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Research Article

A New Method for Centrality Measurement Using Generalized Fuzzy Graphs

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The fuzzy graph is the foundation of many actual structures such as networking, picture planning, and so on. Generalized fuzzy graphs (GFG) are ideal for avoiding certain constraints of fuzzy graphs. Social networks are a useful and powerful way to link citizens worldwide. A central person in a social network is to deny the leading people in it. Different centrality steps have also been established over the years. We evaluated the centrality of the social network through a general fuzzy graph in this article. An application to detect the central person in any online group like WhatsApp is described in this study by using generalized fuzzy graphs. Also, a few important results are established in this study.

1. Introduction

Nowadays, the graph theory is used in all processes, including networks, paths, schedules, photos, etc. A social network could be seen as a graph, with vertices representing an account (person, organization, etc.) and edges representing the relationship between such accounts. The social network is an association between persons or groups related through such relationships. It may be offline or through Internet. Family members establish a network (family), some farmers in a village shape a farmers' network, some business people form a corporate network, and these are typical offline networks. A million people use smartphones and they are pleased to use social apps to communicate and exchange details. SNs such as Twitter, Facebook, and LinkedIn have lately achieved great prominence in human life. The social network involves people, friends, organizations, etc. It is thus a marketing medium, news distribution, etc. The discovery of a significant or key or power node is a fundamental activity to furnish certain works in the social network. Centrality means the primary node or central node within a network. Therefore, the calculation of centrality is an exceedingly critical activity in the social network. Suppose a club is made up of 100 people. The club president assumes to be central. There are several branches of a bank in a region. The bank's headquarters is considered to be key. The head of a college among the teachers is central.

Day by day, a vast range of central initiatives have been adopted and developed to implement real-life issues. Bavelas [1–4] first described centrality measurement for graphs and suggested its implementation for the communication network analysis. Shimbel [5] proposed a centrality measure on the shortest route to calculate the contact number. To measure the relative effect of a node on the network, Katz [6] implemented the Katz centrality. Sabidussi [7] stated Beauchamp's centrality index improvement and described and checked correctly recognized indicators to fulfil that description's requirements. Nieminen [8] revised certain Sabidussi [7] axioms and added the central index based on undirected graph degrees of vertices. Freeman [9] established three form centrality measurements for each concept: first, an absolute calculation and second, a comparative centrality measure for a network position, and third, a centralization measure for a whole device. These measurements have been tested for small groups' experimental culture. Stephenson and Zelen [10] also implemented an information center focused on information conveyed via a linked network between two vertices. Freeman [9] proposed a modern measure to gain a central value through the definition of network flows. This was like the measure of Freeman but separate from the original. Freeman's [9] core indicators for weighted networks contributed tremendously. Brandes [11] implemented a quicker centrality algorithm, shortening time, and space for comparative study. Costenbader explored the stability of all main indicators when the network was sampled, and Costenbader and Valente [12]. A quantitative methodology has been established to address Costenbader and Valente [12] shortcomings, and Borgatti [13] to determine the robustness of broad graphics. Estrada and Rodriguez-Velazquez [14] proposed a centrality in subgraphs to calculate the number of times the vertex has in the numerous connected network subgraphs. Rodriguez et al. [15] extended the centrality as a central feature, based on the assumption that closed walks are sufficiently weighed. Their effect on centrality reduces as the walking order.

Bonacich [16] addressed a few uniquely centralized properties. Opsahl et al. [17] also enhanced weighted network centrality indicators. Joyce et al. [18] implemented a modern leverage centrality and used it to examine the human brain network. Kitsak et al. [19] implemented the decomposition of K shells and indicated that the dominant spreaders remain at the network's center. Zeng and Zhang [20] suggested a process known by residual degree and even depleted degree as "mixed degree decomposition." Liu et al. [21] provided an updated system for the rating list to be seen more clearly. This approach considers the k-shell values and the shortest distance from a network core target node, which is described as the set of k-shell values.

Along with the grade and coreness of the node and its neighbor, Bae and Kim [22] lay down the central location of the neighborhood, which lists all the nodes of the network. Liu et al. [23] suggested locating an influential neighborhood centrality in a diverse network. Wang et al. [24] then suggested a more precise ranking of prominent nodes in a weighted neighborhood centrality. In this paper, we have introduced the new concept of centrality measure [22, 25–29] and established a real-life application by this proposed method. Samanta et al. [30] introduced a new concept of centrality measure in a network.

