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Research Article

The Influence of Vertical Velocity Distribution on the Calculation of Suspended Sediment Concentration

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Based on the advection-diffusion equation of suspended sediment, a general formula of vertical distribution of suspended sediment concentration was derived by considering the influence of vertical velocity. The new formula was tested against over 3000 sets of field data and obtained a reasonable agreement. Comparing with the Rouse equation of concentration, the accuracy of the new formula increases significantly, and the shortcoming of the underestimation of the Rouse profile in practical application is revised. Through the analysis of the new formula with different vertical time-averaged velocity coefficients m, it was found that vertical velocity does have an impact on the accurate estimation of sediment concentration, and the extent of which is related to the value of sediment concentration. Utilizing the regression analysis, it was found the vertical time-averaged velocity coefficient m increases with the height above the bed.

1. Introduction

One of the key points in the basic theory of sediment transport mechanics is the vertical distribution of suspended sediment concentration. In the 1930s, Rouse took the base of the equation derived from the diffusion theory and pioneered the study of the vertical distribution of sediment concentration [1]. Subsequently, many scholars such as Vanoni [2], Ismail [3], Hunt [4], Bagnold [5], Soulsby [6], van Rijn [7], among others, verified and improved their diffusion coefficient, formula structure, and Rouse number. This promoted the wide application of the Rouse equation [8–10]. The Rouse equation is basically correct, but it has the defects of zero sediment concentration on the water surface and infinite sediment concentration on the bed. This is closely related to the special velocity assumption used in the derivation process.

Suspended sediment transport is a geographical phenomenon, where sediment particles move along the flow direction and vertical direction under various forces [11]. Special logarithmic velocity distribution and specific vertical velocity zero assumption are included in velocity assumptions used in the derivation of the Rouse equation. In the former, the logarithmic velocity distribution refers to the horizontal velocity distribution (U) along with water depth. Velikanov [12], Zhang et al. [13], etc., optimized the velocity distribution based on the roughness of the bed surface and Wang Zhide's velocity distribution, respectively, and the effect was obvious. Chien and Wang figured out that this improvement was only for the derivation of the formula and could not reflect the law of sediment transport on the bed surface, making it difficult to conclude [14]. In recent years, Zhang [15], Zheng et al. [16], Li et al. [17], Boudreau et al. [18], and Baronas et al. [19] have improvised the Rouse equation from various aspects and made some remarkable achievements [9, 20–23]. Among them, Li et al. derived the vertical distribution of velocity based on the power function of velocity and friction velocity. The phenomenon of the Rouse formula that suspended sediment mass concentration is zero on the water surface has been eliminated by the vertical distribution of suspended sediment mass concentration. Compared with the early van Rijn formula, the form and the calculation are simpler, with obvious effects [16]. Based on the Larsen hypothesis, Sun et al. [24] deduced the turbulent diffusion coefficient from the logarithmic velocity distribution, combined with the interaction between particles of different sizes. They determined the concentration distribution formula of nonuniform-suspended sediment by eliminating some of the hypothesis of the Rouse equation.

The vertical velocity refers to the velocity component (V) in the upward z-axis direction. The parameter V reflecting the vertical motion state is caused by unstable and inhomogeneous flow, which is ignored in almost all the derivations of suspended sediment motion equations. In the Rouse equation, it has been assumed that the vertical time average velocity is zero, derived from two factual considerations. First, the flow depth is much smaller than the streamwise length [25]. Second, the Rouse equation is established for fine sediment particles and small sediment concentration, ignoring the influence of the interaction between sediment particles and fluid [26]. Based on the energy balance, Velikanov proposed the gravity theory of the vertical distribution of the suspended load and determined that the vertical velocity is linearly related to the sedimentation velocity of sediment particles and sediment concentration. Zhang et al. [27] referred to this proposal and further manifested the vertical time average velocity distribution considering the particle size from the nonslip condition of viscous fluid. This improved the calculation results of sediment concentration distribution. On the contrary, Cao et al. [28] considered that the vertical time average velocity term indicates the downward mass flux of the water-sediment mixture system from the perspective of two-phase flow, and its value must be negative. Zhang's result was considered to be questionable. In recent years, Yang et al. [29] defined the vertical velocity of the continuous equation, indicating that positive and negative vertical velocities are determined by the flow in the direction of water flow. The vertical velocity is positive in accelerating flow and negative in decelerating flow. Unfortunately, they deduced the suspended sediment equation, only assuming that the vertical velocity is related to the sinking velocity so that there is still a certain error in the calculation results of the bottom sediment. For obvious reasons, the influence of the vertical velocity on the calculation of suspended sediment concentration is still controversial, and the specific vertical velocity distribution coefficient is difficult to determine [30]. Currently, very few studies focus on the influence of the vertical velocity on the calculation of suspended sediment concentration.

Based on diffusion theory, the general form of the suspended sediment motion equation considering the vertical velocity is derived, and the influence of different vertical velocity assumptions on the calculation of the vertical distribution of sediment concentration is discussed. The regression analysis based on the measured data is used to calibrate the vertical velocity coefficient. This provides a reference for the accurate calculation of the vertical distribution of sediment concentration and basic theoretical research of sediment diffusion.

2. Vertical Velocity Distribution

2.1. Assuming Zero Value. The Rouse equation is most widely used in the vertical distribution of suspended sediment concentration. The vertical component velocity $\overline{V}$ of the water-sediment mixed system is usually assumed to be zero [31–34]:

$$\overline{V} = 0, \tag{1}$$

where $\overline{V}$ is the time-averaged flow velocity in the z-axis direction.

2.2. Gravity Theory Hypothesis. In the gravity theory of the suspended load vertical distribution, Velikanov explained that when sediment is fine and sediment concentration is not high, the upward time average velocity can be approximately zero [12]. The following hypothesis is proposed from the continuity equation of water-sediment two-phase flow:

$$\overline{V} = \omega\overline{S}, \tag{2}$$

where ω indicates the sediment settling velocity and $\overline{S}$ denotes the mean value of suspended sediment content.

2.3. Viscous Hypothesis. According to the nonslip condition of viscous fluid, when sediment particles sink, part of thewater around them will also sink with them. Water near sediment particles satisfies the continuous condition, so the volume of the rising fluid decreases, and the fluid rising speed caused by downward sediment settlement will boost [30]. Considering the impact of sinking water, Zhang et al. [27] proposed the following equation:

$$\overline{V} = \left(1 + \frac{6l}{d_{50}}\right)\omega\overline{S}, \tag{3}$$

where l indicates the thickness of the water layer attached to the sediment surface and d_{50} is the medium diameter of sediment. For sediment with particle sizes ranging from 0.1 mm to 0.001 mm, the value of l is approximately 0.04 times the particle size [35].

