + P_i + K\left[A_{ij}^{+}\sin\left(\theta_i - \theta_{i+1}\right) + A_{ij}^{-}\sin\left(\theta_i - \theta_{i-1}\right)\right] \quad i = 2, \ldots, N. \end{cases} \tag{15}$$

In what follows, we investigate the equilibria of the ring network, and the Laplacian matrix can be defined as follows

$$\begin{aligned} L_{i,i-1} &= K\cos\left(\theta_i - \theta_{i-1}\right), \\ L_{i,i+1} &= K\cos\left(\theta_i - \theta_{i-1}\right), \\ L_{i,i} &= -K\left[\cos\left(\theta_i - \theta_{i-1}\right) + \cos\left(\theta_i - \theta_{i-1}\right)\right]. \end{aligned} \tag{16}$$

Here, we suppose that the ring network consists of number nodes with half generators and half consumers. Thus, the phase differences between neighbors are as follows:

$$\phi_1 = \theta_1 - \theta_N\,(\mathrm{mod}2\pi), \phi_{i+1} = \theta_{i+1} - \theta_i\,(\mathrm{mod}2\pi). \tag{17}$$

To calculate the equilibrium points, we set $(\theta_i, \dot{\theta}_i) = (\theta^s, 0)$ in (15) from which we obtain the following equations:

$$\sin\phi_i - \sin\phi_{i+1} = \frac{P_i}{K}, \quad i = 1, \ldots, N. \tag{18}$$

Since all phase differences should be restricted to one period of ϕ_i, the following additional requirement holds:

$$\sum_{i=1}^{N}\phi_i = 2k\pi, \quad k \in \left\{-\left[\frac{N}{2}\right], \ldots, -1, 0, 1, \ldots, \left[\frac{N}{2}\right]\right\}. \tag{19}$$

$[N/2]$ represents the floor value of $N/2$, where the largest integer value which is smaller than or equal to $N/2$.

Actually, each equilibrium relates to a synchronous state. Firstly, we focus on the special case with the interconnections between nodes of the ring network being symmetric. That is, half the connections are positive and half interconnections are negative. In the next step, we will investigate the synchronous behavior of the ring oscillatory network with different values of weight coefficient between nodes.

In Figure 5, one can find that the critical value of coupling strength is the smallest when the weight coefficient is equal to zero. From Figure 5(a) and 5(b), we can observe that the order parameter is decreased with the increase of weight coefficient whatever the characteristic of weight coefficient be.

In what follows, we assume that all the interconnections of the ring network are positive or negative, respectively. Then, we observe the evolution process of order parameter in two cases.

Figure 6(a) shows the simulation results of oscillatory ring network with positive interconnection, we can clearly see that weight coefficients are even, and the evolutions of order parameters are unchanged with different values of weight coefficients. Furthermore, and the order parameter is gradually close to one with the increase of the coupling strength. Finally, the ring network realizes synchronization. However, Figure 6(b) exhibits that the weight coefficient is odd, and the order parameter keeps a constant. Moreover, the entire network cannot achieve synchronization with the increasing of coupling strength. It is noticed that the evolutions of order parameters with coupling strength have quite different processes in Figure 5. This is because the weight parameter with different categories and coupling schemes has significant effect on synchronizability of power network.

