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Research Article

Presenting a Multi-Echelon and Multi-Product Model for Inventory Control considering Shortages Using a Heuristic Algorithm

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In the present study, a mathematical model is provided for inventory management in a three-level supply chain including a supplier, a manufacturer, and a distributor, taking into account the potential for product deficiency. Significant parameters such as the optimal order size for raw materials, optimal production rate, optimal allowed deficiencies at each supply chain level, the of the vehicles and the number of times the products are delivered between the supply chain components have been provided aiming at minimizing inventory management costs in the supply chain. The problem was solved using a heuristic algorithm. The proposed model is implemented for 20 different numerical examples. According to the results, the difference between the solutions obtained from the algorithm and the solutions obtained from the optimal values with the real variables is an average of 1.52%.

1. Introduction

The integrated performance of suppliers, manufacturers, distributors, and customers, which make up the components of the supply chain, has been studied significantly during recent years [1–3]. From an operational perspective, supply chain management is to integrate the suppliers, manufacturers, warehouses, and storage centers in an effective manner, so that goods are produced and distributed in the right quantities, at the right time, and in the right place in a way that minimizes the cost of the entire system while maintaining a proper level of customer service [4, 5]. Thus, supply chain coordination is a plan that arranges the operations of supply chain members and enhances system benefits [6–8]. Inventory management problems are very significant in terms of minimization of the total cost of the chain and the cost of the final product [9]. The studies in this area have been so extensive that the search results on the Google Scholar led to 40300 results and on Scopus site, and it led to 2400 related articles [10, 11].

One of the first integrated models that has examined the inventory management for more than two members of the supply chain was provided by Banerjee and Kim [12] who analyzed the ordering of raw materials in the supply chain with one buyer, one manufacturer, and one supplier. This model was generalized by Lee [13] who analyzed and investigated ordering of raw materials in the supply chain, and, unlike the previous model, the author hypothesized that the manufacturer could place an order to the supplier several times its own economic output and could satisfy its demand while preparing multiple orders at different intervals. The Banerjee and Kim [12] model has also been reviewed by

Table 1: Literature review.

Authors	Multi-echelon	Multi-product	Inventory control	Heuristic algorithm	Shortage	Deficiency conditions
Safaei et al. [20]	*	*	*			
Shafipour-Omrani et al. [19]	*		*	*		
Olya et al. [18]			*	*		
Goodarzian et al. [16]			*	*		
Britt et al. [15]			*	*	*	
Banerjee et al. [14]		*	*			
Lee [13]	*	*				
This research	*	*	*	*	*	*

Banerjee et al. [14] in which the supply chain contains several buyers. The producer delivers once to each buyer at regular intervals so that they achieve the required inventory. Britt et al. [15] proposed a stochastic multi-objective model for master surgical scheduling and inventory problem. The intended uncertainty was considered as a scenario. They mainly targeted to maximize the utilization of the operating rooms and minimize the variance of weekly allocation of the surgeons. Genetic approaches and late acceptance hill climbing heuristics were employed for solving the model. Goodarzian et al. [16] developed a bi-objective network design considering route balancing, working time, and inventory problem. They considered two main aims including minimizing the total service time and total costs of HHC service. To solve their model, metaheuristic algorithms called social engineering optimization, firefly algorithm, and artificial bee colony were used and an improved social engineering optimization was developed. Yang et al. [17] presented a multi-objective mathematical model for inventory and scheduling problem. In the presented mathematical model, travel time and service time were taken as uncertain. Their main goal was to minimize the system costs and maximize the service level. The proposed mathematical model was solved using the improved multi-objective artificial bee colony and neighborhood search heuristic. The solution-derived results for the numerical instances indicated that the model performed appropriately. Olya et al. [18] proposed a stochastic mathematical model for resources management and inventory problem in the healthcare system, where the patient demand was uncertain and was satisfied using simulation. They pursued the goal to allocate the nurses to the patients and minimize the service costs. Deep learning and supervised and unsupervised methods were applied for solving the mathematical model. The results suggested that the multi-task learning solution approach operator outperformed other methods. Shafipour-Omrani et al. [19] presented inventory and transportation model for the LNG supply chain. The model minimizes the cost of sending natural gas from the primary port to the destination port. Considering the product variety and the way of presenting the constraint were the contributions of their paper. The results show that if the ships' capacity increases, the product variation decreases.

Safaei et al. [20] presented a new multi-period and multi-echelon mathematical model for closed-loop supply chain. They forecasted demand using time series model using autoregressive integrated moving average (ARIMA). Considering the cost of not satisfying the total amount of demand is the one the contributions of their paper.