2.4. Turbulence Hypothesis. Cao et al. [28] derived the sediment diffusion equation based on the theory of solid-liquid two-phase flow and figured out that the difference between sediment diffusion equations was attributed to the different approximations of the sediment velocity. Assuming that the pulsation value of solid sediment particles is consistent with the overall pulsation value of the water-sediment mixture system and the time-averaged value varies by a relative sedimentation velocity ω, there is

$$\overline{V} = -\frac{\rho_s - \rho_f}{\rho_f}\omega\overline{S}, \tag{4}$$

where ρ_s indicates the sediment density and ρ_f is the fluid density.

Summing up four assumptions, it is evident that the vertical time-averaged velocity of sediment-laden flow is closely related to the sediment concentration, sediment density, and settlement velocity and has the following general forms:

$$\overline{V} = m\omega\overline{S}, \tag{5}$$

where m is the vertical time average velocity coefficient, and when m takes the corresponding coefficient, equations (1) to (4) are obtained, respectively.

3. Derivation of Vertical Distribution of Sediment Concentration

According to the law of mass conservation, the movement of two-dimensional sediment-laden flow on the facade can be described by the diffusion equation:

$$\frac{\partial S}{\partial t} = -\frac{\partial}{\partial x}(uS) - \frac{\partial}{\partial z}(vS) + \omega\frac{\partial S}{\partial z}, \tag{6}$$

where S is the instantaneous concentration of sediment, z denotes the vertical coordinate, and u and v are the instantaneous velocity components in the x-axis direction and z-axis direction, respectively.

Considering the pulsation of sediment-laden flow, we obtain

$$S = \overline{S} + S', u = \overline{u} + u', v = \overline{v} + v', \tag{7}$$

where $\overline{S}$ is the time-averaged component of S, S' is the fluctuation of S, $\overline{u}$ and $\overline{v}$ are the time-averaged velocity components in the x-axes and z-axes, u' and v' are the fluctuations of u and v, respectively.

Substituting (7) into the diffusion equation, we obtain

$$\begin{aligned}\frac{\partial}{\partial t}\left(\overline{S} + S'\right) = &-\frac{\partial}{\partial x}\left(\overline{u}\overline{S} + \overline{u}S' + u'\overline{S} + u'S'\right)\\ &-\frac{\partial}{\partial z}\left(\overline{v}\overline{S} + \overline{v}S' + v'\overline{S} + v'S'\right)\\ &+\omega\frac{\partial}{\partial z}\left(\overline{S} + S'\right).\end{aligned} \tag{8}$$

In Reynolds averaged equation (8), if the long-time average of the pulsation value is zero, then we get

$$\begin{aligned}\frac{\partial\overline{S}}{\partial t} = &-\overline{u}\frac{\partial\overline{S}}{\partial x} - \overline{S}\frac{\partial\overline{u}}{\partial x} - \frac{\partial\left(\overline{u'S'}\right)}{\partial x} - \overline{v}\frac{\partial\overline{S}}{\partial z} - \overline{S}\frac{\partial\overline{v}}{\partial z}\\ &-\frac{\partial\left(\overline{v'S'}\right)}{\partial z} + \omega\frac{\partial\overline{S}}{\partial z}.\end{aligned} \tag{9}$$

According to Prandtl's mixing assumption and the definition of turbulent diffusion coefficient, we get

$$\begin{aligned}\overline{u'S'} &= -\varepsilon_x\frac{\partial\overline{S}}{\partial x},\\ \overline{v'S'} &= -\varepsilon_z\frac{\partial\overline{S}}{\partial z},\end{aligned} \tag{10}$$

where ε_x and ε_z are the sediment diffusivity along the flow direction and perpendicular to the flow direction, respectively.

Substituting (9) and removing the time-average symbol, we obtain

$$\begin{aligned}\frac{\partial S}{\partial t} = &-u\frac{\partial S}{\partial x} - S\frac{\partial u}{\partial x} + \varepsilon_x\frac{\partial^2 S}{\partial x^2} + \frac{\partial\varepsilon_x}{\partial x}\frac{\partial S}{\partial x} - v\frac{\partial S}{\partial z}\\ &-S\frac{\partial v}{\partial z} + \varepsilon_z\frac{\partial^2 S}{\partial z^2} + \frac{\partial\varepsilon_z}{\partial z}\frac{\partial S}{\partial z} + \omega\frac{\partial S}{\partial z}.\end{aligned} \tag{11}$$

Equation (11) is a complete two-dimensional sediment diffusion equation. Considering the situation of fine sediment particles and small sediment content, it has been assumed in the existing studies that the time-averaged value of the flow velocity is zero vertically, ignoring the influence of solid sediment on the flow structure.

When sediment movement reaches equilibrium, the sediment diffusion problem is homogeneous and constant. When the partial terms of time t and distance x are zero, the equation can be simplified as follows:

$$\varepsilon_z\frac{\partial^2 S}{\partial z^2} + \frac{\partial\varepsilon_z}{\partial z}\frac{\partial S}{\partial z} - v\frac{\partial S}{\partial z} - S\frac{\partial v}{\partial z} + \omega\frac{\partial S}{\partial z} = 0. \tag{12}$$

The vertical integral of (12) is the differential equation about z, and the integral constant term is determined to be zero according to the boundary conditions:

$$\varepsilon_z\frac{dS}{dz} + S(\omega - v) = 0. \tag{13}$$

The differential equation defines the vertical distribution of sediment content when the diffusivity, vertical velocity, and settlement velocity in (13) are determined.

For the diffusion coefficient, the Prandtl logarithmic velocity distribution is used as follows:

$$\frac{u}{U_*} = \frac{1}{k}\ln\left(\frac{z}{z_0}\right), \tag{14}$$

where k indicates the Karman constant and U_* denotes the shear velocity.

According to Vanoni, assuming that the diffusion coefficient is related to the momentum exchange coefficient, there is

$$\varepsilon_z = \beta\varepsilon_m = \frac{\beta\tau_0(h - z/h)}{\rho(U_*/k)(1/z)} = kz\beta U_*\frac{h - z}{h}. \tag{15}$$

Among them, ε_m is the momentum exchange coefficient, β is the proportional constant, the shear force is linearly distributed in the vertical direction, $\tau = \tau_0(1 - (z/h))$, and τ_0 denotes the shear force on the bed surface.

