Hindawi
Discrete Dynamics in Nature and Society
Volume 2022, Article ID 3994361, 12 pages
https://doi.org/10.1155/2022/3994361

Research Article

A New Version of Weighted Weibull Distribution: Modelling to COVID-19 Data

Amani Abdullah Alahmadi,[1] Mohammed Alqawba,[2] Waleed Almutiry,[2] A. W. Shawki,[3] Sharifah Alrajhi,[4] Sanaa Al-Marzouki,[4] and Mohammed Elgarhy[5]

[1]*College of Science and Humanities, Shaqra University, Shaqra, Saudi Arabia*
[2]*Department of Mathematics, College of Science and Arts, Qassim University, Ar Rass, Saudi Arabia*
[3]*Central Agency for Public Mobilization & Statistics (CAPMAS), Cairo, Egypt*
[4]*Statistics Department, Faculty of Science, King Abdul Aziz University, Jeddah, Saudi Arabia*
[5]*The Higher Institute of Commercial Sciences, Al Mahalla Al Kubra, Algarbia 31951, Egypt*

Correspondence should be addressed to Mohammed Elgarhy; m_elgarhy85@sva.edu.eg

Received 18 September 2021; Revised 31 October 2021; Accepted 14 March 2022; Published 21 April 2022

Academic Editor: Sovan Samanta

In this study, we will look at a new flexible model known as the new double-weighted Weibull distribution. The new Weibull double-weighted distribution model is highly versatile because numerous submodels are included. The proposed model is very flexible because its density function has many shapes; it can be right skewness, decreasing, and unimodal. Also, the hazard rate function can be increasing, decreasing, up-side-down, and J-shaped. Diverse features of the novel are computed. These qualities include moments, incomplete moments, and Lorenz and Bonferroni curves and quantiles, as well as entropy and order statistics. The maximum likelihood approach is used to estimate the model's parameters. In order to evaluate the accuracy and performance of maximum likelihood estimators, simulation data are presented. The utility and adaptability of the proposed model are demonstrated by utilizing three significant datasets: daily fatalities confirmed cases of COVID-19 in Egypt and Georgia and relief times of twenty patients using an analgesic.

1. Introduction

In 1951, Swedish scientist Walled Weibull created the Weibull (Wei) distribution. The Wei distribution is a frequently used distribution for modelling lifetime data in dependability where the hazard rate function is monotone. However, when the true hazard shape is unimodal or bathtub, the two-parameter Wei distribution is inadequate in many applications, such as lifetime analysis. To deal with bathtub-shaped failure rates, many generalizations of the Wei distribution have been proposed in the statistical literature.

The probability density function (pdf) and cumulative distribution function (cdf) have the following shape:

$$g(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}, \quad x > 0, \alpha, \beta > 0, \tag{1}$$

$$G(x) = 1 - e^{-\alpha x^{\beta}}, \quad x > 0, \tag{2}$$

where β is a positive shape parameter and α is a positive scale parameter.

Weighted (W) distributions may be used to increase comprehension of standard distributions as well as provide techniques for extending distributions for additional flexibility in fitting a dataset. A W distribution can be obtained in a variety of ways. In 1934, Fisher proposed the concept of W distribution. Rao [1] and Patil and Rao [2] discuss applications of a W distribution to biased samples in many disciplines such as medicine, ecology, dependability, and branching processes (1978). Patil and Rao [2] suggested a

Table 1: Some literature of double-weighted distributions.

Model	Authors
DW exponential distribution	Al-khadim and hantoosh [3]
DW Rayleigh distribution	Rishwan [4]
DW inverse Wei distribution	Al-khadim and hantoosh [5]

double-weighted (DW) family of distributions, which has the following characteristics:

$$f_w(x) = \frac{w_1(x)g(x)w_2(x)}{W_D}, \tag{3}$$

where $w_1(x), w_2(x) > 0$ and $W_D = \int_0^\infty w_1(x)g(x)w_2(x)\mathrm{d}x$ is a normalizing constant that forces $f_w(x)$ to integrate to 1. Table 1 shows the literature for authors who use the double-weighted family.

The motivation and limitations of our study to introduce a new double-weighted Weibull (NDWW) is a novel double-weighted distribution. The mathematical features of the proposed model are described in detail in the expectation that it may find wider applications in dependability, engineering, and other study areas. It includes a number of interesting features and allows for greater flexibility in incorporating filtered and uncensored survival data into real-world applications. To illustrate our point, we use three real-life datasets, which correspond to the daily death toll from COVID-19 in Egypt and Georgia and the relief times of twenty patients taking analgesics. NDWW offers a better fit than other statistical distributions, according to a study. Also, our new model serves the data which are positive data.

The remainder of this study can be divided into the following categories. Section 2 defines the NDWW. Section 3 derives some of the distribution's broad statistical features. The maximum likelihood method is used to estimate the distribution's parameters in Section 4. Section 5 conducts a simulation analysis to determine the model parameters of one distribution. Section 6 delves into an illustrated goal based on real-world facts. The study closes with some last thoughts.

2. The New Double-Weighted Weibull

In this section, we introduce a new four-parameter model which is called the NDWW model by using new two transformations $w_1(x) = x^\theta$ and $w_2(x) = \overline{G}(cx)$, and by inserting (1) and (2) in (3), then the pdf and cdf of the NDWW are given by

$$f_w(x) = \frac{\beta\left[\alpha\left(c^\beta + 1\right)\right]^{\theta/\beta+1}}{\Gamma(\theta/\beta + 1)} x^{\theta+\beta-1} e^{-\alpha\left(c^\beta+1\right)x^\beta}, \quad x > 0,\ \alpha, \beta, \theta, c > 0, \tag{4}$$

and

$$F(x) = \frac{\gamma\left(\theta/\beta + 1, \alpha\left(c^\beta + 1\right)x^\beta\right)}{\Gamma(\theta/\beta + 1)}, \quad x > 0, \alpha, \beta, \theta, c > 0, \tag{5}$$

respectively; where the gamma function is defined by

$$\Gamma(a) = \int_0^\infty t^{a-1} e^{-t}\mathrm{d}t, \quad a > 0. \tag{6}$$

and incomplete gamma function is defined by

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t}\mathrm{d}t, \quad a > 0,\ x > 0. \tag{7}$$

On the basis of X, the survival function $(S(x))$, hazard function $(h(x))$, and reversed hazard function $(a(x))$ are given

$$S(x) = 1 - \frac{\gamma\left(\theta/\beta + 1, \alpha\left(c^\beta + 1\right)x^\beta\right)}{\Gamma(\theta/\beta + 1)}, \quad x > 0,$$
$$h(x) = \frac{\beta\left[\alpha\left(c^\beta + 1\right)\right]^{\theta/\beta+1} x^{\theta+\beta-1} e^{-\alpha\left(c^\beta+1\right)x^\beta}}{\Gamma(\theta/\beta + 1) - \gamma\left(\theta/\beta + 1, \alpha\left(c^\beta + 1\right)x^\beta\right)}, \quad x > 0, \tag{8}$$

and

$$a(x) = \frac{\beta\left[\alpha\left(c^\beta + 1\right)\right]^{\theta/\beta+1} x^{\theta+\beta-1} e^{-\alpha\left(c^\beta+1\right)x^\beta}}{\gamma\left(\theta/\beta + 1, \alpha\left(c^\beta + 1\right)x^\beta\right)}. \tag{9}$$

Figure 1 shows the pdf and hazard rate function graphs for the NDWW distribution.

