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*Research Article*

# A Robust Multiobjective Mathematical Model Optimizing Stock Portfolio

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The present paper investigates the problem of capital portfolio selection under uncertain conditions and uses a robust optimization approach for modeling. The model provided in this paper is a three-objective model that aims to maximize returns, maximize liquidity, and minimize risk. The data extracted from the site of the Tehran Stock Exchange are as follows. These data are related to twenty shares from July 2020 to July 2021. The robust approach used in this research has been analyzed by the real data of the Tehran Stock Exchange and then the optimal portfolio for different robust costs has been formed by solving the robust model. In the following section, the relevant model is solved through real stock market data and using the goal programming approach, and the results are investigated and analyzed.

## 1. Introduction

The portfolio selection problem is one of the most significant problems in the field of financial management. In this problem, an effort is done to disseminate the investor's budget among the assets in a way that increases the return on the capital portfolio and decreases its risk. The stock market provides a mechanism through which the small savings of the society are converted into macroeconomic investments, the proportional development of the two main sectors of the economy, i.e., the financial and real sectors, is of special importance [1]. Developed countries have always had and have strong money and capital markets. The lack of proper development of the capital market as an important subset of the financial sector, in addition to creating double pressure on the country's monetary system, has caused production and service units to be deprived of the benefits of an active and dynamic capital market [2]. An effective portfolio means the optimal mixture of assets in a way that the portfolio risk is decreased for a specified rate of return [3]. In fact, the two significant elements for investment decisions are the amount of risk and the return on capital assets [4]. Rational investors consider returns desirable and avoid risk [5]. Furthermore, they act rationally in decision making, which maximizes their desired efficiency [6]. Therefore, the desirability of investors is a function of expected returns and risk, which are the two basic parameters of investment decisions [7]. In other words, in the problem of portfolio optimization, we are looking for a portfolio that produces less standard deviation (risk) and more expected value [8, 9].

The portfolio selection problem is one of the most important problems in the field of finance; various models and methods have been presented in this regard by various researchers [10, 11]. This includes creating a stock portfolio that maximizes investor utility [12]. In these problems, an attempt is made to distribute the specified budget among the assets in such a way as to maximize the return on capital and minimize its risk [13, 14]. However, there are various methods to model an investment portfolio problem [15]. These include single-objective and multiobjective models that can be linear or nonlinear. It should be noted that data uncertainty and the existence of uncontrollable variables in the financial markets and the investor decision-making process are inevitable [16, 17]. In other words, the common thing in all models of portfolio selection in the real world is the uncertainty of some of their parameters [18]. Therefore, it is necessary to consider the data uncertainty with one of the available methods, depending on the characteristics and strengths and weaknesses of each method.

Some parameters in the investment portfolio model are uncertain due to their predictable nature and their randomness, such as the systematic risk parameter. Classical methods for considering parameter uncertainties include sensitivity analysis and stochastic optimization. In sensitivity analysis, first uncertainty is ignored in general, then after solving the problem through sensitivity analysis, the effect of data uncertainty on the problem is investigated. Although sensitivity analysis is a good tool to investigate how good the solution is, it is not a good way to generate solutions that are robust against data changes. On the other hand, it is not possible to use sensitivity analysis in models with high uncertainty parameters. In stochastic optimization, it is assumed that a function of the distribution of input parameters is given. Although the abovementioned model is mathematically strong, it has fundamental problems. It is very unlikely that the definitive distribution function of uncertain parameters can be obtained. Even if the distribution function of these parameters can be obtained, it is difficult to calculate their probability. Also, changing the parameters may confuse the convexity property and complicate the computation of the problem. Considering the abovementioned problems, a useful method for investigating uncertainties in mathematical models is the use of a robust optimization methodology. In the robust optimization approach, we look for near-optimal solutions that are likely to be justified, which are called robust solutions. In addition to maintaining optimality, these solutions also maintain the feasibility of the problem.