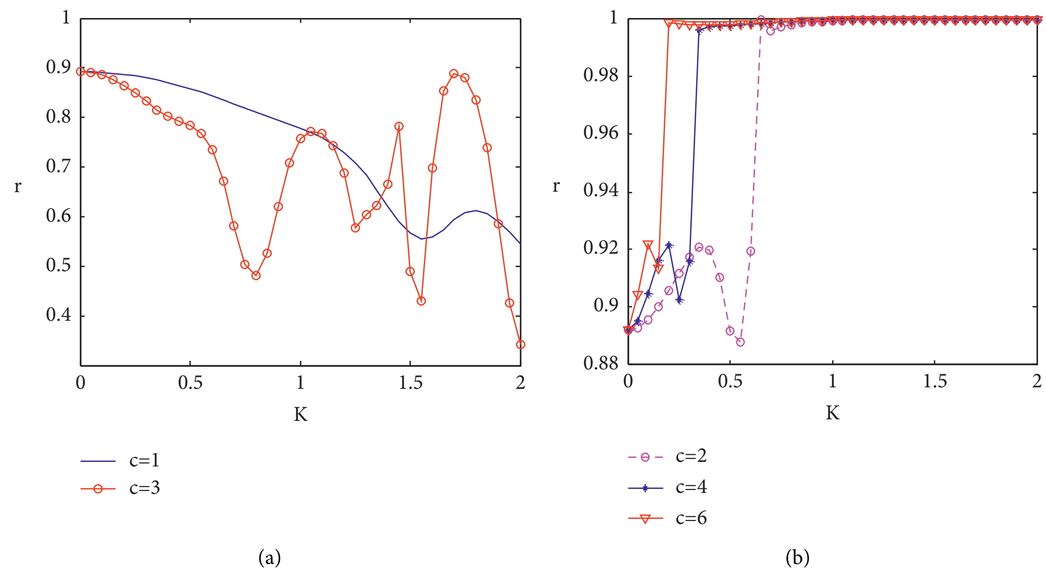

FIGURE 3: The evolutions of order parameters of network with different values of weight coefficients: (a) weight coefficients are odd and (b) weight coefficients are even.

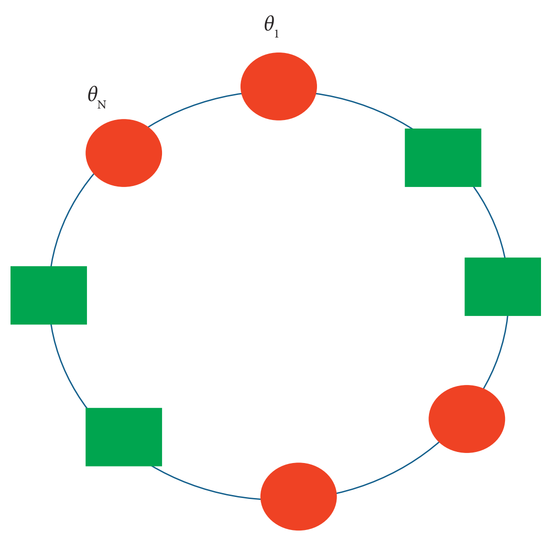

FIGURE 4: The topological structure of a ring network. Circle nodes are generators, and square nodes are consumers.

4. Effects of the Transmission Delay

In the power network, due to the distance between generators and consumers, transmission delay is inevitable in the propagation power through the transmission line. Moreover, time delay can strongly influence the dynamical behavior of the oscillatory network. In presence of time delay, the equations describing the dynamics of the system are given by

$$\begin{cases} \dot{\theta}_i = \omega_i, \\ \dot{\omega}_i = -\alpha_i \omega_i + P_i + K \sum_{j=1}^{N} A_{ij} \sin\left(\theta_j (t - \tau) - \theta_i (t - \tau)\right), \quad i = 1, 2, \ldots, N, \end{cases} \tag{20}$$

where the time delay is τ, and in this section, we assume that the transmission capabilities are equal on all the lines. In addition, we suppose that all the coupling patterns are positive interconnection between oscillators. Similarly, star and ring network structures are considered, respectively. The effects of time delay on the synchronizability are plotted in Figure 7. Figure 7(a) gives the evolution of order parameter versus the coupling strength for ring network under

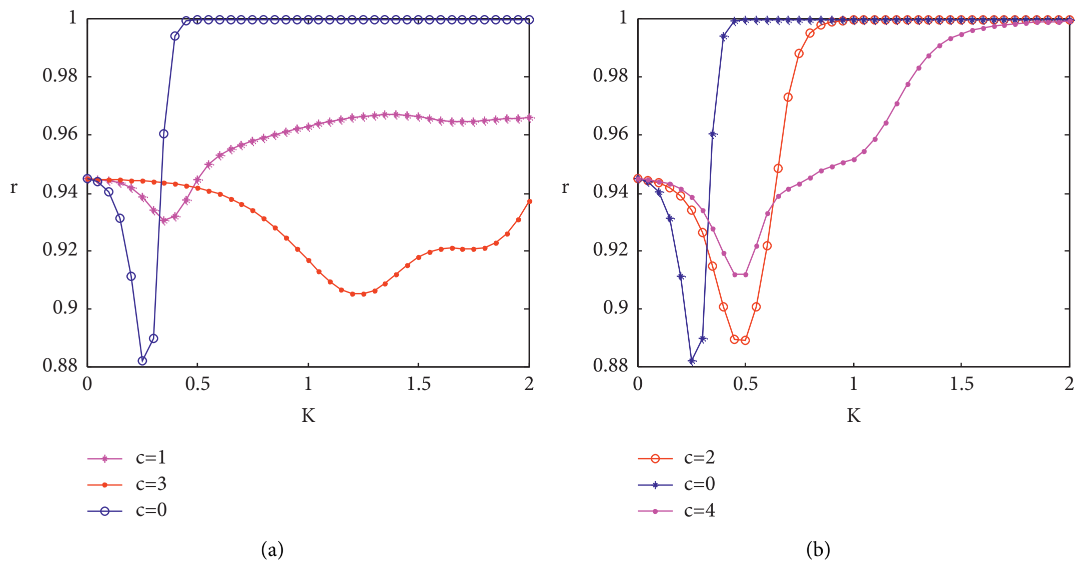

FIGURE 5: The evolutions of order parameters for a ring network with different weight coefficients.