We may deduce from Figure 1 that the pdf of the NDWW distribution can be unimodal, declining, inverted J shaped, and right skewed. In addition, as shown in Figure 1, the hazard rate function of the NDWW distribution might be rising, decreasing, or J-shaped.

It should be noted that α and c remain scale parameters, whereas β and θ are shape parameters. In Table 2, certain distributions appear as special instances of the NDWW distribution.

3. Structural Properties

The quantile function, mean residual life function, moment generating function (MGF), moments, Lorenz and Bonferroni curves, mean, and variance for the NDWW distribution are shown in this section.

3.1. Quantile Function. The quantile function is

$$Q(u) = \sqrt[\beta]{\frac{1}{\alpha\left(c^\beta + 1\right)}\gamma^{-1}\left(\frac{\theta}{\beta} + 1, u\Gamma\left(\frac{\theta}{\beta} + 1\right)\right)}. \tag{10}$$

3.2. Moments. If X has the pdf (4), then r^{th} moment is obtained as follows:

$$\begin{aligned}\acute{\mu}_r &= \int_0^\infty x^r f(x)\mathrm{d}x \\ &= \frac{\beta\left[\alpha\left(c^\beta + 1\right)\right]^{\theta/\beta+1}}{\Gamma(\theta/\beta + 1)} \int_0^\infty x^{r+\theta+\beta-1} e^{-\alpha\left(c^\beta+1\right)x^\beta}\mathrm{d}x.\end{aligned} \tag{11}$$

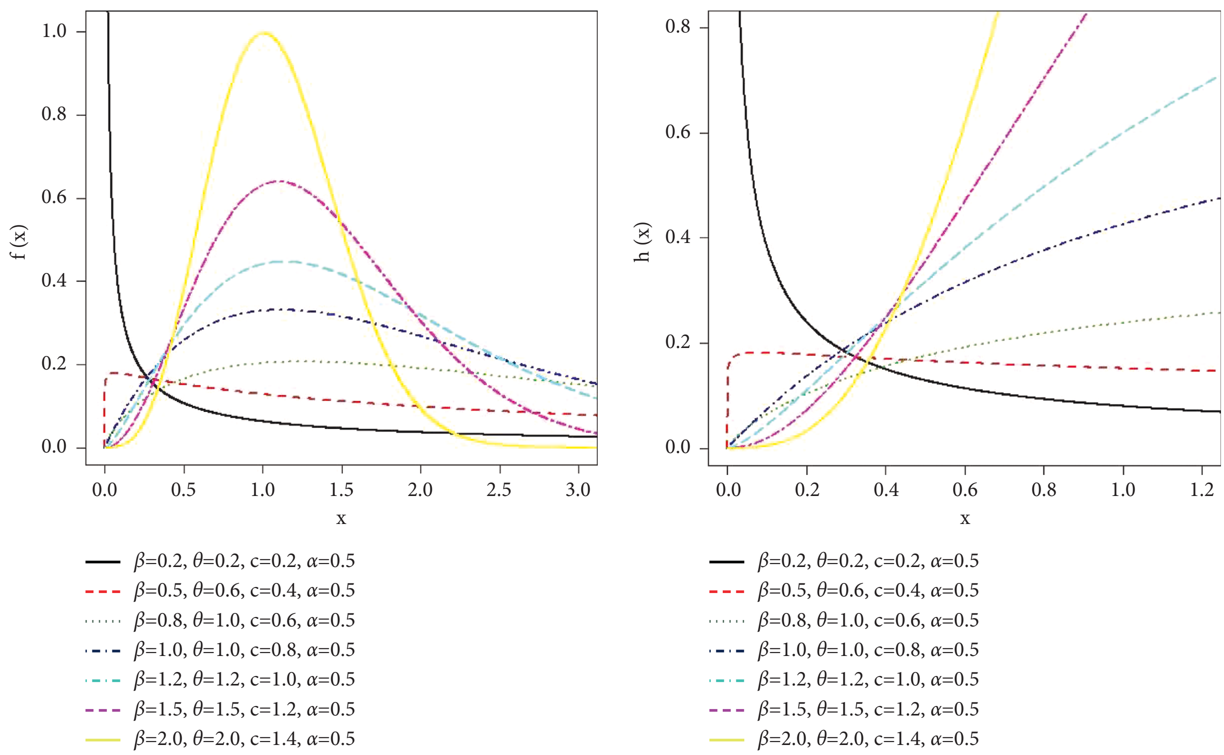


Figure 1: pdf and hazard rate of the NDWW model.

then

$$\mu'_r = E(X^r) = \frac{\Gamma(1 + r/\beta + \theta/\beta)}{\left[\alpha\left(c^\beta + 1\right)\right]^{r/\beta}\Gamma(1 + \theta/\beta)}. \quad (12)$$

The first four rth moments can be obtained by putting $r = 1$, 2, 3, and 4 as

$$\mu'_1 = \frac{\Gamma(1 + 1/\beta + \theta/\beta)}{\left[\alpha\left(c^\beta + 1\right)\right]^{1/\beta}\Gamma(1 + \theta/\beta)},$$

$$\mu'_2 = \frac{\Gamma(1 + 2/\beta + \theta/\beta)}{\left[\alpha\left(c^\beta + 1\right)\right]^{2/\beta}\Gamma(1 + \theta/\beta)}, \quad (13)$$

$$\mu'_3 = \frac{\Gamma(1 + 3/\beta + \theta/\beta)}{\left[\alpha\left(c^\beta + 1\right)\right]^{3/\beta}\Gamma(1 + \theta/\beta)},$$

and

$$\mu'_4 = \frac{\Gamma(1 + 4/\beta + \theta/\beta)}{\left[\alpha\left(c^\beta + 1\right)\right]^{4/\beta}\Gamma(1 + \theta/\beta)}. \quad (14)$$