The vertical time average velocity is a linear function of the sediment content and settlement velocity. The time average symbol of (5) is removed as follows:

$$v = m\omega S. \tag{16}$$

Combining (13) and (15), (16), the two-dimensional sediment diffusion differential equation can be written as

$$\beta kzU_*\frac{h-z}{h}\frac{dS}{dz} + S\omega(1-mS) = 0. \tag{17}$$

By solving (17), the vertical distribution of sediment concentration is

$$\frac{S}{S_a} = \frac{1+m(h-a/a)^{\omega/\beta kU_*}}{1+m(h-z/z)^{\omega/\beta kU_*}}\left(\frac{h-z}{z}\frac{a}{h-a}\right)^{\omega/\beta kU_*}, \tag{18}$$

where S_a is the near-bottom sediment concentration corresponding to the reference height $z = a$[36]. The detailed solution process of (17) is shown in Appendix A.

Considering the relative water depth, (18) can be transformed as

$$\frac{S}{S_a} = \frac{1+m(1-\alpha_a/\alpha_a)^{Z_*}}{1+m(1-\alpha/\alpha)^{Z_*}}\left(\frac{1-\alpha}{\alpha}\frac{\alpha_a}{1-\alpha_a}\right)^{Z_*}, \tag{19}$$

where Z_* is recorded as a Rouse index, $Z_* = \omega/\beta kU_*$; α_a indicates the relative reference height, $\alpha_a = a/h$; and α is the relative height of sediment, $\alpha = z/h$.

To reflect the influence of the vertical velocity on the vertical relative distribution of sediment concentration, (19) can also be expressed as

$$\begin{aligned}\frac{S}{S_a} &= \left(\frac{1-\alpha}{\alpha}\frac{\alpha_a}{1-\alpha_a}\right)^{Z_*}\\ &\quad + \frac{m\left[\alpha^{Z_*}(1-\alpha_a)^{Z_*} - \alpha_a^{Z_*}(1-\alpha)^{Z_*}\right]}{\alpha_a^{Z_*}\left[\alpha^{Z_*} + m(1-\alpha)^{Z_*}\right]}\left(\frac{1-\alpha}{\alpha}\frac{\alpha_a}{1-\alpha_a}\right)^{Z_*}.\end{aligned} \tag{20}$$

In (20), the first term on the right is the relative distribution of the classical Rouse sediment concentration along the water depth and the second item on the right is the change of sediment concentration along the vertical distribution caused by the vertical time average velocity.

Accordingly, the vertical distribution of sediment concentration can be expressed as

$$\frac{S}{S_a} = (1+\delta)\left(\frac{1-\alpha}{\alpha}\frac{\alpha_a}{1-\alpha_a}\right)^{Z_*}, \tag{21}$$

where δ represents the influence coefficient of the vertical velocity on the vertical relative distribution of sediment concentration and can be expressed as

$$\delta = \frac{m\left[\alpha^{Z_*}(1-\alpha_a)^{Z_*} - \alpha_a^{Z_*}(1-\alpha)^{Z_*}\right]}{\alpha_a^{Z_*}\left[\alpha^{Z_*} + m(1-\alpha)^{Z_*}\right]}. \tag{22}$$

Equation (21) is the general form of the suspended sediment vertical relative distribution considering the influence of the vertical velocity. For $m = 0$, the relative distribution described by (21) can be transformed into the classical Rouse distribution. To analyze the influence of the vertical velocity on the suspended sediment distribution separately, the special form of the vertical velocity distribution and the diffusion coefficient cannot be optimized. All symbols involved in the derivation of equations are described in Appendix B.

4. Verification of the Vertical Distribution of Suspended Sediment

4.1. Theoretical Distribution of Suspended Sediment. The theoretical distribution of (21) under different m values can be compared to analyze the influence of the vertical velocity on the vertical distribution of sediment concentration. As demonstrated in Figure 1, the two clusters of curves represent the classical Rouse distribution without considering the influence of the vertical velocity ($m = 0$) and the suspended sediment theoretical distribution considering the influence of the vertical velocity ($m = 0.1$). The Rouse equation is a milestone in research on suspended sediment theory because of its accuracy and progressiveness of times, which is the main reason for selecting the Rouse distribution as a reference. The theoretical distribution of suspended sediment under the condition of $m = 0$ in equation (21) is consistent with the Rouse distribution as verified by Vanoni based on the measured data in the laboratory [37]. However, unlike the indoor experiment, the field environment is complex, and the actual application of the Rouse equation underestimates sediment concentration [32, 36].

Considering the vertical time average velocity coefficient ($m \neq 0$), the theoretical distribution of suspended sediment described by (21) is significantly different from the Rouse theoretical distribution. Comparing the vertical relative distribution of suspended sediment, the vertical velocity does have a strong effect on the vertical distribution of sediment concentration. The greater the Rouse index, the greater the impact of the corresponding vertical velocity on the distribution of sediment concentration.

The higher the value of δ in the sediment concentration vertical distribution (21), the greater the influence of the vertical velocity on the sediment concentration vertical distribution. Furthermore, to separately analyze the influence of different vertical velocity assumptions on the calculation of sediment concentration, the theoretical distribution of δ can be drawn as shown in (22).

As highlighted in Figure 2, the value of δ increases with the value of m on the overall trend. In the bottom area, the vertical velocity has little effect on the vertical distribution of sediment concentration, the value of δ approaches a certain value, and the fixed value is associated with the height of the reference point. In the area closer to the surface, the higher the value of δ, the greater the influence of the vertical velocity on the sediment concentration distribution, which is related to stronger vertical turbulence of sediment-laden flow near the water surface.

