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Research Article

Adaptive Finite-Time Sliding Mode Backstepping Controller for Double-Integrator Systems with Mismatched Uncertainties and External Disturbances

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In this paper, a novel adaptive finite-time sliding mode backstepping (AFSMBS) control scheme is suggested to control a type of high-order double-integrator systems with mismatched disturbances and uncertainties. A robust sliding mode backstepping control method, adaptive control method, and finite-time stability notion are incorporated to provide a better tracking performance over applying them individually and to use their benefits simultaneously. The concept of a sliding mode is used to define a new form of a backstepping controller. The adaptive control method is utilized to adaptively estimate the upper bounds of the disturbances and uncertainties and the estimated data are used in the control low. The notion of the finite-time stability is incorporated with the suggested control scheme to ensure the system's convergence within a finite time. The stability proof is obtained for the closed-loop system in a finite time utilizing the Lyapunov stability theorem. Simulation results are obtained for an example of a remotely operated vehicle (ROV) with three degrees of freedom (3-DOF) to demonstrate the efficacy of the suggested control approach.

1. Introduction

Many practical high-order systems are modeled using nonlinear differential equations due to the stochastic noise, uncertainties in the parameters, and variations in the external environment which are unknown beforehand and may occur in the real system [1]. This makes the control of these systems a challenge and as a result, different nonlinear control methods including the nonlinear stability theory [2], backstepping technique [3], Lyapunov function [4], and sliding mode control (SMC) [5–8] have come into existence. A simple and efficient mathematical framework has been proposed in [9] to tackle nonlinear problems. A novel technique has been suggested in [10] to deal with nonlinear evolutionary issues. A system described as a classical integer-order differential problem has been investigated in [11] to explore the complexities of the human liver.

An effective scheme found in the literature to deal with uncertainties in single or double-integral system is the adaptive control [12, 13] and notable adaptive design methods have been proposed in [14–16] for the control of high-order systems. An adaptive compensation control method has been proposed in [17] to deal with mismatched disturbances as well as uncertain faults. In addition, the finite-time stability is known for its fast transient performance achievement [18]. Hence, adaptive finite-time control encompasses the merits of both control techniques. It guarantees superior disturbance rejection, robustness properties, and faster convergence rates [19, 20].

The downside to finite-time adaptive control is the complications involved in estimating the upper bound of disturbances and uncertainties [21, 22]. Terminal SMC (TSMC) has been incorporated in [23] due to its robustness to obtain adaptive finite-time convergence, fast convergence, improved transient performance, and higher precision for high-order systems. However, singularity issues were present in the controller [24]. Nonsingular TSMC was applied in [25] to solve the singularity issue; however, finite-time convergence was not achieved and the convergence rate to the equilibrium was slow. An integration of adaptive control with nonsingular TSMC was suggested in [26] to tackle the issue of unknown upper boundaries in adaptive control. The resulting control laws were, however, discontinuous across the terminal sliding mode surface when external disturbances were involved. In [27], finite-time SMC has been incorporated with the adaptive control method to provide the estimated data in the controller. However, undesirable chattering phenomenon exists in the control signal of this control method.

Backstepping is a technique introduced in the 1990s [28] to solve regulation or tracking control problems considering uncertainties in nonlinear feedback systems [29, 30]. The control design process in backstepping begins at the source of the high-order system and backs out to new controllers which stabilize each of the outer subsystems a step at a time till the final control law is obtained [31]. The stability analysis is established by selecting a suitable Lyapunov function [32]. Backstepping is generally used as an alternative to feedback linearization [33]. It provides advantages ranging from transient performance improvement, achieving global stability to achieving a model-free control scheme [34–36]. It has the disadvantage of not being applicable to unparameterized systems or nonlinear systems with structural uncertainties or [32, 37]. It can, however, be combined with different control techniques to solve problems relating to parameter uncertainties, unmodeled dynamics, or external disturbances.

Motivated by the aforementioned discussions, a new and enhanced type of sliding mode backstepping control method is proposed where the concept of sliding mode is utilized to define the backstepping controller. It is assumed that there is no information of the upper bounds of disturbances and uncertainties. So, they are adaptively estimated, and the estimated data are provided in the controller. The system's convergence is ensured within a finite time utilizing the Lyapunov stability theorem and the notion of the finite-time stability. The suggested control method is designed for a type of high-order double-integral systems with mismatched uncertainties and external disturbances. Also, an example of ROV with 3-DOF is provided to apply the proposed controller and test its performance. Simulation results reveal the validity of the suggested scheme. The novelties of the research can be highlighted as follows:

(i) A novel incorporation of the robust sliding mode backstepping control method, adaptive control method, and finite-time stability notion is done to provide a superior tracking performance over applying them individually and to use their benefits simultaneously.

