Research Article

Comparison and Forecasting of VaR Models for Measuring Financial Risk: Evidence from China

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With increasing extremal risk, VaR has been becoming a popular methodology because it is easy to interpret and calculate. For comparing the performance of extant VaR models, this paper makes an empirical analysis of five VaR models: simple VaR, VaR based on RiskMetrics, VaR based on different distributions of GARCH-N, GARCH-GED, and GARCH-t. We exploit the daily closing prices of the Shanghai Composite Index from January 4, 2010, to April 8, 2020, and divide the entire sample into two periods for empirical analysis. A rolling window is used to update the daily estimation of risk. Based on the failure rates under different significance levels, we test whether a specific VaR model passes the back-testing. The results indicate that all models, except the RiskMetrics model, pass the test at a 5% level. According to the ideal failure rate, only the GARCH-GED model can pass the test at a 1% level. For the Kupiec confidence interval, the GARCH-t model can also pass the back-testing at all aforementioned levels. Particularly, we find that the GARCH-GED model has the lowest forecasting failure rate in the class of GARCH models.

1. Introduction

With the economic globalization and the continuous innovation of financial products, the financial industry is also facing huge risks while booming, and the complexity, linkage, and uncertainty of financial markets are increasing. After the outbreak of the U.S. subprime mortgage crisis in 2008, it brought a heavy blow to the global financial market, and people’s demand for risk management became more urgent. Financial risk management is an indispensable part of the investment, decision-making, and supervision, which mainly includes four stages: risk identification, risk measurement, risk decision, and risk control. Among them, risk measurement refers to measuring the size and possibility of loss caused by various risks by the quantitative method, which is also the core content of financial risk management. However, the complexity and time-varying of the market factors bring some difficulties to financial risk measurement. Most risk measurement methods involve complex mathematical formulas and models that are less operable. What the investment decision maker needs is a technical method that is easy to grasp and accurately reflect the size of the risk [1–4].

Since the 1990s, the value at risk (VaR) has been introduced into financial risk management and has become a widely used risk measurement and management tool for the regulatory authorities and financial institutions. The earliest method of measuring VaR value is the historical simulation method. Then, the Monte Carlo simulation method and variance-covariance method are gradually used. In 1994, the VaR method was used by J.P. Morgan. The Basel Committee even explicitly stipulated that the VaR should be used as the standard model for determining the bank regulatory capital. Nowadays, the VaR method has been widely used in the field of market risk and credit risk management, however, some scholars still believe that the risk management based on the VaR method cannot avoid the outbreak of financial crisis [5–9].

2. Literature Review

Since the VaR method was proposed, it has attracted the attention of many scholars. They have conducted a lot of research on the theory of VaR and put up with many means to improve the accuracy of the VaR models. Christofersen...
and Errunza [10] found that using the normal distribution to calculate VaR would have a large error. Hence, the GARCH models and their derivative models are used to estimate VaR, which can better describe the distribution characteristics of financial time series and also provide new ideas for calculating VaR. Angelidis et al. [11] studied the performance of the GARCH type models in the daily value at the risk of a diversified portfolio using a large number of distribution assumptions and different sample sizes. The results showed that different distributions would affect the VaR value, however, there was no correlation between model selection and sample size. So and Yu [12] applied seven different GARCH models to the financial data and evaluated the VaR value of each model under different significance levels. To confirm whether the estimation of VaR is affected by the conditional distribution of return, Makiel [13] collected a large amount of data to estimate various types of ARIMA-GARCH (1, 1) models and compared the differences among the models according to the Kupiec test results. Girardi and Ergün [1] used the multidimensional GARCH model to estimate the CoVaR value. Chen et al. [2] researched the variable smooth transition function of the nonlinear GARCH model. Kuhe [3] used the cointegration vector generalized autoregressive conditional heteroscedasticity (VAR-GARCH) model to investigate the dynamic relationship between the crude price and stock market price fluctuation in Nigerians. The results show that there is a reliable relationship between the crude price and stock market price, however, the conditional volatility of the logarithmic rate of the return of the stock market price is stable and predictable, while the conditional volatility of the logarithmic rate of the return of the crude price is unstable and unpredictable.

Manganelli and Engle [14] proposed a new method to calculate the VaR value. They built the conditional value at risk (CVaR) model by autoregressive modeling. CVaR is the conditional mean of the loss exceeding VaR, which reflects the average level of excess loss and the potential loss of the financial assets. Alexander and Baptista [15] studied the change of the portfolio selection behavior of the agents after the CVaR constraints were included in the agent constraint set. They found that compared with the VaR constraint, the CVaR constraint can more effectively inhibit the agent’s mild risk aversion, however, it cannot inhibit or even indulge the agent’s high-risk aversion. Kaut et al. [16] used the CVaR model to analyze the robustness of portfolio management. They found that the CVaR model is sensitive to model parameter setting errors. Abad et al. [17] reviewed the existing literature on the value at risk (VaR). From a practical point of view, the method based on the extreme value theory and filtering history simulation is the best method to predict VaR.