As highlighted in Figure 3, when $Z_* = 1$, the relative distribution of the vertical sediment concentration under

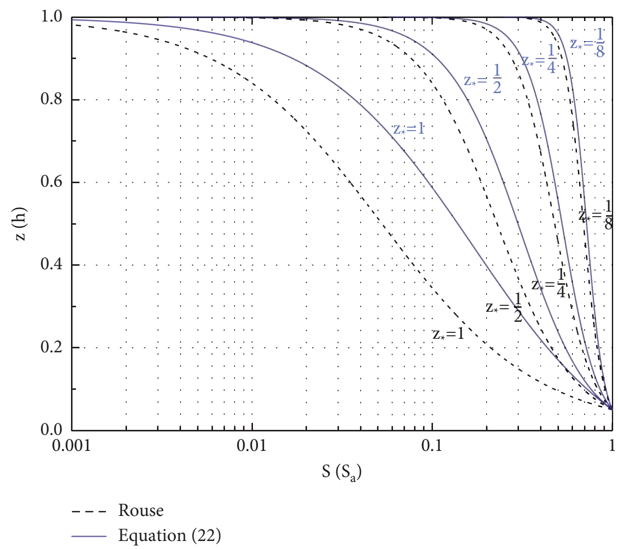

FIGURE 1: Comparison of the vertical relative distribution of sediment concentration.

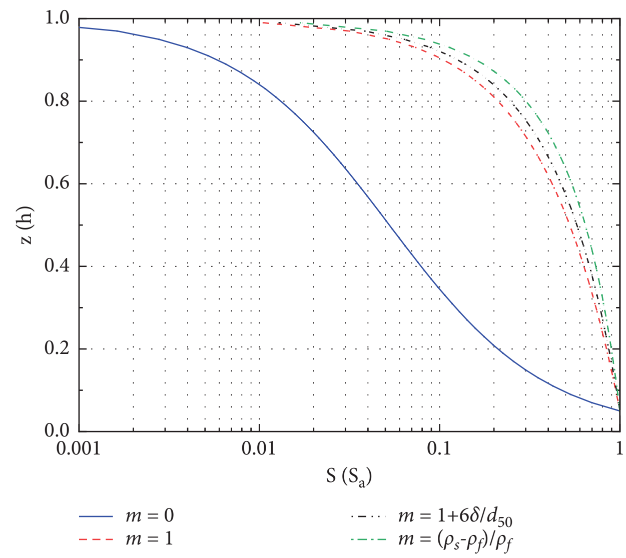

FIGURE 3: Vertical relative distribution of sediment with different m values.

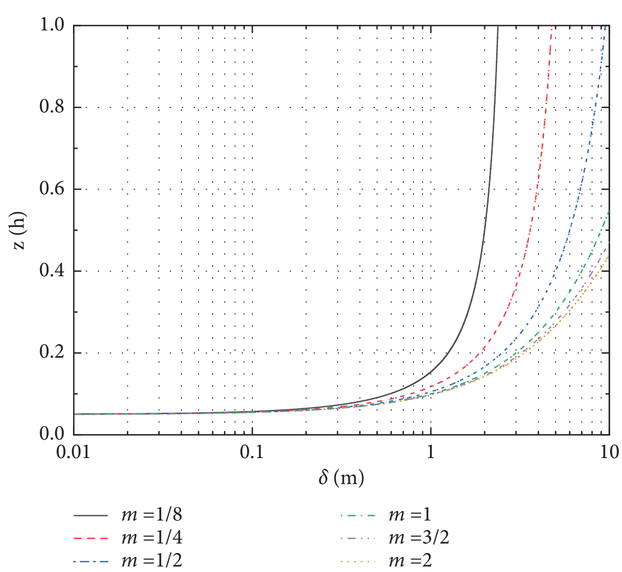

FIGURE 2: Theoretical distribution of the $\delta(m)$ function with different m values.

four assumptions is drawn using equations (1)–(4) in combination with (21). According to the vertical relative distribution of the sediment concentration calculated by Rouse, assuming $m = 0$, the estimated relative sediment concentration theoretical value is on the lower side, ignoring the influence of the vertical velocity on the sediment movement. In addition, the distribution of suspended sediment in (21) is slightly different for different values of the vertical time-averaged velocity coefficient.

4.2. Measured Distribution of Suspended Sediment. Field data are used to further verify and analyze equation (21) and discuss the influence of various vertical velocity distribution assumptions on the computation of the vertical distribution of sediment concentration. These parts of the validation data have been deduced from the long-term monitoring and research work of the experimental team in the coastal area of Jiangsu, China [17, 38–40]. The velocity range is $0.072 \sim 3.12$ m/s, the sediment concentration range is $0.002 \sim 2.27$ kg/m^3, the water depth is $5 \sim 30$ m, and the median particle size is $0.01 \sim 0.2$ mm. Based on assumptions (1)–(4), considering different vertical velocity distributions, the verification results of sediment concentration are highlighted in Figure 4.

Figure 4 illustrates that when different vertical time average velocity distributions are substituted in the sediment concentration estimation, calculation accuracy is different. This is consistent with theoretical distribution results. Compared with the Rouse equation ($m = 0$), using the assumptions of Velikanov, Zhang et al., and Cao et al., ignoring the influence of the vertical time average velocity on sediment concentration calculation, the sediment concentration calculation results considering the influence of the vertical time-averaged velocity are relatively more concentrated ($a > d > c > b$). This indicates that the vertical velocity impacts the accurate estimation of sediment concentration.

However, compared with Figures 4(a)–4(d), when $m \neq 0$, the result of sediment concentration is more accurate, but the overall trend of the calculation result is skewed. The greater the value of m, the more evident the centerline deflection ($\theta_1 < \theta_2 < \theta_3$). This deflection state is related to the position of sediment particles in water and sediment concentration which infers that the deflection angle of low concentration sediment on the surface is small and that of high concentration sediment on the bottom is large. When sediment concentration is high, the vertical velocity significantly impacts the estimation results of sediment concentration, and the m value needs to be selected carefully.

5. Discussion

5.1. Vertical Time-Averaged Velocity Coefficient **m**. From the theoretical analysis of the vertical distribution of sediment concentration and the verification of measured data, it can

