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Research Article

Bipartite Containment Control for Multiagent Systems with Adaptive Quantization Information

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In this study, bipartite containment control for multiagent systems (MASs) with quantitative information is investigated. Communication topology is structurally balanced, and the follower's trajectory is within the area surrounded by the leader by designing distributed error terms. The leader is an external system, and the matrix information of the leader cannot be accessed by followers. Based on an adaptive quantization information distributed observer, the authors introduce an output feedback protocol to study bipartite containment control. Finally, simulations demonstrate the effectiveness of proposed control algorithms.

1. Introduction

In recent years, collaborative control has attracted a lot of attention. It is widely used in vehicle formation [1–3], complex dynamic networks [4, 5], and sensor networks [6–8]. With the development of MASs, many interesting results have been obtained, which makes many control problems become hot issues, including consensus of multiagent problem [9], the output regulation problem [10, 11], contains the problem [12, 13], and an adaptive parameter problem [14]. In the field of control, the MASs are a topical research object. It is composed of a group of single multiagent, and the information transmission between them constitutes the communication topology. The consensus of MASs can be divided into two types by whether there is a leader or not, that is, the MASs without leaders who cannot accept the information of other agents and the leader tracking systems with leaders. In [15], the leaderless continuous and discrete time consistency problem of the first-order MASs is studied by the frequency domain method. Based on the leaderless fixed communication topology [16], the finite-time consensus for MASs with external disturbance is studied. In addition, it is also important to consider the existence of leaders in multiagent research. Tang et al. [17] studied the tracking consistency problem, Hong et al. designed a distributed observer for leading tracking MASs [18], and Ni and Cheng solved leader-following consensus of the MASs through switching topology [19]. This study considers multiple leaders, and the leaders are considered to be the exogenous systems, and the leaders' information cannot be received by the followers.

Inspired by the biological behavior of nature, containment control is adopted for systems with multiple leaders in most cases, and the containment control problem is widely used in practice. In containment control, there is more than one leader. The area surrounded by the leader is called the convex shell, and followers will only move within the boundaries defined by the leader. For example, several robots carry equipment through a dangerous stretch of road. Robots without equipment act as leaders. Other robots transmit a safe travel route range to them through sensors or artificial information, while robots with equipment act as followers to enter the movement area surrounded by the leaders so as to avoid dangerous areas in the path. In [20], authors studied the necessary and sufficient conditions to achieve containment control. This paper studies bipartite output containment; that is, some follower curves converge to the region surrounded by the leader and others converge to the region of symmetry. In [21], the authors solved containment control with an output feedback method. In real life, the state is probabilistically unmeasurable, and external disturbance also affects the stability of the system. In [22, 23], by using the output feedback method, containment control of the MASs with exogenous disturbance is solved. This study also considers the influence of quantitative information on information transmission between agents to meet the practical challenges.

In the existing distributed controller, the information exchange between agent and neighbor is very accurate, but the accurate information exchange is a simplification, which is unreasonable. In practical applications, agents exchange information through digital communication channels. Due to the limitation of their own information storage capacity and communication width, the quantization effect must be considered [24–26], which is also meaningful and interesting for the study of consensus problems. Therefore, many articles on the consensus issue take into account the quantitative impact. In [27], the problem of quantization information consistency with Markov chains for the first-order MASs is solved. Control input problem: in [28], the author uses the distributed variable adjacency matrix method to deal with the quantized second-order consensus problem. In addition, the problem of the digital digraph of MASs is solved by designing an observer, and the consensus of the system in [29] is realized by using the probabilistic quantization method.

Each follower can get the leader's information, which is not realistic in practical application. Therefore, researchers introduced an adaptive algorithm. In [30], the asymptotic output regulation problem was studied by using the Luenberger observer and introducing an adaptive regulation method. In [31], an adaptive controller was proposed to cope with finite-time bipartite consensus for output feedback in the unknown state information. In [21], containment control for the linear MASs was solved. In addition, multiagent networks with competition and cooperation among agents are more common. Such problems can be called bipartite control problems [32, 33]. In topology graphs, the collar matrix has positive and negative weights. A special example of bipartite control is the control that is only cooperative between agents. The bipartite consensus of the MASs is restudied in [34] by means of output feedback and state feedback. In [35], the leader matrix information is obtained by using an adaptive observer, the solution of the mediation equation is obtained by designing an algorithm, and the consensus of the MASs is solved by output feedback. The bipartite containment problem of the MASs is studied in [36]. To solve some problems in actual life, an adaptive quantization information distributed observer is designed to estimate the leader's state, and then, a feedback controller is used to reach output bipartite containment.

The remaining four sections are as follows. Section 2 is the formulation and preliminary work of the problem. Section 3 is the main theoretical results. Numerical simulation results are shown in Section 4 to verify the effectiveness of theoretical results. In the end, Section 5 gives the conclusion.

2. Preliminaries and Model Statement

2.1. Algebraic Graph Theory. To achieve the bipartite output containment, communication topology is described in a topology diagram [37]. Let $G(V, E, A)$ be weighted directed graph and consist of node set $V = \{V_1, V_2, \ldots, V_N\}$ which denotes finite nonempty set of N nodes and edge set $E \subseteq V \times V$ describing transfer of information between nodes. We define $A = (a_{ij}) \in R^{N\times N}$ as weighted adjacency matrix. Let $d_i = \sum_{j=1}^{N} a_{ij}$ be in-degree of vertex i and $D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$ be in-degree matrix of G. Laplace matrix $L = [l_{ij}]$ of weighted directed graph G is defined as $L = D - A$. The point set is divided into two subgroups, v_1 and v_2, and $v_1 \cup v_2 = V$ and $v_1 \cap v_2 = \varnothing$, which is different from the general graph theory. Define $N_i = \{V_j | (V_j, V_i) \in E, i \neq j.\}$ to represent neighbor points of V_i. If there is an edge $(V_i, V_j) \in E$ and V_i and V_j are in the same subgroup, it represents cooperative relationship; then, there is $a_{ij} > 0$; if V_i and V_j are in different subgroups, it represents antagonistic relationship; then, there is $a_{ij} < 0$, otherwise, $a_{ij} = 0$. Define τ_i as a symbolic parameter, where $\tau_i = 1$ is used if $V_i \in v_1$ and $\tau_i = -1$ is used if $V_i \in v_2$.