In this research, an attempt has been made to develop an investment portfolio selection model using this approach. The investor then specifies the investment objectives in the form of a multiobjective model and solves the model using multiobjective planning solution methods such as goal programming, which is one of the most widely used methods in this field.

In this paper, after presenting the introduction, the literature review section has been presented. Then, a multiobjective model of stock portfolio selection has been defined. Solving the model by real data from the Tehran Stock Exchange has been presented in section 4. Finally, the conclusion has been presented in section 5.

## 2. Literature Review

So far, a lot of research has been conducted in the field of stock selection criteria, both in the Tehran Stock Exchange and worldwide. However, studies dedicated to explaining risk criteria have rarely been conducted, and most studies have described risk criteria in addition to stock selection criteria. Research in the specific field of risk criteria has been specific to emerging criteria in the financial literature, such as adverse risk and value at risk, and explaining their significance in pricing theories such as CAPM; therefore, in this research, first, the studies conducted on specific risk criteria are addressed and then the studies in which the risk criteria are mentioned along with the stock selection criteria are considered. In this section, the research conducted in the field of various types of financial risk measures and their application in robust optimization of the stock portfolio selection problem is reviewed, and then, a review of the goal programming literature through which the model in this article has been solved is provided.

The robust conditional value-at-risk model was introduced in 2008 by Quaranta and Zaffaroni [19]. In this model, the objective is to minimize conditional value-at-risk. Conditional value-at-risk is a comprehensive risk measure, also called tail risk. Experimental tests of the above model have been carried out in the Italian financial markets.

The next model to be investigated is the robust optimization model of mean absolute deviations [20]. In this model, a robust model of mean absolute deviations or RMAD for short is presented, which leads to linear programming that reduces computational complexity. In the experimental results of this paper, different conditions that lead to fluctuation and uncertainty of data have been considered.

A robust integrated model of the share selection problem has been developed by Baker et al. [21] in which different uncertainty norms have been used. In their paper, the problem of stock selection is introduced using an integrated model approach in robust optimization; this model is a development based on robust optimization models of uncertain programming problems. In the integrated model, the uncertainty area is estimated with a suitable norm body, and the model, taking into account the parameters considered by the investor, allows the modeler to produce different versions of the integrated model according to the problem conditions. Therefore, the modeler can propose the appropriate model and investment according to the uncertain parameters and consider the utility of the investor. In this model, the Ben-Tal and Nemirovski [22] method has been used for development.

Jarisch et al. [23] presented a robust model for the forestry-avocado portfolio in South Africa. They presented dual discounting when considering time preferences for the market. Considering time preferences for ecosystem services is one of the contributions of this research. Wu et al. [24] presented a multiobjective criteria system for portfolio selection. They use Tomada de Decisão Iterativa Multicritério algorithm for portfolio selection based on the financial performance. Finally, a case study on medical stock investment in the Chinese stock market is examined.

The robust optimization model of the multiperiod financial portfolio using conditional value-at-risk was proposed by Lotfi et al. [25]. In their paper, WCVaR, which stands for the worst-case conditional value-at-risk, is studied when there is only partial information on the probability function of the uncertain parameters. This index, as well as the value-at-risk (VaR) index, is considered by financial managers as a new criterion for calculating financial portfolio risk. The objective is to minimize WCVaR with combined uncertainty, finite partial uncertainty, and elliptical uncertainty for the distribution of asset returns. Robust multiperiod financial portfolio optimization using the WCVaR risk criterion leads to linear and nonlinear programming problems of the second degree that are efficiently solvable. A robust optimization model of the financial portfolio with the approach of the capital asset pricing model was presented by Kuehn et al. [26]. In their research, a robust optimization approach is proposed to solve the problem of multiperiod financial portfolio selection. Robust optimization models consider the future return on assets as uncertain coefficients in the optimization problem and imagine the degree of risk acceptance of investors as the degree of tolerance to the total error of estimating returns.