Also, some system conditions are unclear or uncertain—this type of unclear or uncertainty capture in a fuzzy graph. Kauffman [31] introduced the first description of a fuzzy graph [31]. However, Rosenfeld [32] defined fuzzy relationships with fuzzy sets and established the fuzzy graph theory. Samanta and Pal [33–35] have demonstrated that fuzzy graphs can be used in different real-life applications. Mahapatra et al. [36–42] presented many applications of the fuzzy graph.Both of these fuzzy graphs have a general property: edge membership values are smaller than the minimum of their end vertex membership values. Assume that a social network is defined as fuzzy graphs. Both social units are known to be fuzzy nodes here. The vertical membership values can depend on multiple parameters. Assume that the importance of membership is determined by an information source, and the interaction between such units is described by fuzzy edges. Thus, the membership benefit is determined by information transfer. However, information exchanges can be greater than one social actor/unit, since a more experienced individual tells a less established person. This kind of condition cannot describe by a fuzzy graph. However, generalized fuzzy graph definition cannot be seen as the value of the edge membership should be smaller than the vertices membership values. Thus, the generalized fuzzy graph removed this limitation. Samanta and Sarkar [43] proposed a generalized fuzzy graph. The centrality calculation based on a fuzzy graph can therefore produce a more pertinent result. As the relationships in social networks do not follow the rules of fuzzy graphs, the centrality by using fuzzy social network do not capture the real scenario. The representation of social networks by GFG is thus more meaningful and the centrality by using GFG is more appropriate.

2. Preliminaries

Definition 1. A fuzzy graph [32] $\zeta = (V, \sigma, \mu)$, V is the nonempty vertex set and $\sigma: V \longrightarrow [0, 1]$ and $\mu: V \times V \longrightarrow [0, 1]$ such that $\mu(x, y) \leq \min\{\sigma(x), \sigma(y)\}$, where $\sigma(x)$ is the vertex membership value of the vertex x and $\mu(x, y)$ is the edge membership values of the edge (x, y) in ζ.

Samanta et al. [43] defined generalized fuzzy as follows.

Definition 2. Consider two functions as follows: $\rho: V \longrightarrow [0, 1]$ and $\omega: V \times V \longrightarrow [0, 1]$, V is the nonempty vertex set. We suppose $\mathrm{A} = (\rho(x), \rho(y)) | \omega(x, y) > 0$. Then, (V, ρ, ω) is defined to be GFG if there exists a function $\phi: A \longrightarrow (0, 1]$ such that $\omega(x, y) = \phi(\rho(x), \rho(y)) \forall x, y \in V$. Here, $\rho(x), x \in V$ is the membership value of the vertex x and $\omega(x, y), x, y \in V$ is the membership value of the edge (x, y).

2.1. Degree Centrality. Shaw [44] first introduced a central graduate index, and therefore identified by Nieminen [8] as a significant vertex. Freeman [9] initially developed a centrality mathematical model focused on links to a vertex. A general definition of degree centrality in degree power and centrality has been introduced by Bonacich [16]. Brandes et al. [11], and Newman [45] and expanded the node grade by taking the sums of weights instead of the number of relations into consideration. Albert et al. [46] stated that another feature that many networks have is right-skewed

Table 1: Some basic notations.

Notation	Meaning
ζ	Fuzzy graph
ψ	Generalized fuzzy graph
V	Vertex set
E	Edge set
$\sigma(a)$	Membership value of the vertex a
$\mu(a,b)$	Membership value of the edge (a,b)
$D(a)$	has represented the degree centrality of a vertex a
$D'(a)$	Normalization degree centrality of a vertex a
C_i	Centrality of a vertex v_i by generalised fuzzy graph
$d(a)$	Degree of the vertex a

degree distributions. The previous calculation of the centrality of grades and the power of nodes were generalized by Opsahl et al. [17].

Definition 3. A central node of a network by degree centrality shows the number of directly connected node. $D(a)$ has represented the degree centrality of a vertex a. The equation $D(a) = d(a)$, where $d(a)$ is the degree of the vertex a. Normalization degree centrality is $D'(a) = d(a)/(n-1)$, where n is the number of vertices of the network.

2.2. Some Basic Notations. Some basic notations are shown in Table 1.

3. Centrality Measure by the Generalised Fuzzy Graph

The crisp graph considers all the edges to be equal or equal, but this is not considered in a fuzzy graph. The real social network and the observed network vary greatly. The players (i.e., people, organizations, etc.) and the relationship are unclear in any social network. These kind of uncertainties can be measured by a fuzzy method. The centrality calculation based on fuzzy graph can therefore produce a more pertinent result.

