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Research Article

Synchronization of a Supply Chain Model with Four Chaotic Attractors

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In this article, we construct a three-stage supply chain model using a system of differential equations to reveal the interplay among producers, distributors, and end customers. On the one hand, information about the products and production is exchanged between each stage in the supply chain system. Such information affects the behaviors of each stage. On the other hand, the transport of products between the stages of the supply chain affects the behavior of the system. Our findings are summarized as follows: First, we find that the supply chain model is a system with four chaotic attractors. Second, we explore the synchronization of such a chaotic system. Third, according to the characteristics of the chaotic system, we design a variety of simple control laws to realize the synchronization of two chaotic systems with the same structure. These control laws for the chaotic system are proved to realize local asymptotic synchronization or global exponential synchronization. Numerical simulations are conducted to confirm that the designed controls work well.

1. Introduction

Since the discovery of the first chaotic system, the Lorenz chaotic system [1], chaos theory has received extensive attention. Chaotic systems display complicated dynamical phenomena, sensitive dependence on initial conditions, and nonperiodic and pseudo-random behaviors. Scholars have discovered various classical chaotic systems, such as the Rössler chaotic system [2], Chua's circuit [3], Chen's chaotic system [4], and Lu's chaotic system. The Lorenz system, Chen's system, and Lu's system belong to the family of Lorenz chaotic systems. Chaotic systems are widely used in science and industry. For example, the Lorenz chaotic attractor is generated by a weather forecast model [1], and the Rössler chaotic system can be applied to modeling chemical reactions [2]. Chua's circuit confirmed the existence of chaos in electrical circuits [3]. Investigations have confirmed that there exists chaos in a coagulation model under the constraint of fixed density [5]. Chaos has been discovered in physics, chemistry, computer science, medicine, and engineering [6–9].

Different chaotic systems display different chaotic attractors with unique topologies. The Lorenz chaotic system exhibits an attractor with two wings, the Rössler system has a single-scroll chaotic attractor, and Chua's circuit exhibits a double-scroll chaotic attractor. More complex chaotic systems with multiwing attractors and multiscroll attractors are designed in the literature [9–15]. The highly unpredictable behaviors of chaotic systems pose a challenge to their control and synchronization. In view of this, chaotic systems have been considered uncontrollable for a long time. Since the pioneering works of Pecora and Carroll [16] and Ott et al. [17], the control and synchronization of chaos have attracted the attention of researchers from all over the world. A variety of control and synchronization schemes are reported in the literature [13, 18, 19].

In recent years, researchers have studied various supply chain systems using mathematical modeling. Hou et al. [20]

established a model to study production, inventory, and distribution processes in a supply chain system. Amorim et al. [21] applied this supply chain model to perishable products. Yuan and Hwong [22] discussed the influence of customer behavior and purchase decisions on the stability of a supply chain. Kumar and Tiwari [23] obtained the optimal factory location by studying the risk of safe inventory and operating inventory in a supply chain. Studies show that a variety of supply chains display unpredictable and chaotic behaviors [22, 24]. In this work, we study a supply chain system containing three stages, between which information is exchanged [25]. The behavior of the supply chain is influenced by the transportation of products and information exchange between these stages [26]. Based on systems of ordinary differential equations, we construct a model to study the behavior of the supply chain. We find that the interactions between producers, retailers (distributors), and consumers can lead to chaos. We found that this supply chain system has the same structure as the chaotic system proposed by Nwachioma–Pérez Cruz [27]. We also consider the chaos synchronization of the model. We design several control strategies to realize the chaos synchronization of the system. Mathematical analysis is carried out to prove the effectiveness of these designed control laws. Furthermore, we conduct numerical simulations to illustrate their effectiveness.

The rest of the manuscript is organized as follows: Section 2 considers the modeling of a supply chain. In Section 3, we introduce fundamental theorems and definitions related to chaos synchronization. We present the designed control laws in this section. Mathematical analyses are provided to prove the control schemes. We use numerical simulations to illustrate the application of the designed control laws. The main conclusions are presented in Section 4.