2.2. Quantizer. Quantizer has been added to many literature studies on multiagent research. In this study, we use the quantizer q: $R \longrightarrow R$ as

$$q(x) = \begin{cases} \mu_i, & if \, \dfrac{1}{1+\gamma}\mu_i < x \leq \dfrac{1}{1-\gamma}\mu_i, x > 0, \\ 0, & if \, x = 0, \\ -q(-x), & if \, x < 0, \end{cases} \tag{1}$$

where μ_i is the quantization level and $\gamma \in (0, 1)$ is the quantization accuracy parameter. For the quantization level, one has a set $\overline{\mu} = \{ \pm \mu_i \colon \mu_i = (1-\gamma/\ 1+\gamma)^i \mu_0, i = 0, \pm 1, \pm 2, \ldots, \pm n\} \cup \{0\}$, where $\mu_0 > 0$ is the initial quantization level. $|q(m) - m| \leq \gamma |m|$, $\forall m \in R$can be obtained through the definition of quantizer. It is worth noting that, for any set of vectors$y = [y_1, y_2, \ldots, y_N]^T \in R^n$, we can write$q(y) = [q(y_1), q(y_2), \ldots, q(y_N)]^T \in R^n$. So, obviously there is $q(y) - y = \Gamma y$, where $\Gamma = \mathrm{diag}\{\Gamma_1, \Gamma_2, \ldots, \Gamma_N\}$ and $\Gamma_i \in [-\gamma, +\gamma]$.

2.3. Problem Formulation. In this study, we study the bipartite containment control for a group of MASs composed of Z leaders and N followers. In addition, $Z = \{N+1, N+2, \ldots, N+Z\}$ and $N = \{1, 2, \ldots, N\}$ are defined as leaders' set and followers' set, respectively. The kinetic equation for N followers is as follows:

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i + \displaystyle\sum_{k=N+1}^{N+Z} Q_{ik} V_k, \\ y_i = C_i x_i, i = 1, 2, \ldots, N, \end{cases} \tag{2}$$

where $x_i \in R^l, u_i \in R^l$, and $y_i \in R^l$ are state quantity of i th agent, control input quantity of the system, and the control output quantity of the system, respectively. A_i, B_i, C_i, and Q_{ik} are parameter matrix with suitable dimensions. Then, the kinetic equation of leaders is in the following form:

$$\begin{cases} \dot{V}_k = A_0 V_k, \\ y_k = F V_k, k \in Z, \end{cases} \tag{3}$$

where $V_k \in R^l$ represents the state of the exogenous system, that is, disturbance quantity or reference input of the system.

Definition 1. In this study, a reasonable controller is designed to solve the problem of bipartite containment control, which makes the followers converge to the positive and negative region defined by the leader, namely, the convex hull. So, for follower systems and leader systems (1) and (2), the following features can be used to solve their bipartite containment problem.

(1) Define the convex hull $co(X)$ belonging to set $X = \{x_1, x_2, \ldots, x_N\}$ of the form

$$co(X) = \left\{ \sum_{i=1}^{N} \beta_i x_i x_i \in R, \beta_i \geq 0, \sum_{i=1}^{N} \beta_i = 1 \right\}. \tag{4}$$

(2) For followers and leaders with any initial value, the follower can only enter the positive and negative convex hull defined by the leader:

$$\begin{aligned} &\lim_{t \longrightarrow \infty} \operatorname{dist}(x_i, co(V_k, k \in Z)) = 0, i \in N_1, \\ &\lim_{t \longrightarrow \infty} \operatorname{dist}(x_j, -co(V_k, k \in Z)) = 0, j \in N_2, \end{aligned} \tag{5}$$

in which $N_1 \cup N_2 = N$ and $N_1 \cap N_2 = \varnothing$.

When $e_i = 0$ is used, (2) can be displayed by the following containment error:

$$\begin{aligned} e_i = &\sum_{j \in N_i} |a_{ij}| (y_i - \operatorname{sgn}(a_{ij}) y_j) \\ &+ \sum_{k=N+1}^{N+Z} |a_{ik}| (y_i - \tau_i y_k), i \in N. \end{aligned} \tag{6}$$

Let $e = \operatorname{col}(e_1, e_2, \ldots, e_N)$ and $x = \operatorname{col}(x_1, x_2, \ldots, x_N)$; then,

$$e = (H \otimes I_N) C x - \sum_{k=N+1}^{N+Z} (A_{0k} \otimes F) \widetilde{V}_k, \tag{7}$$

where $\widetilde{V}_k = G 1_N \otimes V_k N$ with $G = \operatorname{diag}\{\tau_1, \tau_2, \ldots, \tau_N\}$ and 1_N is an N-dimensional column vector, $H = \sum_{k=N+1}^{N+Z} (1/Z) L + A_{0k}$, and $A_{0k} = \operatorname{diag}\{|a_{1k}|, |a_{2k}|, \ldots, |a_{Nk}|\}$. Then, if $\lim_{t \longrightarrow \infty} e_i = 0$, we can get the following formula:

$$\lim_{t \longrightarrow \infty} C_i x_i = \lim_{t \longrightarrow \infty} \sum_{k=N+1}^{N+Z} \zeta_{ik} F \tau_i V_k, i \in N, \tag{8}$$

where $\zeta_{ik} \in R^l$ is ith row vector of $H^{-1} A_{0k} 1_N$.

Assumption 1. Graph G is a signed graph with a balanced structure and a spanning tree.

Assumption 2. Eigenvalues of A_0 are in the right half plane.

Assumption 3. (C_i, A_i) is observable and (A_i, B_i) is stabilizable.

3. Main Results

In this section, an adaptive quantitative information observer is designed to observe the information of the leader matrix A_0 so as to solve the problem that followers are unknowable of the information of the leader matrix. The Sylvester equation is presented, and an algorithm for solving it is presented. The bipartite containment control is realized by designing an observer and by using the output feedback method.

