Hindawi
Discrete Dynamics in Nature and Society
Volume 2022, Article ID 3881268, 8 pages
https://doi.org/10.1155/2022/3881268

Research Article

Dual-Channel Dynamic Pricing in the Presence of Low-Carbon Preference

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Received 24 April 2022; Revised 24 July 2022; Accepted 29 July 2022; Published 9 November 2022

Academic Editor: Chunrui Zhang

This paper investigates the dynamic pricing strategy for perishable products sold through online and offline channels with the consideration of consumers' low-carbon preferences. The MNL stochastic utility model is used to describe the purchasing decisions of consumers with different low-carbon preferences. On this basis, we establish a dual-channel dynamic pricing model for perishable products to maximize the firm's expected revenue by using the dynamic programming method. We also study the influence of consumers' low-carbon preferences on optimal prices. The conclusions show that the low-carbon utility and the proportion of consumers with high low-carbon preference have positive effects on the optimal prices of the dual sales channels. Moreover, consumers are more inclined to purchase products through the online channel in the presence of low-carbon preference, so the optimal price of the online channel product is higher than that of the offline channel product.

1. Introduction

With the rapid development of the Internet and communication technology, the traditional offline sales pattern has gradually shifted to the online one and the online sales have increasingly become the most popular sales pattern today. According to the data published by Tmall, the largest e-commerce platform in China, the cumulative sales of Tmall Double 11 Shopping Carnival Season (November 1th–November 11th) in 2021 has reached 540.3 billion yuan, which increased by 8.45% compared with that of the same period in 2020. Because of the importance of the dual-channel sales, e-commerce platforms and offline physical stores actively seek cooperation with each other, which promotes the development of the dual-channel sales pattern driven by service and experience. For example, in addition to more than 1,600 offline physical stores, Suning has also provided an online sales channel. Xiaomi also actively sought cooperation with Suning and announced an alliance with Suning at the product launch in the spring of 2016. Xiaomi 4S mobile phones were exclusively sold by Suning offline physical stores, which shows that Xiaomi has already changed the original single online sales pattern and actively established entities to open up a dual-channel sales pattern.

In recent decades, carbon emission has been regarded as the main inducement of global warming and extreme climate change, which has aroused widespread concern from governments of various countries. Many governments have taken relevant measures and vigorously carried out emission reduction activities. For example, the Chinese government proposed to increase the supply of green and low-carbon products, innovate the supply mode of green services, guide green consumption, and build a new engine for green growth in the "14th Five-Year Plan for Industrial Green Development." With the gradual enhancement of consumers' low-carbon awareness and the improvement of online shopping's convenience, consumers' preferences for consumption channels have also changed. More and more consumers choose online shopping to achieve the purpose of low carbon and environmental protection. Data surveys show that 67% of consumers in the United States consider the importance of low-carbon factors when purchasing products [1] and are even willing to pay extra carbon emission reduction costs for low-carbon products [2]. Considering the rapid increase in consumers' environmental awareness, more and more firms are also spontaneously implementing low-carbon operations to strengthen their brand image and expand their market shares [3]. For example, Gree Group, a Chinese manufacturer, adheres to the core competitiveness of energy conservation and emission reduction and has provided traditional physical stores and online sales channels at the same time. In 2017, the offline revenue of Gree Group was 91 billion with an increase of 26.5% over the previous year, and the online revenue of Gree Group was 10.5 billion with an increase of 218% over the previous year, which indicates that the growth rate of online revenue is much higher than that of offline revenue. Under the dual-channel sales pattern, consumers' purchasing decisions are not only affected by product prices but also by consumers' own preferences. In the meantime, there must be sales conflicts between the online and offline sales channels. If the sellers can set a reasonable price and coordinate the conflicts between the two sales channels, they will ensure the maximization of their own revenues. Therefore, we focus on the dynamic pricing problem based on the characteristics of the sales channel, the green characteristics of the product, and consumers' low-carbon preferences.

The remainder of this paper is organized as follows: In Section 2, we review the related works in the literature. In Section 3, we propose a dynamic pricing decision model for dual sales channels. We then characterize the optimal prices in Section 4 followed by a numerical study presented in Section 5. We finally summarize this paper in Section 6.

2. Literature Review

The proposed research is mainly related to the following streams of the literature: (1) pricing for the dual sales channels, (2) pricing for green products, and (3) pricing in the presence of consumers' low-carbon preferences. Particularly, this study focuses on dynamic pricing for dual sales channels based on consumers' low-carbon preferences. Accordingly, a brief survey of the literature on the basis of these aspects is presented in this section.