(ii) This proposed controller not only ensures the system's finite-time stability but also does not require any knowledge of the upper bound of disturbances and uncertainties for the controller design.

(iii) A new form of the candidate Lyapunov function is defined to obtain the finite-time stability proof for the closed-loop system.

(iv) The proposed control approach is applicable for a wide range of practical applications described by a set of independent double integrator subsystems in the presence of mismatched uncertainties and disturbances.

This article is organized as follows. In Section 2, the system is presented. Mathematical preliminaries and lemmas are given in Section 3. In Section 4, the stability proof is obtained within a finite time utilizing Lyapunov stability theorem. In Section 5, the designed control laws are applied to the ROV with 3-DOF. Section 6 gives the conclusions.

2. Problem Statement

Consider the high-order double-integrator system that includes the mismatched uncertainties and external disturbances.

$$\begin{cases} \dot{x}_1 = x_2 + d_1, \\ \dot{x}_2 = f_1(t,x) + g_1(t,x)u_1 + d_2, \\ \dot{x}_3 = x_4 + d_3, \\ \dot{x}_4 = f_2(t,x) + g_2(t,x)u_2 + d_4, \\ \vdots \\ \dot{x}_{2n-1} = x_{2n} + d_{2n-1}, \\ \dot{x}_{2n} = f_n(t,x) + g_n(t,x)u_n + d_{2n}, \end{cases} \tag{1}$$

where $f_j(t,x), g_j(t,x), j = (1,2,\ldots,n)$ are smooth nonlinear functions; $g_j^{-1}(t,x)$ is available and nonsingular; $d_i, i = (1,2,\ldots,2n)$ is the model of uncertainties and external disturbance; and u_j is the system's control inputs. The system can be rewritten as follows:

$$\begin{cases} \dot{x}_{2j-1} = x_{2j} + d_{2j-1}, \\ \dot{x}_{2j} = f_j(t,x) + g_j(t,x)u_j + d_{2j}. \end{cases} \tag{2}$$

The external disturbances and uncertainties are given as follows:

$$d_i \le h_i \text{ where } h_i \le \widehat{h}_i \le h_i^*. \tag{3}$$

Here, h_i is the uncertainty upper bound (that is assumed to be unknown); $\widehat{h}_i$ is the estimation of their upper bounds; and d_i is the Euclidean norm of disturbances and uncertainties. In the following sections, the finite-time control law is defined utilizing a sliding mode backstepping control scheme. The uncertainty upper bounds are also adaptively estimated and they are utilized in the controller.

Remark 1. A wide range of practical applications can be described by a set of independent double integrator subsystems (given by (1)) including the three-link robotic manipulator [38, 39], ship course system [40], two-link robotic manipulator [41], support structure system for offshore wind turbines [42], etc.

3. Mathematical Preliminaries and Lemmas

Definition 1. Consider a nonlinear system as shown below:

$$\dot{x} = f(t, x), \tag{4}$$

where $x \in \mathbb{R}^n$ is the vector of the system states; $f: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a nonlinear function; and t is considered on the interval $[t_0, \infty)$, where $t_0 \in \mathbb{R}_+ \bigcup \{0\}$. Also, we have $x(t_0) = x_0$.

The origin of (4) has global finite-time stability if it has global asymptotic stability and any solution $x(t, x_0)$ of (4) converges to the origin at some finite time moment for all x_0; i.e., $\forall t \geq T(x_0): x(t, x_0) = 0$, where $T: \mathbb{R}^n \longrightarrow \mathbb{R}_{\geq 0}, \forall x_0 \in \mathbb{R}^n$, is named settling time function, then the origin of (4) has global finite-time stability [43, 44].

Definition 2. The signum function is defined as follows:

$$\operatorname{sign}(a) = \begin{cases} 1; a > 0, \\ 0; a = 0, \\ -1; a < 0. \end{cases} \tag{5}$$

We note that $|a| = a\operatorname{sign}(a)$ is always true.

Definition 3. The function $\operatorname{sig}^a(x)$ is given as follows:

$$\operatorname{sig}^a(x) = |x|^a \operatorname{sign}(x) \tag{6}$$

Thus, we have $x sig^a(x) = |x|^{a+1}$.