3.1. Bipartite Adaptive Quantization Information Observer. The adaptive quantization information distributed observer of the bipartite containment control is as follows:

$$\dot{A}_{0i} = \chi_1 \left(\sum_{j \in N_i} |a_{ij}| (A_{0i} - \operatorname{sgn}(a_{ij}) A_{0j}) + \sum_{k=N+1}^{N+Z} |a_{ik}| (A_{0i} - \tau_i A_0) \right), \tag{9}$$

$$\begin{aligned} \dot{\eta}_i = &\tau_i A_{0i} \eta_i + c_1 \chi_2 \left(\sum_{j \in N_i} |a_{ij}| q((\eta_i - \operatorname{sgn}(a_{ij}) \eta_j)) \right. \\ &\left. + \sum_{k=N+1}^{N+Z} |a_{ij}| q((\eta_i - \tau_i V_k)) \right), \end{aligned} \tag{10}$$

where $\eta_i \in R^n, i \in N$, and $\chi_1, \chi_2 < 0$. $c_1 > 0$ is the compensator parameter.

Lemma 1. *Consider external systems (2) and adaptive quantized information distributed observer (6) and (7). Let* $\widetilde{A}_{0i} = A_{0i} - \tau_i A_0$ *and* $\widetilde{\eta}_i = \eta_i - \sum_{k=N+1}^{N+Z} \zeta_{ik} \tau_i V_k$. *Then, for* $\chi_1, \chi_2 < 0$ *and* $i \in N$, $\lim_{t \longrightarrow \infty} \widetilde{A}_{0i}(t) = 0$ *and* $\lim_{t \longrightarrow \infty} \widetilde{\eta}_i(t) = 0$.

Proof. According to (6), the derivative of $\widetilde{A}_{0i}(t)$ can be obtained as follows:

$$\begin{aligned}\dot{\widetilde{A}}_{0i} &= \dot{A}_{0i} - \tau_i \dot{A}_0\\ &= \chi_1\left(\sum_{j\in N_i}\left|a_{ij}\right|\left(A_{0i} - \operatorname{sgn}\left(a_{ij}\right)A_{0j}\right) + \sum_{k=N+1}^{N+Z}\left|a_{ik}\right|\left(A_{0i} - \tau_i A_0\right)\right)\\ &= \chi_1\left(\sum_{j\in N_i}\left|a_{ij}\right|\left(A_{0i} - \tau_i A_0 + \tau_i A_0 - \operatorname{sgn}\left(a_{ij}\right)\left(A_{0j} - \tau_j A_0 + \tau_j A_0\right)\right) + \sum_{k=N+1}^{N+Z}\left|a_{ik}\right|\left(A_{0i} - \tau_i A_0\right)\right)\\ &= \chi_1\left(\sum_{j\in N_i}\left|a_{ij}\right|\left(\widetilde{A}_{0i} - \operatorname{sgn}\left(a_{ij}\right)\widetilde{A}_{0j}\right) + \sum_{j\in N_i}\left|a_{ij}\right|\left(\tau_i A_0 - \operatorname{sgn}\left(a_{ij}\right)\tau_i A_0\right) + \sum_{k=N+1}^{N+Z}\left|a_{ik}\right|\widetilde{A}_{0i}\right).\end{aligned} \tag{11}$$

Notice if i and j are in the same subset $i, j \in N_1$ or $i, j \in N_2$, there are $a_{ij} > 0, \tau_i = \tau_j$. So, it is easy to get $\tau_i = \operatorname{sgn}(a_{ij})\tau_j$. In another case, i and j in different subgroups, we can get $a_{ij} < 0, \tau_i = -\tau_j\ \tau_i = \operatorname{sgn}(a_{ij})\tau_j$. Through the above discussion, $\tau_i A_0 - \operatorname{sgn}(a_{ij})\tau_j A_0 = 0$ can be obtained. Then, (8) can be written as

$$\dot{\widetilde{A}}_{0i} = \chi_1\left(\sum_{j\in N_i}\left|a_{ij}\right|\left(\widetilde{A}_{0i} - \operatorname{sgn}\left(a_{ij}\right)\widetilde{A}_{0j}\right) + \sum_{k=N+1}^{N+Z}\left|a_{ik}\right|\widetilde{A}_{0i}\right). \tag{12}$$

Let $\widetilde{A}_{0I} = \text{block diag}(\widetilde{A}_{01}, \widetilde{A}_{02}, \ldots, \widetilde{A}_{0N})$; (9) can be written in concise form:

$$\dot{\widetilde{A}}_{0I} = \chi_1 (H \otimes I_n)\widetilde{A}_{0I}, \tag{13}$$

where $H = \sum_{k=N+1}^{N+Z} 1/ZL + A_{0k}$. By assumption 1 and Lemma 2of[29], eigenvalues of matrix L have only positive real part and $1/ZL + A_{0k}$ is nonsingular. Since A_{0k} has no negative eigenvalues, $H = \sum_{k=N+1}^{N+Z} (1/Z)L + A_{0k}$ also has only nonzero positive eigenvalues. That is to say, $\lambda(H) > 0$. $\chi_1 (H \otimes I_n) < 0$ can be obtained by setting the parameter $\chi_1 < 0$, and $\lambda(\chi_1 (H \otimes I_n)) < 0$ with all negative eigenvalues can be obtained. And then, we get $\lim_{t\longrightarrow\infty}\widetilde{A}_{0I}(t) = 0$, which means we get $\lim_{t\longrightarrow\infty}\widetilde{A}_{0i}(t) = 0$. Thus, the following formula can be obtained:

$$\begin{aligned}\dot{\widetilde{\eta}}_i &= \dot{\eta}_i - \sum_{k=N+1}^{N+Z}\zeta_{ik}\tau_i\dot{V}_k = \tau_i A_{0i}\eta_i\\ &+ c_1\chi_2\left(\sum_{j\in N_i}\left|a_{ij}\right|q\left(\left(\eta_i - \operatorname{sgn}\left(a_{ij}\right)\eta_j\right)\right) + \sum_{k=N+1}^{N+}\left|a_{ij}\right|q\left(\left(\eta_i - \tau_i V_k\right)\right)\right)\\ &- \sum_{k=N+1}^{N+Z}\zeta_{ik}\tau_i A_0 V_k.\end{aligned} \tag{14}$$