Table 1 shows the literature review.

In the present paper, an inventory management model in a three-level supply chain is provided, which consists of a supplier, a manufacturer, and a distributor to determine the order size of raw material and optimal production, the capacity of vehicles, and the number of times the products are delivered between chain components in order to minimize the inventory size. The main objective of this research is minimizing the maintenance costs, inventory costs, and ordering and preparation costs. The research gaps are as follows:

(i) Ignoring three-level supply chain system taking into account the deficiency conditions.
(ii) Ignoring multi-echelon and multi-product model for inventory control in presented system.
(iii) Ignoring heuristic algorithm to solve the model.

The contributions of this paper are as follows:

(i) Considering three-level supply chain system taking into account the deficiency conditions.
(ii) Considering multi-echelon and multi-product model for inventory control in presented system.
(iii) Considering heuristic algorithm to solve the model.

Finally, a heuristic algorithm is used to determine the optimal solutions to the problem. In the following, the validity of the model and the proposed solution method are examined. In both cases, the results are satisfactory.

2. Modeling a Three-Level Supply Chain System Taking into Account the Deficiency Conditions

Indices:

i: manufacturer
r: raw material
f: distributor
s: production preparation
w: semi-finished products

Parameters:

P_i: manufacturer's i production rate

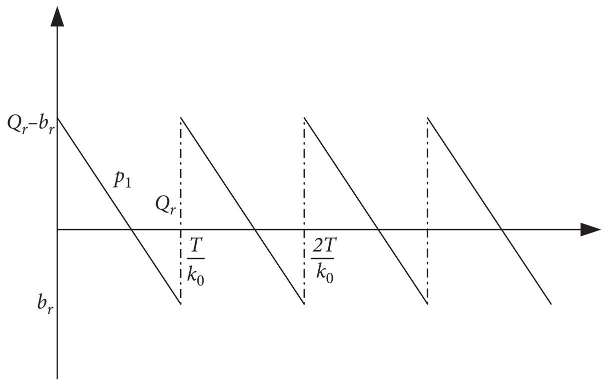

Figure 1: Inventory of raw materials.

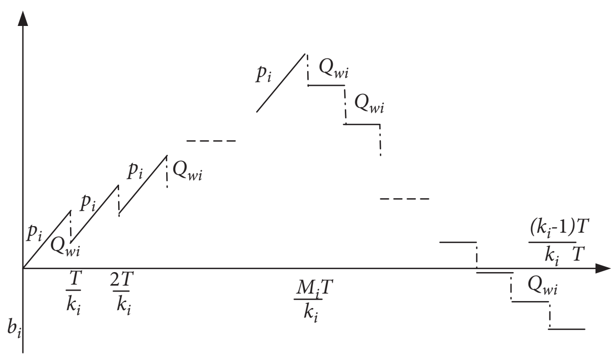

Figure 2: Inventory of manufacturer's products.

D: demand rate
H_r: cost of maintaining the inventory of raw material r for each product
H_{wi}: cost of maintaining each unit of semi-finished products w by the manufacturer i
H_f: cost of maintaining each unit of final product
π_r: cost of deficiency of each unit of raw material r
π_i cost of deficiency of each unit of goods for the manufacturer i
π_f: cost of deficiency of each unit of final product
A_r: cost of ordering raw material r
A_{si}: cost of production preparation s for the manufacturer i
A_{sf}: cost of production preparation s for the distributor f
A_{wi}: preparation costs for sending semi-finished products w to the manufacturer i
A_{wf}: preparation costs for sending semi-finished products w to the distributor f
A_f: preparation costs for sending final products to the final customers
I_t: inventory level at time t
I_{avg}: average inventory
T_{ui}: the time assigned at stage i for production in each production cycle
T_{di}: nonproductive time at stage i in each production cycle
TC_r: cost of maintaining raw material r
TC_r: cost of raw material r storage in the supplier
TC_{wi}: cost of maintaining products w in the manufacturer i
TC_f: cost of maintaining the final products in the distributor f
TC_{wi}: cost of maintaining semi-finished products w by the manufacturer i
TC_f: cost of maintaining the final products by the distributor f
TC: total supply chain costs

Variables:

Q: amount of products manufactured in each production cycle (optimal economic production rate)
Q_r: amount of raw material r that should be delivered from the supplier in each cycle
Q_{wi}: amount of products w in each delivery to the next step for the manufacturer i
Q_f: amount of final products in each delivery to the final customer
b_r: amount of ultimate deficiency for raw material r in the supplier
b_i: amount of ultimate deficiency in each production cycle in the manufacturer i
b_f: amount of ultimate deficiency of the final product in the distributor f
K_i: number of times the manufacturer's i products are sent in each production cycle
K_r: number of times to order delivery of raw material r in the supplier
m_i: number of times the products are sent at the time of manufacturing in each production cycle
K_f: number of times the final products are sent to customers in the distributor f
s: number of times the final products are sent to the final customers at the time of packaging in the distributor

2.1. Supplier Inventory Cost. At the supplier side, it is assumed that the demand rate for raw material inventory equals the rate of production of the manufacturer, i.e., P_1. Orders with a fixed size of Q_r are also sent to the raw material warehouse. The problem for this section to respond to the demand of the production station is to determine the order size of raw materials, number of orders in a production cycle, and the optimal deficiency. The inventory of raw materials is shown in Figure 1.

According to the above model and the costs of maintenance, ordering, and deficiency, the cost of raw materials is calculated as follows:

$$\mathrm{TC}_r = \left(\frac{D}{Q}\right)\left(A_r K_r + H_r I_{\mathrm{avg}} T_r + \pi_r b_{\mathrm{avg}} T_{br}\right). \quad (1)$$

Since the average inventory in each production cycle and at the time of deficiency is equal to $I_{\mathrm{avg}} = Q_r - b_r/2$ and the average deficiency at the time of occurrence is equal to $\pi_{\mathrm{avg}} = b_r/2$ and considering the relevant time relations as $T_r = Q - k_r b_r/\mathrm{D}$ and $T_{br} = k_r b_r/\mathrm{D}$, by substituting the above relations, the cost of raw materials is determined as follows:

$$\mathrm{TC}_r = \frac{D}{Q} A_r K_r + H_r \left(\frac{Q_r - b_r}{2} \right) \left(\frac{Q - k_r b_r}{D} \right) + \pi_r \frac{k_r b_r^2}{2D}. \quad (2)$$

2.2. Inventory Cost of Manufacturer's Products. The products are produced by the manufacturer with the production rate p_i. Products on which the manufacturing process is performed are stored in the warehouse until they are sent to the manufacturer. Products are delivered from the manufacturer to the distributor in Q_{wi} packages by shipping equipment at the ordering times. The manufacturer's inventory level is presented in Figure 2.

In terms of the inventory of semi-finished products in the manufacturer, the following relations are given:

$$T_{wi} = \frac{m_i T}{k_i}. \quad (3)$$

The number of products on which the manufacturing process is performed in station i and in each production, cycle is equal to the amount of economic production Q, which shows the following relation:

$$\begin{aligned} Q &= \int_0^{T_{wi}} P_i \mathrm{d}t, \\ &= P_i T_{wi}, \\ &= P_i \frac{m_i T}{k_i}. \end{aligned} \quad (4)$$

The average amount of products made in the manufacturer is calculated as follows:

$$\begin{aligned} I_{\mathrm{avg}} &= \frac{1}{2} \left(Q_{wi} \left(k_i - m_i + 1 \right) - b_i \right), \\ \pi_{\mathrm{avg}} &= \frac{1}{2} b_i. \end{aligned} \quad (5)$$

Therefore, the inventory cost for the manufacturer's products is calculated as follows:

$$\begin{aligned} \mathrm{TC}_{wi} &= \frac{D}{Q} \left(A_{si} + A_{wi} k_i + H_{wi} \frac{\left(Q_{wi} \left(k_i - m_i + 1 \right) - b_i \right)}{2} \frac{\left(Q_{wi} \left(k_i - m_i + 1 \right) - b_i \right)}{D} + \pi_i \frac{b_i}{2} \frac{b_i}{D} \right) \longrightarrow \mathrm{TC}_{wi}, \\ &= \frac{D}{Q} \left(A_{si} + A_{wi} k_i + H_{wi} \frac{\left(Q_{wi} \left(k_i - m_i + 1 \right) - b_i \right)^2}{2D} + \pi_i \frac{b_i^2}{2D} \right). \end{aligned} \quad (6)$$

2.3. Inventory Cost of Distributor Products. It is assumed that the preparation rate of the final products in the distributor increases with p_f rate. The manufactured products are delivered to end customers in the same sizes. The average inventory of final products in each production cycle is calculated as follows:

$$I_{\mathrm{avg}} = \frac{1}{2} \left(Q_f \left(k_f - m_f + 1 \right) - b_f \right). \quad (7)$$