Single-objective models seek to maximize or minimize the objective function regardless of the decision maker. To solve this problem of single-objective models, thematic literature of multiobjective models and goal programming were developed [27]. Goal programming was first proposed by Charles and Cooper [28]. Goal programming is a special type of linear programming with multiple and conflicting goals in terms of their importance in such a way that low-level goals are considered only when high-level goals are met [29]. In one-goal programming, the objective function is maximized or minimized, but in goal programming, the deviations between the intended goals and the actual results are minimized [30].

In goal programming models such as one-goal programming, the coefficients are assumed to be certain and fixed [31]. While some coefficients are uncertain in nature. This uncertainty can be due to computational error or nature based on coefficient prediction. In the following of this section, an attempt is made to investigate the uncertainty of the input data to the model by a robust approach [32].

Li and Wang [33] proposed a robust model for the multiobjective stock portfolio selection model. In their model, the goal programming approach is used to solve the multiobjective model. Ben-Tal and Nemirovski developed the asset allocation model using the robust approach proposed by Ben-Tal and Nemirovski, [22]. Their model is a multistage model that attempts to invest assets in a way that maximizes returns at the end of the investment period. In this investment period, there are time periods for the redistribution of capital between assets. The return on assets in these time periods is assumed to be uncertain.

Goldfarb and Iyengar [34] developed the stock portfolio selection model using the Ben-Tal and Nemirovski approach. In the model proposed by them, the mean, variance, and value-at-risk are used. Their model has also become a quadratic cone optimization model. This approach requires internal point methods to solve. Kuchta [35] used a robust optimization approach to model the mean-variance problem. They used Ben-Tal and Nemirovski's robust modeling methods for their modeling.

Kawas and Thiele [36] developed a robust optimization model of the capital portfolio selection problem in a situation where asset returns follow a set of interval uncertainty. This set of uncertainties leads to the Bertsimas and Sim model. The objective function proposed in their model is the worst portfolio value that is attempted to be minimized. Their model is a one-period model and short sale is not allowed in it. In their model, the log-normal distribution function is used for the return on assets, and the uncertainty is determined based on this distribution.

Rotella Junior et al. [37] first provide a model called the multiperiod mean-semi-variance-skewness stochastic investment portfolio optimization model considering the transaction cost. Since it is very difficult to solve the multiperiod portfolio problem due to the nonlinearity of the problem, after modeling the problem using a multiobjective and single-objective particle swarm optimization algorithms, they try to solve the proposed model.

Toumazis and Kwon [38] modeled the worst-case conditional value-at-risk model using various uncertainty approaches, which include cubic uncertainty and elliptical uncertainty. In fact, the above two approaches lead to modeling using methods of Ben-Tal, Nemirovski, and Bertsimas and Sim. The models obtained from the above approaches lead to the linear programming model and the second-order cone optimization model, which are easily solvable. Market data in the model presented by them were generated by simulation.

Huang et al. [39] developed the mean and variance model using stochastic optimization (chance constraint) and robust optimization approaches. They assumed the return on assets to be uncertain and used a set of interval uncertainty to develop robust optimization.

Masmoudi and Abdelaziz [40] provided a robust optimization model for the portfolio selection problem. In the model provided by them, the focus is on entering trading authority in the stock portfolio. In this case, the risk is controlled with the help of stock options and available strategies. An interesting point in their model is the use of an elliptical uncertainty set with a common margin.

Huang et al. [41] developed the mean-median absolute deviation with the help of elliptic uncertainty sets. They developed their modeling for single-period and multiperiod problems.

One of the studies on the application of goal programming in the portfolio selection problem is Lee and Olson, [42] who provided the first GP model in the field of finance. Some of the most important studied conducted using GP in the portfolio selection problem include Booth and Dash [43].

Also, some of the studies conducted in the field of multiobjective models of portfolio selection by considering the uncertainty in the parameters include those of Abdelaziz et al. [44].

