Hindawi
Discrete Dynamics in Nature and Society
Volume 2022, Article ID 3990534, 7 pages
https://doi.org/10.1155/2022/3990534

Research Article

Minimizing the Machine Processing Time in a Flow Shop Scheduling Problem under Piecewise Quadratic Fuzzy Numbers

Tingwei Zhou[1] **Hamiden Abd El-Wahed Khalifa**[2,3] **Seyyed Esmaeil Najafi**[4] **and S.A. Edalatpanah**[5]

[1]*School of Mathematics and Physics, Bengbu University, Bengbu 233000, China*
[2]*Department of Operations Research, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza 12613, Egypt*
[3]*Department of Mathematics, College of Science and Arts, Qassim University, Al- Badaya 51951, Saudi Arabia*
[4]*Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran 46818-53617, Iran*
[5]*Department of Applied Mathematics, Ayandegan Institute of Higher Education, Tonekabon, Iran*

Correspondence should be addressed to S.A. Edalatpanah; saedalatpanah@gmail.com

Received 16 June 2022; Accepted 3 August 2022; Published 31 August 2022

Academic Editor: Reza Lotfi

The piecewise quadratic fuzzy number (PQFN) can signify uncertain information that exists in scientific, technological, and engineering fields. Hence, it is a useful tool for describing information in scheduling problems. This study examines structured n-job flow shop scheduling with fuzzy piecewise quadratic processing times and three machines. Close interval approximation of PQFNs is also offered as one of the most effective approximate intervals. Furthermore, the leasing cost of equipment is minimized with the use of a fuzzy style and an inventive algorithm. To demonstrate how the proposed framework can be used, a numerical illustration is provided.

1. Introduction

Scheduling dilemma is concerned with determining the optimal or nearly optimal schedule under certain limitations. Numerous methods have been proposed by several researchers to solve this problem. Scheduling to achieve a specific goal requires a variety of activities by spending time and budget. Flow shop is the most studied production setting in the literature on scheduling. In [1], one of the earliest results in flow shop scheduling is an algorithm for minimizing the completion time of all activities in a two or three-machine shop. Gupta [2] suggested a method for determining the best time to schedule a flow shop scheduling problem (FSSP) with a certain structure.

The method [2] has important considerations and developed by several scholars; see [3–7]. Narian and Bagga [9] investigated the problem of obtaining a sequence that provides the lowest possible cost of renting while minimizing the time spent. Schulz et al. [10] explored a mixture FSSP with varying discrete production speed levels. An upgraded multi-objective algorithm was used by Gheisarihe et al. [11] to solve the flexible FSSP with sequence-based transportation time, a probable network, and setup time.

In the real-world applied scientific problems, due to the complexity of different systems and the inaccuracy of data, classical methods cannot take into account inaccuracies in discussions. Therefore, using tools such as fuzzy perspective [12] can be helpful in managing this important task (see also [13, 14]).

The theory of fuzzy sets and its applications in optimization were proposed by Zimmermann [15]. Kaufmann and Gupta [16] studied several fuzzy mathematical models with their applications to engineering and management sciences.

By using triangular fuzzy sets to describe work processing times, Petrovic and Song [17] studied the task sequence problem in a two-machine flow shop. Multi-product parallel multi-stage cell manufacturing organizations can apply Saracoglu and Suer's methodology [18] to create items on time. They employed this methodology in the case study of a shoe manufacturing plant to produce products on time. Pang et al. [19] presented the FSSP and hybrid flow shop scheduling with the intention of determining the optimal scheduling approach for manufacturing facilities. Shao et al. [20] examined a distributed fuzzy blocking FSSP with processing times represented by fuzzy numbers, with the goal of minimizing the fuzzy makespan across all components. Recently, some papers are introduced to deal with real-world problems in fuzzy environments and their extensions (see [21–26]).

This study aims to investigate a particular *n-job* of scheduling with piecewise quadratic fuzzy number (PQFN). Given the total time elapsed, in which processing times are shown in PQFN, an innovative approach to sequencing tasks is proposed, which minimizes the cost of renting machines.

1.1. Research Gap and Motivation. The following points may lead to motivation of the proposed study.

(1) The piecewise quadratic fuzzy number (PQFN) introduced by Jain [27] is an extended concept of fuzzy set.