The total cost of inventory of final products is obtained as follows:

$$\begin{aligned} \mathrm{TC}_f &= \frac{D}{Q} \left(A_{s(N+1)} + A_f k_f + H_f \frac{\left(Q_f \left(k_f - s + 1 \right) - b_f \right)}{2} \frac{\left(Q_f \left(k_f - s + 1 \right) - b_f \right)}{D} + \pi_f \frac{b_f}{2} \frac{b_f}{D} \right) \longrightarrow \mathrm{TC}_f, \\ &= \frac{D}{Q} \left(A_{s(N+1)} + A_f k_f + H_f \frac{\left(Q_f \left(k_f - s + 1 \right) - b_f \right)^2}{2D} + \pi_f \frac{b_f^2}{2D} \right). \end{aligned} \quad (8)$$

Limitations of the Problem. In order not to face a deficiency in semi-finished products in the manufacturer, conditions should be established where the level of deficiency in the manufacturer is less than the allowable deficiency in the distributor. Therefore, the following conditions must be considered in the problem.

$$\begin{aligned} &\text{if} \quad b_i \le b_f \longrightarrow b_f = b_f, \\ &\text{if} \quad b_f \le b_i \longrightarrow b_f = b_i. \end{aligned} \quad (9)$$

To linearize the above constraints in the problem, it is sufficient to define these equations in a linear manner by defining the binary variable $y \in (0, 1)$ as follows:

$$\begin{aligned}b_f + M(1-y) &\geq b_i,\\ b_f - \mathrm{My} &\leq b_i,\\ b_f &= b_f y + b_i(1-y).\end{aligned} \tag{10}$$

The total cost of a three-level supply chain is as follows. The total cost of the supply chain is calculated as follows by the sum of the costs related to the inventory of the supplier, manufacturer, and distributor.

$$\mathrm{TC}_m = \mathrm{TC}_r + \mathrm{TC}_{wi} + \mathrm{TC}_f. \tag{11}$$

So, we have

$$\begin{aligned}\mathrm{TC}_m = {} & \frac{D}{Q}A_r K_r + H_r\left(\frac{Q_r - b_r}{2}\right)\left(\frac{Q - k_r b_r}{Q}\right) + \pi_r \frac{k_r b_r^{\,2}}{2Q}\\ & + \left(\frac{D}{Q}A_{si} + \frac{D}{Q}A_{wi}k_i + H_{wi}\frac{(Q_{wi}(k_i - m_i + 1) - b_i)^2}{2Q} + \pi_i \frac{b_i^{\,2}}{2Q}\right),\\ & + \left(\frac{D}{Q}A_{sf} + \frac{D}{Q}A_{wf}k_f + H_f\frac{\left(Q_f\left(k_f - s + 1\right) - b_f\right)^2}{2Q} + \pi_f \frac{b_f^{\,2}}{2Q}\right).\end{aligned} \tag{12}$$

The size and number of transferred units in each station are related to the amount of economic production of the supply chain, i.e., according to the following relationships.

$$\begin{aligned}Q &= k_r Q_r\\ &= k_i Q_{wi}\\ &= k_f Q_f.\end{aligned} \tag{13}$$

By substituting the cost of inventory in the relevant equations, the cost of inventory in the entire supply chain will be determined as follows:

$$\begin{aligned}\mathrm{TC}_m = {} & \frac{D}{Q}A_r K_r + H_r\left(\frac{Q/k_r - b_r}{2}\right)\left(\frac{Q - k_r b_r}{Q}\right) + \pi_r \frac{k_r b_r^{\,2}}{2Q}\\ & + \left(\frac{D}{Q}A_{si} + \frac{D}{Q}A_{wi}k_i + H_{wi}\frac{(Q/k_i(k_i - m_i + 1) - b_i)^2}{2Q} + \pi_i \frac{b_i^{2}}{2Q}\right),\\ & + \left(\frac{D}{Q}A_{sf} + \frac{D}{Q}A_{wf}k_f + H_f\frac{\left(Q/k_r\left(k_f - m_f + 1\right) - b_f\right)^2}{2Q} + \pi_f \frac{b_f^{\,2}}{2Q}\right),\end{aligned} \tag{14}$$

subject to

$$b_f + M(1-y) \geq b_i, \tag{15}$$

$$b_f - \mathrm{My} \leq b_i, \tag{16}$$

$$b_f = b_f y + b_i(1-y), \tag{17}$$

$$k_i, k_r, k_f \in \mathrm{int}, \tag{18}$$

$$Q, b_i, b_r, b_f, \in \mathrm{real}, \tag{19}$$

$$y \in (0,1). \tag{20}$$

Of course, it should be noted that in this problem, the variables k_i, k_r, and k_f, i.e., the number of times the materials and products are delivered, are integer variables.