## 3. A Multiobjective Model of Stock Portfolio Selection

As mentioned earlier and considering the materials and explanations provided on the stock portfolio selection problem, it is necessary to consider all aspects affecting the investment in order to form an optimal portfolio. Therefore, in this section, according to the literature review and considering the important aspects of investment, three objectives have been selected in this regard, which are as follows:

(1) A return
(2) A conditional value-at-risk (*CVaR*)
(3) A liquidity

The reason for choosing the first objective is the importance of the return and profitability of a share. The second objective is presented in order to investigate the investment risk in the form of an appropriate risk measure that has the ability to be linear and convex. After the two objectives of risk and return, which are considered in most investments, the objective of liquidity is considered. Because a share may be desirable in terms of return and risk, the ability to sell and convert it into cash is time-consuming or even impossible. Finally, according to the mentioned cases, a multiobjective model of capital portfolio selection with the objectives of mean-conditional value-at-risk-liquidity is presented as the following model:

(1) Objective function 1: Return maximization.

$$\text{Maximize} \sum_{j=1}^{n} r_j x_j. \tag{1}$$

(2) Objective function 2: *CVaR* minimization.

$$\text{Minimize}\, \eta + \frac{1}{s(1-\alpha)} \sum_{i=1}^{s} y_i. \tag{2}$$

(3) Objective function 3: Liquidity maximization.

$$\text{Maximize} \sum_{j=1}^{n} L x_j\,. \tag{3}$$

(4) Constraints related to the amount of investment.

$$\text{subject to} \sum_{j=1}^{n} x_j = 1. \tag{4}$$

(5) Constraints related to CVaR

$$y_i \geq \sum_{j-1}^{n} \left[\left(-r_{ij} x_j\right) - \eta\right],\ i = 1, 2, \ldots, s. \tag{5}$$

(6) Constraints related to the mark.

$$x_j \geq 0. \tag{6}$$

It should be noted that in the above relations we have as follows:

(i): Number of periods index

(j): Number of shares index

$x_j$: Decision variable (percentage of the weight of the jth share)

$\eta$: Value at risk VaR

1-$\alpha$: Confidence level

s: Number of scenarios (periods)

$y_i$: Decision variable to calculate CVaR

$r_j$: Return of the jth share

$L_j$: Liquidity of the jth share

Robust optimization is one of the newest techniques introduced in the field of mathematical modeling and optimization. The main nature of this method is based on the principle that uncertain parameters can be controlled in a mathematical model [45]. The main assumption of mathematical modeling and its optimization in the classic mode is that the values of all parameters are known accurately and definitively. However, in real conditions, some parameters may not be certain. For example, in a mathematical model of production planning, the demand parameter cannot be measured accurately. Therefore, considering the uncertainty in the demand parameter is quite reasonable. The use of methods for dealing with uncertainty helps us to model and then optimize various problems with uncertain parameters [46]. The concept of robustization, which has been introduced by many researchers, refers to the fact that due to changes in uncertain parameter, the value of the objective function also changes and fluctuates. Now, among the different values of this uncertain parameter, we must choose the value that achieves the most appropriate value of the objective function in terms of the decision maker and also the least fluctuation in the value of the objective function. In robust optimization, very simply, an interval for the parameters is introduced first. The lower limit and the upper limit of these parameters can be determined based on numerical estimates. In the next step, by performing the calculations specified by Bertsimas and Sim, the mathematical model is rewritten and a robust model based on Bertsimas and Sim is presented. However, many other models have been proposed to deal with uncertainty.