(2) In real-world scenarios, distinct parameters are further classified into disjoint sets having subparametric values. It presents the optimal selection with the help of suitable parameters. In decision making, the jury may endure some sort of tendency and proclivity while paying no attention to such parametric categorization during the decision.

(3) Inspired from the above literature, new notions of PQFN are conceptualized along with some elementary essential properties and generalized typical results. Moreover, decision-making algorithmic approaches are proposed.

1.2. Main Contributions and Advantages. The following are the main contributions of this proposed study:

(1) The existing relevant models are made adequate with the consideration of multi-argument approximate function through the development of the fuzzy set theory.

(2) The scenario where parameters are further partitioned into sub-parametric values in the form of sets is tackled by using PQFNs.

(3) Some fundamentals like elementary properties and arithmetic operations of PQFNs are characterized.

(4) Decision-making applications are discussed based on the proposal of PQFNS arithmetic operations.

(5) The results of the proposed similarity are compared with relevant existing models.

(6) The proposed structure is compared with relevant models under suitable evaluating indicators.

(7) The advantageous aspects of the proposed structure are discussed. The generalization of proposed structure is presented.

1.3. Paper Organization. This paper is organized as follows. The next section introduces the preliminaries of PQFNs and some notations. A three-stage FSSP model is provided in Section 3. Section 4 provides an efficient method for determining the sequence of jobs that minimizes the cost of equipment rental. Section 5 gives a numerical example for illustration. Section 6 introduces a comparative study with the existing methods. Finally, the conclusions are drawn in Section 7.

2. Prerequisites

Here, we study some preliminaries that we need for the main sections (for more details, see [27]).

Definition 1. A PQFN is denoted by $\widetilde{W}_{PQ} = (w_1, w_2, w_3, w_4, w_5)$, where $w_1 \le w_2 \le w_3 \le w_4 \le w_5$ are real numbers, and its membership function $\mu_{\widetilde{W}_{PQ}}$ is given by

$$\mu_{\widetilde{W}_{PQ}} = \begin{cases} 0, & x < w_1; \\ \dfrac{1}{2}\dfrac{1}{(w_2 - w_1)^2}(x - w_1)^2, & w_1 \le x \le w_2; \\ \dfrac{1}{2}\dfrac{1}{(w_3 - w_2)^2}(x - w_2)^2 + 1, & w_2 \le x \le w_3; \\ \dfrac{1}{2}\dfrac{1}{(w_4 - w_3)^2}(x - w_3)^2 + 1, & w_3 \le x \le w_4; \\ \dfrac{1}{2}\dfrac{1}{(w_5 - w_4)^2}(x - w_4)^2, & w_4 \le x \le w_5; \\ 0, & x > w_5. \end{cases} \quad (1)$$

Figure 1 shows the graphical representation of a PQFN.

Definition 2. Let $\widetilde{U}_{PQ} = (u_1, u_2, u_3, u_4, u_5)$ and $\widetilde{V}_{PQ} = (v_1, v_2, v_3, v_4, v_5)$ be two PQFNs. Then, we have

(i) Addition: $\widetilde{U}_{PQ}(+)\widetilde{V}_{PQ} = (u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4, u_5 + v_5)$.

(ii) Subtraction: $\widetilde{U}_{PQ}(-)\widetilde{V}_{PQ} = (u_1 - v_5, u_2 - v_4, u_3 - v_3, u_4 - v_2, u_5 - v_1)$.

(iii) Scalar multiplication: $k\widetilde{U}_{PQ} = \begin{cases} (ku_1, ku_2, ku_3, ku_4, ku_5), & k > 0, \\ (ku_5, ku_4, ku_3, ku_2, ku_1), & k < 0. \end{cases}$.

Definition 3. For the close interval approximation (CIA) of PQFN of $[U] = [U_\alpha^-, U_\alpha^+]$, we call $\widehat{U} = U_\alpha^- + U_\alpha^+/2$ as the associated real number of $[U]$.

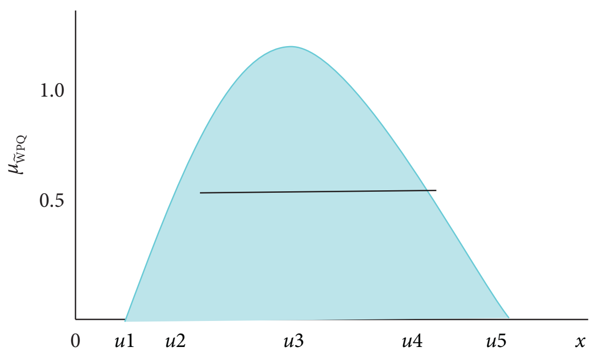

FIGURE 1: Graphical representation of a piecewise quadratic fuzzy number (PQFN).