Lemma 1. *For each value $a_1, a_2, \ldots, a_n \in \mathbb{R}$ and $0 < q < 2$, we have, $|a_1|^q + |a_2|^q + \ldots + |a_n|^q \geq (a_1^2 + a_2^2 + \ldots + a_n^2)^{q/2}$ [45].*

Lemma 2. *Assume there exist two real numbers as $\rho_1 > 0$ and $0 < \rho_2 < 1$ and a continuously differentiable positive function $V(x): \mathbb{R}^n \longrightarrow \mathbb{R}_{\geq 0}$ in such a way that we have $V(x) = 0$ for $x(t) = 0$. If any solution $x(t)$ of (4) satisfies $\dot{V}(x) \leq -\rho_1 V^{\rho_2}(x)$, then the origin of (4) has global finite-time stability and the settling time will be $T \leq V^{1-\rho_2}(x_0)/\rho_1(1-\rho_2)$ [14, 46].*

4. AFSMBS Controller

Here, the control goal is to define the finite-time controller for the system given by (2). Then, the stability proof is obtained by defining a candidate Lyapunov function. The backstepping control law is defined using the sliding control concept as follows:

$$\begin{cases} u_j = g^{-1}\left(-f_j(t, x) - \hat{h}_{2j}\operatorname{sig}^{\alpha_{2j}}\left(Z_j\right) + \dot{x}_{2j}^*\right), \\ x_{2j}^* = -\hat{h}_{2j-1}\operatorname{sig}^{\alpha_{2j-1}}\left(x_{2j-1}\right), \end{cases} \tag{7}$$

where we have $Z_j = x_{2j} - x_{2j}^*$ and $0 < \alpha_i < 1$.

$$\begin{cases} \dot{\hat{h}}_{2j-1} = r_{2j-1}\left|x_{2j-1}\right|^{\alpha_{2j-1}+1}, \\ \dot{\hat{h}}_{2j} = r_{2j}\left|Z_j\right|^{\alpha_{2j}+1}, \end{cases} \tag{8}$$

where we have $0 < r_i < 1$.

The block diagram of the proposed AFSMBS approach is shown in Figure 1.

Theorem 1. *Assume the system given by (2). If the control law (5) and adaptive law (6) are applied to (2), the system's convergence is ensured within a finite time. Also, the uncertainty upper bounds are adaptively estimated within a finite time and the online estimated data are provided in the controller.*

Proof. The stability proof using the backstepping method consists of two phases as follows. □

Phase 1. To prove the first phase, the candidate Lyapunov function is defined as $V_1(x) = 1/2x_{2j-1}^2 + 1/2\tilde{h}_{2j-1}^2$ where $\tilde{h}_{2j-1} = \hat{h}_{2j-1} - h_{2j-1}^*$. Taking its time derivative, we obtain as follows:

$$\begin{aligned} \dot{V}_1(x) &= x_{2j-1}\dot{x}_{2j-1} + \tilde{h}_{2j-1}\dot{\tilde{h}}_{2j-1} \Rightarrow \dot{V}_1(x) \\ &= x_{2j-1}\dot{x}_{2j-1} + \tilde{h}_{2j-1}\dot{\hat{h}}_{2j-1}. \end{aligned} \tag{9}$$

Then, we have as follows:

$$\dot{V}_1(x) = x_{2j-1}\left(x_{2j}^* + d_{2j-1}\right) + \tilde{h}_{2j-1}\dot{\hat{h}}_{2j-1} \Rightarrow \dot{V}_1(x) \leq x_{2j-1}\left(x_{2j}^* + h_{2j-1}\right) + \tilde{h}_{2j-1}\dot{\hat{h}}_{2j-1}. \tag{10}$$

Substituting (5) and (6) into (8) yields as follows:

$$\dot{V}_1(x) \leq x_{2j-1}\left(-\hat{h}_{2j-1} sig^{\alpha_{2j-1}}\left(x_{2j-1}\right) + h_{2j-1}\right) + \tilde{h}_{2j-1} r_{2j-1}\left|x_{2j-1}\right|^{\alpha_{2j-1}+1}. \tag{11}$$

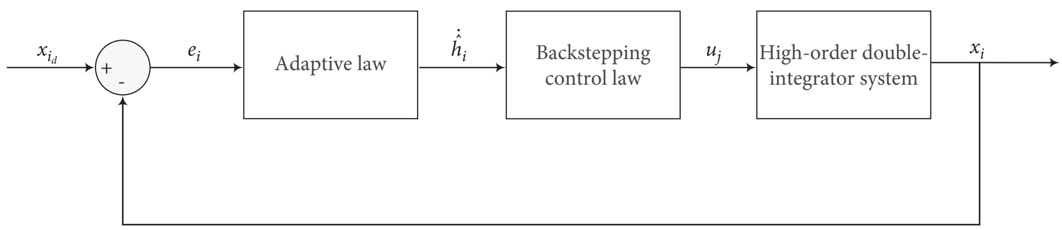

Figure 1: Block diagram of the proposed controller.