In terms of pricing for the dual sales channels, most of the related works discuss static product pricing. Ren et al. researched price and service competition for dual-channel supply chains with consumer returns in both centralized and decentralized scenarios and designed a new contract to enable both the retailer and the manufacturer to be a win-win [4]. Xiao and Shi considered the pricing and sales channel selection problems for online sales and physical retail in the case of supply shortages [5]. Wang et al. studied the pricing and service decisions for complementary products under the dual-channel sales pattern and obtained the analytical equilibrium solution of the decision-making model using reverse induction and game theory [6]. Rahmani and Yavari proposed a pricing model for green products under the dual-channel sales pattern when market demand was interrupted and demonstrated that the optimal price would increase when demand disruption increased market scale or when green cost decreased [7]. Li et al. researched the pricing and return strategies under the dual-channel sales pattern and established a two-stage Stackelberg game model [8]. Ranjan and Jha considered the joint pricing problem for green products and nongreen products under the dual-channel sales pattern and proposed mathematical models under centralized, decentralized, and collaborative scenarios [9]. He et al. explored channel structure and pricing decisions for the manufacturer and government's subsidy policy with competing for new and remanufactured products in a dual-channel closed-loop supply chain [10]. He et al. studied inventory and pricing decisions simultaneously for a dual-channel supply chain with deteriorating products, established models for centralized and decentralized problems, and also proposed proper algorithms to obtain the optimal decisions of prices, ordering frequencies, and ordering quantities [11].

Regarding the pricing for green products, most of the existing literature considers the homogeneity of consumers. Li et al. proposed a Stackelberg game model for green product pricing under the competitive dual-channel sales pattern and concluded that green costs would significantly affect the prices and the green degrees of products [12]. Liu and Yi considered the pricing problem for green products based on the targeted advertising investment and the green cost and demonstrated that the optimal price was negatively related to the green degree and the investment level of targeted advertising [13]. Yang and Xiao examined the impact of channel leadership and government intervention on product prices and the green levels under the uncertainty of manufacturing costs and consumer demand [14]. Jamali and Rasti studied the decision-making problem of the product prices and the green degrees in the case of competition between green products and nongreen products [15].

Due to the different levels of income, education, values, and so on, consumers' low-carbon preferences vary widely. Low-carbon preference will affect consumer's choice behavior and product pricing [16, 17]. Hong et al. investigated the pricing problem for green products considering consumers' environmental awareness and nongreen product references [18]. Gong et al. examined the impact of consumers' low-carbon preferences on the prices of new energy vehicles and the profits of sellers and established pricing game models for centralized and decentralized scenarios [19]. Zhang et al. considered the two-stage product pricing and low-carbon decision-making problems for a dual-channel supply chain based on consumers' low-carbon preferences [20]. Meng et al. proposed a product pricing model for a dual-channel green supply chain by the Stackelberg game theory, in which consumers' green preferences and channel preferences were considered [21]. However, these research studies do not consider the choice behavior of consumers with low-carbon preference.

Different from the above works, this paper studies the multiperiod dynamic pricing problem for the online and offline sales channels in the presence of consumers' low-carbon preferences. We explicitly model the consumer's dynamic choice process by the multinomial logit (MNL) choice model, in which the probability of purchasing a product is specified as a function of the low-carbon preference. At each decision period, the firm dynamically determines the prices for the online and offline sales channels in order to maximize the expected revenue over the selling horizon. Given the consumer choice model, we formulate the dynamic pricing problem using the dynamic programming method based on the consumer's low-carbon preference. Then, we obtain the optimal pricing strategies. Finally, numerical experiments demonstrate the positive effects of the low-carbon utility and the proportion of consumers with high low-carbon preference for dynamic pricing strategies.

3. Dynamic Pricing Formulation for the Dual Sales Channels

We first present a dynamic pricing decision framework to capture consumers' low-carbon preferences. Assume that a risk-neutral firm sells products through online and offline sales channels. The online sales channel is eco-friendly, which can provide consumers with more low-carbon utility. The firm's goal is to maximize the expected revenue from the selling horizon by setting the appropriate price for each sales channel in each period. That is, in each period t, for the current remaining online inventory level x^e and offline inventory level x^c, the firm must determine the prices p_t^e and p_t^c for the online and offline sales channels, respectively. For simplicity, we denote $\mathbf{x} = (x^e, x^c)$ and $\mathbf{p}_t = (p_t^e, p_t^c)$ as the inventory level vector and the price vector, respectively.