Wang et al. [18] focused on the fact that electricity price fluctuation brings more uncertainty and greater risk. They proposed a risk assessment method for the power market based on the EMD algorithm. The impact of different macro policies implemented by the government on the power market risk is also different, such as a feed-in tariff or tax-rebate regulation [19]. The risk assessment of the economic system and financial market can also be analyzed from the perspective of system stability. Ma and Wu [20] established a triopoly price game model based on the derivation of the mathematical models to analyze the pricing of the enterprises in relevant markets. Wang et al. [21] proposed the idea of combining the EMD algorithm with the Copula theory, and hence, they constructed a new EMD-Copula model and applied it to the estimation of value at risk in the actual power market. He et al. [22] proposed a new VaR model based on empirical mode decomposition (EMD) to estimate the downside risk of the power market. Mensi et al. [23] used the wavelet square coherence method and wavelet analysis to build the VaR model to study the portfolio risk and interaction between the BRICS and South Asian border stock markets and each major developed stock market. Berger and Gençay [24] used the wavelet analysis to divide the volatility of the financial conditions into three parts: short-term, medium-term, and long-term, to study VaR. The results showed that the short-term and medium-term information components cover the relevant information necessary to estimate the sufficient daily VaR.

In general, foreign research on the VaR risk management methods is earlier, and the development process of the VaR method is traceable [25–28]. Although domestic research started late, the progress of research on the VaR methods is very fast, and statistics and mathematics are used to optimize the models [29, 30]. At present, the domestic and foreign scholars all agree that the main difficulty of research is how to improve the accuracy and effectiveness of the model. They focus a lot of energy on the establishment of models but neglect the processing and design of the research data [31, 32]. For example, many scholars chose a small sample size and did not consider the update of the estimated sample data [33, 34]. In this paper, 2494 SSE index data are divided into estimated sample and forecast sample. The estimated sample is updated by the R software, and the empirical analysis of the five VaR models based on the variance-covariance method is carried out. Finally, it examines the effectiveness of the models by comparing the VaR values and the actual loss values.

The rest of this article is organized as follows: section 3 describes the VaR method, section 4 describes some models used in this study, section 5 contains the empirical analysis, and section 6 presents the main conclusions of this paper.

### 3. VaR

VaR refers to the maximum possible loss of the value of a certain financial asset or portfolio of assets within a certain period under certain confidence. It is expected to illustrate the potential loss under some extreme conditions. VaR is the quantile of the loss variable with a high probability. Hence, there is just a small chance that the loss can exceed this amount. The statistical nature of VaR also allows the backtesting of this measure so that the practitioners can study the accuracy of the underlying loss model. The specific meaning of VaR can be expressed by,

\[ P(L > \text{VaR}) \leq 1 - c. \]  
(1)
In equation (1), “P” refers to the probability that the actual loss of the asset value is greater than the maximum possible loss, and “L” refers to the loss value of a financial asset in a specific future holding period, whereas “c” refers to the confidence level. Equation (1) demonstrates that in a certain holding period in the future, the probability that the loss does not exceed the VaR value is c.

It can be seen from the definition of VaR that the holding period and confidence level are two important parameters. In addition, the distribution characteristics of the asset prices are also one of the elements of the VaR model. According to different probability distribution models, the VaR value of the financial assets is somewhat different. The basic idea of the VaR method is to use the historical information of the financial assets to infer the future situation, however, what is inferred is a probability distribution. The core of the VaR is to use the historical data to estimate the probability distribution of the value or the return of financial assets, and the key to inferring this probability distribution is to estimate the volatility of the financial assets.

3.1. VaR in General Distribution. If the distribution of the financial asset returns is not specifically set, the initial investment amount is assumed to be $W_0$, and R refers to the random investment return rate. Assuming that the investment position remains unchanged and there is no transaction, the value of the financial assets at the end of the period is as follows:

$$W = W_0 (1 + R).$$ (2)

At a given confidence level of c, assuming the minimum rate of return is $R^*$, the lowest value at the end of the period is as follows:

$$W^* = W_0 (1 + R^*).$$ (3)

VaR measures the maximum possible loss at a given confidence level and is generally expressed as a positive number. VaR is divided into absolute VaR and relative VaR. Absolute VaR does not consider the expected value of the financial assets, and it is expressed by the difference between the initial asset value and the current value. The expression is as follows:

$$\text{VaR (absolute)} = W_0 - W^* = -W_0 R^*, \quad R^* \leq 0.$$ (4)

Relative VaR refers to the maximum possible loss relative to the expected value of the financial assets. Assuming that the expected rate of return is $\mu$, the expression of the relative VaR is as follows:

$$\text{VaR (relative)} = E(W) - W^* = W_0 (1 + \mu) - W_0 (1 + R^*)$$
$$\quad = -W_0 (R^* - \mu).$$ (5)

If the holding period is short and the average yield is low, the VaR value calculated according to the above two definitions is very close. In other cases, especially when measuring the unforeseen loss of the credit risk over a longer period of time, the relative VaR can better reflect the risk that comes from the deviations from the average based on fully considering the time value of the funds.

It can be seen from the VaR calculation formula that the key to estimating the VaR value is to determine the minimum value of the financial asset $W^*$ at the end of the period or the minimum rate of return $R^*$ during the holding period, which is usually a negative number and can be written as $-|R^*|$. When the probability density function of the return rate of the financial assets $f(r)$ is known at a given confidence level, the probability of the actual loss value exceeding VaR is shown as

$$P(R \leq R^*) = \int_{-\infty}^{R^*} f(r)dr = 1 - c.$$ (6)

The value of $R^*$ is called the quantile of the yield distribution that reveals the essence of the VaR value. In addition, equation (6) is applicable under any distribution.