Definition 4. Let $\psi = (V, \sigma, \mu)$ be a generalized fuzzy graph and $|V| = n$ then centrality of a vertex v_i is denoted by C_i and defined by

$$C_i = \frac{\sum_{j=1}^{m} \mu_j}{n-1}, \tag{1}$$

where $i = 1, 2, \ldots, n$ and m is the number of edges connected with the vertex v_i.

Example 1. Let us assume $\psi = (V, \sigma, \mu)$, a generalized fuzzy graph with nine vertices has been considered in Figure 1. The vertex values are shown in Table 2. Also, consider the function $\phi(\sigma(a), \sigma(b)) = ((\sigma(a) + \sigma(b))/2)$ and all edges membership values shown in Table 3.

So, the centrality of a vertex A is $C_A = ((0.55 + 0.55 + 0.65 + 0.40 + 0.65)\ /(9-1)) = 2.8/8 = 0.35$. Similarly, we can easily calculate the centrality of the vertices B, C, D, E, F, G, H, I and all the calculations are shown in Table 4. Also, the central person of this network is vertex A.

3.1. Algorithm to Find the Centrality in GFG

Input:- $\psi = (V, \sigma, \mu)$ be a GFG and $|V'| = n$.

Output:- Centrality of all vertices of this GFG.

Step 1: All the vertices of this GFG are labelled as $1, 2, \ldots, n$.

Step 2: First of all, calculate the centrality of the vertex "1" by the formula $C_1 = \sum_{j=1}^{m} \mu_j/(n-1)$.

where m is the number of vertex connected with the vertex "1" and C_1 denotes the centrality of the vertex "1."

Step 3: Repeat Step 2 until all vertices $2, 3, \ldots, n$.

Step 4: The central node of this network is these node whose centrality value is $\max\{C_1, C_2, C_3, \ldots, C_n\}$.

Lemma 1. *Let $\psi = (V, \sigma, \mu)$ be a generalized fuzzy graph with $|V| = n$ and C_i is the centrality of a vertex v_i then $0 \le C_i \le 1$.*

Proof. ψ be a generalized fuzzy graph and $|V| = n$. So, $0 \le \mu(a,b) \le 1$, $\mu(a,b)$ be the edge membership value of the edge (a,b). Then, the centrality of a vertex v_i is $C_i = \sum_{j=1}^{m} \mu_j/(n-1)$ where $i = 1, 2, \ldots, n$ and m is the number of edges connected with the vertex v_i. So, $0 \le C_i \le 1$ is true. □

Theorem 1. *Let $\psi = (V, \sigma, \mu)$ be a generalized fuzzy graph with $|V| = n$ and C_i is the centrality of a vertex v_i and D'_i be the degree centrality of a vertex v_i of underlaying crisp graph of ψ then $C_i \le D'_i$.*

Proof. ψ be a GFG and C_i is the centrality of a vertex v_i then $C_i = \sum_{j=1}^{m} \mu_j/(n-1)$ where m is the number of edges connected with the vertex v_i and the value of μ_j is $0 \le \mu_j \le 1$. D'_i is the centrality of a vertex v_i of underlaying crisp graph of ψ. Then D_i = degree of the vertex v_i. In the crisp graph, all edges are consider as 1. But, in fuzzy graph maximum value of an edge is 1. So, $C_i \le D'_i$ is true. □

Theorem 2. *Let $\psi = (V, \sigma, \mu)$ be a complete GFG with $|V| = n$ and C_i is the centrality of a vertex v_i and D'_i be the degree centrality of a vertex v_i of underlaying crisp graph of ψ then the value of D'_i is equal for all vertices but the value of C_i may be different for all vertices.*

Proof. ψ be a GFG and C_i is the centrality of a vertex v_i. D'_i is the centrality of a vertex v_i of Figure 2 underlaying the crisp graph of ψ. Then, D_i = degree of the vertex v_i. Since the graph is complete, the degree of all vertices are same. Also, in the crisp graph all edges are consider as 1. So, the value of D'_i is equal for all vertices but in fuzzy graph the edge membership value is not always same. So, the value of C_i may be different for all vertices. □