Suppose that the selling horizon is divided into T discrete time periods such that there is at most one consumer arrival in each time period and each consumer purchases no more than one unit of perishable product. The time period is counted in reverse chronological order. Therefore, the first time period is T and the last time period is 1. At the initial time period T, the inventory levels of the online and offline sales channels are c_1 and c_2, respectively. At the end of the selling horizon, the unsold product has no salvage value. Consumer's return and firm's replenishment are generally not considered. In each time period, consumers arrive independently, and there is one consumer arrival with probability λ.

There are two types of consumers in the market: consumers with high and low low-carbon preferences, the corresponding proportions are $\theta(\theta \in [0,1])$ and $1-\theta$, respectively. Without loss of generality, assume that the low-carbon utility provided by the online sales channel to consumers with high and low low-carbon preferences are τ and 0, respectively. Suppose that the product value ν follows a uniform distribution between $[0,1]$; then, the firm sells the product to consumers through the online and offline sales channels at prices p^e and p^c, respectively.

When the firm sells products through the dual sales channels, for consumers with high low-carbon preference, the utilities gained from purchasing products from the online and offline sales channels are $U_h^e = \nu - p^e + \tau + \xi$ and $U_h^c = \nu - p^c + \xi$, respectively; for consumers with low low-carbon preference, the utilities gained from purchasing products from the online and offline sales channels are $U_l^e = \nu - p^e + \xi$ and $U_l^c = \nu - p^c + \xi$, respectively, where ξ is a random variable following Gumbel distribution with a shift parameter zero and scale parameter one. Suppose that Θ^i $(i = e, c)$ represents the attractiveness of the sales channel variable i [22], and the expected demand of each sales channel variable is proportional to the attractiveness of the sales channel variable. Under the framework of the MNL model, we define Θ^e and Θ^c as

$$\begin{aligned}\Theta^e &= \theta e^{\nu - p_t^e + \tau} + (1-\theta) e^{\nu - p_t^e},\\ \Theta^c &= \theta e^{\nu - p_t^c} + (1-\theta) e^{\nu - p_t^c} = e^{\nu - p_t^c}.\end{aligned} \tag{1}$$

Let q_t^e and q_t^c be the probability that a consumer chooses the online and offline sales channels in each period t. Based on the MNL choice model [23], we obtain the purchase probability q_t^e and q_t^c as

$$\begin{aligned}q_t^e &= \frac{\theta e^{\nu - p_t^e + \tau} + (1-\theta) e^{\nu - p_t^e}}{1 + \theta e^{\nu - p_t^e + \tau} + (1-\theta) e^{\nu - p_t^e} + e^{\nu - p_t^c}},\\ t &= 1, 2, \ldots, T,\\ q_t^c &= \frac{e^{\nu - p_t^c}}{1 + \theta e^{\nu - p_t^e + \tau} + (1-\theta) e^{\nu - p_t^e} + e^{\nu - p_t^c}},\\ t &= 1, 2, \ldots, T.\end{aligned} \tag{2}$$

Therefore, in each period t, the probability that a consumer decides not to purchase any product is $\overline{q}_t = 1 - q_t^e - q_t^c$.

Given the remaining inventory level $\mathbf{x}_t = (x_t^e, x_t^c)$, let $V_t^d(\mathbf{x}_t)$ be the optimal expected revenue from the period t to the last period. Then, we can use the following dynamic programming to formulate the dynamic pricing problem for the dual sales channels.

$$V_t^d(x_t) = \max_{\mathbf{p}_t} \left\{ \lambda \left[q_t^e \left(p_t^e + V_{t-1}^d (x_t - \varepsilon^e) \right) + q_t^c \left(p_t^c + V_{t-1}^d (x_t - \varepsilon^c) \right) + \overline{q}_t V_{t-1}^d (x_t) \right] + (1-\lambda) V_{t-1}^d (x_t) \right\}. \tag{3}$$

There are boundary conditions $V_0^d(\mathbf{x}_t) = 0$, for all $\mathbf{x}_t$, and $V_t^d(0) = 0$, for $t = 1, 2, \ldots, T$, where $\varepsilon^e = (1,0)$ and $\varepsilon^c = (0,1)$.

For the expected revenue function $V_t^d(\mathbf{x}_t)$, we define the marginal revenue of the dual sales channels as

$\Delta^i V_{t-1}^d(\mathbf{x}_t) = V_{t-1}^d(\mathbf{x}_t) - V_{t-1}^d(\mathbf{x}_t - \varepsilon^i), (i = e, c)$. Using this notation, we can rewrite (3) as follows:

$$V_t^d(\mathbf{x}_t) = \max_{\mathbf{p}_t}\left\{\lambda \sum_{i=e,c} q_t^i\left(p_t^i - \Delta^i V_{t-1}^d(\mathbf{x}_t)\right) + V_{t-1}^d(\mathbf{x}_t)\right\}. \quad (4)$$