We obtain as follows:

$$\dot{V}_1(x) \le \left|x_{2j-1}\right| h_{2j-1} - \hat{h}_{2j-1}\left|x_{2j-1}\right|^{\alpha_{2j-1}+1} + \tilde{h}_{2j-1} r_{2j-1}\left|x_{2j-1}\right|^{\alpha_{2j-1}+1}. \tag{12}$$

Adding $\pm|x_{2j-1}|^{\alpha_{2j-1}+1} h_{2j-1}^*$ to (10) yields as follows:

$$\dot{V}_1(x) \le \left|x_{2j-1}\right| h_{2j-1} - \hat{h}_{2j-1}\left|x_{2j-1}\right|^{\alpha_{2j-1}+1} + \tilde{h}_{2j-1} r_{2j-1}\left|x_{2j-1}\right|^{\alpha_{2j-1}+1} \pm \left|x_{2j-1}\right|^{\alpha_{2j-1}+1} h_{2j-1}^*. \tag{13}$$

As a result, we have as follows:

$$\begin{aligned}
\dot{V}_1(x) &\le -\left|x_{2j-1}\right|\left(\left(\left|x_{2j-1}\right|^{\alpha_{2j-1}} h_{2j-1}^* - h_{2j-1}\right) - \tilde{h}_{2j-1}\left|x_{2j-1}\right|^{\alpha_{2j-1}+1} + \tilde{h}_{2j-1} r_{2j-1}\left|x_{2j-1}\right|^{\alpha_{2j-1}+1}\right. \\
&\Rightarrow \dot{V}_1(x) \le -\left|x_{2j-1}\right|\left(\Delta_{1_1}\right) - \tilde{h}_{2j-1}\left(\left(1 - r_{2j-1}\right)\left|x_{2j-1}\right|^{\alpha_{2j-1}+1}\right) \\
&\Rightarrow \dot{V}_1(x) \le -\left|x_{2j-1}\right|\Delta_{1_1} - \tilde{h}_{2j-1}\Delta_{1_2} \\
&\Rightarrow \dot{V}_1(x) \le -\Delta_m\left(\left|x_{2j-1}\right| + \tilde{h}_{2j-1}\right),
\end{aligned} \tag{14}$$

where $\Delta_{m_1} = \min(\Delta_{1_1}, \Delta_{1_2})$, and according to Lemma 1, we obtain as follows:

$$\dot{V}_1(x) \le -\Delta_{m_1}\left(\left|x_{2j-1}\right|^2 + \tilde{h}_{2j-1}^2\right)^{1/2} \Rightarrow \dot{V}_1(x) \le -\Delta_{m_1}\left(2V_1(x)\right)^{1/2}. \tag{15}$$

Choosing $\rho_{1_1} = \sqrt{2}\Delta_{m_1}$ *and* $\rho_{2_1} = 1/2$, we have, $\dot{V}_1(x) \le -\rho_{1_1} V_1^{\rho_{2_1}}(x)$, and based on Lemma 2, the stability proof of the first phase is guaranteed. Thus, the settling time upper bound T_1 will be $T_1 \le V^{1-\rho_{2_1}}(x_0)/\rho_{1_1}(1 - \rho_{2_1})$.

Phase 2. To obtain the second phase of the proof, the candidate Lyapunov function is considered as $V_2(x) = 1/2 Z_j^2 + 1/2\tilde{h}_{2j}^2$. Taking its time derivative, we obtain the following:

$$\begin{aligned}
\dot{V}_2(x) &= Z_j \dot{Z}_j + \tilde{h}_{2j}\dot{\hat{h}}_{2j} \Rightarrow \dot{V}_2(x) \\
&= Z_j\left(\dot{x}_{2j} - \dot{x}_{2j}^*\right) + \tilde{h}_{2j}\dot{\hat{h}}_{2j}.
\end{aligned} \tag{16}$$

Applying the control law and simplifying it yields the following:

$$\begin{aligned}
\dot{V}_2(x) &= Z_j\left(f_j(t,x) + g_j(t,x)u_j + d_{2j} - \dot{x}_{2j}^*\right) + \tilde{h}_{2j}\dot{\hat{h}}_{2j} \\
&\Rightarrow \dot{V}_2(x) \le -\hat{h}_{2j}\left|Z_j\right|^{\alpha_{2j}+1} + h_{2j}\left|Z_j\right| + \tilde{h}_{2j}\dot{\hat{h}}_{2j}.
\end{aligned} \tag{17}$$