Given that, in this paper, we consider the share return as an uncertain parameter, using the robust approach of Bertsimas and Sim, we change the model and so-called make it robust. We also use the goal programming method to solve the model. Therefore, the model presented above changes as follows:

Table 1: Summary of the problem data.

| Share | Period | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Hormozgan Cement | 0 | 0.0401 | −0.040 | 0.2029 | −0.0478 | −0.043 | −0.0032 | 0.0082 | 0.0300 | −0.0133 | −0.0193 | −0.0398 |
| Darab Cement | 0.15147 | 0.4437 | 0.226 | 0.09387 | 0.0457 | −0.1042 | 0.10099 | −0.0075 | 0.1898 | −0.1006 | 00.1038 | −0.0100 |
| Gharb Cement | 0.0003 | 0.3328 | 0.410 | 0.3359 | 0.3500 | 0.07138 | 0.09771 | −0.0185 | −0.070 | 0.2166 | −0.1343 | 0.025 |
| Ilam Cement | 0.01788 | 0.146 | 0.461 | 0.0758 | 0.0435 | −0.0758 | 0.11442 | 0.0644 | 0.1819 | −0.0331 | −0.0516 | −0.0018 |
| Karun Cement | 0.5683 | 0.3007 | −0.003 | 0.2538 | 0.0247 | −0.097 | 0.02869 | 0.0176 | 0.0827 | −0.0702 | −0.0553 | −0.0244 |
| Kerman Cement | 0.01981 | 0.7258 | 0.2483 | 0.05047 | −0.03 | −0.162 | 0.05532 | 0.1901 | 0.3412 | 0.0686 | −0.01693 | −0.1275 |
| Khuzestan Cement | 0.12572 | 0.4185 | 0.5595 | 0.03365 | 0.2658 | −0.172 | 0.01867 | −0.0051 | 0.3949 | −0.0383 | −0.0358 | −0.0019 |
| Shargh Cement | 0.01795 | 0.294 | 0.1484 | 0.9713 | 0.0813 | −0.0029 | −0.0981 | 0.2146 | 0.2296 | 0.02841 | −0.0638 | −0.0003 |
| Doroud Cement | 0.04691 | 0.0576 | 0.0584 | 0.2055 | −0.070 | −0.1401 | 0.11341 | −0.0624 | 0.2884 | −0.1809 | −0.03907 | −0.0975 |
| Sepahan Cement | 0.40194 | 0.0529 | 0.3802 | 0.13824 | −0.023 | −0.1363 | −0.0209 | 0.04915 | 0.5443 | −0.1348 | −0.03857 | −0.0128 |
| Tehran Cement | 0.156371 | 0.3694 | 0.1206 | 0.212 | −0.048 | −0.0257 | 0.05228 | 0.1025 | 0.4123 | −0.1819 | −0.1483 | −0.0848 |
| Hegmatan Cement | 0.09024 | 0.0398 | 0.0408 | 0.4595 | −0.083 | −0.108 | 0.08073 | −0.0435 | 0.24077 | −0.0377 | −0.1389 | 0.00931 |
| Khash Cement | 0.1513 | 0.311 | 0.3094 | 0.2587 | −0.0132 | −0.1324 | 0.1310 | 0.3644 | 0.1784 | −0.1034 | −0.1064 | −0.1270 |
| Sufiyan Cement | 0.1207 | 0.4287 | 0.0103 | 0.1107 | 0.00065 | −0.1259 | 0.2044 | 0.0333 | 0.2297 | −0.1257 | −0.0881 | −0.006 |
| Urmia Cement | 0.1573 | 0.2454 | 0.1714 | −0.1176 | −0.0294 | −0.1217 | 0.09182 | −0.029 | −0.0607 | −0.1112 | −0.1096 | −0.0034 |
| Shomal Cement | −0.27628 | 0.1750 | 0.462 | 0.1939 | 0.096 | −0.1804 | 0.132 | 0.2021 | 0.1761 | −0.1982 | −0.072 | −0.739 |
| Shahroud Cement | 0.1069 | 0.0017 | 0.0001 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0002 |
| Cement of Kurdistan | 0.002 | −0.2566 | · | 0.4235 | 0.3671 | −0.160 | 0.1203 | 0.1057 | −0.108 | 0.04205 | −0.0173 | −0.0008 |
| Qaen Cement | 0 | 0 | −0.053 | 0.1976 | 0 | 0 | 0 | 0 | 0.2588 | −0.178 | −0.1720 | −0.1243 |
| Lar Sabzevar Cement | 0.0326 | 0.0253 | 0.0003 | 0.05328 | 0.0033 | 0.0072 | 0 | 0.0044 | 0 | 0 | 0.0174 | 0.01446 |