4. Optimal Pricing Strategy for the Dual Sales Channels

We next investigate the optimal pricing strategy. To facilitate a description of the structural properties, we define

$$\varphi(\mathbf{p}_t, \mathbf{x}_t) = \sum_{i=e,c} q_t^i\left(p_t^i - \Delta^i V_{t-1}^d(\mathbf{x}_t)\right). \quad (5)$$

This can be proved to be a unimodal function with respect to $\mathbf{p}_t = (p_t^e, p_t^c)$ in Theorem 1. Intuitively, $\varphi(\mathbf{p}_t, \mathbf{x}_t)$ is the expected additional revenue realized in the period t by selling one unit of the remaining inventory level $\mathbf{x}_t$ at price $\mathbf{p}_t$. We interpret $\varphi(\mathbf{p}_t, \mathbf{x}_t)$ as the marginal revenue of time at the remaining inventory level $\mathbf{x}_t$ in the period t when prices are set at $\mathbf{p}_t$. Consequently, we set the optimal prices in order to maximize $\varphi(\mathbf{p}_t, \mathbf{x}_t)$. Therefore, exploring the structural properties of $\varphi(\mathbf{p}_t, \mathbf{x}_t)$ is the key to find the optimal prices. We discuss the structural properties of $\varphi(\mathbf{p}_t, \mathbf{x}_t)$ as follows.

Theorem 1. $\varphi(\mathbf{p}_t, \mathbf{x}_t)$ is unimodal with respect to $\mathbf{p}_t$.

Proof. Proof of this theorem is carried out in Appendix A. □

Theorem 1. *implies the uniqueness of the optimal price vector for the dual sales channels. Furthermore, the optimal policy can be characterized in an easier way.*

Theorem 2. *For all $\mathbf{x}_t > 0$, the optimal price for each sales channel is to set the price*

$$p_t^i(\mathbf{x}_t) = \eta_t + \Delta^i V_{t-1}^d(\mathbf{x}_t), \quad (i = e, c), \quad (6)$$

where $\eta_t = 1/\overline{q}_t$. The expected revenue is given by

$$V_t^d(\mathbf{x}_t) = \lambda \sum_{t_1=1}^{t}\left(\eta_{t_1} - 1\right). \quad (7)$$

Proof. Proof of this theorem is carried out in Appendix B.

Theorem 2 gives the simplified forms of the optimal prices and the expected revenue for the dual sales channels. The optimal price for the channel i can be expressed as $p_t^i(\mathbf{x}_t) = \eta_t + \Delta^i V_{t-1}^d(\mathbf{x}_t)(i = e, c)$, where η_t represents the marginal revenue from selling one unit of the product for the channel i in the period t and $\Delta^i V_{t-1}^d(\mathbf{x}_t)$ represents the revenue loss caused by one unit of the remaining product for the channel i in the period $t-1$, which can be understood as the future value of one unit of the remaining product for the channel i.

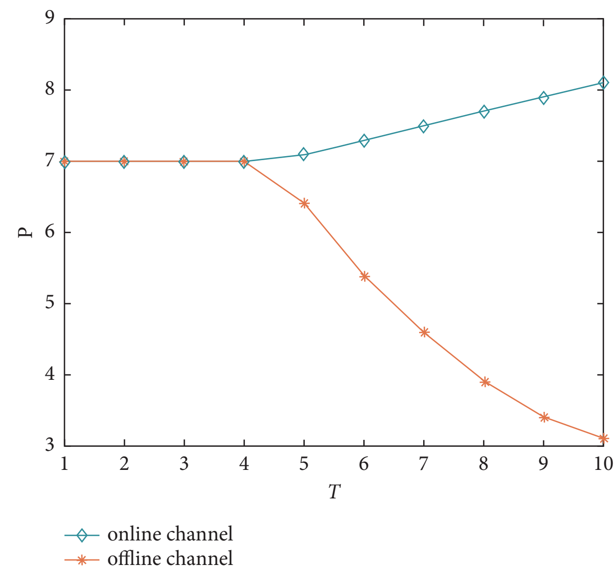

Figure 1: Optimal prices for the dual sales channels in the time remaining (t).