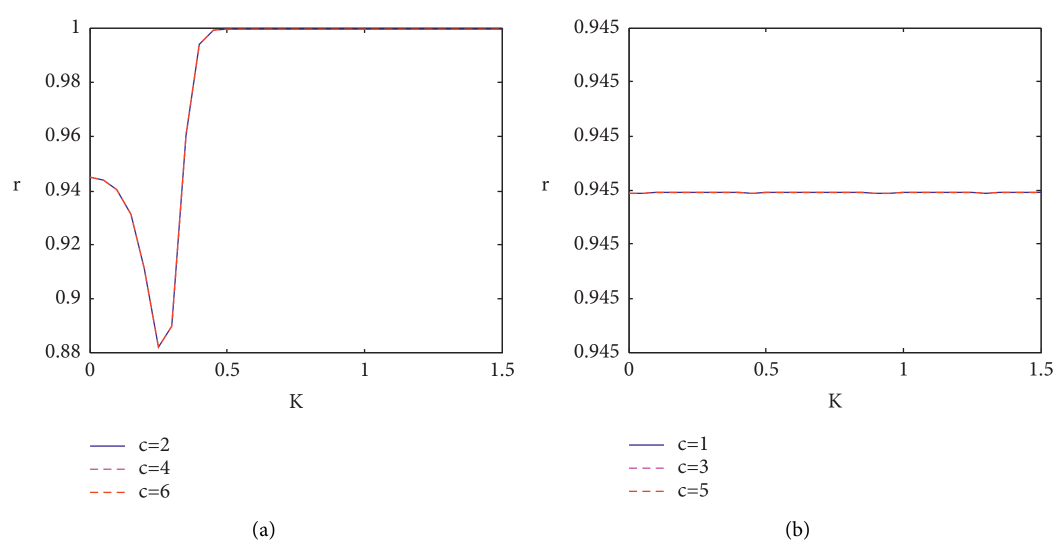

FIGURE 6: The evolutions of order parameters for a ring network with different weight coefficients: (a) the interconnections of network are positive; (b) the interconnections of network are negative.

different values of time delays while Figure 7(b) depicts the same order parameter of the chain network for different values of time delay.

According to Figure 7, the synchronizability of networks is related to the values of time delay. We notice that, there is one critical value of the order parameter where the network keeps synchronous state. For the time delay equal to $\tau \leq 1$, the order parameter increases with the increase of the coupling strength. In addition, one can find that when the time delay $\tau > 1$, the order parameter decreases with the increase of coupling strength whatever the topological structures of networks be.

This is because the phase difference of the oscillators gets larger with the increase of time delay under the same coupling strength. Thus, time delay $\tau = 1$ can be considered as a critical value with parameters used in Figure 7. This is a critical time delay, above which the network cannot achieve synchronization whatever the value of the coupling strength be.

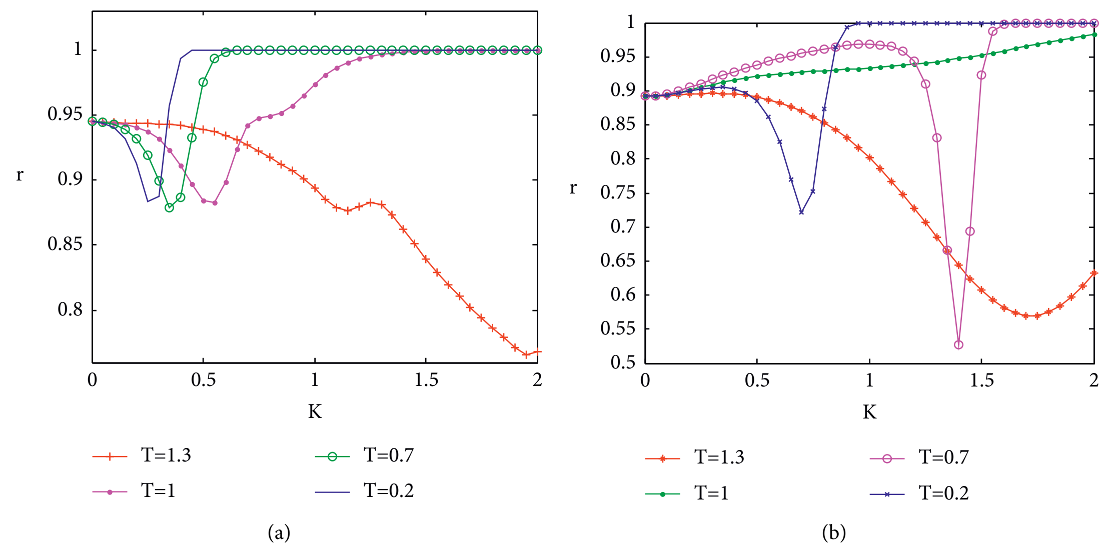

FIGURE 7: The order parameter evolution of oscillatory network under different values of time delay: (a) ring network and (b) chain network.