Adding $\pm|Z_j|^{\alpha_{2j}+1} h_{2j}^*$ to (15), we yield as follows:

$$\begin{aligned}\dot{V}_2(x) &\leq -\hat{h}_{2j}\left|Z_j\right|^{\alpha_{2j}+1} + h_{2j}\left|Z_j\right| + \tilde{h}_{2j}\dot{\hat{h}}_{2j} \pm \left|Z_j\right|^{\alpha_{2j}+1} h_{2j}^* \\ &\Rightarrow \dot{V}_2(x) \leq -\left|Z_j\right|\left(\left|Z_j\right|^{\alpha_{2j}} h_{2j}^* - h_{2j}\right) - \tilde{h}_{2j}\left|Z_j\right|^{\alpha_{2j}+1} + \tilde{h}_{2j} r_{2j}\left|Z_j\right|^{\alpha_{2j}+1} \\ &\Rightarrow \dot{V}_2(x) \leq -\left|Z_j\right|\Delta_{2_1} - \tilde{h}_{2j}\left(\left(1-r_{2j}\right)\left|Z_j\right|^{\alpha_{2j}+1}\right) \\ &\Rightarrow \dot{V}_2(x) \leq -\left|Z_j\right|\Delta_{2_1} - \tilde{h}_{2j}\left(\Delta_{2_2}\right) \\ &\Rightarrow \dot{V}_2(x) \leq -\Delta_{m_2}\left(\left|Z_j\right| + \tilde{h}_{2j}\right),\end{aligned} \tag{18}$$

where $\Delta_{m_2} = \min(\Delta_{2_1}, \Delta_{2_2})$ and according to Lemma 1, we have as follows:

$$\dot{V}_2(x) \leq -\Delta_{m_2}\left(\left|Z_j\right|^2 + \tilde{h}_{2j}^2\right)^{1/2} \Rightarrow \dot{V}_2(x) \leq -\Delta_{m_2}\left(2V_2(x)\right)^{1/2}. \tag{19}$$

Choosing $\rho_{1_2} = \sqrt{2}\Delta_{m_2}$, $\rho_{2_2} = 1/2$, we have $\dot{V}_2(x) \leq -\rho_{1_2} V_2^{\rho_{2_2}}(x)$ and based on Lemma 2, the stability proof of the second phase is guaranteed. Consequently, the settling time upper bound T_2 is as $T_2 \leq V^{1-\rho_{2_2}}(x_0)/\rho_{1_2}(1-\rho_{2_2})$.

As a result, the stability proof of the system (2) is completed and the settling time upper bound will be as $T = T_1 + T_2$. n

Remark 2. The proof shows that in a finite time, we have $Z_j \longrightarrow 0$. Consequently, in a finite time, we have $x_{2j} \longrightarrow x_{2j}^*$ as well as all the system states reach zero in a finite time and remains zero. Also, the uncertainty upper bounds are estimated in a finite time.

5. Application Example

In [47–50], the ROV model with 3-DOF has been presented as follows:

$$\begin{cases} p_1\ddot{x} + V_x|V|\left(p_2|\cos(\phi)| + p_3|\sin(\phi)|\right) + p_4 x - p_5 V_{cx}\left\|V_c\right\| = T_x, \\ p_1\ddot{x} + V_y|V|\left(p_2|\sin(\phi)| + p_3|\cos(\phi)|\right)p_4 y - p_5 V_{cy}\left\|V_c\right\| = T_y, \\ p_6\ddot{\phi} + p_7\dot{\phi}|\dot{\phi}| + p_8\left\|V_c\right\|^2 \sin\left(\dfrac{\phi-\phi_c}{2}\right) + p_9 = M_z, \end{cases} \tag{20}$$

where $V = [V_x, V_y]^T = [(\dot{x} - V_{cx}), (\dot{y} - V_{cy})]^T$ and $V_c = [V_{cx}, V_{cy}]^T$ are the vectors of speed in directions x, y that are constants; $p_i, i = (1, 2, \ldots, 9)$ are constants which are provided in Table 1 with their uncertainties; $(T_x, T_y, M_z) = (u_1, u_2, u_3)$ are control inputs that need to be designed; ϕ_c is the current angle between the x axis and the speed direction.

To obtain the system state equations, state variables are defined as follows, $X = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [x, \dot{x}, y, \dot{y}, \phi, \dot{\phi}]$. Then, the state equations are rewritten as follows:

$$\begin{cases} \dot{x}_1 = x_2 + d_1, \\ \dot{x}_2 = -p_1^{-1}\left(V_x\|V\|\left(p_2\left|\cos(x_5)\right| + p_3\left|\sin(x_5)\right|\right)p_4 x_1, \\ \dot{x}_3 = x_4 + d_3, \\ \dot{x}_4 = -p_1^{-1}\left(V_y\|V\|\left(p_2\left|\sin(x_5)\right| + p_3\left|\cos(x_5)\right|\right) + p_4 x_3 - p_5 V_{cy}\left\|V_c\right\|\right) + d_4 + p_1^{-1}u_2, \\ \dot{x}_5 = x_6 + d_5, \\ \dot{x}_6 = -p_6^{-1}\left(p_7 x_6\left|x_6\right| + p_8\left\|V_c\right\|^2 \sin\left(\dfrac{x_5 - \varnothing_c}{2}\right) + d_6 + p_6^{-1}u_3\right), \end{cases} \tag{21}$$