The variance can be calculated as

$$\mathrm{var}(x) = \mu'_2 - \left(\mu'_1\right)^2 = \frac{\Gamma(1 + 2/\beta + \theta/\beta)}{\left[\alpha\left(c^\beta + 1\right)\right]^{2/\beta}\Gamma(1 + \theta/\beta)} - \left[\frac{\Gamma(1 + 1/\beta + \theta/\beta)}{\left[\alpha\left(c^\beta + 1\right)\right]^{1/\beta}\Gamma(1 + \theta/\beta)}\right]^2. \quad (15)$$

By definition of moment generating function of X and using (4), we have

$$M_x(t) = \sum_{r=0}^{\infty}\frac{t^r}{r!}\mu'_r = \sum_{r=0}^{\infty}\frac{t^r}{r!}\frac{\Gamma(1 + r/\beta + \theta/\beta)}{\left[\alpha\left(c^\beta + 1\right)\right]^{r/\beta}\Gamma(1 + \theta/\beta)}. \quad (16)$$

3.3. Conditional Moments. The s^{th} conditional moment, say $\tau_s(t)$, is defined by

$$\tau_s(t) = \int_t^\infty x^s f(x)\mathrm{d}x. \quad (17)$$

Table 2: Submodels of the NDWW distribution.

Θ	β	α	c	Distribution	Author
—	—	—	—	NDWW	NEW
$k+\lambda$	—	—	0	Weighted Weibull distribution (WWW)	NEW
$k+3$	—	—	0	Volume biased weighted Weibull distribution (VBWW)	Ahmed et al. [6]
$k+2$	—	—	0	Area biased weighted Weibull distribution (ABWW)	Perveen et al. [7]
k+1	—	—	0	Size biased weighted Weibull distribution (SBWW)	Ahmad [6]
Kβ	—	—	0	Weighted Weibull distribution (WW)	Dey et al. [8]
—	1	—	0	Weighted exponential distribution (WE)	Ahmed et al. [6]
1	1	—	0	Length biased (LB) exponential distribution (LBE)	Dara and ahmad [9].
—	2	1/(2k)	0	Weighted Rayleigh distribution (WR)	Ajami and jahanshahi [10]
1	2	1/(2k)	0	LB Rayleigh distribution (LBR)	NEW
2	2	1/(2k)	0	Area biased Rayleigh distribution (ABR)	NEW
0	1	—	0	One parameter exponential distribution (E)	Sea ahmed et al. [6]
0	2	—	0	One parameter Rayleigh distribution (R)	Sea ajami and jahanshahi [10]
0	—	—	0	Weibull distribution (W)	Sea dey et al. [8]
—	1	—	0	LB gamma distribution (LBG)	Ahmed et al. [6]
—	1	1	0	One-parameter gamma distribution (G)	Sea Ahmed et al. [6]
$\beta(\mathrm{k}-1)$	—	λ^{β}	0	Generalized gamma distribution (GG)	Khodabin and Ahmadabadi [11]
k-2	2	1	1	Chi distribution	
2(k-1)	2	λ^2	0	Generalized normal distribution (GN)	Saralees [12]
-1	2	σ^2	1	Half-normal distribution	Cooray and ananda [13]
$\beta k+1$	—	λ^{β}	0	LB weighted weibull distribution (LBWW)	Kishore and tanusree [14]
k-2	2	—	0	Generalized Rayleigh distribution	See das and roy [15]
2	2	—	0	LB Rayleigh distribution (LBR)	Bashir and rasul [16]
2(k-1)	2	kc^2	—	Weighted generalized Rayleigh distribution (WGR)	Das and roy [15]
2k-1	2	kc^2/c^2+1	—	LB weighted generalized Rayleigh distribution (LBWGR)	Das and roy [15]
$\beta k+1$	—	$1/\lambda^{\beta}$	0	LB weighted Weibull distribution (LBWW)	Das and roy [17]
1	1	$\lambda-k$	0	LB weighted exponential distribution (LBWE)	Al-kadim and hussein [18]
2	2	—	$1/\sqrt{k}-1$	LB weighted Rayleigh distribution (LBWR)	Al-kadim and hussein [18]
1	2	$\lambda/2$	0	Maxwell distribution (M)	See dar et al. [19]
1	2	$\lambda-k/2$	0	Weighted Maxwell distribution (WM)	Joshi and modi [20]
$k+1$	2	$\lambda/2$	0	Weighted Maxwell distribution (WM1)	Dar et al. [19]
2	2	$\lambda/2$	0	LB Maxwell distribution (LBM)	NEW
3	2	$\lambda/2$	0	Area biased Maxwell distribution (ABM1)	NEW
$2(\lambda-1)$	2	λ/k	0	Nakagami distribution (N)	See mudasir and ahmed [21]
$2\lambda+\sigma-2$	2	λ/k	0	Weighted Nakagami distribution (WN)	Mudasir and ahmed [21]
2λ	2	λ/k	0	Area biased Nakagami distribution (ABN)	NEW
$2\lambda-1$	2	λ/k	0	LB Nakagami distribution (LBN)	Mudasir and ahmad [21]
$\lambda+1$	1	—	1	Weighted Ailamujia distribution (WA)	
1	1	—	1	Ailamujiadistribution (A)	See Jan et al. (2017)
2	1	—	1	LB Ailamujia distribution (LBA)	NEW
3	1	—	1	Area biased Ailamujia distribution (ABA)	NEW

Using (3), then $\tau_s(t)$ will be as follows:

$$\tau_s(t)=\frac{\beta\left[\alpha\left(c^{\beta}+1\right)\right]^{\theta/\beta+1}}{\Gamma(\theta/\beta+1)}\int_t^{\infty}x^{s+\theta+\beta-1}e^{-\alpha\left(c^{\beta}+1\right)x^{\beta}}\mathrm{d}x. \tag{18}$$

Then,

$$\tau_s(t)=\frac{\Gamma\left(1+s+\theta/\beta,\alpha\left(c^{\beta}+1\right)x^{\beta}\right)}{\left[\alpha\left(c^{\beta}+1\right)\right]^{s/\beta}\Gamma(1+\theta/\beta)}. \tag{19}$$

3.4. Incomplete Moments. The s^{th} incomplete moment, say $\varphi_s(t)$, is defined by

$$\varphi_s(t)=\int_0^t x^s f(x)\mathrm{d}x. \tag{20}$$

Using (3), then, $\varphi_s(t)$ will be as follows:

$$\varphi_s(t)=\frac{\beta\left[\alpha\left(c^{\beta}+1\right)\right]^{\theta/\beta+1}}{\Gamma(\theta/\beta+1)}\int_0^{t}x^{s+\theta+\beta-1}e^{-\alpha\left(c^{\beta}+1\right)x^{\beta}}\mathrm{d}x. \tag{21}$$