3.2. VaR under Normal Distribution. When the returns of the financial assets follow a normal distribution, the standard deviation $\sigma$ and mean of the returns $\mu$ can be used to estimate the VaR. This method is also called the parameter method. Under the assumption of normal distribution, there are $R \sim N(\mu, \sigma^2)$. After standard normalization of the variables, the new variable $Z = (R - \mu)/\sigma$ follows the standard normal distribution. $Z_{\alpha}$ is defined as the upper quantile under the confidence level of $c$, and $Z_{\alpha} > 0$. It is shown as follows:

$$-Z_{\alpha} = \frac{-|R^*| - \mu}{\sigma}. \quad (7)$$

The probability of financial asset loss exceeding the VaR value can be expressed as follows:

$$\int_{-\infty}^{-|R^*|} f(r)dr = \int_{-\infty}^{-Z_{\alpha}} \phi(\epsilon)d\epsilon = 1 - c. \quad (8)$$

Note: $\phi(\epsilon)$ represents the probability density function of a standard normal distribution.

According to equation (7), the minimum critical rate of return can be expressed as $R^* = -Z_{\alpha}\sigma + \mu$, where $Z_{\alpha}$ is determined by the confidence level, $\mu$ is the average of the sample data, and $\sigma$ represents the volatility. Under the assumption of normal distribution, the expressions of absolute VaR and relative VaR are, respectively, shown as equations (9) and (10).

$$\text{VaR (absolute)} = W_0 - W^* = -W_0 R^* = W_0 (Z_{\alpha}\sigma - \mu),$$ (9)

$$\text{VaR (relative)} = E(W) - W^* = W_0 [E(R) - R^*] = W_0 (\mu - R^*) = W_0 Z_{\alpha}\sigma.$$ (10)

When the observations of the return rate of the financial assets meet the three assumptions of an efficient market, an independent and identical distribution, and a random walk, the volatility of the annual return rate can be derived from
the volatility of the daily return rate. The VaR of different time lengths can be converted into each other, which is called “time aggregation” in quantitative economics. Assuming that the average annual return rate and the variance of the financial asset are, respectively, expressed as \( \mu \) and \( \sigma^2 \), the average return rate of year \( T \) is shown as \( \mu_T \), and the volatility rate is \( \sigma_T \). Then, according to the mathematical properties of independent and identical distribution, it is easy to get the relationships in equations (11) and (12).

\[
\mu_T = \mu^T, \quad (11)
\]

\[
\sigma_T = \sigma \sqrt{T}. \quad (12)
\]

Equation (12) shows that under the condition that the return rate series are uncorrelated when the observation period becomes longer, the volatility increases by the square root of the time. If only the relative VaR is considered, the VaR of period \( T \) can be expressed by the VaR of period 1.

\[
\text{VaR}_T = W_0 a \sigma_T = W_0 a \sigma \sqrt{T} = \sqrt{T} \text{VaR}_1. \quad (13)
\]

4. Methods of Calculating VaR

This paper mainly introduces the historical simulation method, Monte Carlo simulation method, and variance-covariance method for calculating the VaR value. The first two methods adopt the complete valuation method and use the statistical method to simulate the distribution of the future return of the financial assets without assuming the distribution of the return of the financial assets. The variance-covariance method adopts the local valuation method and mainly uses the sample data to estimate certain parameters to calculate the VaR value.

4.1. Historical Simulation. The historical simulation method refers to a nonparametric method that calculates the VaR value using the historical data to simulate the future return of the financial assets at a certain confidence level, assuming that the future fluctuations of the financial assets are the same as the historical fluctuations. The historical simulation method does not need to make assumptions about the distribution of the return rate, nor does it need to estimate the parameters of the model. It can be calculated based on the historical data and the definition of VaR, and hence, it is very simple and intuitive, and its simulation results depend largely on historical data selection. However, this method assumes that history will repeat itself, but the capital market, in reality, is very volatile. If the historical data contains fewer extreme values, it will not be able to simulate well the extreme events in reality.

4.2. Monte Carlo Simulation. Compared with the variance-covariance method and the historical simulation method, the biggest difference of the Monte Carlo simulation method is that this method can simulate the model by establishing a random process model and using a computer to generate a large number of random numbers from outside the sample. This method is also similar to the historical simulation method. They both do not need to make assumptions about the distribution of returns, and the basic principle is to use statistical methods to simulate the distribution of the future returns of the financial assets. This model can simulate various possible situations of the future changes in the financial asset prices and is not constrained by the capacity and quality of historical data. Therefore, its simulation results are often closer to the true value and have higher accuracy, however, the method is complicated to operate and requires a large number of calculations. Besides, the cost is high, and it takes a long time. The choice of random model has a great influence on the simulation results, and hence, there is a certain model risk.

4.3. Variance-Covariance Method. Based on the traditional delta-normal method and gamma-normal method, some simpler and easier-to-operate variance-covariance methods have evolved, e.g., the RiskMetrics model proposed by J. P. Morgan and the VaR method based on the GARCH model.