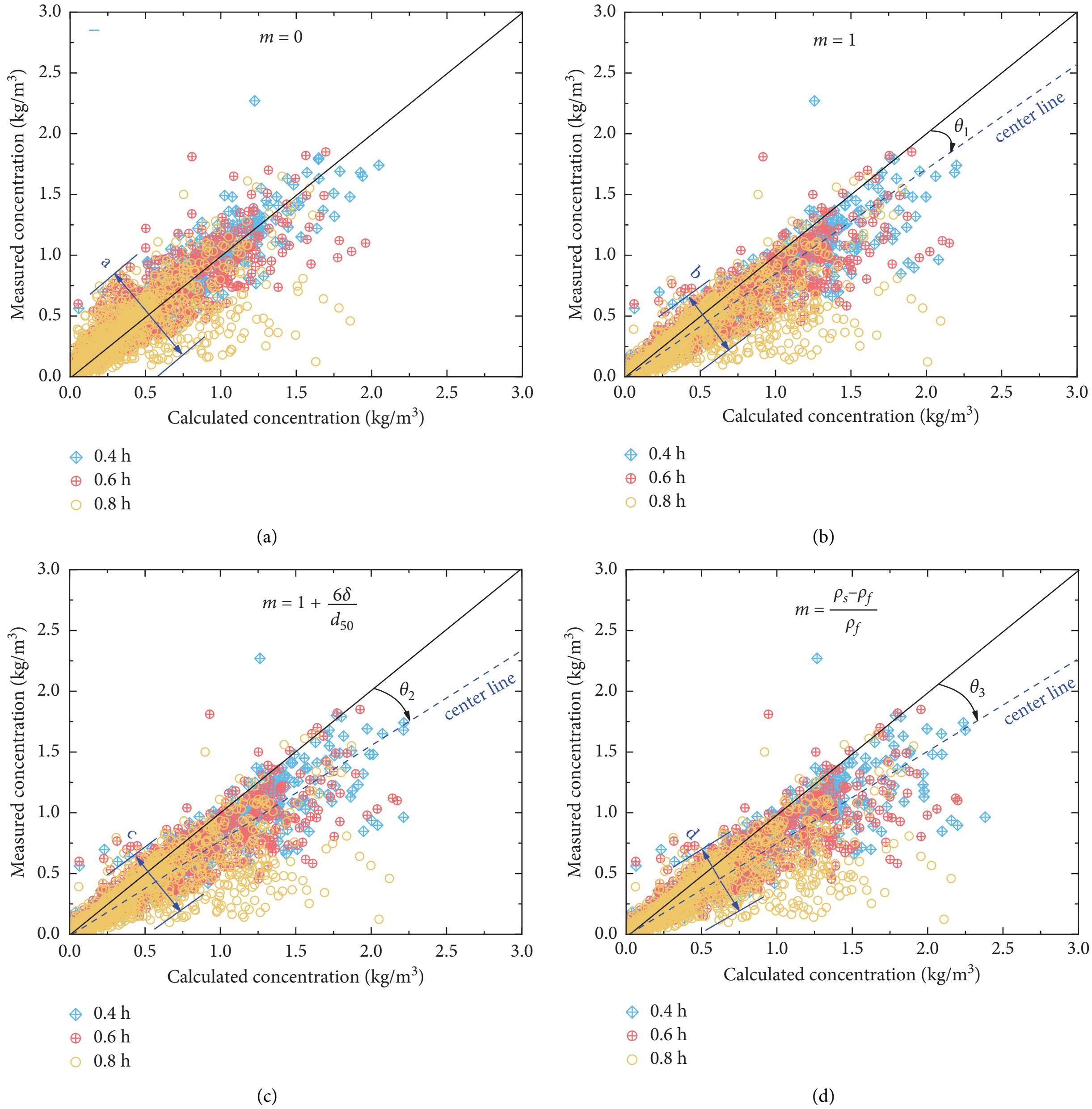

FIGURE 4: Verification of sediment concentration with different vertical time-averaged velocities. Among them, (a) the verification of sediment concentration with Rouse's assuming zero value, (b) the verification of sediment concentration with Velikanov's gravity theory hypothesis, (c) the verification of sediment concentration with Zhang's viscous hypothesis, and (d) the verification of sediment concentration with Cao's turbulence hypothesis. In all figures, the black solid line is a 45° diagonal line, and the blue-dotted line is the linear fitting center line of scattered points. The smaller the angle between the two lines, the higher the calculation accuracy; the width of the strip area with scattered points indicates the concentration of calculation accuracy. The smaller the width, the better the concentration.

be inferred that the vertical time average velocity does have an impact on the accurate calculation of sediment concentration. This impact is reflected in the specific value of the vertical time-averaged velocity coefficient m. It is related to the position of sediment particles in water and sediment concentration.

The influence of reasonable calibration of the vertical time-averaged velocity coefficient on sediment concentration calculation is discussed. Furthermore, 3,000 groups of measured data are randomly categorized into two parts, including 2,100 groups of training data to calibrate the vertical time average velocity coefficient and 900 groups of test data to verify its correctness. The results are enlisted in Table 1.

The coefficients in (21) are calibrated locally based on different water depths. The values of vertical time-averaged velocity coefficients calibrated at 0.4 h, 0.6 h, and 0.8 h are 0.18, 0.31, and 0.44, respectively. The minimum correlation coefficient between the calculated and measured values is 0.92. Thus, it is feasible to calibrate the value by regression analysis.

Comparing the vertical time-averaged velocity coefficient of different water depths, the maximum value is achieved at the surface (0.8 h), and the m value is closely related to the depth, as shown in equation (24). Figure 5 shows the verification results of sediment concentration using the regression calibration coefficient. Compared with Figure 4, the concentration of sediment concentration estimation accuracy is better.

$$m = 0.672\alpha - 0.093. \tag{23}$$

To facilitate the practical engineering application and ignore the influence of water depth conditions, the global coefficient in (21) is calibrated. The vertical time-averaged velocity coefficient is $m = 0.34$, and the calculation error of sediment concentration is $R^2 = 0.825$. The relative error analysis diagram is drawn by comparing the coefficient calibration effects of local regression considering the water depth and global regression without considering the water depth. As represented in the box diagram in Figure 6(b), the vertical time-averaged velocity coefficient of global calibration relative to local calibration is used for sediment concentration calculation. Although the average error increases ($0.224 > 0.212$), it is always better than the result of the Rouse equation without considering the influence of the vertical velocity (0.236). The calculation accuracy of sediment concentration equation (21) is higher than the error distribution frequency of the Rouse equation (Figure 6(a)). From the perspective of accuracy, the results are quite useful.

5.2. Suspended Sediment Profiles with Different Formulas. Based on the field sampling data of the Jiangsu coastal area, different methods are used to calculate the vertical distribution of suspended sediment, and the difference between equation (21) and the existing vertical distribution formula of suspended sediment is analyzed. Among them, 8 groups of data samples are measured by the onsite six-point method. There was no extreme weather impact on the sampling time, and the sampling environment was stable. The samples consist mainly of silts. The nominal diameters of samples range between 5.5 ϕ and 6.5 ϕ, and a majority of sorting coefficients are around 1. Sand samples are very homogeneous. The specific parameters are shown in Table 2.