Definition 4. For $[U] = [U_\alpha^-, U_\alpha^+]$ and $[V] = [V_\alpha^-, V_\alpha^+]$, we have the following properties:

(1) Addition: $[U](+)[V] = [U_\alpha^- + Vb_\alpha^-, U_\alpha^+ + V_\alpha^+]$.

(2) Subtraction: $[U](-)[V] = [U_\alpha^- - V_\alpha^+, U_\alpha^+ - V_\alpha^-]$.

(3) Scalar multiplication: $k[U] = \begin{cases} [kU_\alpha^-, kU_\alpha^+], & k > 0 \\ [kU_\alpha^+, kU_\alpha^-], & k < 0 \end{cases}$.

(4) Multiplication: $[U](\times)[V]$, $[U_\alpha^+ V_\alpha^- + U_\alpha^- V_\alpha^+/2, U_\alpha^- Vb_\alpha^- + U_\alpha^+ V_\alpha^+/2]$.

(5) Division: $[U](\div)[V]$, $\begin{cases} [2(U_\alpha^-/V_\alpha^- + V_\alpha^+), \\ 2(U_\alpha^+/V_\alpha^- + V_\alpha^+)], & [V] > 0, V_\alpha^- + V_\alpha^+ \neq 0 \\ [2(U_\alpha^+/V_\alpha^- + V_\alpha^+), \\ 2(U_\alpha^-/V_\alpha^- + V_\alpha^+)], & [V] < 0, V_\alpha^- + V_\alpha^+ \neq 0 \end{cases}$.

(6) The order relations:

(i) $[U](\underset{\sim}{\le})[V]$ if $U_\alpha^- \le V_\alpha^-$ and $U_\alpha^+ \le V_\alpha^+$ or $U_\alpha^- + U_\alpha^+ \le V_\alpha^- + V_\alpha^+$.

(ii) $[U]$ is preferred to $[V]$ if and only if $U_\alpha^- \ge V_\alpha^-, U_\alpha^+ \ge V_\alpha^+$.

2.1. Symbolization. Table 1 shows the symbols of our work.

3. Methodology

Before we discuss the issue formulation, let us define the rental cost.

3.1. Cost of Renting. The machines are rented out if needed and returned if they are no longer needed. For example, the first machine is rented at the beginning of the work process, the second machine is rented when the first work is completed in the first machine, and so on.

Suppose that some tasks $i, i = \overline{1, n}$ under the definite rental policy L are managed on three machines $M_j, j = 1, 2, 3$. Let $\widetilde{a}_{ij}^{PQ}$ be the PQFPT of i-th task on j-th machine (see Table 2). Let $S_{ij}, i = \overline{1, n}; j = 1, 2, 3$. Determine the related processing times with crisp number on devices M_1, M_2, and M_3 in such a way that either $\widehat{a}_{j2} \le \widehat{a}_{i1}$ or $\widehat{a}_{j2} \le \widehat{a}_{i3}; \forall i, j$. Our objective is to determine $\{S_k\}$ of the tasks that minimizes the cost of renting the equipment.

The problem may be expressed mathematically as follows:

$$\min \widetilde{R}^{PQ}(S_k) = \sum_{i=1}^{n} \widetilde{a}_{i1}^{PQ} \times C_1 + \widetilde{U}_2^{PQ}(S_k) \times C_2 + \widetilde{U}_3^{PQ}(S_k) \times C_3 \text{ Subject to rental policy L}. \quad (2)$$

Using the CIA of PQFN, model (10) may be reformulated as follows:

$$\min[R_\alpha^-(S_k), R_\alpha^+((S_k)] = \sum_{i=1}^{n} [(a_{i1})_\alpha^-, (a_{i1})_\alpha^+] \times C_1 + [U_{2\alpha}^-(S_k), U_{2\alpha}^+(S_k)] \times C_2 + [U_{3\alpha}^-(S_k), U_{3\alpha}^+(S_k)] \times C_3 \text{ Subject to rental policy L}. \quad (3)$$

Table 2 may be recreated in CIA of PQFN format as shown in Tables 3 and 4.