Then,

$$\varphi_s(t)=\frac{\gamma\left(1+s+\theta/\beta,\alpha\left(c^{\beta}+1\right)x^{\beta}\right)}{\left[\alpha\left(c^{\beta}+1\right)\right]^{s/\beta}\Gamma(1+\theta/\beta)}. \tag{22}$$

3.5. Mean Residual Life Function. The mean residual life function may be calculated by using the following formula:

$$\mu(x)=E(X-x|X>x)=\frac{\int_x^{\infty}yf(x)dy}{\overline{F}(x)}-x, \tag{23}$$

Table 3: MLE and MSE of NDWW distribution for set 1and set 2.

n	Parameters	Init	MLE	MSE	Init	MLE	MSE
100	α	0.500	0.933	0.340	0.500	0.534	0.262
	β	1.500	1.004	0.280	0.500	0.504	0.003
	θ	1.000	0.804	0.915	1.500	1.215	0.255
	c	0.500	0.386	0.438	0.300	0.469	0.048
500	α	0.500	0.887	0.324	0.500	0.494	0.077
	β	1.500	1.336	0.209	0.500	0.514	0.003
	θ	1.000	0.855	0.887	1.500	1.367	0.124
	c	0.500	0.411	0.266	0.300	0.444	0.037
1000	α	0.500	0.557	0.097	0.500	0.503	0.076
	β	1.500	1.457	0.100	0.500	0.517	0.002
	θ	1.000	0.920	0.424	1.500	1.537	0.095
	c	0.500	0.520	0.142	0.300	0.338	0.032

Table 4: MLE and MSE of NDWW distribution for set 3 and set 4.

n	Parameters	Init	MLE	MSE	Init	MLE	MSE
100	α	0.100	0.475	0.191	1.300	1.087	1.346
	β	0.700	0.439	0.072	1.100	1.019	0.187
	θ	0.700	0.937	0.379	1.100	1.486	0.726
	c	0.300	0.360	0.021	2.800	2.683	0.205
500	α	0.100	0.240	0.157	1.300	1.180	0.789
	β	0.700	0.542	0.070	1.100	1.071	0.123
	θ	0.700	0.846	0.333	1.100	1.444	0.687
	c	0.300	0.356	0.019	2.800	2.698	0.136
1000	α	0.100	0.137	0.0606	1.300	1.337	0.597
	β	0.700	0.660	0.062	1.100	1.077	0.056
	θ	0.700	0.724	0.184	1.100	1.291	0.504
	c	0.300	0.310	0.018	2.800	2.753	0.081

In the NDWW distribution, we obtain the MRL function as follows:

$$\mu(x) = \frac{\Gamma\left(1+1+\theta/\beta, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)/\left[\alpha\left(c^{\beta}+1\right)\right]^{1/\beta}\Gamma(1+\theta/\beta)}{1-\gamma\left(\theta/\beta+1, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)/\Gamma(\theta/\beta+1)} - x. \tag{24}$$

3.6. Lorenz and Bonferroni Curves. The Lorenz (*Lo(x)*) and Bonferroni (*Bo(x)*) curves are inequality metrics that are widely employed in income and wealth distributions [22]. They are gained in the following order:

$$\begin{aligned} Lo(x) &= \frac{\int_0^t xf(x)dx}{\mathrm{E}(X)} \\ &= \frac{\gamma\left(1+1+\theta/\beta, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)}{\Gamma(1+1+\theta/\beta)}, \end{aligned} \tag{25}$$

and

$$\begin{aligned} Bo(x) &= \frac{Lo(x)}{F(x)} \\ &= \frac{\gamma\left(1+1+\theta/\beta, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)}{\Gamma(1+1+\theta/\beta)\gamma\left(1+\theta/\beta, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)}. \end{aligned} \tag{26}$$

Table 5: MLE and MSE of NDWW distribution for set 5 and set 6.

n	Parameters	Init	MLE	MSE	Init	MLE	MSE
100	α	1.800	2.119	0.874	0.500	0.534	0.262
	β	0.800	0.750	0.124	0.500	0.514	0.003
	θ	0.800	1.142	0.9841	1.500	1.215	0.755
	c	3.000	3.630	0.948	0.300	0.469	0.048
500	α	1.800	2.018	0.767	0.500	0.494	0.177
	β	0.800	0.764	0.056	0.500	0.514	0.003
	θ	0.800	1.082	0.591	1.500	1.367	0.524
	c	3.000	3.272	0.456	0.300	0.404	0.037
1000	α	1.800	1.8613	0.511	0.500	0.503	0.076
	β	0.800	0.774	0.041	0.500	0.507	0.001
	θ	0.800	0.853	0.417	1.500	1.537	0.395
	c	3.000	3.144	0.394	0.300	0.338	0.032

3.7. Order Statistics. Let $X_{1:\,n} < X_{2:n} < \ldots < X_{n:\,n}$ be the matching ordered random sample from an n-person population. The s^{th} order statistic's pdf is defined as

$$f_{s:n}(x) = \frac{n!}{(s-1)!(n-s)!}f(x)F(x)^{s-1}(1-F(x))^{n-s}. \tag{27}$$

By entering (4) and (5) into (27), the pdf of the s^{th} order statistic for the NDWW distribution may be derived:

$$\begin{aligned} f_{s:n}(x) &= \frac{n!}{(s-1)!(n-s)!}\left(\frac{\beta\left[\alpha\left(c^{\beta}+1\right)\right]^{\theta/\beta+1}}{\Gamma(\theta/\beta+1)}x^{\theta+\beta-1}e^{-\alpha\left(c^{\beta}+1\right)x^{\beta}}\right) \\ &\left(\gamma\left(\theta/\beta+1, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)\right)^{s-1} \times \left(1-\gamma\left(\theta/\beta+1, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)\right)^{n-s}. \end{aligned} \tag{28}$$

The smallest pdf of order statistics can be obtained at $s=1$ as

$$\begin{aligned} f_{1:n}(x) &= \frac{n\beta\left[\alpha\left(c^{\beta}+1\right)\right]^{\theta/\beta+1}}{\Gamma(\theta/\beta+1)}x^{\theta+\beta-1}e^{-\alpha\left(c^{\beta}+1\right)x^{\beta}} \\ &\left(1-\gamma\left(\frac{\theta}{\beta}+1, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)\right)^{n-1}. \end{aligned} \tag{29}$$

The greatest pdf of order statistics can be obtained at $s=n$ as

$$\begin{aligned} f_{n:n}(x) &= \frac{n\beta\left[\alpha\left(c^{\beta}+1\right)\right]^{\theta/\beta+1}}{\Gamma(\theta/\beta+1)}x^{\theta+\beta-1}e^{-\alpha\left(c^{\beta}+1\right)x^{\beta}} \\ &\left(\gamma\left(\frac{\theta}{\beta}+1, \alpha\left(c^{\beta}+1\right)x^{\beta}\right)\right)^{n-1}. \end{aligned} \tag{30}$$