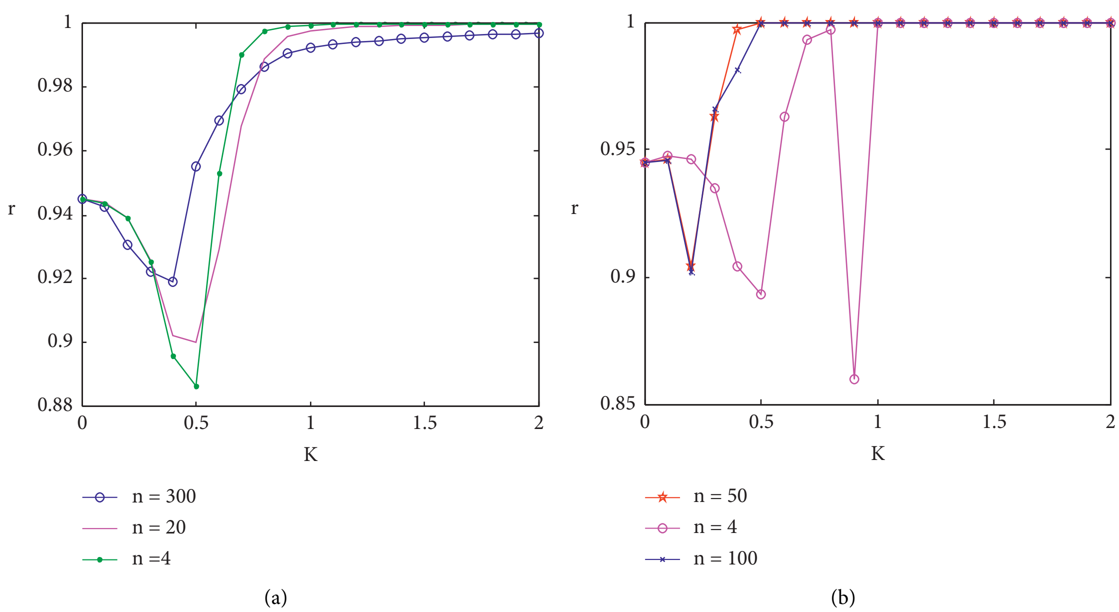

FIGURE 8: The influence of nodes number on dynamics behavior of network with different structures: (a) ring-like topology structure and (b) network with WS topology.

5. Discussion and Conclusion

5.1. The Influence of Node Numbers. In this subsection, we will investigate the synchronous behavior of the power network with different node numbers. Here, we select the weight parameter $c = 0$. Numerical simulations are shown in Figure 8.

From Figure 8(a), it is clearly seen that the smaller the nodes number is, the closer the order parameter is to one. That is, the synchronizability of the ring network decreases with the increase of nodes number. However, Figure 8(b) shows that there is a counterintuitive range of coupling strength values where the synchronous ability increases as the nodes number increases, based on a WS topology structure. This is because synchronous behavior will be influenced many factors besides topology structure. Therefore, it is difficult to obtain a qualitative result for this perspective.

5.2. Conclusions. This paper presents a more practical power network model with positive and negative coupling strategies to analyze the collective behavior. Especially, the proposed model takes into consideration the contribution of transmission line; moreover, the role of the weight coefficient on the synchronous state of the power network is studied. Simualtion results show that oscillatory ring network with positive interconnection and even weight coefficients. Furthermore, the order parameter is gradually close to one with the increase of the coupling strength. However, when weight coefficient is odd, the order parameter keeps a constant.

In addition, there exists a critical value of the time delay $\tau = 1$, above which the network cannot achieve synchronization whatever the value of the coupling strength be. When the time delay $\tau \leq 1$, the order parameter increases with the increase of the coupling strength. However, when the time delay $\tau > 1$, the order parameter decreases with the increase of coupling strength whatever the topological structures of networks be.

These simulation results reveal that coupling schemes, weight coefficient, and time delay play a crucial role in dynamical behavior of coupled oscillatory networks [31, 32].

Data Availability

The data used to support the findings of this study are included within the paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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