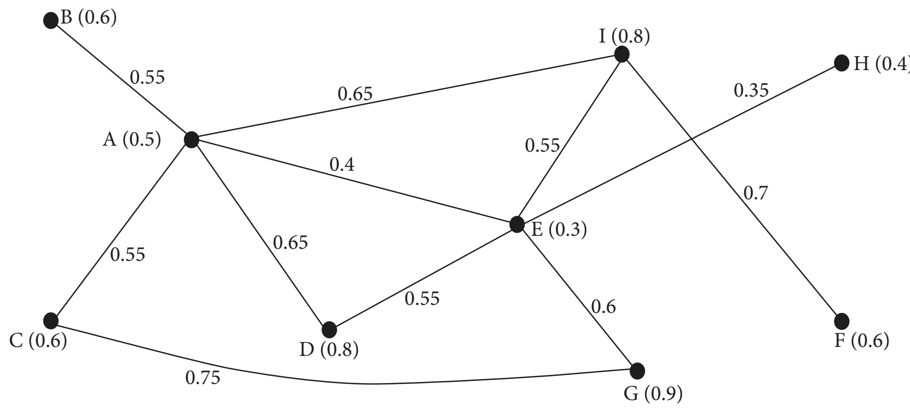

Figure 1: A generalized fuzzy graph.

Table 2: Vertex membership value of Figure 1.

Vertex	Membership value
A	0.5
C	0.6
E	0.3
G	0.9
I	0.8
B	0.6
D	0.8
F	0.6
H	0.4

Table 3: Edges membership value of Figure 1.

Edge	Membership value
(A, B)	0.55
(A, D)	0.65
(A, I)	0.65
(D, E)	0.55
(E, I)	0.55
(F, I)	0.7
(A, C)	0.55
(A, E)	0.4
(C, G)	0.75
(G, E)	0.6
(H, E)	0.35

Table 4: Centrality of all vertices of Figure 1.

Vertex	Centrality
A	0.35
C	0.16
E	0.31
G	0.17
I	0.24
B	0.07
D	0.15
F	0.09
H	0.04

4. Application

Nowadays, online social networks are very useful in human life. At present, everyone may use social network. Social networking services are an online platform used by users to establish social networks or social links with others with similar interests, professions or backgrounds, or real-life relationships. Social networking services enable users to exchange ideas, digital images and videos, posts, and inform others within their network on online. In 2017, Facebook has almost 2.13 billion active monthly members and an average of 1.4 billion active daily users.

But, a new communication rule for WhatsApp and WhatsApp calls (voice and video calls) will be implemented by the Indian government.

01. All calls will be recoded.

02. All call recordings will be saved.

03. WhatsApp, Facebook, Twitter, Instagram, and all social media will be monitored.

04. Your device will connect to the Minister's system.

Also, all government department maintain an account in SNs. They are try to spread a news in common people through social network. So, to find out a central node of a social network is very important task.

Here, we consider a network of some WhatsApp's users and every users are consider as nodes of a networks. Also, there exists an edge if they have at least one common WhatsApp group. The membership value of a vertex is calculated based on number of massage shared in last seven days. Also, the vertex membership value is consider as the normalized score of this number of massage shared last seven days. All the calculations are shown in Table 5. Also, consider the function $\phi(\sigma(a), \sigma(b)) = ((\sigma(a) + \sigma(b))/2)$ and all edges membership values shown in Table 6. In Figure 2, a generalized fuzzy graph is shown. Now, the centrality of a vertex A is $((0.66 + 0.54 + 0.8)/(10 - 1)) = 2/9 = 0.22$. Similarly, we can easily calculate the centrality of the vertices $B, C, D, E, F, G, H, I, J$, and centrality of all the vertices of Figure 2 are shown in Table 7. So, the central node of this network is D. So, the node D can spread a message very quickly in this network.

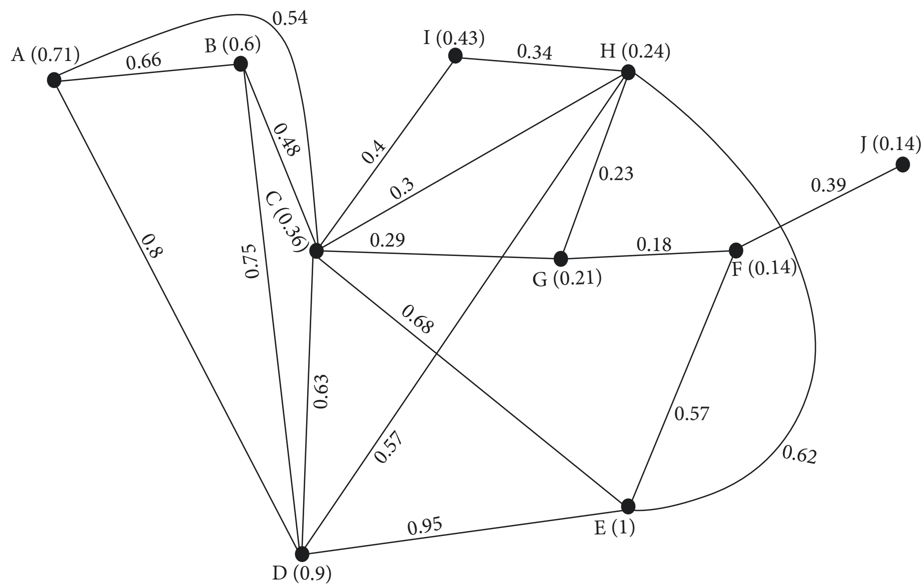

Figure 2: A generalized fuzzy graph.