4. Maximum Likelihood Estimation

If n independent items are tested and the lifetime distribution of each component is supplied by (3), then the log likelihood (LL) function based on the sample that was seen $x = (x_1, x_2, \mathrm{K}, x_n)$ is defined:

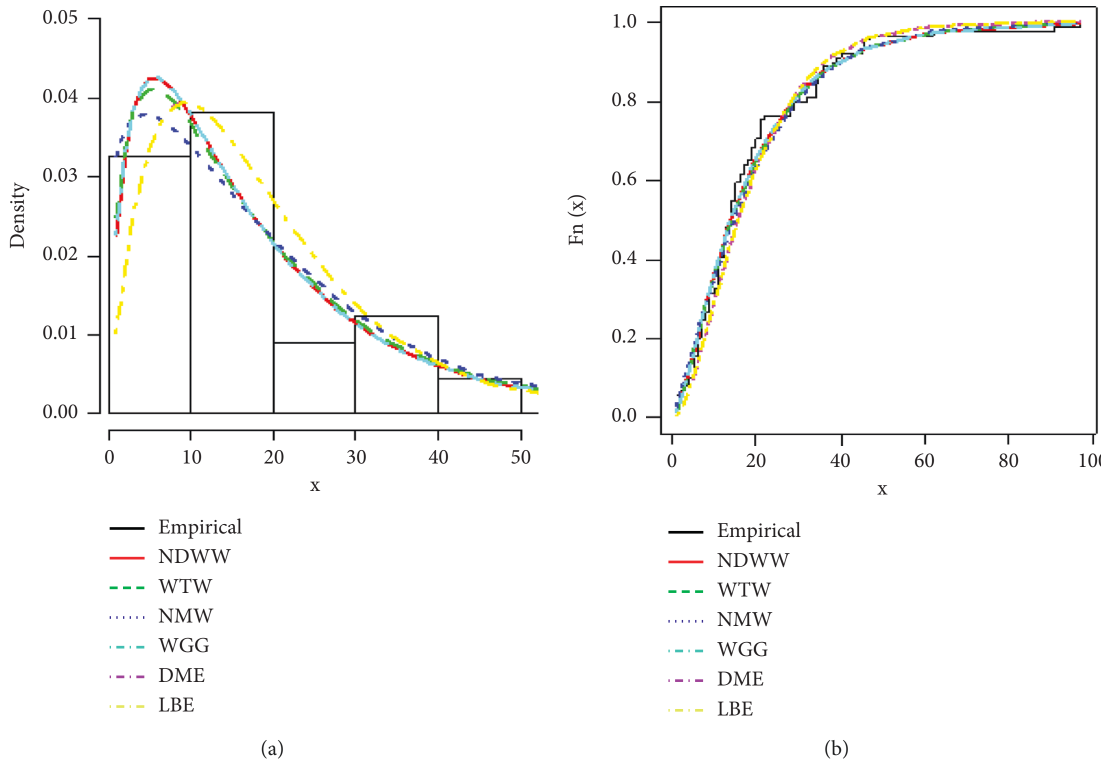


Figure 2: The fitted pdfs (a) and cdfs (b) for the dataset (1).

$$\ln L = n \ln \beta + n\left(\frac{\theta}{\beta}+1\right)\ln(\alpha) + n\left(\frac{\theta}{\beta}+1\right)\ln\left(c^{\beta}+1\right) - n \ln\left[\Gamma\left(\frac{\theta}{\beta}+1\right)\right] + (\theta+\beta-1)\sum_{i=1}^{n}\ln x_i - \alpha\left(c^{\beta}+1\right)\sum_{i=1}^{n} x_i^{\beta}. \tag{31}$$

The first partial derivatives of the parameters are

$$\frac{\partial \ln L}{\partial \theta} = \frac{n}{\beta}\ln(\alpha) + \frac{n}{\beta}\ln\left(c^{\beta}+1\right) - \frac{n}{\beta}\psi\left(1+\frac{\theta}{\beta}\right) + \sum_{i=1}^{n}\ln x_i = 0, \tag{32}$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha}\left(\frac{\theta}{\beta}+1\right) - \left(c^{\beta}+1\right)\sum_{i=1}^{n} x_i^{\beta} = 0, \tag{33}$$

$$\frac{\partial \ln L}{\partial c} = n\beta c^{\beta-1}\left(\frac{\theta}{\beta}+1\right)\psi\left(c^{\beta}+1\right) - \alpha\beta c^{\beta-1}\sum_{i=1}^{n} x_i^{\beta} = 0, \tag{34}$$

$$\begin{aligned}\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} - \frac{n\theta}{\beta^2}\ln\left(\alpha\left(c^{\beta}+1\right)\right) + \frac{nc^{\beta}\ln c\,(\theta/\beta+1)}{c^{\beta}+1} + \frac{n\theta}{\beta^2}\psi\left(1+\frac{\theta}{\beta}\right) \\ + \sum_{i=1}^{n}\ln x_i - \alpha\left(c^{\beta}+1\right)\sum_{i=1}^{n} x_i^{\beta}\ln x_i - \alpha c^{\beta}\ln c\sum_{i=1}^{n} x_i^{\beta} = 0,\end{aligned} \tag{35}$$

where $\psi(x) = \Gamma(x)/\overline{\Gamma}(x)$ is the digamma function.

Because the LL equations (32-35) do not have analytic solutions, we will utilize iterative techniques to compute these equations' numerical solutions to provide maximum LL estimators of θ,α, c, and β, say $\widehat{\theta}_{\mathrm{MLE}}$, $\widehat{\alpha}_{\mathrm{MLE}}$, $\widehat{c}_{\mathrm{MLE}}$, and $\widehat{\beta}_{\mathrm{MLE}}$, respectively.