4. Proposed Algorithm

In this part, we show our strategy for minimizing the time and, consequently, the cost of renting a three-stage FSSP with PQF-based processing time while ignoring the makespan.

Step 1. Find the associated ordinary number for all tasks.

Step 2. If $\widehat{a}_{j2} \le \widehat{a}_{i1}$ or $\widehat{a}_{j2} \le \widehat{a}_{i3}; \forall i, j$, i.e., $\max\{\widehat{a}_{i1}\} \ge \min\{\widehat{a}_{j2}\}$ or $\max\{\widehat{a}_{i3}\} \ge \min\{\widehat{a}_{j2}\}; \forall i, j$, go to next step; otherwise, break.

Table 1: List of symbols.

Abbreviations	Descriptions
S	Arrangement of jobs, $i = \overline{1, n}$
S_k	Sequence obtained through the method [1], $k = 1, 2, \ldots, n$
M_h	Machine $h, h = 1, 2, 3$
M	Minimum makespan
$\tilde{a}_{lh}^{P}$	PQF processing time (PQFPT) for the l-th task on $M_h, h = 1, 2, 3$
$[a_{ij}^{PQ}]$	Close interval estimate of the PQFPT of the i-th task in sequence S_k running on M_j
$t_{ij}(S_k)$	Time of i-th task for S_k on M_j
$U_j(S_k)$	Consumption time for M_j that is necessary for S_k
$CT(S_k)$	Whole completion time
$I_{ij}(S_k)$	Idle time
$\hat{a}_{ij}$	Corresponding normal time of the i-th task on M_j
$R(S_k)$	Whole rental payment
C	Cost of renting

Table 2: Description of the problem with the PQFN matrix.

Tasks	M_1	M_2	M_3
i	$\tilde{a}_{i1}^{PQ}$	$\tilde{a}_{i2}^{PQ}$	$\tilde{a}_{i3}^{PQ}$
1	$\tilde{a}_{11}^{PQ}$	$\tilde{a}_{12}^{PQ}$	$\tilde{a}_{13}^{PQ}$
2	$\tilde{a}_{21}^{PQ}$	$\tilde{a}_{22}^{PQ}$	$\tilde{a}_{23}^{PQ}$
...	...	...	...
n	$\tilde{a}_{n1}^{PQ}$	$\tilde{a}_{n2}^{PQ}$	$\tilde{a}_{n3}^{PQ}$

Table 3: The problem with CIA matrix.

Tasks	M_1	M_2	M_3
i	$[(a_{i1})_\alpha^-, (a_{i1})_\alpha^+]$	$[(a_{i2})_\alpha^-, (a_{i2})_\alpha^+]$	$[(a_{i3})_\alpha^-, (a_{i3})_\alpha^+]$
1	$[(a_{11})_\alpha^-, (a_{11})_\alpha^+])$	$[(a_{12})_\alpha^-, (a_{12})_\alpha^+]$	$[(a_{13})_\alpha^-, (a_{13})_\alpha^+]$
2	$[(a_{21})_\alpha^-, (a_{21})_\alpha^+]$	$[(a_{22})_\alpha^-, (a_{22})_\alpha^+]$	$[(a_{23})_\alpha^-, (a_{23})_\alpha^+]$
...	...	...	...
n	$[(a_{n1})_\alpha^-, (a_{n1})_\alpha^+]$	$[(a_{n2})_\alpha^-, (a_{n2})_\alpha^+]$	$[(a_{n3})_\alpha^-, (a_{n3})_\alpha^+]$

Table 4: The problem with the corresponding crisp matrix form.

Tasks	M_1	M_2	M_3
i	$(a_{i1})_\alpha^- + (a_{i1})_\alpha^+/2$	$(a_{i2})_\alpha^- + (a_{i2})_\alpha^+/2$	$(a_{i3})_\alpha^- + (a_{i3})_\alpha^+/2$
1	$(a_{11})_\alpha^- + (a_{11})_\alpha^+/2$	$(a_{12})_\alpha^- + (a_{12})_\alpha^+/2$	$(a_{13})_\alpha^- + (a_{13})_\alpha^+/2$
2	$(a_{21})_\alpha^- + (a_{21})_\alpha^+/2$	$(a_{22})_\alpha^- + (a_{22})_\alpha^+/2$	$(a_{23})_\alpha^- + (a_{23})_\alpha^+/2$
...	...	...	...
n	$(a_{n1})_\alpha^- + (a_{n1})_\alpha^+/2$	$(a_{n2})_\alpha^- + (a_{n2})_\alpha^+/2$	$(a_{n3})_\alpha^- + (a_{n3})_\alpha^+/2$

Step 3. Define dummy machines H_1 and H_2, and their processing times H_1^i and H_2^i are as follows: $H_1^i = \hat{a}_{i1} + \hat{a}_{i2}, H_2^i = \hat{a}_{i2} + \hat{a}_{i3}; \forall i$.