From Theorem 2, we get the optimal price for channel i.

$$p_t^i(\mathbf{x}_t) = 1 + (\theta e^{\tau} + 1 - \theta)e^{\nu - p_t^e} + e^{\nu - p_t^c} + \Delta^i V_{t-1}^d(\mathbf{x}_t), \quad (i = e, c). \quad (8)$$

When the channel i has surplus inventory $(x^i \geq t)$, the future revenue of a surplus unit is 0, that is, $\Delta^i V_{t-1}^d(\mathbf{x}) = 0$, if $x^i \geq t$. Therefore, we conclude that all products with inventory surplus should be set at the same price. However, when the channel i has inventory shortfalls $(x^i < t)$, the future revenue of a surplus unit is greater than 0, that is, $\Delta^i V_{t-1}^d(\mathbf{x}) > 0$, if $x^i < t$. Therefore, it is not optimal to set a uniform pricing scheme for products with inventory shortfalls $(x^i < t)$, and any product that has an inventory shortfall is set to a higher price than that with an inventory surplus, from which the following conclusion can be summarized. □

Theorem 3. *In the presence of the consumer's low-carbon preference, the optimal price for each sales channel has the following properties:*

(i) The optimal prices of products with surplus inventories are the same, i.e., for all $x^e \geq t$, $x^c \geq t$, $p_t^e(\mathbf{x}) = p_t^c(\mathbf{x})$

(ii) The optimal price of a product with inventory shortfall is set to be higher than that with the inventory surplus, i.e., for all $x^i \geq t$, $x^j < t\,(i, j = e, c)$ and $p_t^i(\mathbf{x}) < p_t^j(\mathbf{x})$.

Theorem 3. *reveals that in the presence of consumers' low-carbon preferences, the price difference between the online and offline sales channels is closely related to whether there is a surplus inventory, regardless of the proportion of various consumers and the low-carbon utility of consumers with high low-carbon preferences.*

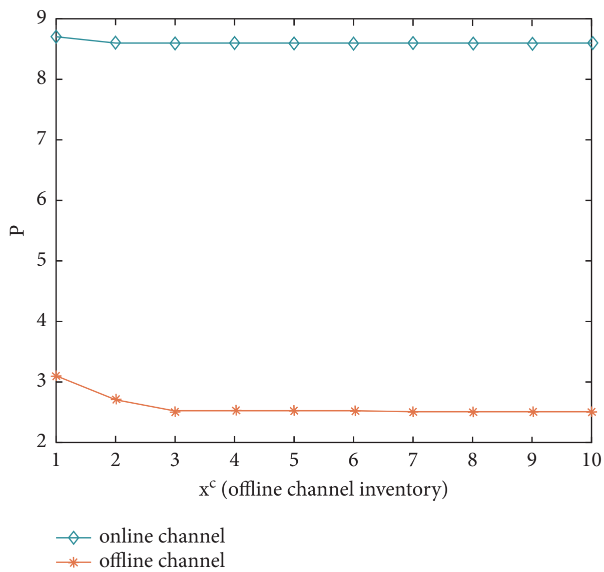

Figure 2: The optimal prices for the dual sales channels in the remaining inventory level of the offline channel.

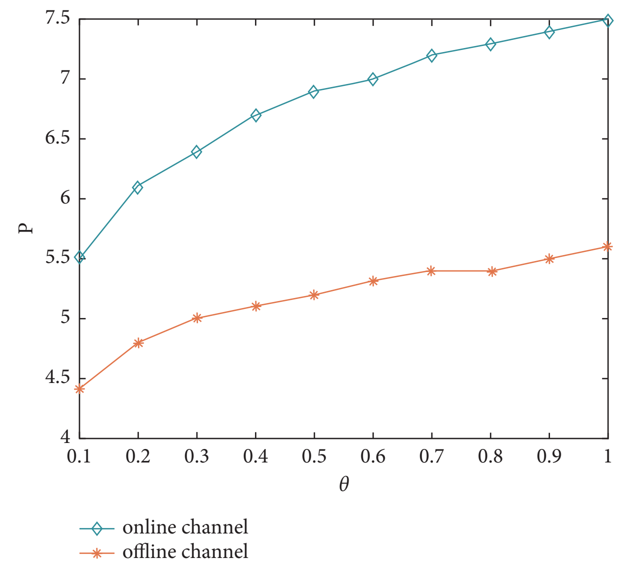

Figure 3: Effect of the proportion of consumers with a high low-carbon preference on the optimal prices.

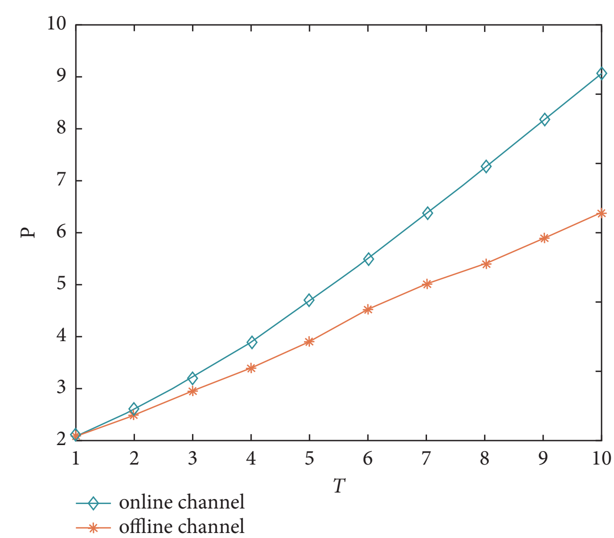

Figure 4: Effect of the low-carbon utility on the optimal prices.