3. Solution Algorithm

Step 1. Determination of the optimal solutions of the three-level supply chain with real variables.

The optimal values of deficiency variables in different stations are determined as follows as the first step:

$$\begin{aligned}\frac{\partial \mathrm{TC}_m}{\partial b_r} &= -\frac{H_r}{2Q}\left(Q - k_r b_r + Q - k_r b_r\right) + \frac{\pi_r}{2Q}2k_r b_r,\\ &= 0 \longrightarrow 2k_r b_r \pi_r,\\ &= 2H_r\left(Q - k_r b_r\right) \longrightarrow b_r^*,\\ &= \frac{Q}{k_r\left(\pi_r/H_r + 1\right)},\\ \frac{\partial \mathrm{TC}_m}{\partial b_i} &= -\frac{2\pi_i b_i}{2Q} - \frac{2H_{wi}\left(Q\left(1 + \mathrm{D}/P_i + 1/k_i\right) - b_i\right)}{2Q},\\ &= 0 \longrightarrow b_i\left(\pi_i + H_{wi}\right)\\ &= H_{wi}\left(Q\left(1 - \frac{D}{P_i} + \frac{1}{k_i}\right) - b_i\right) \longrightarrow b_i^*\\ &= \frac{H_{wi}Q\left(1 - D/P_i + 1/k_i\right)}{\pi_i + 2H_{wi}},\\ \frac{\partial T\ C_m}{\partial b_f} &= -\frac{2\pi_f b_f}{2Q} - \frac{2H_f\left(Q\left(1 - D/P_f + 1/k_f\right) - b_f\right)}{2Q},\\ &= 0 \longrightarrow b_f\left(\pi_f + H_f\right)\\ &= H_f\left(Q\left(1 - \frac{D}{P_f} + \frac{1}{k_f}\right) - b_f\right) \longrightarrow b_f^*,\\ &= \max\left(\frac{H_f\left(2 - D/P_f + 1/k_f\right)Q}{2\left(\pi_f + H_f\right)}, b_i^*\right).\end{aligned} \tag{21}$$

By substituting the optimal deficiencies calculated in the objective function of the problem and simplification of the objective function, we have

$$\begin{aligned}
\mathrm{TC}_m &= \frac{D}{Q}A_r K_r + H_r\left(\frac{Q/k_r - Q/k_r\left(1/\left(1+\pi_r/H_r\right)\right)}{2}\right)\left(\frac{Q - k_r Q/k_r\left(1/\left(1+\pi_r/H_r\right)\right)}{Q}\right) + \pi_r\frac{k_r Q^2/k_r^2\left(1/\left(1+\pi_r/H_r\right)\right)^2}{2Q}\\
&+ \frac{D}{Q}A_{si} + \frac{D}{Q}A_{wi}k_i + \frac{H_{wi}Q}{2}\left(\left(1-\frac{D}{P_i}+\frac{1}{k_i}\right)\left(\frac{\pi_i + H_{wi}}{\pi_i + 2H_{wi}}\right)\right)^2 + \frac{\pi_i Q}{2}\left(\left(1-\frac{D}{P_i}+\frac{1}{k_i}\right)\frac{H_{wi}}{\pi_i + 2H_{wi}}\right)^2\\
&+ \frac{D}{Q}A_{sf} + \frac{D}{Q}A_{wf}k_f + \frac{H_f Q}{2}\left(\left(1-\frac{D}{P_f}+\frac{1}{k_f}\right)\left(\frac{\pi_f + H_f}{\pi_f + 2H_f}\right)\right)^2 + \frac{\pi_f Q}{2}\left(\left(1-\frac{D}{P_f}+\frac{1}{k_f}\right)\frac{H_f}{\pi_f + 2H_f}\right)^2\\
\longrightarrow \mathrm{TC}_m &= \frac{D}{Q}A_r K_r + \frac{H_r Q}{2k_r}\left(\frac{\pi_r}{H_r + \pi_r}\right)^2 + \frac{\pi_r Q}{2k_r}\left(\frac{H_r}{H_r + \pi_r}\right)^2\\
&+ \frac{D}{Q}A_{si} + \frac{D}{Q}A_{wi}k_i + \frac{H_{wi}Q}{2}\left(\left(1-\frac{D}{P_i}+\frac{1}{k_i}\right)\left(\frac{\pi_i + H_{wi}}{\pi_i + 2H_{wi}}\right)\right)^2 + \frac{\pi_i Q}{2}\left(\left(1-\frac{D}{P_i}+\frac{1}{k_i}\right)\frac{H_{wi}}{\pi_i + 2H_{wi}}\right)^2\\
&+ \frac{D}{Q}A_{sf} + \frac{D}{Q}A_{wf}k_f + \frac{H_f Q}{2}\left(\left(1-\frac{D}{P_f}+\frac{1}{k_f}\right)\left(\frac{\pi_f + H_f}{\pi_f + 2H_f}\right)\right)^2 + \frac{\pi_f Q}{2}\left(\left(1-\frac{D}{P_f}+\frac{1}{k_f}\right)\frac{H_f}{\pi_f + 2H_f}\right)^2.
\end{aligned} \tag{22}$$