Step 4. Use the existing algorithm [1] on H_i and get S_1.

Step 5. Put the $2^{\text{nd}}, \ldots, n^{th}$ tasks of the S_1 in the first position and all other tasks of S_1 in the same order.

Step 6. For all possible sequences $S_k, k = \overline{1, n}$, calculate: $\hat{R}(S_k) = \sum_{i=1}^{n} \hat{a}_{ij} \times C_1 + \hat{U}_2(S_k) \times C_2 + \hat{U}_3(S_k) \times C_3$.

Step 7. Set $\min\{\hat{R}(S_k)\}, k = \overline{1, n}$ as the optimal solution.

5. Numerical Example

Consider Table 5 as the problem. Now, we solve this problem by our model.

At first, in Tables 6 and 7, we compute the related interval and crisp numbers for each PQFPT.

Then, using Step 3 of our algorithm, the processing times can be computed as shown in Table 8.

Using procedure [1], S_1:2-4-5-1-3.

The subsequent viable sequences correspond to the minimal rental cost: S_2: $4 - 2 - 5 - 1 - 3$; S_3: $1 - 2 - 4 - 5 - 3$; S_4: $3 - 2 - 4 - 5 - 1$.

Tables 9 and 10 illustrate the in-out flow for the sequence S_1 in the PQFNs and CIA forms.

For S_1, we get the following.

The completion time for S_1 is $\widetilde{CT}^{PQ}(S_1) = (40, 49, 56, 62, 76)$, $[CT(S_1)] = [49, 62]$, and $\widehat{CT}(S_1) = 55.5$.

The consumption time for machine M_2 is $\tilde{U}_2^{PQ}(S_1) = (11, 23, 30, 38, 53)$, $[U_2(S_1)] = [23, 38]$, and $\hat{U}_2(S_1) = 30.5$.

The consumption time for machine M_3 is $\tilde{U}_3^{PQ}(S_1) = (14, 28, 37, 45, 61)$, $[U_3(S_1)] = [28, 45]$, and $\hat{U}_3(S_1) = 36.5$.

$$\begin{aligned} \tilde{R}^{PQ}(S_1) &= \sum_{i=1}^{5} \tilde{a}_{i1} \times C_1 + U_2(S_1) \times C_2 + U_3(S_1) \times C_3 \\ &= (572, 726, 831, 935, 1145), [R(S_1)] \\ &= [726, 935] \text{ and } \hat{R}(S_1) = 830.5. \end{aligned} \tag{4}$$

Similarly, we have the following.

For S_2:

$$\begin{aligned} &\widetilde{CT}^{PQ}(S_2) = (37, 47, 54, 61, 73), \\ &[CT(S_2)] = [47, 61], \text{and } \widehat{CT}(S_2) = 54, \end{aligned} \tag{5}$$

$$\begin{aligned} \tilde{U}_2^{PQ}(S_2) &= (12, 23, 30, 38, 52), [U_2(S_2)] \\ &= [23, 38], \text{and } \hat{U}_2(S_2) = 30.5, \end{aligned} \tag{6}$$

Table 5: PQFN processing times for machines.

Tasks	M_1	M_2	M_3
i	$\tilde{a}_{i1}^{PQ}$	$\tilde{a}_{i2}^{PQ}$	$\tilde{a}_{i3}^{PQ}$
1	(6,7,8,9,10)	(5,6,7,8,10)	(2,3,4,5,7)
2	(11,12,13,14,17)	(4,5,6,7,9)	(3,4,5,6,8)
3	(7,8,10,12,14)	(3,4,5,6,9)	(5,6,7,8,9)
4	(9,10,11,12,14)	(3,5,6,7,9)	(10,11,12,13,15)
5	(7,9,10,11,13)	(2,5,6,8,10)	(5,8,9,10,11)

Table 6: PQFN processing times with interval format.