Using the Rouse equation and considering the vertical flow velocity (21), the calculated vertical distribution of suspended sediment is calculated as shown in Figure 7, where the vertical time-averaged velocity coefficient in (21) is taken as 0.34. The distribution of suspended sediment described in (21) is more consistent with the field-measured distribution compared with the calculated results by Rouse, indicating that calculation accuracy can be improved by considering the influence of the vertical flow velocity on sediment movement.

In the overall trend, (21) considers the effect of the vertical flow velocity, which effectively improves the low calculated value of sediment concentration in the Rouse equation. In the results of equation (22), the accuracy of calculation varies for different water depths at some points. The calculated values of sediment concentration in the surface layer are low (Figure 7(e)), and those near the bottom are high (Figure 7(d)). However, deviation errors are minimal, and the results are sufficient for adoption. The deviation of the measured values of the theoretical distribution at some points may be related to adopting a global coefficient rate determination method for vertical time-averaged velocity coefficients, in addition to errors in the actual measurement process in the field. Considering the accuracy and convenience of equation (21), it is still recommended to use the global rate of the vertical time-averaged velocity coefficient for engineering applications.

In order to analyze the specific accuracy of (21), the results of the calculation of suspended sediment profiles were analyzed using four indicators to evaluate the effectiveness of the fit: the sum of squares due to error (SSE), root mean squared error (RMSE), coefficient of determination (R^2), and Pearson correlation coefficient (ρ). The four evaluation indexes are calculated by the following formula:

$$\begin{aligned} \mathrm{SSE} &= \sum_{i=1}^{n} w_i \left(S_i - \widehat{S}_i\right)^2, \\ \mathrm{RMSE} &= \sqrt{\frac{1}{n}\sum_{i=1}^{n} w_i \left(S_i - \widehat{S}_i)^2, R^2 = 1 - \frac{\mathrm{SSE}}{\sum_{i=1}^{n} w_i \left(S_i - \overline{S}_i)^2, \rho_{\left(S_i, \widehat{S}_i\right)} = \dfrac{Cov\left(S_i, \widehat{S}_i\right)}{\sqrt{Var\left(S_i\right) Var\left(\widehat{S}_i\right)}}, \right.} \right.} \end{aligned} \tag{24}$$

Table 1: Regression results of the vertical time-averaged velocity coefficient m.

Depth (h)	Sediment concentration (kg m^{-3})	Training array	Validate array	m	Correlation coefficient[1]
0.4	0.01 ~ 1.79	693	351	0.18	0.97
0.6	0.01 ~ 1.85	695	325	0.31	0.97
0.8	0.002 ~ 1.65	712	224	0.44	0.92

1The correlation coefficient is the Pearson correlation coefficient, and the calculation range is 3,000 groups of measured data (including training and validation groups).

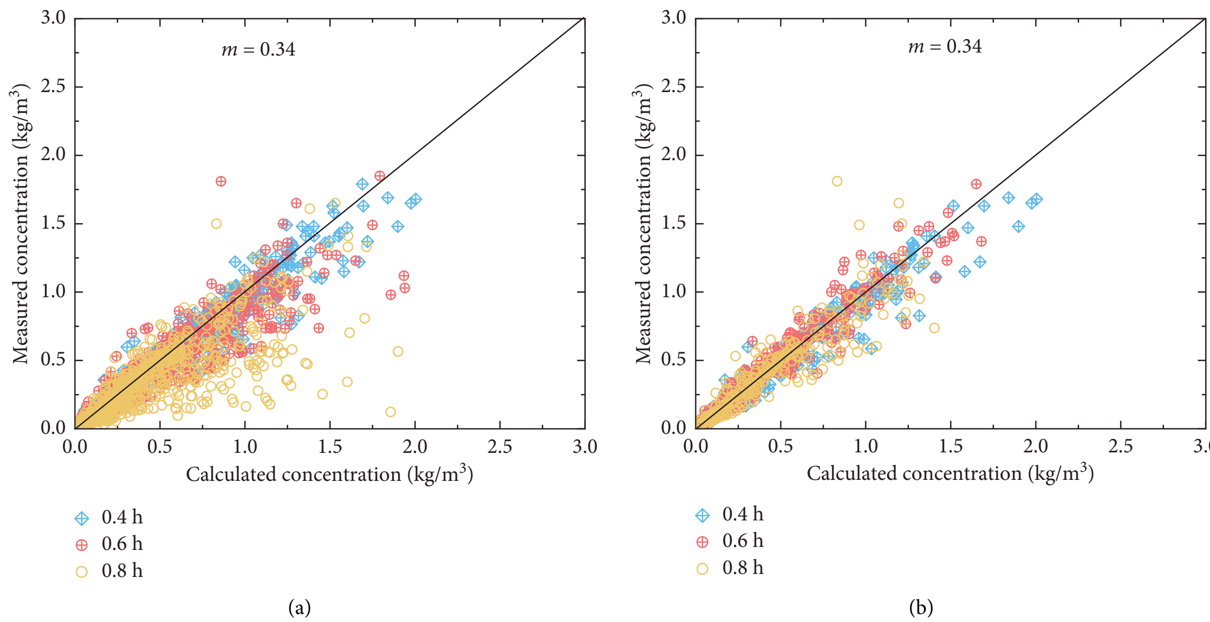

Figure 5: Validation of sediment concentration using the regression coefficient **m**. Among them, (a) the sediment concentration verification with 2100 groups of training data and (b) the calculation accuracy of sediment content with 900 groups of test data.