Table 1: System parameters with their uncertainties.

p_1	12670	$Kg \pm 10\%$	p_2	2667	$\mathrm{Kg} \cdot m^{-1} \pm 10\%$	p_3	4934	$\mathrm{Kg} \cdot m^{-1} \pm 10\%$
p_4	417	$N \cdot m^{-1} \pm 5\%$	p_5	46912	$\mathrm{Kg} \cdot m^{-1} \pm 10\%$	p_6	18678	$\mathrm{Kg} \cdot m^2 \pm 10\%$
p_7	9200	$\mathrm{Kg} \cdot m^2 \pm 10\%$	p_8	−308.4	$Kg \pm 5\%$	p_9	1492	$N \cdot m \pm 5\%$

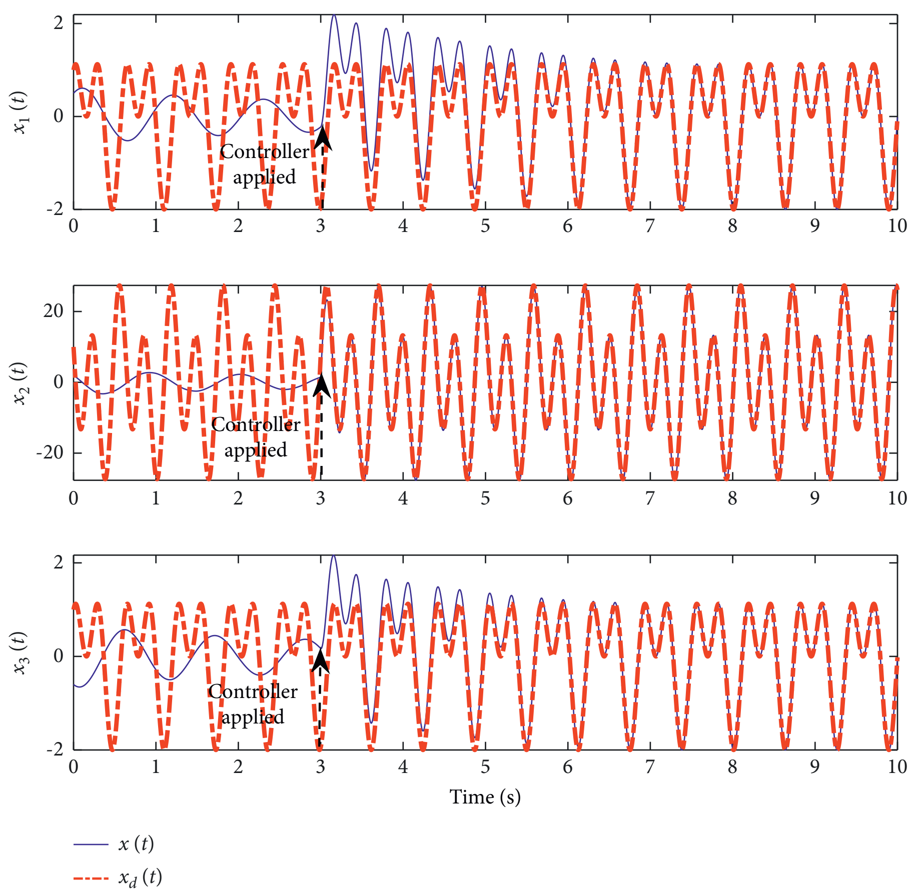

Figure 2: Time responses of x_1, x_2, x_3 and x_{1_d}, x_{2_d}, x_{3_d} using AFSMBS.

where $\|V_c\| = \sqrt{V_{cx}^2 + V_{cy}^2}$, $\|V\| = \sqrt{V_x^2 + V_y^2}$, and d_j, $j = (1, 2, \ldots, 6)$ is the system uncertainty model.