Tasks	M_1	M_2	M_3
i	$[a_{i1}^{PQ}]$	$[a_{i2}^{PQ}]$	$[a_{i3}^{PQ}]$
1	[7,9]	[6,8]	[3,5]
2	[12,14]	[5,7]	[4,6]
3	[8,12]	[4,6]	[6,8]
4	[10,12]	[5,7]	[11,13]
5	[9,11]	[5,8]	[8,10]

Table 7: PQFN processing times with crisp format.

Tasks	M_1	M_2	M_3
i	$\hat{a}_{i1}$	$\hat{a}_{i2}$	$\hat{a}_{i3}$
1	8	7	4
2	13	6	5
3	10	5	7
4	11	6	12
5	10	6.5	9

Table 8: The related crisp numbers of the processing times.

Tasks	H_1	H_2
1	15	11
2	19	11
3	15	12
4	17	18
5	16.5	15.5

Table 9: The in-out flow for S_1 in the PQFNs.

Tasks	M_1	M_2	M_3
i	In- out	In- out	In- out
2	(11,12,13,14,17)	(15,17,19,21,26)	(18,21,24,27,34)
4	(19,22,24,26,31)	(18,22,25,28,35)	(28,32,36,39,49)
5	(26,31,34,37,44)	(20,27,31,36,45)	(33,40,45,49,60)
1	(32,38,42,46,54)	(25,33,38,44,55)	(35,43,49,54,67)
3	(39,46,52,58,68)	(28,37,43,50,64)	(40,49,56,62,76)

$$\tilde{U}_3^{PQ}(S_2) = (14, 28, 37, 45, 61), [U_3(S_2)] = [28, 45], \text{and } \hat{U}_2(S_2) = 36.5, \quad (7)$$

Table 10: The in-out flow for S_1 in CIA.

Jobs	Machine M_1	Machine M_2	Machine M_3
2	[12,14]	[17,21]	[21,27]
4	[22,26]	[22,28]	[32,39]
5	[31,37]	[27,36]	[40,49]
1	[38,46]	[33,44]	[43,54]
3	[46,58]	[37,50]	[49,62]

$$\tilde{R}^{PQ}(S_2) = \sum_{i=1}^{5} \tilde{a}_{i1}^{PQ} \times C_1 + \tilde{U}_2^{PQ}(S_2) \times C_2 + \tilde{U}_3^{PQ}(S_2) \times C_3 = (582, 718, 769, 927, 1131), [R(S_2)] = [718, 927], \text{and } \hat{R}(S_1) = 822.5. \quad (8)$$

For S_3, we have

$$\widetilde{CT}^{PQ}(S_3) = (36, 45, 52, 59, 70), [CT(S_3)] = [45, 59], \text{and } \widehat{CT}(S_3) = 52, \quad (9)$$

$$\tilde{U}_2^{PQ}(S_3) = (13, 23, 30, 38, 49), [U_2(S_3)] = [23, 38], \text{and } \hat{U}_2(S_3) = 30.5, \quad (10)$$

$$\tilde{U}_3^{PQ}(S_3) = (16, 28, 37, 46, 59), [U_3(S_3)] = [28, 46], \text{and } \hat{U}_2(S_3) = 37, \quad (11)$$

$$\tilde{R}^{PQ}(S_3) = \sum_{i=1}^{5} \tilde{a}_{i1}^{PQ} \times C_1 + \tilde{U}_2^{PQ}(S_3) \times C_2 + \tilde{U}_3^{PQ}(S_3) \times C_3 = (562, 686, 791, 896, 1075), [R(S_3)] = [686, 896], \text{and } \hat{R}(S_3) = 791. \quad (12)$$

For S_4, we have

$$\widetilde{CT}^{PQ}(S_4) = (35, 44, 52, 60, 73), [CT(S_4)] = [44, 60], \text{and } \widehat{CT}(S_4) = 52, \quad (13)$$

$$\tilde{U}_2^{PQ}(S_4) = (10, 11, 30, 40, 54), [U_2(S_4)] = [11, 40], \text{and } \hat{U}_2(S_4) = 25.5, \quad (14)$$

$$\tilde{U}_3^{PQ}(S_4) = (12, 26, 37, 48, 63), [U_3(S_4)] = [26, 48], \text{and } \hat{U}_2(S_4) = 37, \quad (15)$$

$$\tilde{R}^{PQ}(S_4) = \sum_{i=1}^{5} \tilde{a}_{i1}^{PQ} \times C_1 + \tilde{U}_2^{PQ}(S_4) \times C_2 + \tilde{U}_3^{PQ}(S_4) \times C_3 = (560, 672, 823, 956, 1161), [R(S_4)] = [672, 956], \text{and } \hat{R}(S_4) = 814. \quad (16)$$

Table 11: Comparison of different researchers' contributions.