Table 5: Vertex membership value of Figure 2.

Vertex	Number of massage share last 7 days	Membership value
A	50	0.71
C	25	0.36
E	70	1.0
G	15	0.21
I	30	0.43
B	42	0.6
D	63	0.9
F	10	0.14
H	17	0.24
J	45	0.64

5. Comparison of Degree Centrality (N) and Centrality by GFG

Here, a network of Figure 2 has been considered for the comparison between the degree centrality (N) and centrality by GFG. Also, the degree centrality (N) of all vertices of this network has been shown in Table 8 and the centrality by GFG has been shown in the same table. Also, the comparison graph of degree centrality (N) and centrality by GFG of all vertices of Figure 2 is shown in Figure 3. The degree centrality of all vertices of Figure 2 is shown in Table 9.

5.1. Analysis of the Result. It is observed that the degree centrality (N) is giving higher values of predictions compared to centrality by GFG. Also, the central node of this network by degree centrality (N) is C and the value is 0.78 but the value of C is 0.33 by the GFG. But the central node of this network is D by centrality by GFG. It is true that the node C is connected with the maximum number of nodes but this node is node active the node D is connected with five vertices but the vertex C is connected with seven vertices. But the node D is very much active rather than the node C in this network. So, our proposed method gave more accurate results.

Table 6: Edges membership value of Figure 2.

Edge	Membership value
(A, B)	0.66
(A, D)	0.80
(B, C)	0.48
(C, E)	0.68
(C, H)	0.30
(D, E)	0.95
(E, F)	0.57
(F, G)	0.18
(F, J)	0.39
(H, I)	0.34
(A, C)	0.54
(B, D)	0.75
(C, D)	0.63
(C, G)	0.29
(C, I)	0.40
(D, H)	0.57
(E, H)	0.62
(F, H)	0.19
(G, H)	0.23

Table 7: Centrality of all vertices of Figure 2.

Vertex	Centrality
A	0.22
C	0.37
E	0.31
G	0.08
I	0.08
B	0.21
D	0.41
F	0.13
H	0.23
J	0.04

Table 8: Comparison of degree centrality (N) and centrality by GFG of all vertices of Figure 2.

Vertex	Centrality by GFG	Degree centrality (N)
A	0.22	0.33
C	0.37	0.78
E	0.31	0.44
G	0.08	0.33
I	0.08	0.22
B	0.21	0.33
D	0.41	0.56
F	0.13	0.33
H	0.23	0.56
J	0.04	0.11

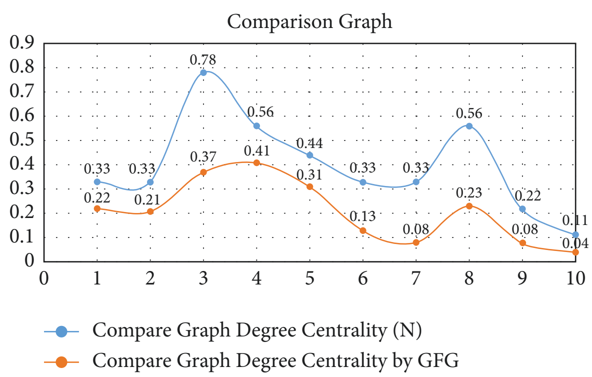

Figure 3: Comparison graph of degree centrality (N) and centrality by GFG of all vertices of Figure 2.

Table 9: Degree centrality of all vertices of Figure 2.

Vertex	Degree centrality	Degree centrality (N)
A	3	0.33
C	7	0.78
E	4	0.44
G	3	0.33
I	1	0.22
B	3	0.33
D	5	0.56
F	3	0.33
H	5	0.56
J	1	0.11

6. Conclusion

All the previous methods of centrality were prepared based on the number of nodes that are connected with this node. However, in real-life, centrality is dependent on the characteristic/personality of the nodes. A new method of centrality was introduced in this article. Also, this method depended on this characteristic/personality of node. This study makes a comparison of centrality by GFG with the existing method of degree centrality. However, for simplicity, the proposed method is well expected.

Data Availability

There are no data associated with this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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