Since the signature parameter $\tau_i = 1$ or $\tau_i = -1$, multiply both sides of $\widetilde{A}_{0i} = A_{0i} - \tau_i A_0$ by τ_i to obtain $\tau_i\widetilde{A}_{0i} = \tau_i A_{0i} - A_0$. Then, formula can be written as

$$\begin{aligned}\dot{\widetilde{\eta}}_i &= \left(\tau_i\widetilde{A}_{0i} + A_0\right)\eta_i\\ &+ c_1\chi_2\left(\sum_{j\in N_i}\left|a_{ij}\right|q\left(\left(\eta_i - \operatorname{sgn}\left(a_{ij}\right)\eta_j\right)\right) + \sum_{k=N+1}^{N+Z}\left|a_{ij}\right|q\left(\left(\eta_i - \tau_i V_k\right)\right)\right)\\ &- \sum_{k=N+1}^{N+Z}\zeta_{ik}\tau_i A_0 V_k,\end{aligned} \tag{15}$$

and (11); from Section 2.2, quantizer definition can be written in the following concise form as

$$\begin{aligned}\dot{\widetilde{\eta}} &= \left((G\otimes I_n)\widetilde{A}_{0I} + I_N \otimes A_0\right)\eta\\ &+ c_1(1-\gamma)\chi_2(H\otimes I_n)\left(\eta - \sum_{k=N+1}^{N+Z}\left(H^{-1}A_{0k}\otimes I_n\right)\widetilde{V}_k\right)\\ &- \sum_{k=N+1}^{N+Z}\left(H^{-1}A_{0k}\otimes I_n\right)(I_N\otimes A_0)\widetilde{V}_k\\ &= (G\otimes I_n)\widetilde{A}_{0I}\eta + (I_N\otimes A_0)\widetilde{\eta}\\ &= (G\otimes I_n)\widetilde{A}_{0I}\left(\widetilde{\eta} + \sum_{k=N+1}^{N+Z}\left(H^{-1}A_{0k}\otimes I_n\right)\widetilde{V}_k\right)\\ &+ (I_N\otimes A_0)\widetilde{\eta} + c_1(1-\gamma)\chi_2(H\otimes I_n)\widetilde{\eta}\\ &= (I_N\otimes A_0 + c_1(1-\gamma)\chi_2(H\otimes I_n))\widetilde{\eta}\\ &+ (G\otimes I_n)\widetilde{A}_{0I}\widetilde{\eta} + (G\otimes I_n)\widetilde{A}_{0I}\sum_{k=N+1}^{N+Z}\left(H^{-1}A_{0k}\otimes I_n\right)\widetilde{V}_k.\end{aligned} \tag{16}$$

Under the condition of assumption 3, χ_2 is negative and small enough; that is, $\chi_2 \ll 0$. So, $I_N\otimes A_0 + \chi_2(H\otimes I_n)$ is Hurwitz. We can get $\lim_{t\longrightarrow\infty}\widetilde{\eta}(t) = 0$ and $\lim_{t\longrightarrow\infty}\widetilde{\eta}_i(t) = 0$ through Lemma 1of[30]. The proof is completed. □

3.2. Solution of Sylvester Equations. In this study, some followers cannot get information from the leader's matrix A_0. A_0 is estimated by A_{0i}, and the generalized Lyapunov equation of the Sylvester equation is used to prove the stability of MASs. Therefore, in this section, the Sylvester equations in this paper are solved by the method in [30]. The Sylvester equations are as follows

$$X_k(I_N \otimes A_0) = \widehat{A}X_k + \widehat{B}_k, \tag{17}$$

$$0 = \widehat{C}X_k + \widehat{D}, \tag{18}$$

where X_k is solution to the above equation and $\widehat{A}$, $\widehat{B}_k$, $\widehat{C}$, and $\widehat{D}$ are given in the simulation examples in Section 4.

Lemma 2. *By Lemma 3in[30], we give a matrix $P(t)$; $\lim_{t\longrightarrow\infty}(P(t)-p)=0$ can be obtained. For any $\varepsilon>0$,*

$$\dot{X} = -\varepsilon P^T(t)P(t)X, \tag{19}$$

has and only has a unique solution $X(t)$. There are several X^* that make $pX^*=0$ and $\lim_{t\longrightarrow\infty}(X(t)-X^*)=0$.

Proof. The orthogonal matrix M guarantees $pM=(\overline{p}0)$ and

$$M^T p^T pM = \begin{pmatrix} \overline{p}^T\overline{p} & 0 \\ 0 & 0 \end{pmatrix}, \tag{20}$$

$$M^T p^T = \begin{pmatrix} \overline{p} \\ 0 \end{pmatrix}. \tag{21}$$

Let $X^* = M\begin{pmatrix} \overline{X}_1^* \\ \overline{X}_2^* \end{pmatrix}$ and $\overline{p}\overline{X}_1^*=0$. Then, the formula is established as follows:

$$pX^* = pMM^TX^* = (\overline{p}0)\begin{pmatrix} \overline{X}_1^* \\ \overline{X}_2^* \end{pmatrix} = 0. \tag{22}$$

Let $\overline{X}=M^TX$; it yields

$$\begin{aligned}\dot{\overline{X}} &= -M^T\varepsilon P^T(t)P(t)X \\ &= -\varepsilon M^Tp^TpX + \varepsilon M^Tp^TpX - \varepsilon M^TP^T(t)P(t)X \\ &= -\varepsilon M^Tp^TpX + \varepsilon M^T\left(p^Tp - P^T(t)P(t)\right)X \\ &= -\varepsilon M^Tp^TpM\overline{X} + f(t),\end{aligned} \tag{23}$$

in which $f(t)=\varepsilon M^T(p^Tp-P^T(t)P(t))M\overline{X}$. Let $\overline{X}=\begin{pmatrix} \overline{X}_1 \\ \overline{X}_2 \end{pmatrix}$ and $f(t)=\begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$. By Lemma 3 and Remark 2 in [30], $\lim_{t\longrightarrow\infty}f(t)=0$ and $\lim_{t\longrightarrow\infty}f_1(t)=0$. Then, from (18), we can get the following formula:

$$\begin{pmatrix} \dot{\overline{X}}_1 \\ \dot{\overline{X}}_2 \end{pmatrix} = -\varepsilon\begin{pmatrix} \overline{p}^T\overline{p} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \overline{X}_1 \\ \overline{X}_2 \end{pmatrix} + \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}, \tag{24}$$

and it is concluded that

$$\begin{aligned}\dot{\overline{X}}_1 &= -\varepsilon\overline{p}^T\overline{p}\overline{X}_1 + f_1(t), \\ \dot{\overline{X}}_2 &= f_2(t),\end{aligned} \tag{25}$$

and there is $\overline{X}_2^*$ to make $\lim_{t\longrightarrow\infty}(\overline{X}_2-\overline{X}_2^*)=0$. Let $\widetilde{X}_2=\overline{X}_1-\overline{X}_1^*$; derivative of $\widetilde{X}_2$ is

$$\begin{aligned}\dot{\overline{X}}_1 &= f_1(t) - \varepsilon\overline{p}^T\overline{p}\overline{X}_1 \\ &= f_1(t) - \varepsilon\overline{p}^T\overline{p}\overline{X}_1^* - \varepsilon\overline{p}^T\overline{p}\widetilde{X}_1 \\ &= f_1(t) - \varepsilon\overline{p}^T\overline{p}\widetilde{X}_1.\end{aligned} \tag{26}$$

By Lemma 1in[30], $\lim_{t\longrightarrow\infty}\widetilde{X}_1=0$ because $\overline{X}=M^TX$ has

$$\begin{aligned}\lim_{t\longrightarrow\infty}(X(t)-X^*) &= \lim_{t\longrightarrow\infty}\left(M\overline{X} - M\begin{pmatrix} \overline{X}_1^* \\ \overline{X}_2^* \end{pmatrix}\right) \\ &= \lim_{t\longrightarrow\infty}M\begin{pmatrix} \overline{X}_1-\overline{X}_1^* \\ \overline{X}_2-\overline{X}_2^* \end{pmatrix} = 0.\end{aligned} \tag{27}$$

Proof is done.

Now, the feedback protocol is designed:

$$\begin{aligned}\dot{\xi}_i &= A_i\xi_i + B_iu_i + \sum_{k=N+1}^{N+Z}E_{ik}V_k + L_i(C_i\xi_i - y_i), \\ u_i &= K_{1i}\xi_i + \eta_i\end{aligned} \tag{28}$$

in which K_{1i} is gain matrix. (1), (2), and (19) give us the following formula:

$$\begin{aligned}\dot{x}_i &= A_ix_i + B_iK_{1i}\xi_i + B_i\eta_i + \sum_{k=N+1}^{N+Z}E_{ik}V_k, \\ \dot{\xi}_i &= A_i\xi_i + B_iK_{1i}\xi_i + B_i\eta_i + \sum_{k=N+1}^{N+Z}E_{ik}V_k + L_i(C_i\xi_i - C_ix_i),\end{aligned} \tag{29}$$

Let $X=\begin{pmatrix} x \\ \xi \end{pmatrix}$ and $V_k=\begin{pmatrix} \overline{V}_k \\ \eta \end{pmatrix}$, where

$$\begin{aligned}x &= \mathrm{col}(x_1, x_2, \ldots, x_N), \\ \xi &= \mathrm{col}(\xi_1, \xi_2, \ldots, \xi_N), \\ \overline{V}_k &= \mathrm{col}(V_k, V_k, \ldots, V_k) \in R^{Nn}, \\ \eta &= \mathrm{col}(\eta_1, \eta_2, \ldots, \eta_N).\end{aligned} \tag{30}$$

So, (20) can be changed as

$$\dot{X} = \widehat{A}X + \sum_{k=N+1}^{N+Z}\widehat{B}_kV_k, \tag{31}$$

where

$$\begin{aligned}\widehat{A} &= \begin{pmatrix} A & BK_1 \\ -LC & A+BK_1+LC \end{pmatrix}, \\ \widehat{B}_k &= \begin{pmatrix} E_k & \frac{1}{Z}B \\ & \\ E_k & \frac{1}{Z}B \end{pmatrix},\end{aligned} \tag{32}$$

and

$$\begin{aligned} A &= \text{block diag}\{A_1, A_2, \ldots, A_N\},\\ B &= \text{block diag}\{B_1, B_2, \ldots, B_N\},\\ C &= \text{block diag}\{C_1, C_2, \ldots, B_N\},\\ E_k &= \text{block diag}\{E_{1k}, E_{2k}, \ldots, E_{3k}\},\\ K_1 &= \text{block diag}\{K_{11}, K_{12}, \ldots, K_{1N}\},\\ K_2 &= \text{block diag}\{K_{21}, K_{22}, \ldots, K_{2N}\}. \end{aligned} \tag{33}$$

Through (13) and (14), where $\widehat{D} = (0 - I_N \otimes 1/ZF)$ and $\widehat{C} = (C\,0)$. And then, we solve (13) and (14) through the lemma derived from [30]. □

Lemma 3. *Consider equation as*

$$\dot{Y} = -\varepsilon\psi^T(t)\psi(t)Y, \tag{34}$$

where $\varepsilon > 0$ is large enough and $\psi(t) = (I_{2N} \otimes A_{0i}(t))^T \otimes \begin{pmatrix} I_{2Nn} & 0 \\ 0 & 0 \end{pmatrix} - I_{2Nn} \otimes \begin{pmatrix} \widehat{A} & \widehat{B}_k \\ \widehat{C} & \widehat{D} \end{pmatrix}$. Let $\Xi(t) = m_{3Nn}^{2Nn}(Y) = \begin{pmatrix} X_k(t) \\ I_{2Nn} \end{pmatrix}$. Then, X_k^* is the solution of (13) and (14):

$$\lim_{t\longrightarrow\infty}\left(\Xi(t) - \begin{pmatrix} X_k^* \\ I_{2Nn} \end{pmatrix}\right) = 0, \tag{35}$$

which goes to zero no slower than $\widetilde{A}_{0I}$.