4.3.1. Delta-Normal Model and Gamma-Normal Model. The core idea of the delta-normal model is to use the approximate relationship between the financial asset portfolio value function and the market risk factor under the assumption of normal distribution and infer the probability distribution of the financial asset portfolio value function from the statistical distribution of risk factors. According to Taylor’s first-order expansion, we approximate the value of VaR to simplify the calculation of VaR. The establishment process of the gamma-normal model is similar to that of the delta-normal model. The difference is that the gamma-normal model describes the nonlinear change characteristics of the financial assets, and it approximates the VaR value according to Taylor’s second-order expansion.

4.3.2. RiskMetrics Model. The RiskMetrics model was proposed by J. P. Morgan in 1994. The core of this model is to assume that the logarithmic returns of the financial assets follow a conditional normal distribution. It uses the exponentially weighted moving average (EWMA) model to estimate the standard deviation sequence of returns. The biggest feature of this model is that when estimating the volatility of the return rate, it is assumed that the weight of the volatility of each period decreases exponentially with the increase of the retrospective time, that is to say, the closer to the current period, the greater the weight. The assumption can better reflect the recent change trend of volatility, and the formula for measuring the rate of return volatility is shown as equation (14).

\[
\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1 - \lambda) (r_{n-1} - \bar{r})^2. \quad (14)
\]

In equation (14), \( \lambda \) represents the attenuation factor, and \( 0 < \lambda < 1 \). \( \sigma_n \) represents the volatility of the \( n \)th day and \( r_{n-1} \) represents the rate of return of the previous period. \( \bar{r} \) represents the mean market rate of the return of the previous period, usually assuming \( \bar{r} = 0 \), and \( (r_{n-1} - \bar{r}) \) represents the
changes in the previous period’s yield. Based on this, equation (14) can be simplified as,
\[ \sigma_n^2 = \lambda \sigma_{n-1}^2 + (1 - \lambda) r_{n-1}^2. \]  

When there are \( m \) historical data, equation (16) can be obtained by \( m \) iterations.
\[ \sigma_n^2 = \lambda m \sigma_{n-m}^2 + (1 - \lambda) \sum_{i=1}^{m} \lambda^{i-1} r_{n-i}^2. \]  

As \( 0 < \lambda < 1 \), when \( m \) approaches infinity, the term \( \lambda m \sigma_{n-m}^2 \) approaches 0, and the volatility expression can be further simplified as equation (17).
\[ \sigma_n^2 = (1 - \lambda) \sum_{i=1}^{m} \lambda^{i-1} r_{n-i}^2. \]  

The weight of the data during period \( i \) is \( (1 - \lambda) \lambda^{i-1}. \) The weight of each period is the product of the weight of the previous period and \( \lambda. \) By repeated research, J. P. Morgan company believes that it is more appropriate to assume \( \lambda = 0.94 \) when estimating the volatility of daily yield.

4.3.3. Variance-Covariance Method Based on GARCH Model. Engle proposed the autoregressive conditional heteroscedasticity model (also known as the ARCH model). This model can better describe the heteroscedasticity in a financial time series, however, it belongs to a nonlinear function. Its parameter estimation cannot be obtained by ordinary methods. In addition, for some financial time series with a long memory, a high-order ARCH model is required. When the lag order is too high, the difficulty of estimating the model parameters will increase, and it will also lead to problems, such as multicollinearity. However, the GARCH model proposed by Bollerslev on the basis of the ARCH model can effectively overcome the shortcomings of the ARCH model. It defines \( \sigma_n^2, \) the conditional variance of the disturbance term, as the variance that depends on the first \( m \) disturbance terms and the first \( n \) conditional variances. An high-order ARCH model can be represented by a simple GARCH model. In practical applications, usually, only low-order GARCH models, such as GARCH (1, 1), GARCH (1, 2), and GARCH (2, 1) are used. The expressions of the GARCH (1, 1) model are as follows:
\[ r_t = \mu + \alpha \varepsilon_t, \]  
\[ \alpha_t = \sigma^2 \varepsilon_t, \]  
\[ \sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 \varepsilon_{t-1}^2, \]  
\[ 0 \leq \alpha_1, \beta_1 \leq 1; \alpha_1 + \beta_1 < 1. \]  

Of these, equation (18) is the mean yield equation, and \( r_{t} \) represents the return rate of the financial assets, where \( \varepsilon_t \) represents the residual term of the mean equation. \( \{ \varepsilon_t \} \) is a sequence of independent and identically distributed random variables with mean 0 and variance 1. In equation (20), \( \alpha_0 \) is the constant term, and \( \sigma_t^2 \) is the conditional variance of the disturbance term. \( (t-1) \) represents the first-order lag. In addition, \( \alpha_1 \) and \( \beta_1 \), respectively, represent the coefficients of the yield disturbance term lagging by one order and the conditional variance.

After fitting the GARCH (1, 1) model based on the data in the estimated sample, the prediction model can be used to estimate the volatility of the return rate of the next holding period as in equation (21).
\[ \sigma_{h+1}^2 = \alpha_0 + \alpha_1 \sigma_h^2 + \beta_1 \sigma_{h-1}^2. \]  

Note: \( h \) is the origin for predicting. \( \alpha_1 \) and \( \beta_1 \) are estimated by the estimated samples before the origin.