The control goal is to fulfill a trajectory tracking problem for the ROV; hence, the dynamic error is defined as $e_j = x_j - x_{j_d}$. Accordingly, the dynamic error is given as follows:

$$\begin{cases} \dot{e}_1 = e_2 + d_1, \\ \dot{e}_2 = f_1 + d_2 - \dot{x}_{2_d} + p_1^{-1} u_1, \\ \dot{e}_3 = e_4 + d_3, \\ \dot{e}_4 = f_2 + d_4 - \dot{x}_{4_d} + p_1^{-1} u_2, \\ \dot{e}_5 = e_6 + d_5, \\ \dot{e}_6 = f_3 + d_6 - \dot{x}_{6_d} + p_6^{-1} u_3, \end{cases} \quad (22)$$

where

$$\begin{cases} f_1 = -P_1^{-1}\left(V_x \|V\| \left(p_2 |\cos(x_5)| + p_3 |\sin(x_5)|\right) + p_4 x_1 - p_5 V_{cx} \|V_c\|\right), \\ f_2 = -P_1^{-1}\left(V_y \|V\| \left(p_2 |\sin(x_5)| + p_3 |\cos(x_5)|\right) + p_4 x_3 - p_5 V_{cy} \|V_c\|\right), \\ f_3 = -P_6^{-1}\left(p_7 x_6 |x_6| + p_8 V_c^2 \sin\left(\dfrac{x_5 - \phi_c}{2}\right) + p_9\right). \end{cases} \quad (23)$$

The desired trajectory tracking is considered as follows:

$$x_{i_d} = \cos(20t) + \sin(10t). \quad (24)$$

In this section, the designed controller in previous section is applied to the ROV with 3-DOF given by (18). It is to be noted that $\dot{x}_{2j_d}$ given in the control law (5) should track the desired trajectory. To obtain simulation results, the

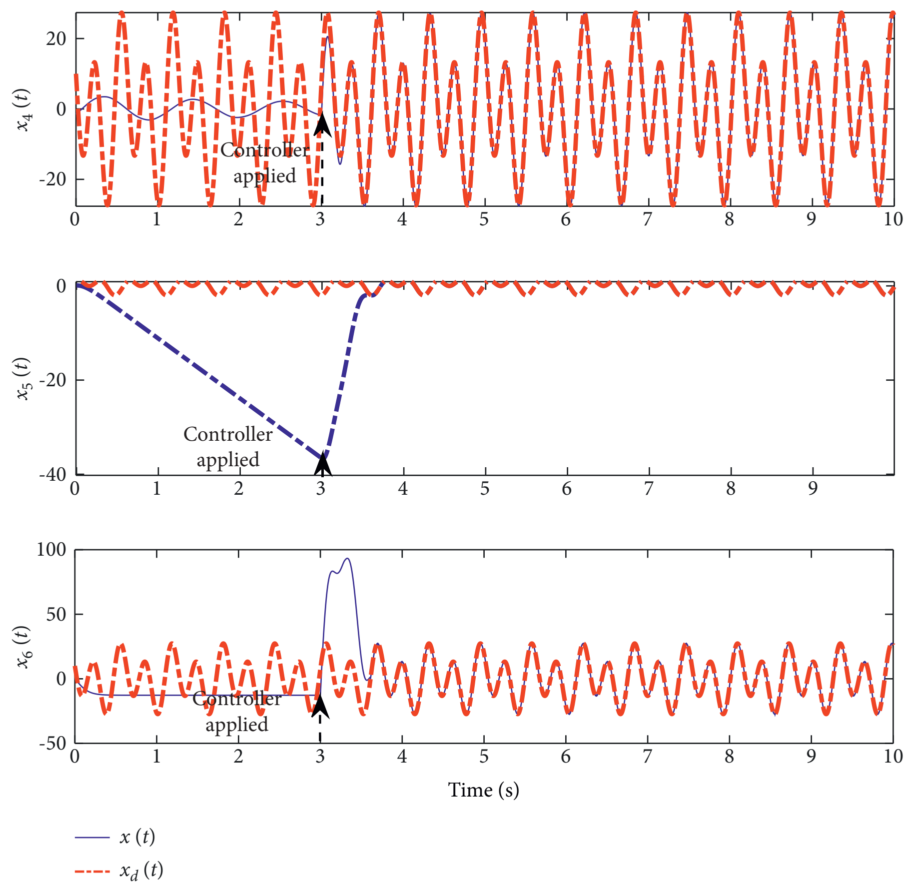

Figure 3: Time responses of x_4, x_5, x_6 and x_{4_d}, x_{5_d}, x_{6_d} using AFSMBS.

Simulink/MATLAB is utilized with the numerical method of ode4 and the step-size of 0.001. Also, the control input is applied after 3 seconds of start up of the system. In (23), the value of the selected design parameters is given.

$$\alpha_{2j-1} = \alpha_{2j} = \frac{5}{11}, \quad r_i = 0.1. \tag{25}$$

Figures 2 to 4 show the simulation results of the AFSMBS method for ROV with 3-DOF. Figures 2 and 3 show the tracking performance before and after applying the controller, where the controller is applied to the system at $t = 3\,(s)$. It can be seen that the system states reach the desired trajectories after applying the controller to the system. The efficacy of the controller can be demonstrated by comparing the behavior of the system states before and after applying the controller to the system. The controller effectively drives the system states to their references.