2. Model Formulation

In this paper, we establish a supply chain model using a system of differential equations to study a supply chain system with three layers. This supply chain includes producers, distributors, and consumers, and the relationships between them are shown in Figure 1. In this system, the producers and distributors obtain information about the demand for the product from consumers. Distributors place orders based on consumer demand and current inventory. The order is passed on to the producers. Then, the producers make decisions about production based on the orders placed by distributors and information from consumers. The producers send the products to distributors, and then, the distributors sell the products to the consumers.

Here, we use x to denote the number of products produced by the manufacturers, y the quantity demanded by retailers, and z the quantity demanded by consumers. We assume that the rate of change in the quantity of products produced by the manufacturers is positively correlated with the retailer's demand and the demand of the consumers. The defective rate of the product is a1. We use a differential equation to model the interactions between the quantity of products produced by the manufacturers, the retailer's demand, and the demand of the consumers. Then, we have

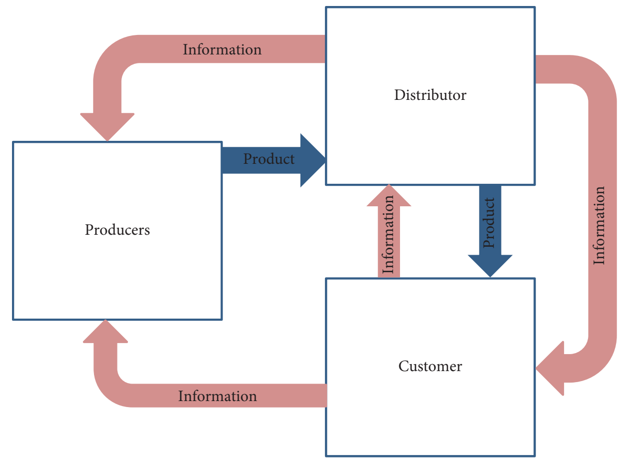


Figure 1: Schematic diagram of the supply chain model.

$$\dot{x} = -a_1 x + a_2 xz + a_3 yz. \tag{1}$$

The rate of change of the demand of distributors has a negative correlation with the quantity of products produced by the manufacturers. That is to say, when producers produce enough products, distributors will not place large orders because they know that there is a sufficient quantity of products. When manufacturers reduce production, distributors start to place more orders due to the worry of insufficient products to sell to consumers. The rate of change of the demand of distributors has a positive correlation with the current demand for distributors. We then obtain the following differential equation:

$$\dot{y} = a_4 y - a_5 xz. \tag{2}$$

In this model, the rate of change of the demand of consumers increases with the quantity of products produced by manufacturers and the quantity demanded by retailers. The rate of change of the demand of consumers will decrease with the increase of the demand of consumers. We then get

$$\dot{z} = -a_6 z + a_7 xy + a_8 z^2. \tag{3}$$

Summarizing the above discussion, we construct the following model:

$$\begin{aligned} \dot{x} &= -a_1 x + a_2 xz + a_3 yz, \\ \dot{y} &= a_4 y - a_5 xz, \\ \dot{z} &= -a_6 z + a_7 xy + a_8 z^2. \end{aligned} \tag{4}$$

where $a_1 = 1$, $a_2 = 1$, $a_3 = 2.3$, $a_4 = 2$, $a_5 = 1$, $a_6 = 6$, $a_7 = 1$, and $a_8 = -0.25$. We notice that the model has the same structure as the chaotic system proposed by Nwachioma and Pérez-Cruz [27]. Nwachioma and Pérez-Cruz showed that system (1) can be synchronized. Here, we use numerical simulation to verify the chaotic behavior (see Figure 2). As shown in Figure 2, the dynamic behavior of the system shows a chaotic trajectory, which is sensitive to initial conditions.