5. Numerical Examples

In this section, we numerically illustrate the effects of consumers' low-carbon preferences on the optimal price for each sales channel. We examine sets of examples with the dual sales channels in which the product value $v = 1$, consumer arrival probability $\lambda = 0.7$, initial inventory levels $c_1 = 4$ and $c_2 = 6$, and the proportion of consumers with high low-carbon preference $\theta = 0.8$ and low-carbon utility $\tau = 8$.

Figure 1 depicts the optimal prices for the dual sales channels with respect to the remaining time, at fixed inventory levels $x^e = 4$, $x^c = 6$. In Figure 1, we observe time monotonicity, the optimal price for the online sales channel increases in the remaining time t; on the contrary, the optimal price for the offline sales channel decreases in the remaining time t. Furthermore, the optimal prices for both sales channels converge to an identical value. This means that the uniform pricing strategy begins to take effect when the remaining inventory level is abundant relative to the consumer demand.

Specifically, when the remaining time $t \leq 4$, the online and offline sales channels have surplus inventory. Therefore, during this period, the optimal prices for the dual sales channels are equal. When the remaining time $4 < t \leq 6$, the online sales channel has inventory shortfalls and the offline sales channel has surplus inventory. Therefore, during this period, the optimal price for the online sales channel is higher than that for the offline sales channel. When the remaining time $t > 6$, the online and offline sales channels both have inventory shortfalls. Since the initial inventory level of the online sales channel is less than that of the offline sales channel, the optimal price for the online sales channel is still higher than that for the offline sales channel.

Figure 2 illustrates the optimal prices for the dual sales channels as a function of the remaining inventory level of the offline sales channel in period 6, with the remaining inventory level of the online sales channel fixed as 4. The optimal prices for the online and offline sales channels are nonincreasing with respect to the remaining inventory level of the offline sales channel.

Next, we examine how the proportion of consumers with high low-carbon preference affects the optimal prices for the dual sales channels. Specifically, we depict the change of the optimal prices for the dual sales channels as the proportion of consumers with high low-carbon preference when $t = 6$, $x^e = 4$, and $x^c = 6$. We keep the low-carbon utility constant; that is, $\tau = 8$. The proportion of consumers with high low-carbon preference can be chosen from 0.1 to 1. In Figure 3, we observe that the proportion of consumers with high low-carbon preference significantly affects the optimal prices for the dual sales channels. Given the remaining inventory level and time, the optimal prices for the dual sales channels are nondecreasing with respect to the proportion of consumers with high low-carbon preference. Therefore, when the proportion of consumers with high low-carbon preference is greater, the firm can set a higher price for the product.

Finally, the effect of the low-carbon utility on the optimal prices is numerically examined using Figure 4. The parameters in Figure 4 are the same as those in Figure 3 except that the proportion of consumers with high low-carbon preference is now set to be 0.8 and the low-carbon utility is chosen from 1 to 10. Figure 4 illustrates that the optimal prices for the dual sales channels are nondecreasing with the low-carbon utility. Therefore, when the low-carbon utility is greater, the firm can set a higher price for the product. As shown in Figures 3 and 4, in the presence of the low-carbon preference, consumers are more inclined to purchase products from the online sales channel, so the optimal price for the online sales channel is higher than that for the offline sales channel.

The study results provide some guidelines on dual-channel dynamic pricing when consumers have low-carbon preferences. First, if firms adopt the dual-channel operation, consumers with high low-carbon preference will purchase products through the online sales channel, so the offline sales channel should reduce carbon emissions or improve service quality to reduce revenue loss. Second, the optimal price increases with the proportion of consumers with high low-carbon preference and low-carbon utility after firms adopt dual-channel operations. Therefore, firms should take steps to promote consumers' low-carbon awareness and create a low-carbon supply chain to achieve economic goals and take on social responsibility.