Author	Processing time	Piecewise quadratic fuzzy numbers	Close approximate interval	Minimum rental cost
Ruiz et al. [28]	↓	↓	↓	↑
Liang et al. [29]	↑	↓	↓	↑
Sanchez-Herrera et al. [30]	↑	↓	↓	↑
Our proposed approach	↑	↑	↑	↑

For S_5, we have

$$\widehat{R}(S_5) = 2066.6257. \quad (17)$$

Thus,

$$\widetilde{R}^{PQ}(S_3) = \sum_{i=1}^{5} \widetilde{a}_{i1}^{PQ} \times C_1 + \widetilde{U}_2^{PQ}(S_3) \times C_2 + \widetilde{U}_3^{PQ}(S_3) \times C_3 = (562, 686, 791, 896, 1075), \quad (18)$$

$$[R(S_3)] = [686, 896]. \quad (19)$$

Therefore, S_3: $1-2-4-5-3$ is the optimal sequence subject to the minimum rental cost, and $\widehat{R}(S_3) = 791$ is the minimum rental cost irrespective of the total time passed.

6. Comparative Study

In this section, the proposed approach is compared with some existing studies to illustrate the advantages of the proposed approach. The results for this analysis are summarized in Table 11. The symbol "↓" or "↑" shown in the table represents whether the associated feature satisfies or not.

7. Conclusions and Future Works

In this paper, the problem of minimizing the cost of renting machines for flow shop scheduling with a specific structure is investigated. An innovative approach to solve it is then proposed in which the processing times are fragmented as piecewise quadratic fuzzy numbers. The result shows that the proposed method has its advantage in flexible decision making corresponding to favorite priorities of alternatives. This study may be extended to additional fuzzy-like structures, such as interval-valued fuzzy set, Pythagorean fuzzy set, spherical fuzzy set, intuitionistic fuzzy set, picture fuzzy set, neutrosophic set, and so on, in future work.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors would like to thank the Deanship of Scientific Research, Qassim University, for funding the publication of this project.

References

[1] S. M. Johnson, "Optimal two- and three-stage production schedules with setup times included," *Naval Research Logistics Quarterly*, vol. 1, no. 1, pp. 61–68, 1954.
[2] J. N. D. Gupta, "Optimal schedules for special structure flowshops," *Naval Research Logistics Quarterly*, vol. 22, no. 2, pp. 255–269, 1975.
[3] W. Szware, "Special cases of the flow shop problems," *Naval Research Logistics*, vol. 22, no. 3, pp. 483–492, 1971.
[4] P. C. Bagga, "Sequencing in a rental situation," *Journal of Canadian Operation Research Sociey*, vol. 7, pp. 152-153, 1969.
[5] J. N. D. Gupta, "Two-stage, hybrid flowshop scheduling problem," *Journal of the Operational Research Society*, vol. 39, no. 4, pp. 359–364, 1988.
[6] P. L. Maggu and G. Das, "On n×2 sequencing problem with transportation time of jobs," *Pure and Applied Mathematical Sciences*, vol. 3, pp. 12–16, 1981.
[7] T. P. Singh, K. Rajindra, and G. Deepak, "Optimal three stage production schedule the processing time and set times associated with probabilities including job block criteria," *Proceedings of National Conference FACM*, pp. 463–492, 2005.
[8] E. Ignall and L. Schrage, "Application of the Branch and Bound Technique to some flow-shop scheduling problems," *Operations Research*, vol. 13, no. 3, pp. 400–412, 1965.
[9] L. Narian and K. Bagga, "Two machine flow shop problem with availability constraints on each machine," *JISSOR*, vol. XXIV, no. 1-4, pp. 17–24, 2005.
[10] S. Schulz, U. Buscher, and L. Shen, "Multi-objective hybrid flow shop scheduling with variable discrete production speed levels and time-of-use energy prices," *Journal of Business Economics*, vol. 90, no. 9, pp. 1315–1343, 2020.
[11] E. Gheisarihe, M. Tavana, F. Jolai, and M. Rabiee, "A simulation- optimization model for solving flexible flow shop scheduling problems with rework and transportation," *Mathematics and Computers in Simulation*, vol. 180, pp. 152–177, 2021.
[12] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
[13] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, 1980.
[14] R. Bellmann and L. Zadeh, "Decision making in a fuzzy environment," *Management Science*, vol. 17, pp. 141–164, 1970.
[15] H.-J. Zimmermann, "Fuzzy programming and linear programming with several objective functions," *Fuzzy Sets and Systems*, vol. 1, no. 1, pp. 45–55, 1978.