Proof. (13) and (14) can be written as follows:

$$\begin{pmatrix} I_{2Nn} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} X_k \\ I_{2Nn} \end{pmatrix}(I_{2N} \otimes A_0) - \begin{pmatrix} \widehat{A} & \widehat{B}_k \\ \widehat{C} & \widehat{D} \end{pmatrix}\begin{pmatrix} X_k^* \\ I_{2Nn} \end{pmatrix} = 0. \tag{36}$$

Then, we have

$$\psi\Theta = 0, \tag{37}$$

in which

$$\psi = (I_{2N} \otimes A_0)^T \otimes \begin{pmatrix} I_{2Nn} & 0 \\ 0 & 0 \end{pmatrix} - I_{2Nn} \otimes \begin{pmatrix} \widehat{A} & \widehat{B}_k \\ \widehat{C} & \widehat{D} \end{pmatrix}. \qquad \Theta = \begin{pmatrix} X_k^* \\ I_{2Nn} \end{pmatrix} \tag{38}$$

$\lim_{t\longrightarrow\infty}(\psi(t) - \psi) = 0$ goes to zero no slower than $\widetilde{A}_{0I}$. Based on Lemma 2, $\lim_{t\longrightarrow\infty}(\Xi(t) - \Theta) = 0$ can be obtained, where there is a X_k^* satisfying (21). The proof is completed. □

3.3. Collaborative Analysis. By the above lemma and proof, we prove (4) and solve the problem of the bipartite containment control. In addition, $X_k^* = \begin{pmatrix} X_{k11}^* & X_{k12}^* \\ X_{k21}^* & X_{k22}^* \end{pmatrix}$ is the solution of equations (13) and (14), where $X_{k11}^*, X_{k12}^*, X_{k21}^*, X_{k22}^* \in R^{Nn\times Nn}$.

Theorem 1. *For (1) and (2), the MASs composed of the follower and the leader are described, respectively. Under assumption two, the existence of K_{1i} and L_i makes $A_i + B_iK_{1i}$ and $A_i + L_iC_i$ are Hurwitz. Then, a matrix $T = \begin{pmatrix} I_{Nn} & 0 \\ -I_{Nn} & I_{Nn} \end{pmatrix}$ controller (20) can be used to achieve the bipartite containment and the distributed error (4) converges to zero.*

Proof. Let $\overline{A} = T\widehat{A}T^{-1}$; we obtain

$$\overline{A} = \begin{pmatrix} A_i + BK_1 & BK_1 \\ 0 & A + LC \end{pmatrix}. \tag{39}$$

In assumption 2, we can see that $\overline{A}$ is Hurwitz. Then, we know $\widehat{A}$ is Hurwitz. If $\widetilde{X} = X - \sum_{k=N+1}^{N+Z} X_k^* V_k$ is set, the derivative of $\widetilde{X}$ can be obtained:

$$\begin{aligned} \dot{\widetilde{X}} &= \dot{X} - \sum_{k=N+1}^{N+Z} X_k^*\dot{V}_k = \widehat{A}X + \sum_{k=N+1}^{N+Z}\widehat{B}_kV_k - \sum_{k=N+1}^{N+Z} X_k^*\begin{pmatrix} \dot{\overline{V}}_k \\ \dot{\eta} \end{pmatrix} \\ &= \widehat{A}X + \sum_{k=N+1}^{N+Z}\widehat{B}_kV_k \\ &\quad - \left(\sum_{k=N+1}^{N+Z} X_k^*\begin{pmatrix} (I_N \otimes A_0)V_k \\ (I_N \otimes A_0)\eta \end{pmatrix} + \sum_{k=N+1}^{N+Z} X_k^*\begin{pmatrix} 0 \\ (G\otimes I_N)\widetilde{A}_{0I}\,\eta \end{pmatrix} + \sum_{k=N+1}^{N+Z} X_k^*\begin{pmatrix} 0 \\ c_1(1-\gamma)\chi_2(H\otimes I_N) \end{pmatrix}\widetilde{\eta}\right) \\ &= \widehat{A}X + \sum_{k=N+1}^{N+Z}\widehat{B}_kV_k - \sum_{k=N+1}^{N+Z} X_k^*(I_{2N}\otimes A_0)V_k \\ &\quad - \left(\sum_{k=N+1}^{N+Z} X_k^*\begin{pmatrix} 0 \\ (G\otimes I_N)\widetilde{A}_{0I}\,\eta \end{pmatrix} + \sum_{k=N+1}^{N+Z} X_k^*\begin{pmatrix} 0 \\ c_1(1-\gamma)\chi_2(H\otimes I_N) \end{pmatrix}\widetilde{\eta}\right). \end{aligned} \tag{40}$$

Then, formula (13) is used to rewrite (23) into the following form:

$$\dot{\tilde{X}} = \widehat{A}X - \sum_{k=N+1}^{N+Z} \widehat{A}X_k^* V_k - \left(\sum_{k=N+1}^{N+Z} X_k^* \begin{pmatrix} 0 \\ (G\otimes I_N)\widetilde{A}_{0I}\eta \end{pmatrix} + \sum_{k=N+1}^{N+Z} X_k^* \begin{pmatrix} 0 \\ c_1(1-\gamma)\chi_2(H\otimes I_N) \end{pmatrix} \widetilde{\eta} \right) = \widehat{A}\widetilde{X} + \Lambda_1\widetilde{\eta} + \Lambda_2, \tag{41}$$

in which

$$\begin{aligned} \Lambda_1 &= -\sum_{k=N+1}^{N+Z} X_k^* \begin{pmatrix} 0 \\ c_1((1-\gamma))\chi_2(H\otimes I_N) \end{pmatrix}, \\ \Lambda_2 &= -\sum_{k=N+1}^{N+Z} X_k^* \begin{pmatrix} 0 \\ (G\otimes I_N)\widetilde{A}_{0I}\eta \end{pmatrix}, \end{aligned} \tag{42}$$

and $\lim_{t\longrightarrow\infty}\Lambda_2 = 0$ exponentially at least as fast as $\lim_{t\longrightarrow\infty}\widetilde{A}_{0I} = 0$. Therefore, through Lemma 1in[30] and Lemma 1 of this paper, we get $\lim_{t\longrightarrow\infty}\widetilde{X} = 0$. Now, consider distributed error (4), which has the following form:

$$\begin{aligned} e &= (H\otimes I_N)Cx - (H\otimes I_N)(I_N\otimes F)\eta \\ &\quad + (H\otimes I_N)(I_N\otimes F)\eta - \sum_{k=N+1}^{N+Z} (A_{0k}\otimes F)\widetilde{V}_k \\ &= (H\otimes I_n)\sigma + (H\otimes F)\widetilde{\eta}, \end{aligned} \tag{43}$$

where $\sigma = Cx - (I_N\otimes F)\eta$, and σ has this

$$\begin{aligned} \sigma &= Cx - (I_N\otimes F)\eta \\ &= (C \quad 0)X + \sum_{k=N+1}^{N+Z} \left(0 - I_N\otimes \frac{1}{Z}F\right)\begin{pmatrix} \overline{V}_k \\ \eta \end{pmatrix} \\ &= \widehat{C}X + \sum_{k=N+1}^{N+Z} \widehat{D}V_k \\ &= \widehat{C}\overline{X} + \sum_{k=N+1}^{N+Z} \widehat{C}XV_k + \sum_{k=N+1}^{N+Z} \widehat{D}V_k. \end{aligned} \tag{44}$$

Through (14), we have

$$\sigma = \widehat{C}\widetilde{X}. \tag{45}$$

Because of $\lim_{t\longrightarrow\infty}\widetilde{X} = 0$, we can get $\lim_{t\longrightarrow\infty}\sigma = 0$. Under Lemma 1, $\lim_{t\longrightarrow\infty}\widetilde{\eta} = 0$. So, bipartite containment error converges to zero, that is, $\lim_{t\longrightarrow\infty}e = 0$.

The proof is completed. □

4. Numerical Simulations

In this section, a numerical example is used to illustrate the effectiveness of the bipartite containment control. Figure 1 can be viewed as a topology of (1) and (2), showing the communication relationship between six agents. Since the symbolic graph is considered in this study, we select 5 and 6 as the leader and the other agents as the followers and then divide 1 and 3, 2, and 4 into two different subgroups N_1, N_2, including $N_1 \cup N_2 = N$ and $N_1 \cap N_2 = \varnothing$. It can be concluded from Figure 1 that the Laplace matrix L and matrix A have the following forms:

$$\begin{aligned} A &= \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \\ L &= \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}. \end{aligned} \tag{46}$$

In addition, $A_{0k} = \mathrm{diag}\{|a_{1k}|, |a_{2k}|, \ldots, |a_{Nk}|\}$ and $H = \sum_{k=N+1}^{N+Z}(1/Z)L + A_{0k}$ have the following forms:

$$A_{05} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, A_{06} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{47}$$

$$H = L + A_{05} + A_{06} = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & -1 \\ -1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}. \tag{48}$$

Considering the follower and leader systems (1) and (2), the correlation matrix is given as follows:

$$\begin{aligned} A_0 &= \begin{pmatrix} 1 & -2 \\ 1.5 & -1 \end{pmatrix}, F = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}, \\ A_i &= \begin{pmatrix} 0\,0.5 * i \\ 0\,0 \end{pmatrix}, B_i = \begin{pmatrix} 0\,1 \\ 0.5 * i\,0 \end{pmatrix}, \\ C_1 &= \begin{pmatrix} 8 & -10.5 \\ 2 & -7 \end{pmatrix}, C_2 = \begin{pmatrix} 2 & -6 \\ -3 & -5.5 \end{pmatrix}, \\ C_3 &= \begin{pmatrix} 1.5 & -8 \\ -3 & -3.8 \end{pmatrix}, C_4 = \begin{pmatrix} -4 & -15 \\ -4 & -2 \end{pmatrix}, \\ E_{i5} &= \begin{pmatrix} 0 & 0 \\ 0 & 0.1 \end{pmatrix}, E_{i6} = \begin{pmatrix} 0 & 0 \\ 0.0 & 50 \end{pmatrix}. \end{aligned} \tag{49}$$

Based on Section 3.3, it can be known that $A_i + B_iK_{1i}$ and $A_i + L_iC_i$ are Hurwitz, and then, K_{1i} and L_i can be calculated according to the above matrix. $c_1 = 3.6$ is also given:

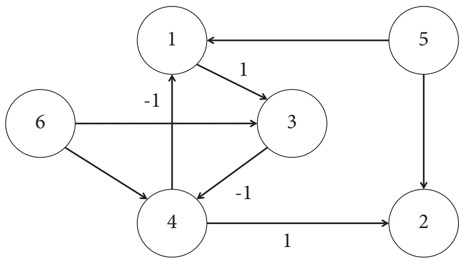

Figure 1: Communication topology of 2 leaders and 4 followers.

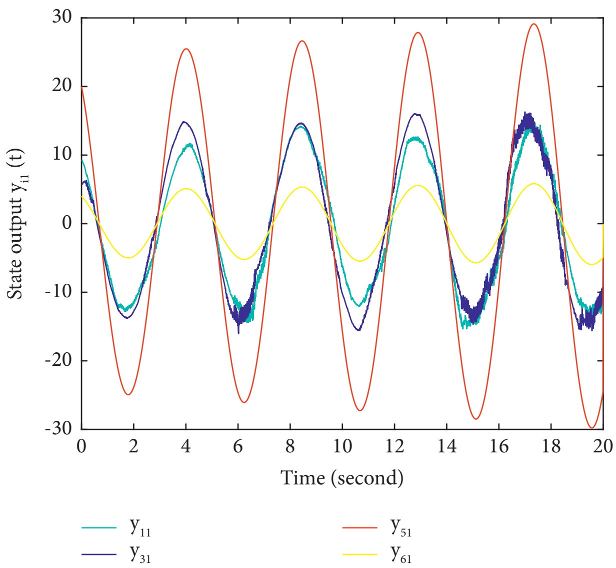

Figure 2: Output curves of leader 51 and 61 and follower 11 and 31.