At this step, we must determine the optimal problem variables using the partial derivatives of the objective function relative to the problem variables as follows:

$$\begin{aligned}
&\frac{\partial \mathrm{TC}_m}{\partial K_r} = 0 \longrightarrow K_r^*,\\
&\quad = \sqrt{\frac{H_r \pi_r}{2\mathrm{DA}_r\left(H_r + \pi_r\right)}Q},\\
&\frac{\partial \mathrm{TC}_m}{\partial K_i} = 0 \longrightarrow k_i^*,\\
&\quad = \sqrt{\frac{H_{wi}\left(\pi_i + H_{wi}/\pi_i + 2H_{wi}\right)^2 + \pi_i\left(H_{wi}/\pi_i + 2H_{wi}\right)^2}{2\mathrm{DA}_{wi}}Q},\\
&\frac{\partial \mathrm{TC}_m}{\partial K_f} = \frac{D}{Q}A_{wf} - \frac{Q}{2k_f^2}\left(H_f\left(\frac{\pi_f + H_f}{\pi_f + 2H_f}\right)^2 + \pi_f\left(\frac{H_f}{\pi_f + 2H_f}\right)^2\right) = 0,\\
&k_f^* = \sqrt{\frac{H_f\left(\pi_f + H_f/\pi_f + 2H_f\right)^2 + \pi_f\left(H_f/\pi_f + 2H_f\right)^2}{2\mathrm{DA}_{wf}}Q}.
\end{aligned} \tag{23}$$

Table 2: The input parameters of a three-level supply chain.

D	P_1	P_2	H_r	H_{wi}	H_f	π_r	π_i	π_f	A_r	A_{si}	A_{wi}	A_{wf}	A_{sf}
36000	42000	45000	400	500	700	2000	1000	800	400	1300	350	300	1250

Table 3: Solution result.

Q	Q_r	Q_{wi}	Q_f	b_r	b_i	b_f	K_i	K_r	K_f	TC
22161	7387	4432	3693	1622	1899	2585	3	5	6	3128606

As observed, the optimal number of orders at different levels is determined based on a function of the amount of economic production. In this section, by obtaining the optimal economic production using the partial derivative of the objective function relative to Q, the optimal value of all problem variables will be determined using the above relations. So, we have