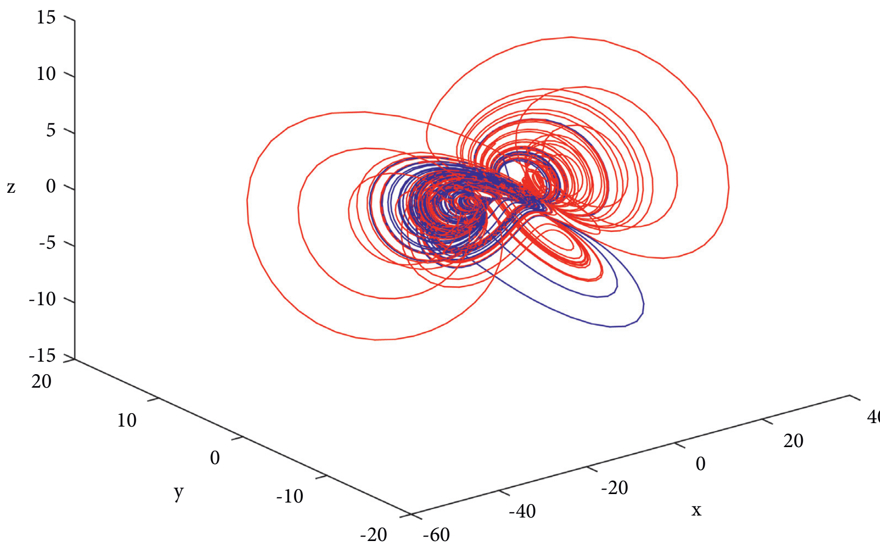


FIGURE 2: Simulated phase portrait of system (4) with $a_1 = 1$, $a_2 = 1$, $a_3 = 2.3$, $a_4 = 2$, $a_5 = 1$, $a_6 = 6$, $a_7 = 1$, and $a_8 = -0.25$ in the x-y-z space with initial conditions (1.2, 1.0, or 3.0) (blue) and (0.1, 0.2, 0.01) (red).

To demonstrate its sensitivity to initial conditions, we simulate the system with different initial conditions. For initial conditions (1.2, 1.0, and 3.0), system (4) displays the blue-colored trajectory. If we choose the initial conditions (0.1, 0.2, or 0.01), the system displays the red-colored trajectory. As can be seen from Figure 1, the two trajectories are completely different. System (4) displays complicated dynamical behavior within a wide range of parameter values. Here, we present numerical simulation results to confirm the complex behavior of the model. Figure 3 shows that system (4) has chaotic behavior for $a_1 \in (0, 5.5)$ and $a_1 \in (7.8, 8.5)$. Figure 4 shows that the system has chaotic behaviours for $a_2 \in (0, 0.42)$ and $a_1 \in (0.6, 1.75)$.

3. Synchronization of the Chaotic System with Four Attractors

The main objective of this section is to establish chaos control laws to achieve the synchronization of system (4). In view of the sensitivity of chaotic systems to initial conditions, two chaotic systems with the same structure will exhibit completely different behaviors due to different initial values. With appropriate controls, such two systems can be synchronized. Here, we name the two chaotic systems as the driving system and the receiving system, respectively. The driving system takes the form of

$$\begin{aligned} \dot{x}_d &= -a_1 x_d + a_2 x_d z_d + a_3 y_d z_d, \\ \dot{y}_d &= a_4 y_d - a_5 x_d z_d, \\ \dot{z}_d &= -a_6 z_d + a_7 x_d y_d + a_8 z_d^2, \end{aligned} \tag{5}$$

where subscript d stands for the driving system. The corresponding receiving system is then given by

$$\begin{aligned} \dot{x}_r &= -a_1 x_r + a_2 x_r z_r + a_3 y_r z_r + u_1, \\ \dot{y}_r &= a_4 y_r - a_5 x_r z_r + u_2, \\ \dot{z}_r &= -a_6 z_r + a_7 x_r y_r + a_8 z_r^2 + u_3, \end{aligned} \tag{6}$$

where subscript r represents the receiving system. Here, u_1, u_2, and u_3 are controls to be constructed. We thus obtain the error system