$$\begin{aligned}
K_{11} &= \begin{pmatrix} -0.1604 & -1.1036 \\ -0.9871 & -0.3207 \end{pmatrix}, \\
K_{12} &= \begin{pmatrix} -0.4142 & -1.2872 \\ -0.9102 & -0.4142 \end{pmatrix}, \\
K_{13} &= \begin{pmatrix} -0.6074 & -1.4321 \\ -0.7944 & -0.4049 \end{pmatrix}, \\
K_{14} &= \begin{pmatrix} -0.7310 & -1.5259 \\ -0.6824 & -0.3655 \end{pmatrix}, \\
L_{1} &= \begin{pmatrix} -0.9321 & 0.5586 \\ 0.4413 & 0.8974 \end{pmatrix}, \\
L_{2} &= \begin{pmatrix} -0.5706 & 0.8181 \\ 0.7434 & 0.6689 \end{pmatrix}, \\
L_{3} &= \begin{pmatrix} -0.3307 & 0.9681 \\ 0.8798 & 0.4754 \end{pmatrix}, \\
L_{4} &= \begin{pmatrix} 0.1999 & 0.8685 \\ 0.9989 & -0.0451 \end{pmatrix}.
\end{aligned} \tag{50}$$

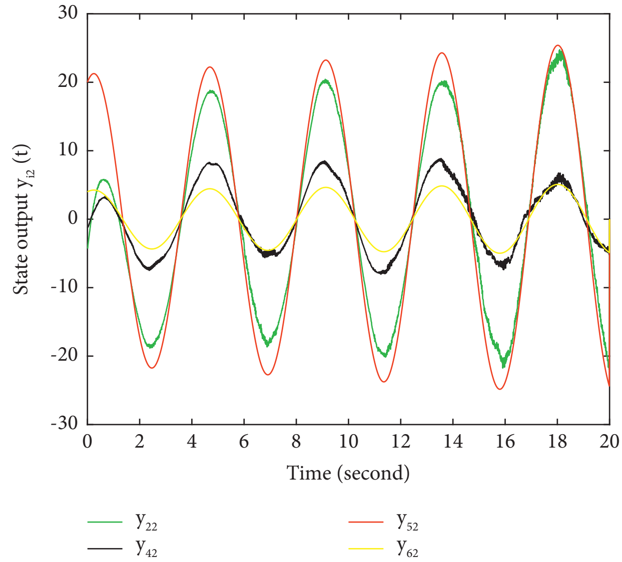

Figure 3: Output curves of leader 52 and 62 and follower 22 and 42.

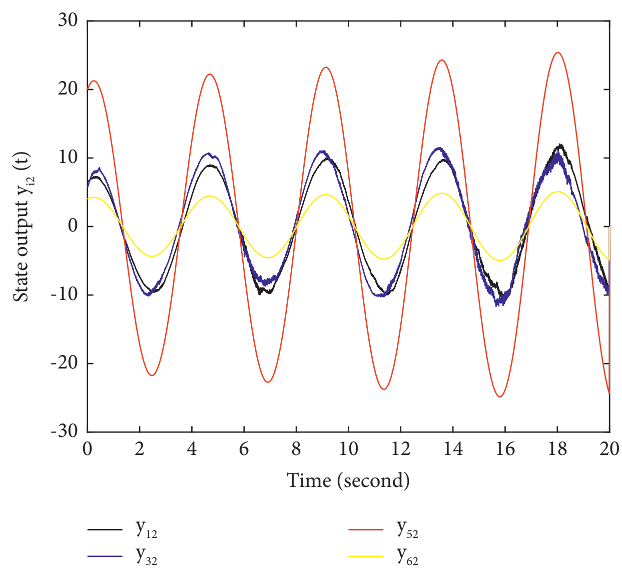

Figure 4: Output curves of leader 52 and 62 and follower 12 and 32.

Based on the above matrix, Figures 2–5 show the bilateral output curve of the agent. The red line represents leader 5, the yellow line represents leader 6, and the remaining colors are followers. Figures 2 and 5 show the first row contained in the leader bilateral output with 5 and 6. Specifically, y_{51} is the first line of $y_5 = FV_5$ and y_{61} is the first line of $y_6 = FV_6$. y_{11} is the first line of $y_1 = C_1x_1$, y_{21} is the first line of $y_2 = C_2x_2$, y_{31} is the first line of $y_3 = C_3x_3$, and y_{41} is the first line of $y_4 = C_4x_4$. Similarly, Figures 3 and 4 show the output containing the second row, the second component. To sum up, it can be observed that some followers converge to the region bounded by leaders 5 and 6, while the rest converge to the opposite region. Moreover, follower information transmission takes into account the

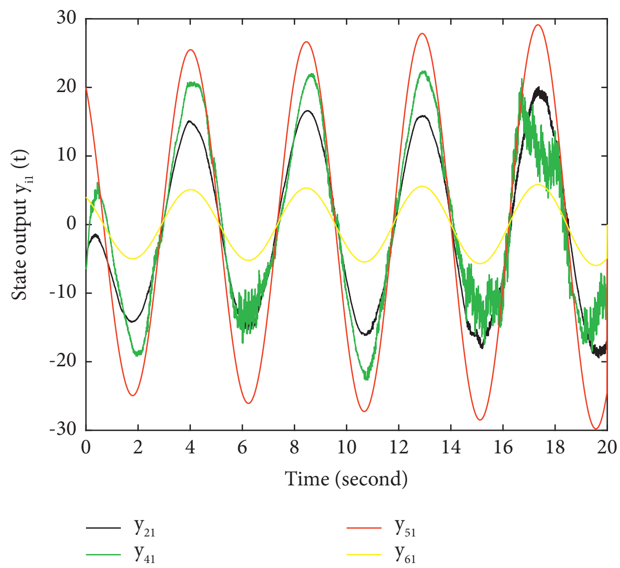

Figure 5: Output curves of leader 51 and 61 and follower 21 and 41.

influence of quantification, which is more interesting and challenging.

5. Conclusions

In this study, bipartite containment control is discussed. The matrix A_0 of the external leader system is observed by using an adaptive quantization observer, and the adaptive quantization information observer is proved to be effective by the eigenvalue analysis method without M-matrix. A controller with only one gain matrix and an appropriate protocol for the control evolution algorithm is designed. At the same time, considering the influence of quantization on follower information transmission, a quantizer is added. Finally, the bipartite containment control is realized by using the feedback method.

In the future, we will further study the bipartite containment control with adaptive quantization information distributed observer under switched topology or in finite time.

Data Availability

The data that support the findings of this study can be obtained from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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