6. Conclusion

In this paper, we study the dynamic pricing problem for dual sales channels over the finite selling horizon in the presence of consumers' low-carbon preferences. We use the Bellman equation to obtain the optimal prices. Furthermore, we present numerical experiments to investigate how consumers' low-carbon preferences affect the optimal prices for the dual sales channels. According to the numerical examples, we conclude that consumer's low-carbon preference has a positive impact on optimal prices. Meanwhile, in the presence of low-carbon preferences, consumers are more inclined to purchase products from the online sales channel, so the optimal price for the online sales channel is higher than that for the offline sales channel.

Despite our research's several major contributions to the literature on consumer's low-carbon preference and dual-channel dynamic pricing, there are still some limitations and deficiencies for future research to address. Firstly, our paper assumes deterministic arrival probability. So, further research can consider the nonhomogeneous Poisson arrival in modeling and analysis. Secondly, our results are established by assuming the firm's risk neutrality. In practice, some firms are risk-averse. Therefore, it is also important to consider the sales firm's risk aversion in the dynamic pricing problem. Finally, our paper focuses on discussing the dual-channel dynamic pricing problem. Hence, future research can study the strategy combining dynamic pricing and inventory control in our model.

Appendices

A. Proof of Theorem 1

The first-order condition of $\varphi(\mathbf{p}_t, \mathbf{x}_t)$ is as follows:

$$\frac{\partial \varphi(\mathbf{p}_t, \mathbf{x}_t)}{\partial p_t^j} = \sum_{i=e,c} \frac{\partial q_t^i}{\partial p_t^j}\left(p_t^i - \Delta^i V_{t-1}^d(\mathbf{x}_t)\right) + q_t^j = 0, \quad (j = e, c). \tag{A.1}$$

Then, we derive

$$\mathbf{p}_t = \mathbf{h}_t + \mathbf{\Delta}_{t-1}, \tag{A.2}$$

where $\mathbf{h}_t = -\mathbf{q}_t(\partial \mathbf{q}_t/\partial \mathbf{p}_t)^{-1}$, $\Delta_{t-1} = (\Delta^e V_{t-1}^d(\mathbf{x}_t), \ \Delta^c V_{t-1}^d(\mathbf{x}_t))$, $\mathbf{q}_t = (q_t^e, q_t^c)$, $(\partial \mathbf{q}_t/\partial \mathbf{p}_t)$ is the Jacobian matrix of $\mathbf{q}_t$ with $\partial q_t^i/\partial p_t^j$ as its element (i, j), and $(\partial \mathbf{q}_t/\partial \mathbf{p}_t)^{-1}$ is the inverse of $(\partial \mathbf{q}_t/\partial \mathbf{p}_t)$.

From $\partial q_t^i/\partial p_t^j = \begin{cases} -q_t^i(1-q_t^i), & i = j, \\ q_t^i q_t^j, & i \neq j. \end{cases}$, we have

$$\left(\frac{\partial \mathbf{q}_t}{\partial \mathbf{p}_t}\right)^{-1} = -\begin{pmatrix} \dfrac{1 + \theta e^{\nu - p_t^e + \tau} + (1-\theta)e^{\nu - p_t^e}}{q_t^e} & \dfrac{1}{\overline{q}_t} \\ \dfrac{1}{\overline{q}_t} & \dfrac{1 + e^{\nu - p_t^c}}{q_t^c} \end{pmatrix}. \tag{A.3}$$

It follows that

$$\begin{aligned} \mathbf{h}_t &= (q_t^e, q_t^c)\begin{pmatrix} \dfrac{1 + \theta e^{\nu - p_t^e + \tau} + (1-\theta)e^{\nu - p_t^e}}{q_t^e} & \dfrac{1}{\overline{q}_t} \\ \dfrac{1}{\overline{q}_t} & \dfrac{1 + e^{\nu - p_t^c}}{q_t^c} \end{pmatrix}, \\ &= \left(1 + \theta e^{\nu - p_t^e + \tau} + (1-\theta)e^{\nu - p_t^e} + e^{\nu - p_t^c}, 1 + \theta e^{\nu - p_t^e + \tau} \right. \\ &\quad \left. + (1-\theta)e^{\nu - p_t^e} + e^{\nu - p_t^c}\right). \end{aligned} \tag{A.4}$$

Substituting the above expression into (A.2), we get

$$p_t^i(\mathbf{x}_t) \quad (i = e, c). \tag{A.5}$$

(A.5) guarantees a positive optimal price for each product. Next, we show that the optimal price is unique.