Table 2: Forming an optimal stock portfolio considering various robust costs.

| Stock | Robust cost | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 10 | 15 | 20 |
| Darab Cement | — | — | — | — | — | — | — | — | — |
| Gharb Cement | — | — | — | — | — | — | — | — | — |
| Ilam Cement | 0.3522 | 0.34236 | 0.33306 | 0.72377 | 0.72556 | 0.72556 | 0.73435 | 0.3273 | 0.73435 |
| Karun Cement | — | — | — | 0.00085 | 0.00052 | 0.0005 | — | — | — |
| Kerman Cement | — | — | — | — | — | — | — | — | — |
| Khuzestan Cement | — | — | — | — | — | — | — | — | — |
| Shargh Cement | — | — | — | — | — | — | — | — | — |
| Doroud Cement | 0.19211 | 0.18674 | 0.18167 | 0.00425 | 0.0026 | 0.00251 | — | 0.2321 | — |
| Sepahan Cement | — | — | — | — | — | — | — | — | — |
| Tehran Cement | 0.38824 | 0.40533 | 0.42149 | 0.26963 | 0.26907 | 0.26924 | 0.26565 | 0.4407 | 0.26565 |
| Hegmatan Cement | — | — | — | — | — | — | — | — | — |
| Khash Cement | — | — | — | — | 0.00134 | 0.0013 | — | — | — |
| Sufiyan Cement | — | — | — | — | — | — | — | — | — |
| Urmia Cement | — | — | — | — | — | — | — | — | — |
| Shomal Cement | 0.06744 | — | 0.06378 | 0.00149 | 0.00091 | 0.00088 | — | — | — |
| Shahroud Cement | — | — | — | — | — | — | — | — | — |
| Cement of Kurdistan | — | — | — | — | — | — | — | — | — |
| Qaen Cement | — | — | — | — | — | — | — | — | — |
| Lar Sabzevar Cement | — | — | — | — | — | — | — | — | — |
| Obj | — | — | — | — | — | — | — | — | — |
| Return | · | · | · | · | · | · | · | · | · |
| Liquidity | 0.0319 | 0.03231 | 0.0327 | 0.0322 | 0.0322 | 0.0322 | 0.03218 | 0.0322 | 0.03218 |
| *CVaR* | 0.084189 | 0.8089 | 0.8401 | 0.87115 | 0.87123 | 0.87123 | 0.87237 | 0.8459 | 0.87237 |

$$
\begin{aligned}
&\text{Minimize } w_1 d_1^+ + w_2 d_2^+ + w_3 d_3^+ \\
&\text{Subject to.} - \sum_{j=1}^{n} r_j x_j - \overset{+}{d_1} + \overline{d_1} + Z\Gamma + \sum_{j=1}^{n} P_j = -b_1 \\
&\eta + \frac{1}{s(1-\alpha)} \sum_{i=1}^{s} (y_i) - d_2^+ + d_2^- = b_2 \\
&- \sum_{i=1}^{n} L_j x_j - \overset{+}{d_3} + \overline{d_3} = -b_3 \\
&\text{Subject to} \sum_{j=1}^{n} x_j = 1 \\
&\sum_{j=1}^{n} \left[ \left( -r_{ij} x_j \right) - \eta \right] - y_i \le 0 \; i = 1, 2, \ldots, s \\
&Z + P_j \ge \hat{r}_j \, f_j \; j = 1, \ldots, n \\
&-f_j \le x_j \le f_j \; j = 1, \ldots, n \\
&P_j \ge 0 \; j = 1, \ldots, n \\
&f_j \ge 0 \; j = 1, \ldots, n \\
&Z \ge 0 \\
&\overset{+}{d_i}, \overset{-}{d_i} \ge 0 \; i = 1, \ldots, 3 \\
&x_j \ge 0 \; j = 1, \ldots, n \\
&y_i \ge 0. \; i = 1, \ldots, s.
\end{aligned}
\tag{7}
$$