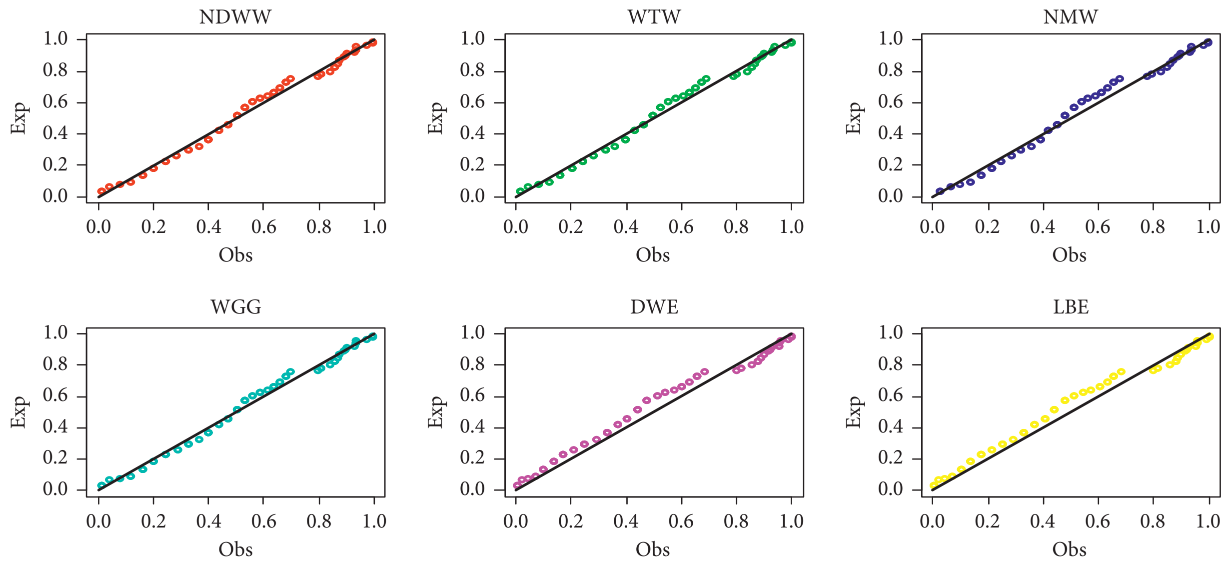


Figure 3: The pp plots for the dataset (1).

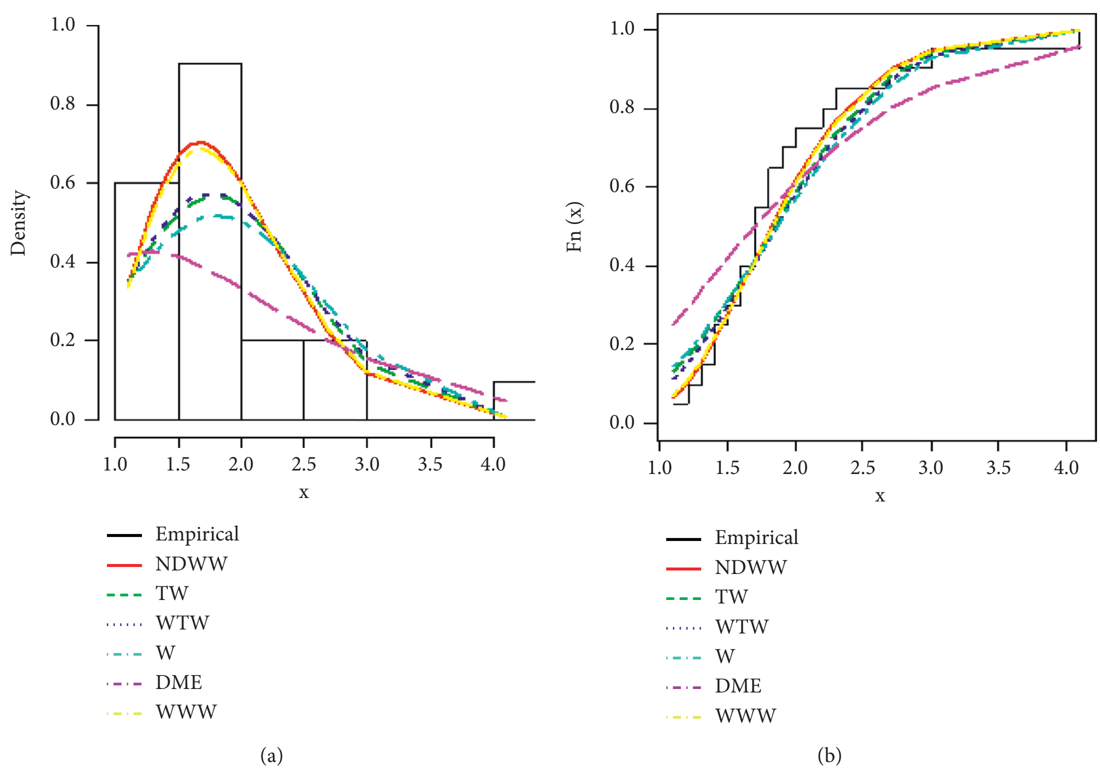


Figure 4: The fitted pdfs (a) and cdfs (b) for the dataset (2).

5. Numerical Results

In this section, we evaluate the ML estimators' performance in terms of sample size n. A numerical evaluation of the performance of ML estimators for the NDWW distribution is performed. Estimates are X_1, X_2, K, and X_n and are evaluated using the Mathematica program based on the following quantities for each sample size: empirical mean square errors (MSEs). The following are the numerical procedures:

(i) A random sample of sizes $n = 100$, 500, and 1000 is taken into account; these random samples are produced from the NDWW distribution using the inversion approach.

(ii) Six sets of parameters are taken into account.