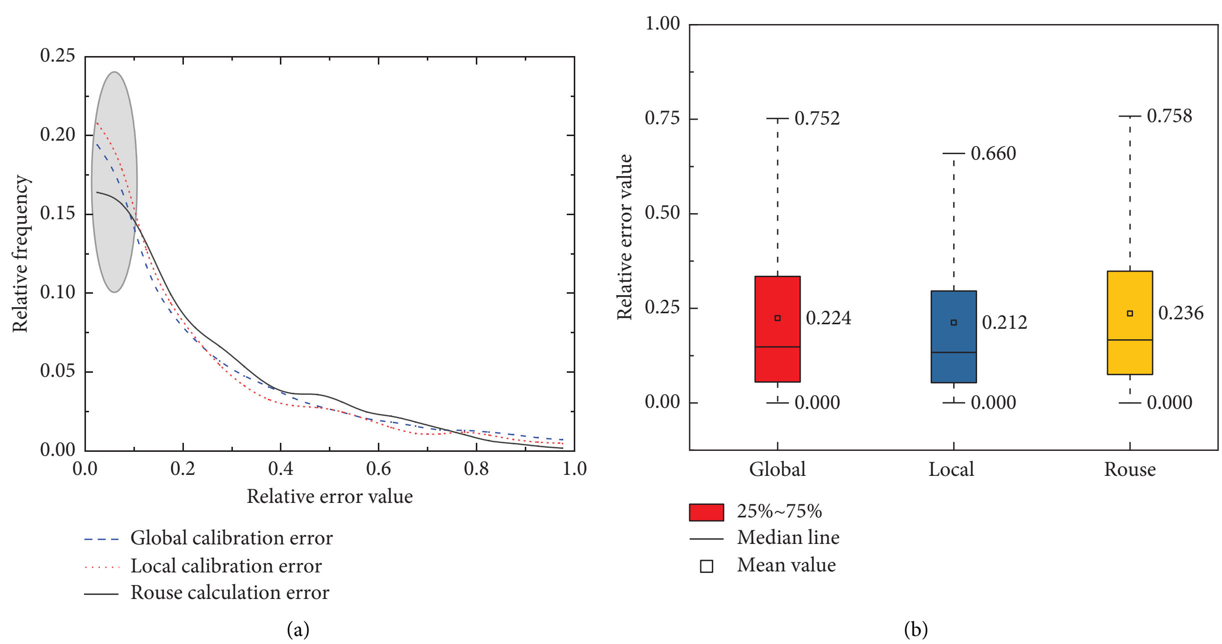

Figure 6: Calculation error of sediment concentration with different m values. Among them, (a) the probability distribution of the relative error of different m values, the black solid line represents the calculation result of the Rouse equation with $m = 0$, the red-dashed line represents the result calculated by equation (21) with m values (Table 1) determined at different water depths, and the blue-dotted line represents the result of equation (21) calculated with the global coefficient rate $m = 0.34$. (b) Boxplots of relative error distributions for different m values.

Table 2: List of measured field data.

Sample numbers	Date (m/d/y)	Depth (m)	Average velocity (m/s)	D_{50} (mm)	Sediment concentration (kg/m^3)					
					Surface	0.8 h	0.6 h	0.4 h	0.2 h	Bottom
a	8/24/2006	12.80	0.27	0.010	0.121	0.160	0.209	0.215	0.233	0.283
b	8/31/2006	7.00	0.06	0.010	0.150	0.187	0.365	0.452	0.703	1.060
c	8/31/2006	6.00	0.15	0.010	0.206	0.269	0.419	0.553	0.642	0.864
d	8/31/2006	17.20	0.10	0.010	0.283	0.401	0.616	0.937	1.010	2.190
e	12/19/2006	8.70	0.24	0.060	0.199	0.224	0.237	0.274	0.306	0.367
f	1/3/2007	8.70	0.13	0.060	0.055	0.081	0.095	0.114	0.124	0.196
g	1/11/2007	13.70	0.59	0.085	0.502	0.526	0.528	0.574	0.605	0.651
h	1/11/2007	11.20	0.10	0.020	0.100	0.106	0.151	0.264	0.331	0.464

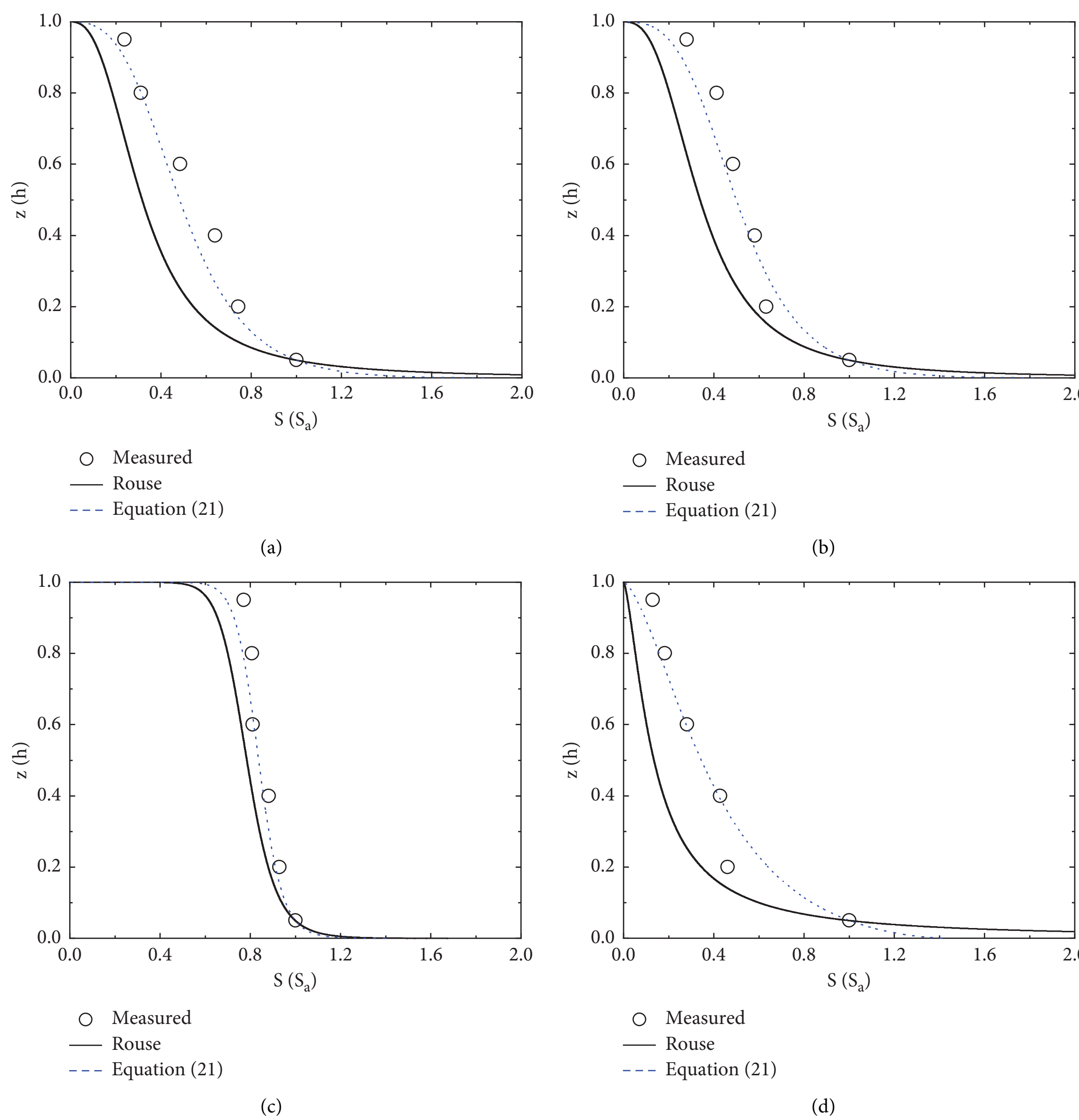

Figure 7: Continued.