## 4. Solving the Model by Real Data of Tehran Stock Exchange

The data extracted from the site of the Tehran Stock Exchange are as follows. These data are related to twenty shares from July 2020 to July 2021. Information is monthly. The shares are related to the cement industry. The details of the shares are in Table 1.

Following this section, the optimal stock portfolio is determined by giving the value of the problem parameters to the model and assuming different robust costs. The weights of all three objective functions are considered the same. It should also be noted that the value of goals is equal to:

The conditional value-at-risk (*CVaR*) goal = 0.1.

The return goal = 0.03.

The liquidity goal = 0.8.

Table 2 shows the optimal stock portfolio considering various robust costs. As you can see the robust value for liquidity are 0.0319, 0.03231, 0.0327, 0.0322, 0.0322, 0.03218, 0.0322, and 0.03218, respectively. Also, the robust values for *CVaR* are 0.084189, 0.8089, 0.8401, 0.87115, 0.87123, 0.87123, 0.87237, 0.8459, and 0.87237.

## 5. Conclusion

In the past, the models proposed for development were devoid of any uncertainty, and all problem inputs were assumed to be certain, which is an incomplete assumption. Classical approaches to entering this uncertainty into mathematical models are very difficult and inefficient. These approaches, like sensitivity analysis, have many problems. This paper investigates the optimization approach under conditions of uncertainty. In this research, the robust goal programming approach has been used for modeling. The model presented in this paper includes three objective functions of return, liquidity, and conditional value-at-risk. The reason for choosing the first objective is the importance of the return and profitability of a share. The second objective is presented in order to investigate the investment risk in the form of an appropriate risk measure that has the ability to be linear and convex. After the two objectives of risk and return, which are considered in most investments, the objective of liquidity is considered. As it is clear from the results of the tables, the optimal stock portfolio has been reported and it is clear that the most productive factories can be selected among the cement factories at the time of the study. Robust Therefore values for liquidity are 0.0319, 0.03231, 0.0327, 0.0322, 0.0322, 0.03218, 0.0322, and 0.03218, respectively. Also, the robust values for *CVaR* are 0.084189, 0.8089, 0.8401, 0.87115, 0.87123, 0.87123, 0.87237, 0.8459, and 0.87237.

The robust approach used in this research has been analyzed by the real data of the Tehran Stock Exchange and then the optimal portfolio for different robust costs has been formed by solving the robust model. Thus, the investor can easily choose the desired portfolio in the real world, where many effective factors such as stock returns are uncertain, in different conditions of uncertainty. Considering a robust multiobjective mathematical model and optimizing stock portfolio are the superiority of this study over Meng-Ren et al. [47]; Liu et al. [48]; and Luo et al. [49]. Also, considering the objectives such as maximizing returns, maximizing liquidity, and minimizing risk are the superiorities of this study over those of Wu et al. [50] and Teng et al. [51].

The results of this research can be useful for organizations such as stock exchange organizations, and factory shareholders. Minimizing risks also helps investors and decision makers choose the best portfolio with more precision and less risk. This maximizes investor confidence and can pave the way for attracting more investors. [52].

The future directions as presented as follows:

(i) Considering uncertainty in input parameters as fuzzy demand or fuzzy costs

(ii) Considering alternative goods in stock exchange

(iii) Considering other objectives such as minimizing cost

## Data Availability

The data used in the article are available in the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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