$$\begin{aligned} \dot{e}_x &= -a_1 e_x + a_2 x_d z_d - a_2 x_r z_r + a_3 y_d z_d - a_3 y_r z_r - u_1, \\ \dot{e}_y &= a_4 e_y - a_5 x_d z_d + a_5 x_r z_r - u_2, \\ \dot{e}_z &= -a_6 e_z + a_7 x_d y_d - a_7 x_r y_r + a_8 z_d^2 - a_8 z_r^2 - u_3, \end{aligned} \tag{7}$$

where $e_x = x_d - x_r$, $e_y = y_d - y_r$, and $e_z = z_d - z_r$. In order to make the behaviors of driving system (5) and receiving system (6) exactly the same (synchronized), we designed appropriate controls (u_1, u_2, u_3, u_4). Obviously, if the zero solution of system (7) is stabilized, then the driving system and the receiving system are synchronized.

Definition 1. $\forall\, x_d(0), y_d(0), z_d(0) \in R^3$ and $x_r(0), y_r(0), z_r(0) \in R^3$, if the zero solution of error system (7) is locally asymptotically stable, driving system (5) and receiving system (6) are said to be locally asymptotically synchronized. Now, we construct a set of controls to realize the local asymptotic synchronization between systems (5) and (6).

Next, we will design a set of control laws and present analytic results.

Theorem 1. *The following control laws stabilize the zero solution of error system (7) locally asymptotically, i.e., the set of control laws realizes the local asymptotic synchronization between systems (5) and (6):*

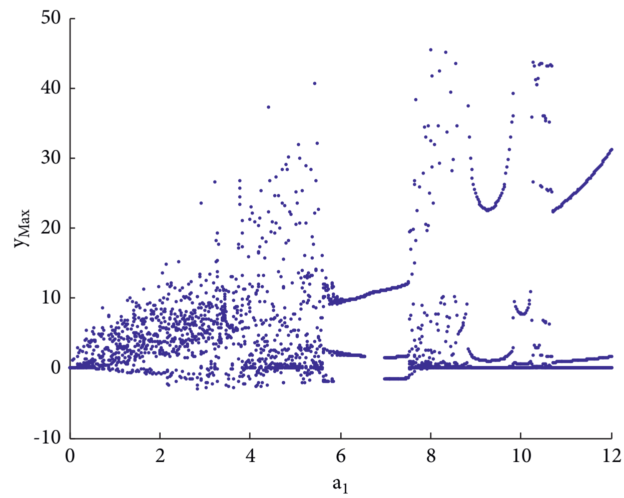


Figure 3: Bifurcation diagram of the model with the variation of a_1.

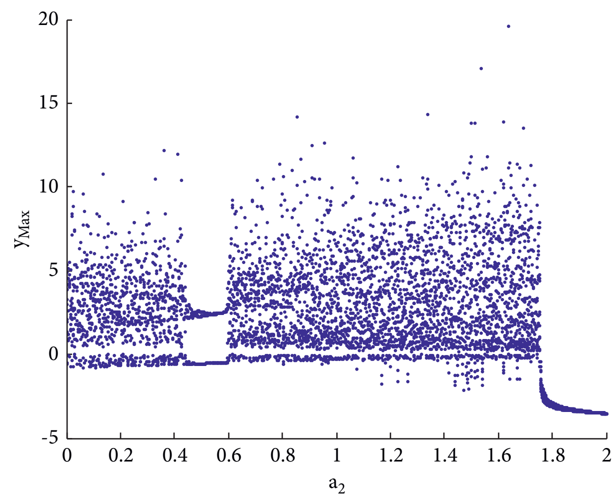


Figure 4: Bifurcation diagram of the model with the variation of a_2.

$$\begin{aligned} u_1 &= a_2 x_d z_d - a_2 x_r z_r + a_3 y_d z_d - a_3 y_r z_r, \\ u_2 &= -a_5 x_d z_d + a_5 x_r z_r + k_{12} e_y, \\ u_3 &= a_7 x_d y_d - a_7 x_r y_r + a_8 e_z (z_d + z_r), \end{aligned} \tag{8}$$

where $k_{12} > a_4$.