$$
\begin{aligned}
\frac{\partial \mathrm{TC}_m}{\partial Q} &= -\frac{D}{Q^2}A_rK_r + \frac{1}{2k_r}\left(\frac{H_r\pi_r}{H_r+\pi_r}\right)\\
&\quad -\frac{D}{Q^2}A_{si} - \frac{D}{Q^2}A_{wi}k_i + \frac{(1-D/P_i+1/k_i)}{2}\left(H_{wi}\left(\frac{\pi_i+H_{wi}}{\pi_i+2H_{wi}}\right)^2 + \pi_i\left(\frac{H_{wi}}{\pi_i+2H_{wi}}\right)^2\right) - \frac{D}{Q^2}A_{si} - \frac{D}{Q^2}A_{wi}k_i\\
&\quad + \frac{(1-D/P_f+1/k_f)}{2}\left(H_f\left(\frac{\pi_f+H_f}{\pi_f+2H_f}\right)^2 + \pi_f\left(\frac{H_f}{\pi_f+2H_f}\right)^2\right) \longrightarrow\\
&\frac{D}{Q^2}\left(A_rK_r + A_{si} + A_{wi}k_i + A_{sf} + A_{wf}k_f\right)\\
&= \frac{1}{2k_r}\left(\frac{H_r\pi_r}{H_r+\pi_r}\right) + \frac{(1-D/P_i+1/k_i)}{2}\left(H_{wi}\left(\frac{\pi_i+H_{wi}}{\pi_i+2H_{wi}}\right)^2 + \pi_i\left(\frac{H_{wi}}{\pi_i+2H_{wi}}\right)^2\right),\\
&\quad + \frac{(1-D/P_i+1/k_i)}{2}\left(H_f\left(\frac{\pi_f+H_f}{\pi_f+2H_f}\right)^2 + \pi_f\left(\frac{H_f}{\pi_f+2H_f}\right)^2\right) \longrightarrow\\
Q^* &= \sqrt{\frac{D\left(A_rK_r + A_{si} + A_{wi}k_i + A_{sf} + A_{wf}k_f\right)}{1/2k_r\left(H_r\pi_r/H_r+\pi_r\right) + (1-\mathrm{D}/P_i+1/k_i)/2\left(H_{wi}\left(\pi_i+H_{wi}/\pi_i+2H_{wi}\right)^2 + \pi_i\left(H_{wi}/\pi_i+2H_{wi}\right)^2\right) + \left(1-D/P_f+1/k_f\right)/2\left(H_f\left(\pi_f+H_f/\pi_f+2H_f\right)^2 + \pi_f\left(H_f/\pi_f+2H_f\right)^2\right)}},\\
Q^* &= \frac{2D\left(A_{si}+A_{sf}\right)}{\sqrt{\left(1-D/P_f\right)\left(H_{wi}\left(\pi_i+H_{wi}/\pi_i+2H_{wi}\right)^2 + \pi_i\left(H_{wi}/\pi_i+2H_{wi}\right)^2\right) + \left(1-D/P_f\right)\left(H_f\left(\pi_f+H_f/\pi_f+2H_f\right)^2 + \pi_f\left(H_f/\pi_f+2H_f\right)^2\right)}}.
\end{aligned}
\tag{24}
$$

With the replacement of K_i^*, K_f^*, and K_r^* in the above equation and after calculation and simplification, the optimal value of Q variable will be obtained as follows:

$$
Q^* = \frac{2D\left(A_{si}+A_{sf}\right)}{\sqrt{(1-D/P_i)\left(H_{wi}\left(\pi_i+H_{wi}/\pi_i+2H_{wi}\right)^2 + \pi_i\left(H_{wi}/\pi_i+2H_{wi}\right)^2\right) + \left(1-D/P_f\right)\left(H_f\left(\pi_f+H_f/\pi_f+2H_f\right)^2 + \pi_f\left(H_f/\pi_f+2H_f\right)^2\right)}}.
\tag{25}
$$

After determining the optimal economic production according to (25) and substituting the value obtained in (15)–(20), the optimal real values of the other problem variables and the optimal real value of the objective function

Table 4: Efficiency of the proposed heuristic method.

Problem	Z = optimal real value of the problem	TC = optimal integer value of the problem	% = percentage difference between the optimal real solution and optimal integer solution to the problem (%)
1	244301.3	245867.2	0.64
2	246162.7	244085.6	0.84
3	248188.5	246202.6	0.80
4	247622.5	255067.1	3.01
5	277597.1	279114.3	0.55
6	281391	281906.4	0.18
7	282395.6	285566.8	1.12
8	281966	289377	2.63
9	281116.2	290394.9	3.30
10	306972.5	314815.5	2.55
11	307402.1	316513.7	2.96
12	206642.8	205731.3	0.44
13	209231.6	210423	0.57
14	211189.3	210961.8	0.11
15	213020.4	215275	1.06
16	212720.8	218395	2.67
17	223065.9	228946.5	2.64
18	245184.9	241843.7	1.36
19	258218	256284.7	0.75
20	262500	268350.2	2.23
Average			1.52

of the problem are determined.