4.4. Back-Testing. Back-testing means that after predicting the VaR value, the VaR value is compared with the actual loss value to test the validity of the VaR model. The back-testing of VaR is the coverage degree of the model results to the actual loss, and one feasible method is the failure rate test introduced by Kupiec. The definition of “failure rate” is the probability that the actual maximum loss value exceeds the VaR value. By statistical “failure rate,” the effectiveness of the VaR models can be quickly and accurately tested. Assuming that the size of the samples is \( T \) and the number of failure days is \( N \), the failure rate is \( \kappa = N/T. \) Ideally, the failure rate is \( p^* \), \( p^* = 1 - c, \) and \( c \) is the given confidence level. If the failure rate is greater than \( p^* \), the VaR value is underestimated. On the contrary, when the failure rate is less than \( p^* \), the model estimation is too conservative and the VaR value is overestimated. The original hypothesis in the test is shown as \( H_0. \)
\[ H_0: \kappa = \frac{N}{T} = p^*. \]  

Test by constructing the likelihood statistic LR (the source and derivation of formula (23) are placed in the appendix at the end of the text).
\[ LR = -2 \ln \left( (1 - p)^{T-N} p^N \right) + 2 \ln \left( (1 - \kappa)^{T-N} (\kappa)^N \right). \]  

The ideal number of expected failures is represented as \( N^* = p^* T. \) However, Kupiec gives the acceptance confidence interval for back-testing at various confidence levels as shown in Table 1.

5. Empirical Analysis

5.1. Data Acquisition and Processing. The Shanghai Composite Index was officially released on July 15, 1991. The calculated sample stocks in the Shanghai Composite Index include all stocks listed on the Shanghai Stock Exchange. It can not only reflect the changes in the price of the listed stocks on the Shanghai Stock Exchange but also reflect the economic conditions of various industries. It is of great significance to measure and manage risks. This article collects the daily closing prices of the Shanghai Composite Index from January 4, 2010, to April 8, 2020, a total of 2494 sample data. The entire sample is divided into two parts: an
estimated sample, the 1456 closing prices data from January 4, 2010, to December 31, 2015, and a prediction sample, the 1038 closing prices data from January 4, 2016, to April 8, 2020.

The stock holding period studied in this paper is one day. The data on that day is represented by \( t \), and the previous day’s data is represented by \( t - 1 \). The arithmetic rate of return of a stock is obtained by dividing the stock price change during the holding period plus the dividend during this period by the stock price at the beginning of the period.

\[
rt = \frac{P_t + D_t - P_{t-1}}{P_{t-1}}. \tag{24}
\]

The geometric yield of the stock \( R_t \) represents the continuous compound interest rate of the return on day \( t \), which is the logarithm of the stock price ratio as shown in equation (24).

\[
R_t = \ln \left( \frac{P_t + D_t}{P_{t-1}} \right). \tag{25}
\]

In the research, to simplify the analysis, it is often assumed that the dividend \( D_t \) is 0 and the logarithmic rate of return is simplified to equation (26). When \( r_t \) is very small, \( R_t \approx r_t \). Therefore, for a small rate of return such as the daily rate of return, the difference between the geometric rate of return and arithmetic rate of return is very small. The geometric rate of return is usually used instead of the arithmetic rate of return in calculations. All calculations in this article use the logarithmic rate of return as follows:

\[
R_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \approx \ln (1 + r_t). \tag{26}
\]

5.2. Forecasting of VaR. According to the VaR calculation formula under the assumption of normal distribution, the key to estimating the VaR value is to predict the standard deviation of the rate of return. The prediction of the return rate fluctuation is very complicated. To forecast the VaR of a certain holding period simply and quickly, the average value and variance of the historical data can be used to approximate the mean and variance of return rate in the forecast period by simple statistics of the existing historical data. There are some unscientific and irrational processes in the estimation process, however, it still has a certain reference function for financial asset risk management. We select the Shanghai Composite Index data to conduct an empirical analysis. According to the variance and mean of the initially estimated sample, the VaR value on January 4, 2016, can be estimated. Then, the estimated sample is rolled back one day, in turn, to forecast the daily VaR of the entire forecast sample. The final forecasting result is shown in Figure 1. The red curve represents the absolute VaR value at 99% confidence level, and the blue curve represents the absolute VaR value under 95% confidence level. The black line represents the actual loss, which is the difference between the closing price of the previous day and the current day. The red line in the figure is above the blue line, indicating that the VaR value under high confidence is larger. The two VaR curves have a certain deviation from the actual loss value, and they are both above the actual loss value, which shows that the simple prediction method of the VaR value has a certain effect on avoiding risk loss, however, the prediction results have a large error and are not scientific.

5.3. VaR Forecasting Based on RiskMetrics Model. Unlike the simple VaR forecasting method, the RiskMetrics model uses an exponentially weighted moving average model (EWMA) to forecast the volatility of returns, which is more scientific. According to equation (17), we can use the estimated sample of the Shanghai Composite Index to estimate the volatility of the return on the next trading day and roll toward one day to update the estimated sample and estimate the daily VaR values of the entire forecast sample. The result is shown in Figure 2. Compared with Figure 1, the absolute VaR value estimated based on the RiskMetrics model has a lower deviation from the actual loss value. The absolute VaR value curve is close to the upper edge of the actual loss value curve, which shows that the RiskMetrics model may be weaker to resist risks than the simple estimation model of VaR. However, the VaR value estimated based on the RiskMetrics model is closer to the true loss situation, which can avoid the loss caused by the conservative investment strategies to a certain extent.