Proof. By linearizing error system (7) with the control law proposed in Theorem 1 at equilibrium (0, 0, 0), we then have

$$J_1 = \begin{bmatrix} -a_1 & 0 & 0 \\ 0 & a_4 - k_{12} & 0 \\ 0 & 0 & -a_6 \end{bmatrix}. \tag{9}$$

It is easy to see that the eigenvalues of J_1 are $\lambda_1 = -a_1$, $\lambda_2 = a_4 - k_{12}$, and $\lambda_3 = -a_6$. Note that all of these eigenvalues are negative. It thus follows that the zero solution of system (7) is locally asymptotically stable.

Therefore, control law (8) synchronizes systems (5) and (6). Next, numerical simulation is carried out to illustrate the application of the control law in Theorem 1. Here, we use numerical simulation to explore the behavior of the error system. As shown in Figure 5, the zero solution of the error system is stable under the designed control. Thus, the behavior of the driving system and the receiving system has achieved complete synchronization. Next, we design a control law to achieve global exponential synchronization. □

Definition 2. $\forall\, x_d(0), y_d(0), z_d(0) \in R^3$ and $x_r(0), y_r(0), z_r(0) \in R^3$, if the zero solution of error system (7) is globally exponentially stable, then driving system (5) and receiving system (6) are said to be globally exponentially synchronized.

Theorem 2. *The control law*

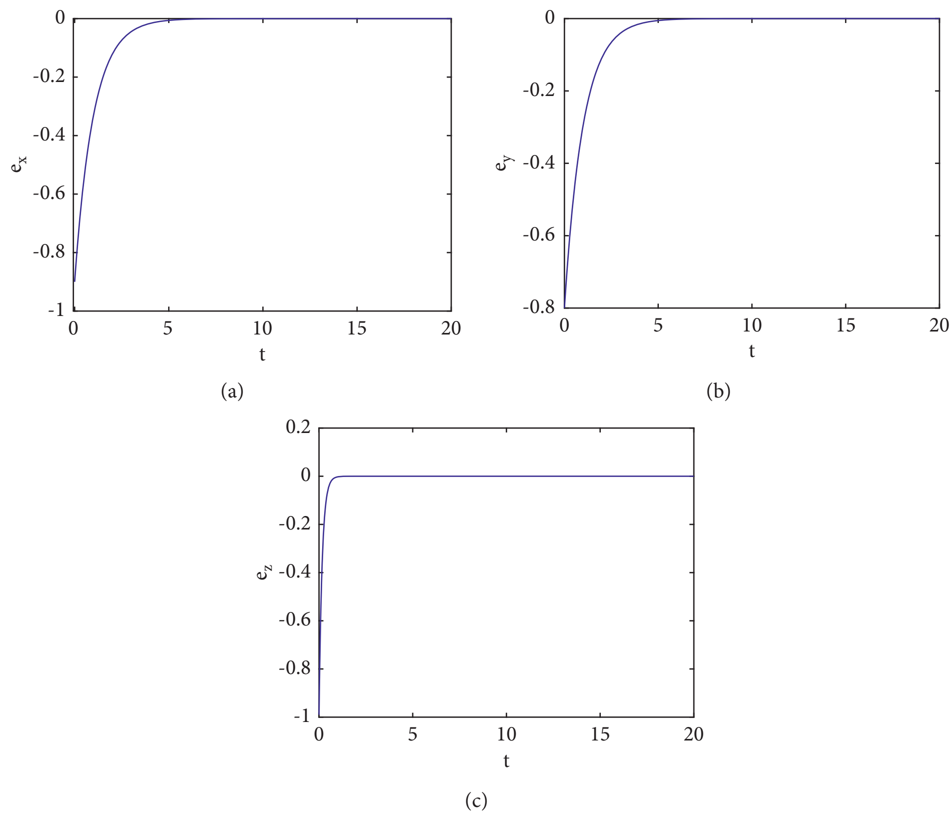


Figure 5: Time history of error system (7) with $a_1 = 1$, $a_2 = 1$, $a_3 = 2.3$, $a_4 = 2$, $a_5 = 1$, $a_6 = 6$, $a_7 = 1$, and $a_8 = -0.25$ under control law (8). (a) Time history of ex under control law (8). (b) Time history of ey under control law (8). (c) Time history of ez under control law (8).

$$
\begin{aligned}
u_1 &= a_2 x_d z_d - a_2 x_r z_r, \\
u_2 &= k_{22} e_y, \\
u_3 &= a_8 z_d^2 - a_8 z_r^2,
\end{aligned}
\tag{10}
$$

where $k_{22} > a_4$, applied to receiving system (6), stabilizes the zero solution of (7) globally exponentially. That is to say, systems (5) and (6) are globally exponentially synchronized under control law (10).