[16] A. Kaufmann and M. M. Gupta, *Fuzzy Mathematical Models in Engineering and Management Science*, Elsevier Science Publishing Company INC, New York, 1988.
[17] S. Petrovic and X. Song, “A new approach to two-machine flow shop problem with uncertain processing times,” *Optimization and Engineering*, vol. 7, no. 3, pp. 329–342, 2006.
[18] I. Saracoglu and G. A. Sure, “Multi- objective fuzzy flow shop scheduling model in a manufacturing company,” *Procedia Manufacturing*, vol. 17, pp. 214–221, 2018.
[19] X. Pang, H. Xue, M.-L. Tseng, M. K. Lim, and K. Liu, “Hybrid flow shop scheduling problems using Improved fireworks algorithm for Permutation,” *Applied Sciences*, vol. 10, no. 3, pp. 1174–1189, 2020.
[20] Z. Shao, W. Shao, and D. Pi, “Effective heuristics and metaheuristics for the distributed fuzzy blocking scheduling problem,” *Swarm and Evolutionary Computation*, vol. 59, Article ID 100747, 2020.
[21] A. Akgül, “A novel method for a fractional derivative with non-local and non-singular kernel,” *Chaos, Solitons & Fractals*, vol. 114, pp. 478–482, 2018.
[22] F. Dayan, N. Ahmed, M. Ahmed et al., “Construction and numerical analysis of a fuzzy non-standard computational method for the solution of an SEIQR model of COVID-19 dynamics,” *AIMS Mathematics*, vol. 7, no. 5, pp. 8449–8470, 2022.
[23] S. Ahmad, A. Ullah, A. Akgul, and T. Abdeljawad, “Semi-analytical solutions of the 3 order fuzzy dispersive partial differential equations under fractional operators,” *Alexandria Engineering Journal*, vol. 60, no. 6, pp. 5861–5878, 2021.
[24] S. Ahmad, A. Ullah, A. Akgul, and T. Abdeljawad, “Computational analysis of fuzzy fractional order non-dimensional Fisher equation,” *Physica Scripta*, vol. 96, no. 8, Article ID 084004, 2021.
[25] D. Behera, K. Peters, S. A. Edalatpanah, and D. Qiu, “New methods for solving imprecisely defined linear programming problem under trapezoidal fuzzy uncertainty,” *Journal of Information and Optimization Sciences*, vol. 42, no. 3, pp. 603–629, 2021.
[26] A. U. Rahman, M. Saeed, H. A. Saeed, and W. A. Khalifa, “Decision making algorithmic techniques based on aggregation operations and similarity measures of possibility intuitionistic fuzzy hypersoft sets,” *AIMS Math*, vol. 7, no. 3, pp. 3866–3895, 2022.
[27] S. Jain, “Close interval approximation of piecewise quadratic fuzzy numbers for fuzzy fractional program,” *Iranian journal of operations research*, vol. 2, no. 1, pp. 77–88, 2010.
[28] R. Ruiz, C. Maroto, and J. Alcaraz, “Solving the follow shop scheduling problem with sequence dependent setup times using advanced metaheuristics,” *European Journal of Operational Research*, vol. 165, no. 1, pp. 34–54, 2005.
[29] Z. Liang, P. Zhong, M. Liu, C. Zhang, and Z. Zhang, “A computational efficient optimization of flow shop scheduling problems,” *Scientific Reports*, vol. 12, p. 845, 2022.
[30] S. Sanchez- Herrera, J. R. Montoya- Torres, and E. L. Solano-Charris, “Flow shop scheduling problem with position- dependent processing times,” *Computers & Operations Research*, vol. 111, pp. 325–345, 2019.