5.4. VaR Forecasting Based on GARCH (1, 1) Model

5.4.1. Analysis of Basic Statistical Characteristics of Logarithmic Return

(1) Descriptive Analysis. The basic statistical analysis of the daily logarithmic return data of the initial estimation sample of the Shanghai Composite Index is shown in Table 2. The mean value is close to 0, which is consistent with the assumption that the mean daily return is 0, and the standard deviation is 0.015. According to the characteristics of the
normal distribution, the skewness of the normal distribution is 0, and the skewness of the sample data is less than 0, indicating that the distribution of the sample data has a long tail on the left. In addition, the kurtosis of the normal distribution is 3, however, the kurtosis of the sample data is 5.072, indicating that the tail of the sample data distribution decays faster than the normal distribution and the sample distribution and the sample data has a thick tail. When the normal distribution test is performed on the estimated sample, the JB statistic is 1727.344 and the p-value is 0.000, indicating that the daily logarithmic return rate of the Shanghai Composite Index does not obey the normal distribution.

(2) Stationarity Test. The stationarity test should be carried out before the study of the time series, otherwise, it is unavoidable that the phenomenon of “pseudoregression” in a nonstationary series leads to unreliable research conclusions. From Figure 3, we find that the daily logarithmic return of the Shanghai Composite Index fluctuates around 0, which is stable from the graph. To further examine its stability, we use the R software to conduct the ADF unit root test. As shown in Table 3, the daily logarithmic rate of return and its lag time series are stable.

(3) Autocorrelation Analysis. The autocorrelation of the daily logarithmic return rate of the Shanghai Composite Index is tested, and its autocorrelation graph and partial autocorrelation graph are obtained. From Figure 4, it can be seen that the autocorrelation of the daily logarithmic return of the Shanghai Composite Index is more significant. The autocorrelation coefficient is 0-order truncated, and the partial autocorrelation coefficient fluctuates within twice the standard deviation after the third order. It can be estimated that the sample logarithmic return rate meets the third-order autoregressive model.

5.4.2. Mean Equation

(1) Establishment of the Mean Value Equation. From the autocorrelation graph and partial autocorrelation graph of the estimated sample data, the ARIMA (3, 0, 0) model can be initially established, and the expression is as follows:

\[ R_t = \mu + \gamma_1 R_{t-1} + \gamma_2 R_{t-2} + \gamma_3 R_{t-3} + \sigma_i. \quad (27) \]

The coefficients of \( \mu \) and \( R_{t-3} \) in the fitting result of ARIMA (3, 0, 0) are not significant, and the values are close to 0. Hence, this article will eliminate these two variables and construct ARIMA (2, 0, 0) model without a constant term. The model expression is as follows:

\[ R_t = \gamma_1 R_{t-1} + \gamma_2 R_{t-2} + \sigma_i. \quad (28) \]

The fitting results of the ARIMA (2, 0, 0) model are as shown in equation (29), where the coefficients of \( R_{t-1} \) and \( R_{t-2} \) are significant. Hence, equation (28) is the mean equation finally determined in this paper, and this model is also a follow-up study foundation.

\[ R_t = 0.0471R_{t-1} - 0.0389R_{t-2}. \quad (29) \]

(2) Residual Analysis of Mean Equation. The residual diagram in Figure 5 shows that there is a “fluctuation clustering phenomenon” in the volatility of the return. In other words, the variance is small in some periods, and the variance is larger in other periods. For example, between the 200th and 500th sample data, the absolute value of the residual is larger, however, the absolute value of the residuals between the 800th and 1000th samples is smaller. This clustering effect of the residuals indicates that the residuals have autocorrelation, and there may be heteroscedasticity.

(3) ARCH Effect Test. It can be judged that the residuals have an autocorrelation through the graphical characteristics of the residuals. To further verify this characteristic, this paper uses the R software to test the ARCH effect of the residual series. The basic principle of the test is if the residual sequence has an ARCH effect, the residual squared sequence generally will have autocorrelation. Therefore, the test of the ARCH effect can be transformed into the autocorrelation test of the residual squared sequence. There are two test methods, namely the Q test and the LM test.

(a) Q test.

The Q test was proposed by Box and Pierce in 1970, and in 1978, Ljung and Box improved the Q statistic. The null hypothesis of Q test is as follows: the residual square sequence is a pure random sequence, and there is no ARCH effect. If \( \rho_l \) is used to represent the autocorrelation coefficient of the lag order, its null hypothesis can be expressed as,

\[ H_0: \rho_1 = \rho_2 \cdots = \rho_l = 0. \quad (30) \]

The improved Q statistic expression is as follows:

\[ Q = T(T+2) \sum_{i=1}^{l} \frac{\rho_i^2}{T-i}. \quad (31) \]

(b) LM test.