$$
\begin{aligned}
b_r^* &= \frac{Q^*}{k_r\left(\pi_r/H_r+1\right)},\\
b_i^* &= \frac{H_{wi}\left(1-D/P_i+1/k_i\right)}{\pi_i+2H_{wi}}Q^*,\\
b_f^* &= \max\left(\frac{H_f\left(2-D/P_f+1/k_f\right)Q}{2\left(\pi_f+H_f\right)}, b_i^*\right),\\
K_r^* &= \sqrt{\frac{H_r\pi_r}{2\mathrm{DA}_r\left(H_r+\pi_r\right)}Q^*},\\
k_i^* &= \sqrt{\frac{H_{wi}\left(\pi_i+H_i/\pi_{wi}+2H_{wi}\right)^2+\pi_i\left(H_i/\pi_{wi}+2H_{wi}\right)^2}{2\mathrm{DA}_{wi}}Q^*},\\
k_i^* &= \sqrt{\frac{H_{wi}\left(\pi_i+H_i/\pi_{wi}+2H_{wi}\right)^2+\pi_i\left(H_i/\pi_{wi}+2H_{wi}\right)^2}{2\mathrm{DA}_{wi}}Q^*},\\
k_f^* &= \sqrt{\frac{H_f\left(\pi_f+H_f/\pi_f+2H_f\right)^2+\pi_f\left(H_f/\pi_f+2H_f\right)^2}{2\mathrm{DA}_{wf}}Q^*}.
\end{aligned}
\tag{26}
$$

Step 2. Method of determining the optimal integer solution of the problem.

Step 3. Method of determining the variable.

In this case, too, the concepts of high and low-bound false cost method have been used to determine the variable in each stage. Therefore, at each decision-making level to determine the variable on which branching should be performed, first the increase or decrease level of the objective function is determined for all remaining variables in case branching is performed on the variable in each of the following problems. Then, a variable is selected for branching that has the maximum average difference between the values of the objective function in the two branches at all stages.

Step 4. Assigning the probability of selection to the decision variables in each step.

In order to use all the data generated at all different levels in the decision tree, the probability of selecting each variable is calculated as follows.

The probability of selecting variable i in each step = AVG|costup$_i$ − costdown$_i$|/$\sum_i$AVG|costup$_i$ − costdown$_i$|.

According to the assigned probability, in each stage, a variable will be selected that has the average maximum difference between the costs of the lower and upper branches up to that stage.

Step 5. Determining the value of selected variable.

After determining the desired variable in the step, a value is considered for the variable that induces the least increase in the objective function.

Example 1. The input parameters of a three-level supply chain are shown in Table 2.

Modeling and solving the model using the proposed method based on the branch and bound algorithm, the resulting solutions to the problem are determined as shown in Table 3.

4. Efficiency of the Proposed Heuristic Method

The proposed model is implemented for 20 different numerical examples. In Table 4, the efficiency of the method used is shown in comparison with the real optimal values.

According to the results, the difference between the solutions obtained from the algorithm and the solutions obtained from the optimal values with the real variables is an average of 1.52%.

5. Managerial Insights

The proposed model can be used for products with high maintenance costs or those subject to special conditions, including chemical and hazardous products, sensitive electronic and telecommunication products, and perishable products. As there was no official database for some parts of cost, the experts' estimations were sought to help. The questions about the inventory costs for each route have been categorized and the estimated costs have been entered into the mathematical model. The model is presented as a separate run, which paves the ground for the simulation to be run several times and to report the average of the obtained results. This minimizes simulation errors as much as possible.

6. Conclusion

In this paper, first, the proposed mathematical models related to the application of inventory control system in three-level supply chains were investigated, and then, an inventory management model was presented considering the conditions of deficiency. Since the model provided is in a mixed nonlinear programming form, the branch and bound algorithm is used to determine the solutions to the problem.

The results are as follows:

(i) According to the results, the difference between the solutions obtained from the algorithm and the solutions obtained from the optimal values with the real variables is an average of 1.52%.

(ii) According to the assigned probability, in each stage, a variable will be selected that has the average maximum difference between the costs of the lower and upper branches up to that stage.

To evaluate the efficiency of the proposed model, examples for different supply chains were examined and the values of the objective function obtained using the branch and bound algorithm were measured with the optimal real values of the objective function, which indicates the appropriate efficiency of the algorithm. To expand the proposed problem, any of the following conditions can be considered in this problem:

(i) Taking into account the possible conditions in relation to parameters such as delivery, production, loading, and unloading times.

(ii) Expanding the models offered in multi-product mode and considering the relationships and limitations between different products.

(iii) Limiting the number of orders and transfers of products between different stations.

(iv) Considering limitations such as storage space, amount of capital, and so on in the problem.

(v) Considering specific conditions for demand, such as the dependence of demand on price and inventory, and considering advertising-related activities to increase final demand.

(vi) Considering more than one member at each level of the supply chain.

(vii) Considering cooperation agreements between supply chain components.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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