It can be observed from Figure 2 that the states converge to their references after applying the controller as follows. It is to be noted that the controller is applied to the system at $t = 3\,(s)$.

(i) $x_1 \longrightarrow x_{1_d}$ within $t \approx 2.2\,(s)$ using AFSMBS

(ii) $x_2 \longrightarrow x_{2_d}$ within $t \approx 0.2\,(s)$ using AFSMBS

(iii) $x_3 \longrightarrow x_{3_d}$ within $t \approx 1.7\,(s)$ using AFSMBS

Figure 3 shows that the states reach their references after applying the controller as follows. Note that the controller is applied to the system at $t = 3\,(s)$.

(i) $x_4 \longrightarrow x_{4_d}$ within $t \approx 0.3\,(s)$ using AFSMBS

(ii) $x_5 \longrightarrow x_{5_d}$ within $t \approx 0.5\,(s)$ using AFSMBS

(iii) $x_6 \longrightarrow x_{6_d}$ within $t \approx 0.6\,(s)$ using AFSMBS

Figure 4 shows the control signals u_1, u_2, and u_3 using the AFSMBS controller. It can be seen that the AFSMBS controller is applied to the system at $t = 3\,(s)$.

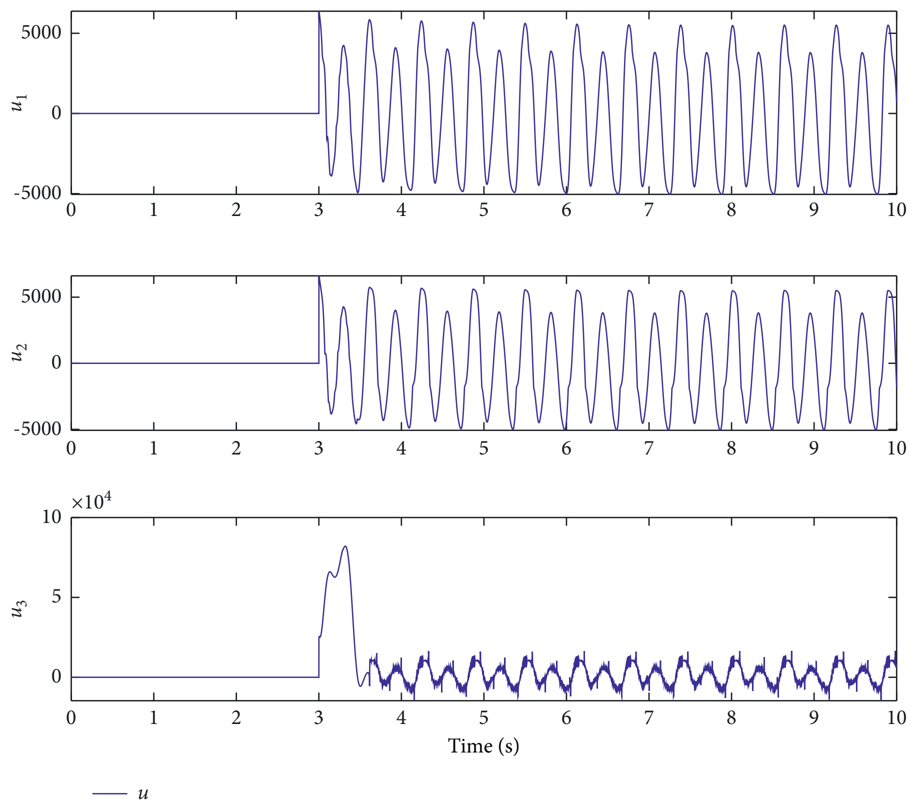

Figure 4: Time responses of the control signals u_1, u_2, and u_3 using AFSMBS.

6. Conclusion

In this paper, a novel AFSMBS controller is proposed by incorporating the robust sliding mode backstepping control scheme, adaptive control method, and finite-time stability notion for a type of high-order double-integrator systems considering mismatched uncertainties. The backstepping control law is defined utilizing the concept of sliding mode. The upper bound of the uncertainties and external disturbances is adaptively estimated within a finite time and the online estimated data are provided in the controller. The finite-time stability notion is used to guarantee the system's convergence in a finite time. The stability proof is obtained for the closed-loop system in the two phases utilizing a backstepping method and by defining proper candidate Lyapunov functions. The proposed method is applied and simulated for an example of ROV with 3-DOF. The efficacy of the suggested method is demonstrated in the simulation results. For future works, the optimization of the design parameters is recommended.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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