The LM test is the abbreviation of Lagrange multiplier test. The basic principle of the LM test is to construct an autoregressive auxiliary model of the residual squared sequence and test whether the
residual squared sequence has the ARCH effect by checking whether the regression coefficients in the autoregressive model are all 0. The auxiliary model expression is as follows:

$$a_t^2 = \varphi_0 + \varphi_1 a_t^2 + \varphi_2 a_{t-1}^2 + \cdots + \varphi_q a_{t-q}^2 + \varepsilon_t,$$

(32)

where \(\{a_t\}\) is the residual sequence, \(q\) is the lag order, \(\varphi_i\) is the coefficient \((i = 1, 2, 3, \cdots, q)\), and \(\varepsilon_t\) is the residual of the regression model. Based on equation (32), the null hypothesis of LM test can be expressed as follows:

$$H_0: \varphi_1 = \varphi_2 = \cdots = \varphi_q = 0.$$

(33)

The statistical expression of the LM test is as follows:

$$LM = TR^2$$

(34)

\(T\) is the sample size of the regression model, and \(R^2\) is the goodness of fit of the auxiliary model in equation (32).

It can be seen from Table 4 that the results of the Q test and the LM test reject the null hypothesis, indicating that the residual sequence has an ARCH effect and meets the research conditions of the GARCH model.

5.4.3. Fitting Results of GARCH (1, 1) Model. The fitting results of the GARCH (1, 1) model under the assumptions of normal distribution, \(t\) distribution, and GED distribution are as follows:

The fitting results of the GARCH (1, 1) model show that under any distribution, the coefficients of the ARCH term \((a_{t-1}^2)\) and GARCH term \((\sigma_t^2)\) are significant at a significance level of 99%, while the constant terms are not significant. The value of the constant term is very small, and hence, this article ignores the influence of the constant term when predicting the fluctuation of the yield. Under the three distribution assumptions, there is little difference in the estimated values of regression coefficients, however, the coefficients are the most significant under the GED distribution assumption, followed by the \(t\) distribution. The coefficients are the weakest under the normal distribution assumption. From the value of AIC, the model fits best under the GED distribution assumption, followed by the \(t\) distribution, and the model fits the worst under the normal distribution assumption. It shows that the \(t\) distribution and GED distribution can better characterize the thick-tailed characteristics of the logarithmic return of the Shanghai Composite Index.

It can be seen from Table 5 that the \(p\) values of the LM statistic and the \(Q(20)\) statistic are greater than 0.05, and the null hypothesis of the ARCH test should be accepted. In other words, the residuals of the GARCH (1,1) model under the three hypothetical distributions have no autocorrelation, and the ARCH effect has disappeared.

5.4.4. Forecasting and Calculate VaR Value. By the initially estimated sample, a specific GARCH model can be estimated, and the volatility on January 4, 2016, can be predicted and estimated to obtain the VaR value of the day. Rolling the estimated sample toward one day and keeping its sample size unchanged, it can continue to estimate another GARCH model. Then, estimate the volatility on January 5, 2016, and calculate the VaR value of the day, and so on. The estimated samples are rolled in sequence for one day, and bring the predicted volatility of return and the corresponding quantiles of each distribution (as shown in Table 6) into the VaR calculation formula to estimate the VaR value of each trading day during the period from January 4, 2016, to April 8, 2020. The series of operations are completed by the R software.

The R software is used to plot the absolute VaR value and the actual loss value of the Shanghai Composite Index from January 4, 2016, to April 8, 2020. Figures 6–8, respectively, show the daily absolute VaR value of the Shanghai Composite Index under the assumptions of normal distribution, \(t\) distribution, and GED distribution under the 95% significance level. The black curve represents the true daily loss value, expressed as the difference between the closing price of the previous day and that of the day. For convenience, we denote the GARCH (1, 1) models under the three distributions as GARCH-N, GARCH-t, and GARCH-GED, respectively. From the comparison of the absolute VaR value and the actual loss value, it can be seen that the actual loss value fluctuates around 0, and the actual loss value is
negative, indicating the profit of the day. The VaR curve is almost above the actual loss value, and there are only a few VaR values falling on the upper edge of the actual loss curve. Figure 9 shows that the absolute VaR values predicted under the three distribution assumptions overlap, however, it can be seen that the VaR curve predicted by the GARCH-GED model is the highest, followed by the GARCH-t model, and the VaR curve predicted by the GARCH-N model is the lowest.

5.5. Back-Testing Results. According to the comparison chart of the VaR forecast value and the actual loss value of the
Shanghai Composite Index, the VaR value is mostly above the actual loss value, indicating that the estimated VaR value is effective, and the risk can be controlled to a certain extent. In addition, this paper refers to the situation where the actual loss value exceeds the VaR value as a prediction failure. The number of VaR value prediction failures and the failure rate in 1038 prediction samples are counted by the R software. The return test results of each VaR model are shown in Table 7.

The results in Table 7 show that there is no difference between the return test results of the relative VaR and absolute VaR. With 95% confidence, only the RiskMetrics model cannot pass the back-testing. Under 99% confidence, compared with the ideal failure rate, only the GARCH-GED model can pass the strict back-testing. However, when the number of samples is 1000, according to the confidence interval given by Kupiec (Table 1), the failure times of the GARCH-t model are within the confidence interval, and it is considered to be able to pass back-testing. In addition, for the prediction results based on the GARCH (1, 1) model, the VaR forecasting failure rate under the GED distribution assumption is the lowest, followed by the t distribution, and the failure rate under the normal distribution assumption is higher.