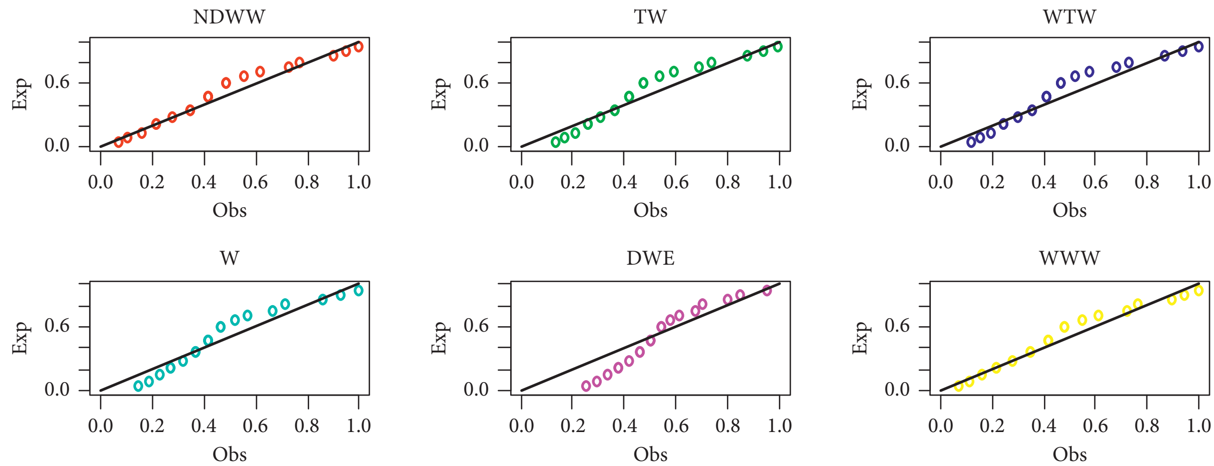
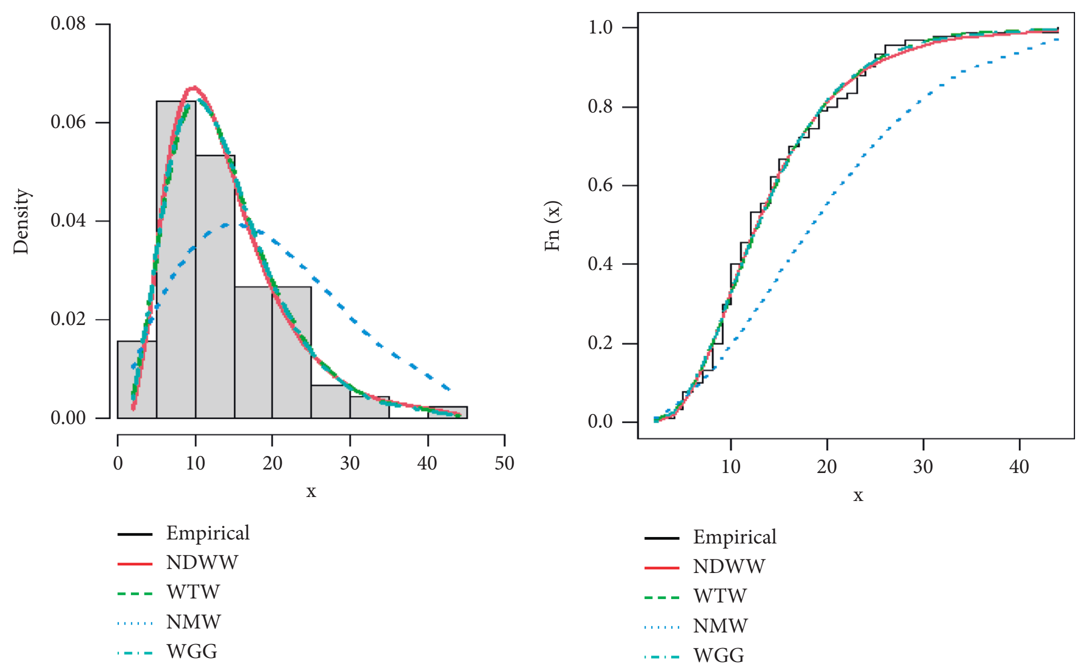


Figure 5: The pp plots for the dataset (2).



(a) (b)

Figure 6: The fitted pdfs (a) and cdfs (b) for the dataset (3).

(iii) The NDWW model's ML estimates (MLEs) are assessed for each parameter value and sample size.

(iv) Repeat this process 10000 times to obtain the means and MSEs of the MLE for various parameter values in both models and for each sample size.

(v) Tables 3–5 present empirical findings. These tables show that the estimates are fairly consistent and near to the real value of the parameters as sample sizes grow.

6. Applications

In this section, we apply the NDWW distribution to illustrate our point; we use three real-life datasets, which correspond to the daily death toll from COVID-19 in Egypt and Georgia and the relief times of twenty patients taking analgesics. To compare the NDWW distribution to other fitted distributions, five, four, three, two, and one parameters are used. The NDWW distribution is compared to the (DWE), (LBE), (W), new modified Weibull distribution (NMW), weighted generalized gamma distribution (WGG), weighted transmuted Weibull distribution (WTW), transmuted Weibull distribution (TW), and weighted Weibull distribution (WWW).

The first dataset proposes a concrete application with an actual dataset to assess the interest in the NDWW distribution. The considered data, called the daily deaths, confirmed cases of COVID-19 in Egypt from 19 March to 15 June 2020. The dataset was obtained from the following

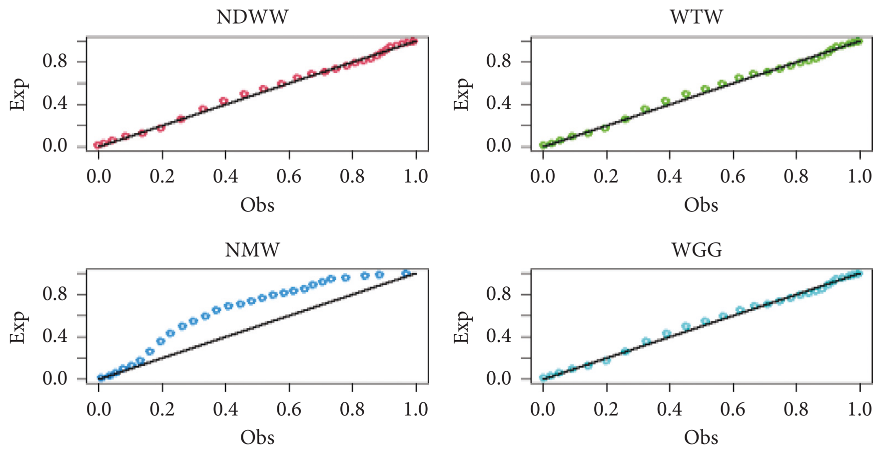


FIGURE 7: The pp plots for the dataset (3).

TABLE 6: MLEs and (SErs) for the dataset (1) COVID-19.

Distribution	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\theta}$	$\widehat{c}$	$\widehat{\lambda}$
NDWW	1.194 (0.966)	0.69 (0.295)	0.146 (0.00016)	2.107 (0.00005)	— —
WTW	0.000065 (0.158)	0.835 (0.112)	0.114 (0.863)	0.176 (0.119)	— —
NMW	1.195 (0.094)	0.000056 (0.0001)	0.028 (0.009)	0.02 (5.051)	1.006 (26.457)
WGG	0.716 (2789)	0.69 (0.295)	0.524 (2079)	1.158 (4232)	1.522 (1434)
DWE	— —	— —	0.107 (0.008)	61.943 (153.721)	— —
LBE	— —	— —	0.107 (0.008)	— —	— —

TABLE 7: Numerical values of Z1, Z2, Z3, Z4, Z5, Z6, Z7, and Z8 for the dataset (1) COVID-19.

Distribution	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8
NDWW	700.22	700.696	700.017	704.232	0.62155	0.0947	0.07511	0.69687
WTW	700.507	700.984	700.305	704.52	0.63119	0.09925	0.08151	0.59543
NMW	704.86	705.583	704.607	709.876	1.49564	0.103505	0.09823	0.35689
WGG	702.22	702.943	701.967	707.235	0.62193	0.09481	0.07511	0.69687
DWE	703.964	704.103	703.863	705.97	1.55629	0.261494	0.11959	0.15677
LBE	701.995	702.041	701.945	702.999	1.5351	0.25794	0.11958	0.1568

TABLE 8: MLEs and (SErs) for the dataset (2).