Proof. For error system (7) with control (10), we construct the positive definite, radially unbounded Lyapunov function.

$$
V = \frac{e_x^2}{a_3} + \frac{e_y^2}{a_5} + \frac{e_z^2}{a_7}. \tag{11}
$$

We then calculate the derivative of V along the trajectory of system (7) with control (10) to get

$$
\begin{aligned}
\frac{dV}{dt} &= \frac{2}{a_3} e_x \dot{e}_x + \frac{2}{a_5} e_y \dot{e}_y + \frac{2}{a_7} e_z \dot{e}_z \\
&= 2 \frac{\left(-\left(-e_y a_4 + k_{33} e_y\right)\left(y_d - y_r\right) a_7 - a_5 a_6 e_z \left(z_d - z_r\right)\right) a_3 - a_1 a_5 a_7 e_x \left(x_d - x_r\right)}{a_3 a_5 a_7}, \\
&= \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}^T \begin{pmatrix} -\dfrac{2a_1}{a_3} & 0 & 0 \\ 0 & \dfrac{2\left(a_4 - k_{22}\right)}{a_3 a_5} & 0 \\ 0 & 0 & -\dfrac{2a_6}{a_3 a_7} \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}.
\end{aligned}
\tag{12}
$$

Thus, the zero solution of system (7) is globally exponentially stabilized, i.e., driving system (5) and receiving system (6) are globally exponentially synchronized. Now, we conduct numerical simulations to show the applicability of the control law proposed in Theorem 2. In order to show the synchronization between the driving system and the receiving system, we plot the time histories of the two systems in the same figure. Here, the initial condition for the drive system is $x_d(0) = 0.3$, $y_d(0) = -6$, and $z_d(0) = -0.1$, and the initial condition of the receiving system is $x_r(0) = 2$, $y_r(0) = 3$, and $z_r(0) = 5$. We choose $k_{22} = 4$, which satisfies the conditions of Theorem 2. The time history of driving system (5) is plotted in blue, and the time history of corresponding receiving system (6) is plotted in red. As shown in Figure 6, as $t \longrightarrow \infty$, the driving system and the receiving system are synchronized. □

Theorem 3. *Apply the control law*

$$\begin{aligned} u_1 &= a_2 x_d z_d - a_2 x_r z_r, \\ u_2 &= k_{33} e_y, \\ u_3 &= a_8 z_d^2 - a_8 z_r^2 + a_7 (y_d + y_r)(x_d - x_r), \end{aligned} \tag{13}$$

where $k_{33} > a_4$, to receiving system (6). Then, the zero solution of (7) is globally exponentially stabilized. That is to say, we realize the global exponential synchronization of the drive system (5) and the receiving system (6) with control law (13).

Proof. Construct the positive definite, radially unbounded Lyapunov function

$$V = \frac{e_x^2}{a_3} + \frac{e_y^2}{a_5} + \frac{e_z^2}{a_7}, \tag{14}$$

for error system (7) with control (13). Calculating the derivative of V along the trajectory of system (7) with control (13) yields

$$\begin{aligned} \frac{dV}{dt} &= \frac{2}{a_3} e_x \dot{e}_x + \frac{2}{a_5} e_y \dot{e}_y + \frac{2}{a_7} e_z \dot{e}_z \\ &= 2\frac{\left(-\left(-e_y a_4 + k_{33} e_y\right)(y_d - y_r) a_7 - a_5 a_6 e_z (z_d - z_r)\right) a_3 - a_1 a_5 a_7 e_x (x_d - x_r)}{a_3 a_5 a_7}, \\ &= \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}^T \begin{pmatrix} -\dfrac{2a_1}{a_3} & 0 & 0 \\ 0 & \dfrac{2(a_4 - k_{33})}{a_3 a_5} & 0 \\ 0 & 0 & -\dfrac{2a_6}{a_3 a_7} \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}, \end{aligned} \tag{15}$$