Comparing the prediction results of the RiskMetrics model and the GARCH (1, 1) model in Table 7, the failure rate of the VaR prediction results of the former is higher than that of the latter. Both models consider the dynamic and time-varying of the logarithmic return when calculating the conditional variance. However, when calculating VaR in the RiskMetrics model, the weight of the index weighted moving average model uses a constant $\lambda = 0.94$, which is not necessarily suitable for every market. Therefore, the GARCH (1, 1) model is more accurate and scientific in predicting VaR.

### 6. Conclusion

To further study the effectiveness of the VaR model, this paper conducts an empirical analysis of the Shanghai Composite Index data based on the variance-covariance method. We selected the daily log return rate of the Shanghai Composite Index from January 4, 2010, to April 8, 2020, as the sample and used the data of the first 1456 days as the estimated sample and the data of the other 1038 days as the forecast sample to update the estimated sample data by rolling backward one day, in turn. Then, the daily VaR value
of the forecast sample is predicted, and some conclusions are drawn.

The VaR model has a certain risk prevention function. From the research in this paper, it can be seen that the VaR curves obtained by the five models are generally above the actual loss value. The greater the deviation between the VaR curve and the actual loss curve, the lower the failure rate of the model and the stronger the effectiveness. Compared with other models, the VaR curve estimated based on the RiskMetrics model has a lower deviation from the actual loss curve, and hence, its back-testing failure rate is significantly higher than the other models.

Different distribution assumptions will affect the validity of the VaR prediction results based on the GARCH (1, 1) model. When estimating the VaR value based on the GARCH (1, 1) model, it is found that the daily logarithmic return rate of the Shanghai Composite Index does not obey the normal distribution and has characteristics of sharp peaks and thick tails. The residuals of the mean equation have an obvious ARCH effect. After introducing the GARCH (1, 1) model under the assumption of t distribution and GED distribution, the new model has better fitting results and can predict the volatility of the return more accurately. From the back-testing results, it can be seen that the GARCH-GED model has the lowest failure rate. The second is the GARCH-t model, and the GARCH-N model has a relatively high failure rate.

The VaR model is easier to pass the back-testing at the 95% confidence level. Comparing the VaR values under two different confidence levels, it can be seen that the higher the confidence level, the greater the corresponding VaR values. The VaR curve at the 99% confidence level is higher than the VaR curve at the 95% confidence level. At the 95% confidence level, all the five models except the RiskMetrics model, can pass the back-testing. However, at the 99% confidence level, comparing the ideal failure rate, only the GARCH-GED model can pass the back-testing. According to the confidence given by the Kupiec interval, the GARCH-t model can also pass the back-testing.

Appendix

The likelihood ratio test is a test method that uses the ratio of two opposite likelihood functions, i.e., likelihood ratio, to test whether a hypothesis is valid.

Definition 1. Generalized likelihood Ratio

Let \( X_1, \ldots, X_n \) be the sample from the density function \( p(x; \theta) \in \Theta \), and consider the test problem,

\[
H_0: \theta \in \Theta_0 \leftrightarrow H_1: \theta \in \Theta_1 = \Theta - \Theta_0.
\]

Let

\[
\lambda(X) = \frac{\max_{\theta \in \Theta} p(x_1, \ldots, x_n; \theta)}{\max_{\theta \in \Theta_0} p(x_1, \ldots, x_n; \theta)}.
\]

Then, we call it the generalized likelihood ratio of the hypothesis testing problem. The failure ratio test proposed by Kupiec (1995) can measure whether the number of failure values in the VaR estimation sequence is consistent with the given confidence level \( c \). Under the original assumption, the model is "correct," and the number of failure values \( N \) follows a binomial distribution. Therefore, the known variables for the failure rate test are the sample size \( T \), number of failure values \( n \), and confidence level \( c \).

The original hypothesis of the failure rate test is

\[
H_0: \kappa = N/T = p^*.
\]

As the number of failure values \( N \) follows binomial distribution, its probability mass function is,

\[
P_T[X = N] = \binom{T}{N} p^N (1-p)^{T-N}, \quad N = 1, 2, \ldots
\]

Then, the generalized likelihood ratio is,

\[
\lambda(N) = \frac{(1-p)^{T-N} p^N}{\left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N}.
\]

\[
\ln \lambda(N) = \ln \left( \frac{(1-p)^{T-N} p^N}{\left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N} \right) - 2 \ln \lambda(X)
\]

\[
= -2 \ln \left( \frac{(1-p)^{T-N} p^N}{\left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N} \right)
\]

\[
-2 \ln \lambda(N) = -2 \ln (1-p)^{T-N} p^N
\]

\[
+ 2 \ln \left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N.
\]

Let \(-2 \ln \lambda(N) = LR\), and \(\kappa = N/T = p^*\). Then, the aforementioned formula is equal to,

\[
LR = -2 \ln p^N (1-p)^{T-N} + 2 \ln (1-\kappa)^{T-N} (\kappa)^N.
\]

Under the null hypothesis, the LR statistic obeys the \( \chi^2 \) distribution with one degree of freedom. Therefore, the confidence interval of \( \chi^2 \) distribution and the formula of LR statistics can be used to calculate the interval of the failure rate under any confidence level.

Data Availability

The data used to support the findings of the study are included within the supplementary information files.

Conflicts of Interest

The authors declare no conflicts of interest.

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Supplementary Materials

The authors have accessed the data from Sina Finance. The website is as follows: https://finance.sina.com.cn/realstock/company/sh000001/nc.shtml. (Supplementary Materials)

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