From (A.5), we derive $e^{\nu - p_t^i(\mathbf{x}_t)} = e^{-[1+(\theta e^{\tau}+1-\theta)e^{\nu-p_t^e}+e^{\nu-p_t^c}]-}$ $[\Delta^i V_{t-1}^d(\mathbf{x}_t) - \nu](i = e, c)$. It follows that

$$\begin{aligned} & e^{(\theta e^{\tau}+1-\theta)e^{\nu-p_t^e}+e^{\nu-p_t^c}}\left[(\theta e^{\tau}+1-\theta)e^{\nu-p_t^e}+e^{\nu-p_t^c}\right] \\ & = (\theta e^{\tau}+1-\theta)e^{-1-\Delta^e V_{t-1}^d(\mathbf{x}_t)+\nu}+e^{-1-\Delta^c V_{t-1}^d(\mathbf{x}_t)+\nu}. \end{aligned} \tag{A.6}$$

Let $z = (\theta e^{\tau} + 1 - \theta)e^{\nu - p_t^e} + e^{\nu - p_t^c}$; then, (A.6) can be rewritten as follows:

$$f(z) = ze^{z} = (\theta e^{\tau}+1-\theta)e^{-1-\Delta^e V_{t-1}^d(\mathbf{x}_t)+\nu}+e^{-1-\Delta^c V_{t-1}^d(\mathbf{x}_t)+\nu}. \tag{A.7}$$

It is clear that $(\theta e^{\tau} + 1 - \theta)e^{-1-\Delta^e V_{t-1}^d(\mathbf{x}_t)+\nu} + e^{-1-\Delta^c V_{t-1}^d(\mathbf{x}_t)+\nu}$ is independent of $\mathbf{p}_t$ and z and only depends on the marginal revenue $\Delta^e V_{t-1}^d(\mathbf{x}_t)$ and $\Delta^c V_{t-1}^d(\mathbf{x}_t)$. We denote the Lambert function as W; then, we get the inverse of the function $f(z) = ze^{z}$, i.e.,

$$\begin{aligned} z &= (\theta e^{\tau}+1-\theta)e^{\nu-p_t^e}+e^{\nu-p_t^c} \\ &= W\left((\theta e^{\tau}+1-\theta)e^{-1-\Delta^e V_{t-1}^d(\mathbf{x}_t)+\nu}+e^{-1-\Delta^c V_{t-1}^d(\mathbf{x}_t)+\nu}\right). \end{aligned} \tag{A.8}$$

Note that $W(\cdot)$ is single-valued and injective in its domain, if its domain is real and greater than or equal to $-1/e$. Since $(\theta e^{\tau} + 1 - \theta)e^{-1-\Delta^e V_{t-1}^d(\mathbf{x}_t)+\nu} + e^{-1-\Delta^c V_{t-1}^d(\mathbf{x}_t)+\nu} \geq 0$, it is deduced that $z = (\theta e^{\tau} + 1 - \theta)e^{\nu - p_t^e} + e^{\nu - p_t^c}$ is the unique solution of (A.6). Substituting (A.8) into (A.5), it yields that

$$\begin{aligned} & p_t^i(\mathbf{x}_t) \\ & + \Delta^i V_{t-1}^d(\mathbf{x}_t)(i = e, c). \end{aligned} \tag{A.9}$$

We claim that $\mathbf{p}_t(\mathbf{x}_t) = (p_t^e(\mathbf{x}_t), p_t^c(\mathbf{x}_t))$ is the unique solution to (A.1), and $\varphi(\mathbf{p}_t, \mathbf{x}_t)$ is unimodal with respect to $\mathbf{p}_t$.

B. Proof of Theorem 2

Let $\eta_t = 1/\overline{q}_t$. From (A.5), we have

$$p_t^i(\mathbf{x}_t) = \eta_t + \Delta^i V_{t-1}^d(\mathbf{x}_t). \tag{B.1}$$

Substituting (B.1) into (4), we deduce

$$V_t^d(\mathbf{x}_t) = \lambda\eta_t(1-\overline{q}_t) + V_{t-1}^d(\mathbf{x}_t) = \lambda(\eta_t - 1) + V_{t-1}^d(\mathbf{x}_t). \tag{B.2}$$

Applying the boundary conditions, we get the following:

$$V_t(\mathbf{x}_t) = \lambda\sum_{t_1=1}^{t}\left(\eta_{t_1} - 1\right). \tag{B.3}$$

Data Availability

The method in the numerical examples is computer mathematical simulation. No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Yusheng Hu and Wensi Zhang contributed equally to this work.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (11801556 and 11971458), Beijing Social Science Foundation (19YJB015 and 20JJC023), Humanities and Social Science Fund of the Ministry of Education (20YJC630056), General Science and Technology Project of Beijing Municipal Education Commission (KM202111232017), Qinxin Talents Cultivation Program of Beijing Information Science and Technology University (QXTCPC202116), and the Yueqi Youth Scholar Research Funds for the China University of Mining and Technology-Beijing (8000150A113).

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