Thus, the zero solution of system (7) is globally exponentially stabilized. Thus, we use the control law to link driving system (5) and receiving system (6) to achieve global exponential synchronization. We use numerical simulation to verify the correctness of the designed control law in Theorem 3. Here, we compare the time history of driving system (5) with that of the driven system by plotting them in the same figure. We choose $k_{33} = 5$, which guarantees that the condition of Theorem 3 is satisfied. The initial condition for the driving system is $x_d(0) = 3$, $y_d(0) = 2$, and $z_d(0) = 3$, and the initial condition of the receiving system is $x_r(0) = 0.3$, $y_r(0) = 0.5$, and $z_r(0) = 0.5$. The time history of the driving system is plotted in blue, and the time history of the receiving system is plotted in red. As shown in Figure 7, though the initial conditions of the drive system and those of the driven systems are different, under control law (13), the behavior of the driving system is synchronized with that of the driven system. □

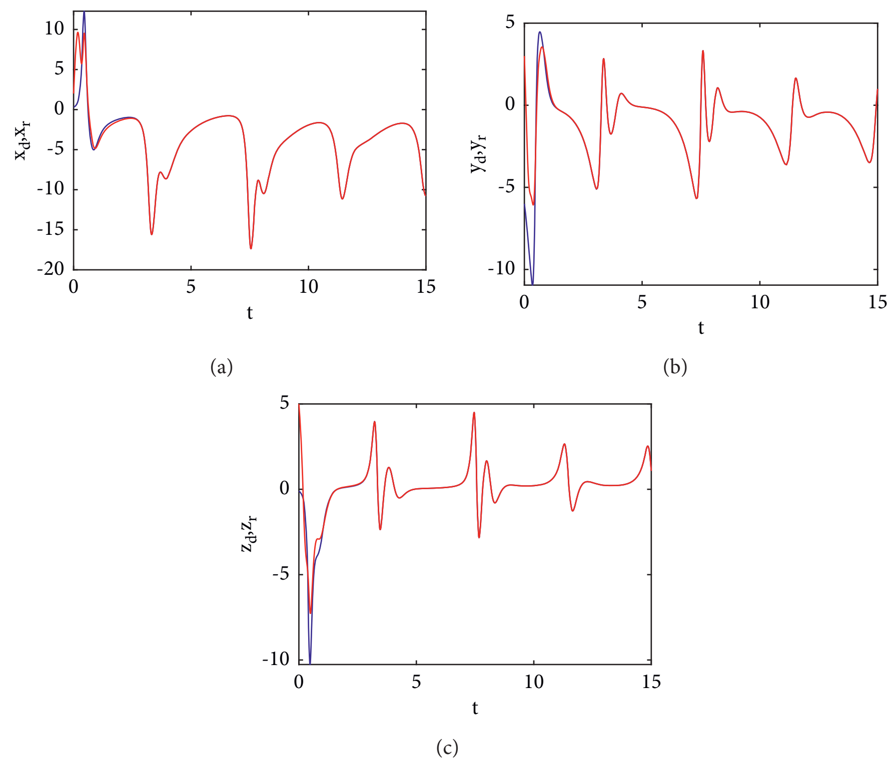


Figure 6: Time histories of driving system (5) (blue curve) and receiving system (6) (red curve) under control law (10) when $a_1 = 1$, $a_2 = 1$, $a_3 = 2.3$, $a_4 = 2$, $a_5 = 1$, $a_6 = 6$, $a_7 = 1$, and $a_8 = -0.25$ with initial conditions $x_d(0) = 0.3$, $y_d(0) = -6$, $z_d(0) = -0.1$, $x_r(0) = 2$, $y_r(0) = 3$, and $z_r(0) = 5$ under control law (10) with $k_{22} = 4$. (a) Time histories of x_d and x_r for systems (5) and (6) under control law (10). (b) Time histories of y_d and y_r for systems (5) and (6) under control law (10). (c) Time histories of z_d and z_r for systems (3.1) and (6) under control law (10).