Distribution	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\theta}$	$\widehat{c}$
NDWW	19.994 (18.594)	0.473 (0.398)	5.861 (18.786)	24.27 (136.949)
TW	0.063 (0.033)	3.096 (0.47)	0.711 (0.316)	- -
WTW	0.018 (7.738)	2.062 (0.211)	0.102 (0.632)	0.451 (0.074)
W	- -	2.787 (0.427)	0.122 (0.056)	- -
DWE	- -	- -	0.0002 (0.573)	1.579 (0.496)
WWW	5.53 (1.799)	0.745 (0.448)	10.643 (16.104)	6.395 (1.799)

Table 9: Numerical values of Z1, Z2, Z3, Z4, Z5, Z6, Z7, and Z8 for the dataset (2).

Distribution	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8
NDWW	42.457	45.123	39.661	43.234	0.51694	0.08775	0.15626	0.71323
TW	45.495	46.995	43.398	46.078	0.94357	0.1572	0.17266	0.59002
WTW	46.646	49.313	43.85	47.423	0.87024	0.14441	0.18335	0.51201
W	45.173	45.879	43.775	45.562	1.06815	0.176	0.21196	0.33003
DWE	49.775	50.48	48.377	50.163	2.55133	0.21707	0.25259	0.15575
WWW	43.054	45.721	40.258	43.832	0.5721	0.09684	0.16494	0.64802

Table 10: MLEs and (SErs) for the dataset (3) COVID-19.

Distribution	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{c}$	$\hat{\lambda}$
NDWW	12.76	0.276	6.77	25.318	-
	(12.545)	(0.267)	(21.414)	(177.294)	-
WTW	2.86	1.793	0.589	0.0062	-
	(1.887)	(0.252)	(0.665)	(0.0066)	-
NMW	1.889	0.0029	0.0025	0.00009	1.889
	(0.283)	(0.00288)	(0.00229)	(0.27)	(0.28)
WGG	0.0074	0.686	3.758	1.519	2.86
	(18.343)	(0.00079)	(188.427)	(30.49)3	(134.32)

Table 11: Numerical values of Z1, Z2, Z3, Z4, Z5, Z6, Z7, and Z8 for the dataset (3) COVID-19.

Distribution	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8
NDWW	600.834	601.305	600.651	604.867	0.4168	0.06454	0.0692	0.78194
WTW	601.129	601.6	600.946	605.161	0.45796	0.08109	0.08356	0.5561
NMW	610.64	611.355	610.412	615.681	1.22455	0.21458	0.10933	0.23224
WGG	603.278	603.993	603.05	608.319	0.4525	0.07863	0.0796	0.61856

electronic address: https://www.care.gov.eg/(1, 1, 2, 4, 5, 1, 1, 3, 6, 6, 4, 1, 5, 6, 6, 8, 5, 7, 7, 9, 9, 15, 17, 11, 13, 5, 14, 5, 13, 9, 19, 15, 11, 14, 12, 11, 7, 13, 10, 20, 22, 21, 12, 14, 9, 14, 7, 16, 17, 13, 21, 11, 11, 8, 11, 12, 15, 21, 20, 18, 15, 14, 21, 16, 11, 28, 29, 19, 14, 19, 29, 34, 34, 46, 46, 47, 36, 38, 40, 32, 39, 34, 35, 36, 35, 45, 62, 91, 97).

Tables 6 and 7 provide the ML estimates of the distribution parameters together with their standard errors (SErs). The analytical measures include the Akaike information criterion (Z1), the correct Akaike information criterion (Z2), the Bayesian information criterion (Z3), and the Hannan-Quinn information criterion (Z4), the Anderson Darling statistic (Z5), the Cramér-von Mises statistic (Z6), the Kolmogorov–Smirnov test (Z7), and the p value (Z8).

The second dataset is taken from the work of Gross and Clark [23]. The values of Z1, Z2, Z3, Z4, Z5, Z6, Z7, and Z8 are given in Tables 8 and 9.

The third dataset proposes a concrete application with an actual dataset to assess the interest in the NDWW distribution. The considered data, called the daily deaths, confirmed cases of COVID-19 in Georgia from 1 Jan to 31 March 2021. The dataset was obtained from the following electronic address: https://github.com/CSSEGISandData/COVID-19/(23, 44, 31, 25, 18, 20, 28, 34, 22, 23, 23, 24, 26, 21, 26, 23, 17, 25, 15, 14, 11, 24, 16, 17, 16, 25, 12, 19, 21, 11, 19, 16, 14, 13, 19, 18, 11, 14, 15, 8, 5, 10, 15, 7, 9, 11, 14, 13, 9, 7, 19, 10, 12, 10, 6, 12, 10, 14, 11, 10, 12, 9, 12, 10, 4, 9, 15, 10, 12, 9, 12, 9, 5, 2, 8, 7, 9, 9, 8, 9, 10, 4, 8, 10, 6, 8, 5, 14, 8, 5). The values of Z1, Z2, Z3, Z4, Z5, Z6, Z7, and Z8 are given in Tables 10 and 11. Many studies proposed COVID-19 datasets, such as, the works of Alanazi et al. [24], Abdul Latif et al. [25], and Kaur et al. [26].

We observe the NDWW distribution gives the best fit among the other selected models for the three datasets. The fitted pdfs and cdfs plots and pp plots of mentioned distributions for the first, second, and third data are represented in Figures 2–7 for the purpose visual comparison. According to these plots, we observe that the NDWW distribution is better than other distributions.

7. Conclusions

NDWW (new double-weighted Weibull distribution) is a new four-parameter double-weighted Weibull distribution that is suggested. As a result, more data may be examined using the model. Moments, moment generating function (MGF), and incomplete moments are addressed. Quantile function, mean residual life, AND Lorenz and Bonferroni curves are also examined. The maximum likelihood approach is used to estimate the parameters of the models. The behavior of MLE estimations is examined using simulation. The suggested model's utility is demonstrated by three real-world datasets: daily fatalities confirmed instances of COVID-19 in Egypt and Georgia and relief times of twenty patients getting an analgesic. In the future, we plan to use Neutrosophic statistics, as many authors use this new branch, such as, Mahapatra et al. [27] and Mahapatra et al. [28].

Abbreviations

Wei:	Weibull
pdf:	The probability density function
cdf:	Cumulative distribution function
W:	Weighted
DW:	Double weighted
NDWW:	New double-weighted Weibull
MGF:	Moment generating function
LL:	Log likelihood
MSEs:	Mean square errors
MLEs:	Maximum likelihood estimates
SErs:	Standard errors.

Data Availability

The data used to support the findings of the study can be obtained from the corresponding author upon request.

Conflicts of Interest

There are no conflicts of interest in this paper's publishing.

Acknowledgments

The researchers would like to thank the Deanship of Scientific Research, Qassim University for funding the publication of this project.

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