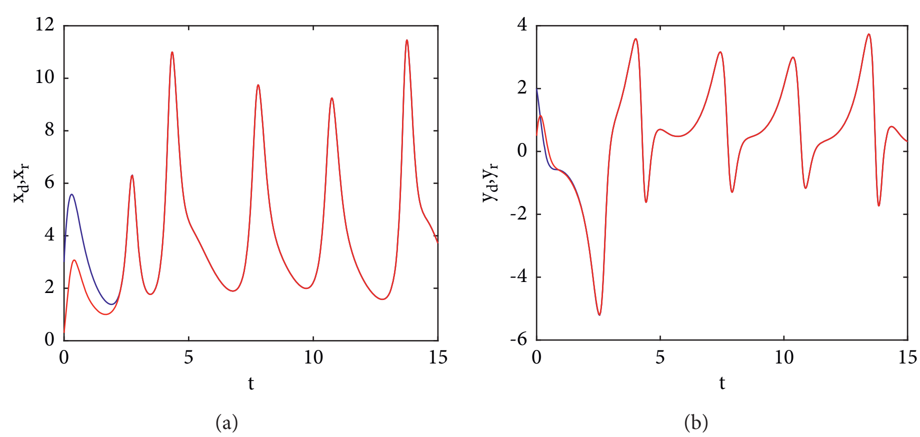
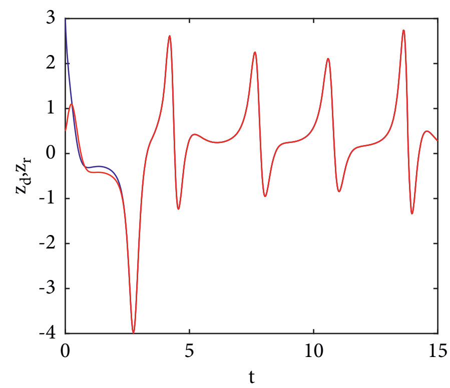


Figure 7: Continued.



(c)

Figure 7: Time histories of driving system (5) (blue curve) and receiving system (6) (red curve) under control law (10) when $a1 = 1$, $a2 = 1$, $a3 = 2.3$, $a4 = 2$, $a5 = 1$, $a6 = 6$, $a7 = 1$, and $a8 = -0.25$ with initial conditions $x_d(0) = 3$, $y_d(0) = 2$, $z_d(0) = 3$, $x_r(0) = 0.3$, $y_r(0) = 0.5$, and $z_r(0) = 0.5$ under control law (10) with $k_{33} = 5$. (a) Time histories of x_d and x_r for systems (5) and (6) under control law (10). (b) Time histories of y_d and y_r for systems (5) and (6) under control law (10). (c) Time histories of z_d and z_r for systems (5) and (6) under control law (10).

4. Conclusions

This paper established a mathematical model to study a supply chain with a three-stage structure. This model has the same structure as the chaotic system proposed by Nwachioma and Pérez-Cruz [27]. We studied the chaotic behavior of the system using numerical simulations. The chaotic behaviors of the model are the results of the interactions of various stages in the system. The unpredictability of chaotic systems poses difficulty in their application. The realization of chaos synchronization provides a theoretical basis for its further application. We design a series of controls to realize the synchronization of the chaotic system. We prove the correctness of these control laws by means of mathematical analysis. To illustrate that these control laws are applicable, we perform numerical simulations. These numerical simulation results show that our control laws are effective. Chaos in supply chains often causes loss and risk. The realization of chaos synchronization provides a theoretical basis for controlling chaos in supply chains. In particular, when there are several interconnected supply chains in an economic system, synchronizing these chains can improve their efficiency, reduce uncertainty, and minimize potential risks and disruptions.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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