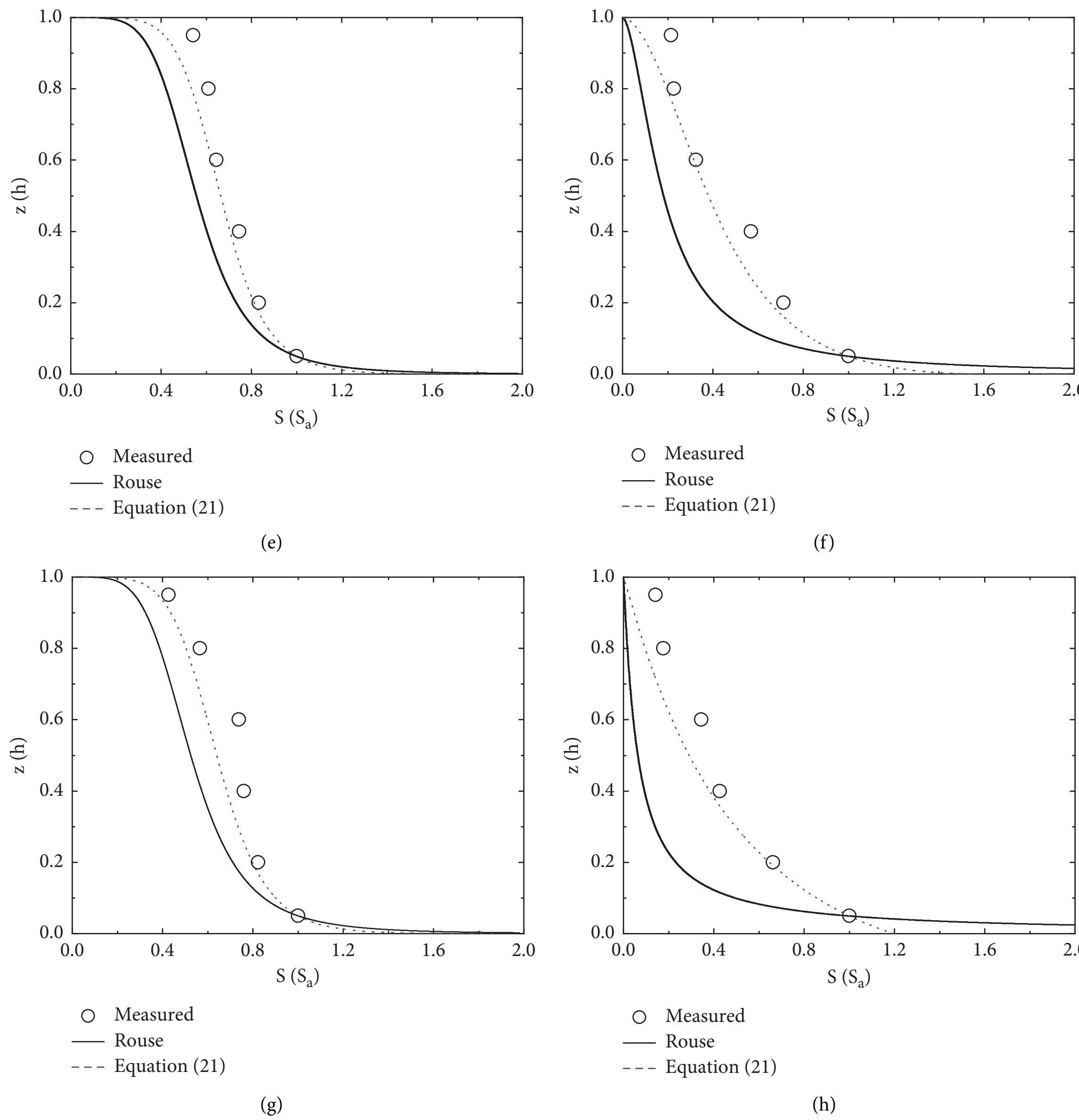

Figure 7: Calculation-suspended sediment profiles calculated by different methods.

where S_i is the field measured sediment concentration, $\widehat{S}_i$ is the sediment concentration calculated by the formula, w_i is the weight, where w_i is taken as 1, $Cov(S_i, \widehat{S}_i)$ is the covariance between the field-measured sediment concentration and the formula-calculated sediment concentration, $Var(S_i)$ is the statistical variance of the field-measured sediment concentration, and $Var(\widehat{S}_i)$ is the statistical variance of the formula-calculated sediment concentration.

The vertical distribution of suspended sediment calculated by the Rouse equation and (21) correlates well ($\rho > 0.95$) with the measured suspended sediment in the field. Still, the suspended sediment distribution described in (21) is closer to the field-measured-suspended sediment distribution than the Rouse distribution. As shown in Table 3, the RMSE of (21) is only 0.056, which is reduced by 64% compared with the calculation result of the Rouse formula. At the same time, the R^2 is 0.945 and the accuracy has increased by 66.67%. It can be seen that a reasonable consideration of the influence of the vertical flow velocity factor in the suspended sediment distribution equation can significantly improve the accuracy of calculation.

5.3. Effect of Settling Velocity on Suspended Sediment Distribution. Settling velocity of the sediment particle is an important physical quantity to describe the characteristics of sediment movement. The settling velocity of particles used in (21) is calculated by (25), which is the famous Stokes formula in still water. However, according to Richarson and Zaki [41], the sediment settling velocity ω_s in sediment-laden water is smaller than that in clear water which is known as a hindered settling effect, and they suggested (26), which was verified by Mohan and Kumbhakar in their semianalytical solution of the suspended sediment transport model [42, 43].

Table 3: Verification error of suspended sediment vertical distribution.

Methods	SSE	RMSE	R^2	ρ
Rouse	1.560	0.156	0.567	0.965
Equation (21)	0.200	0.056	0.945	0.984

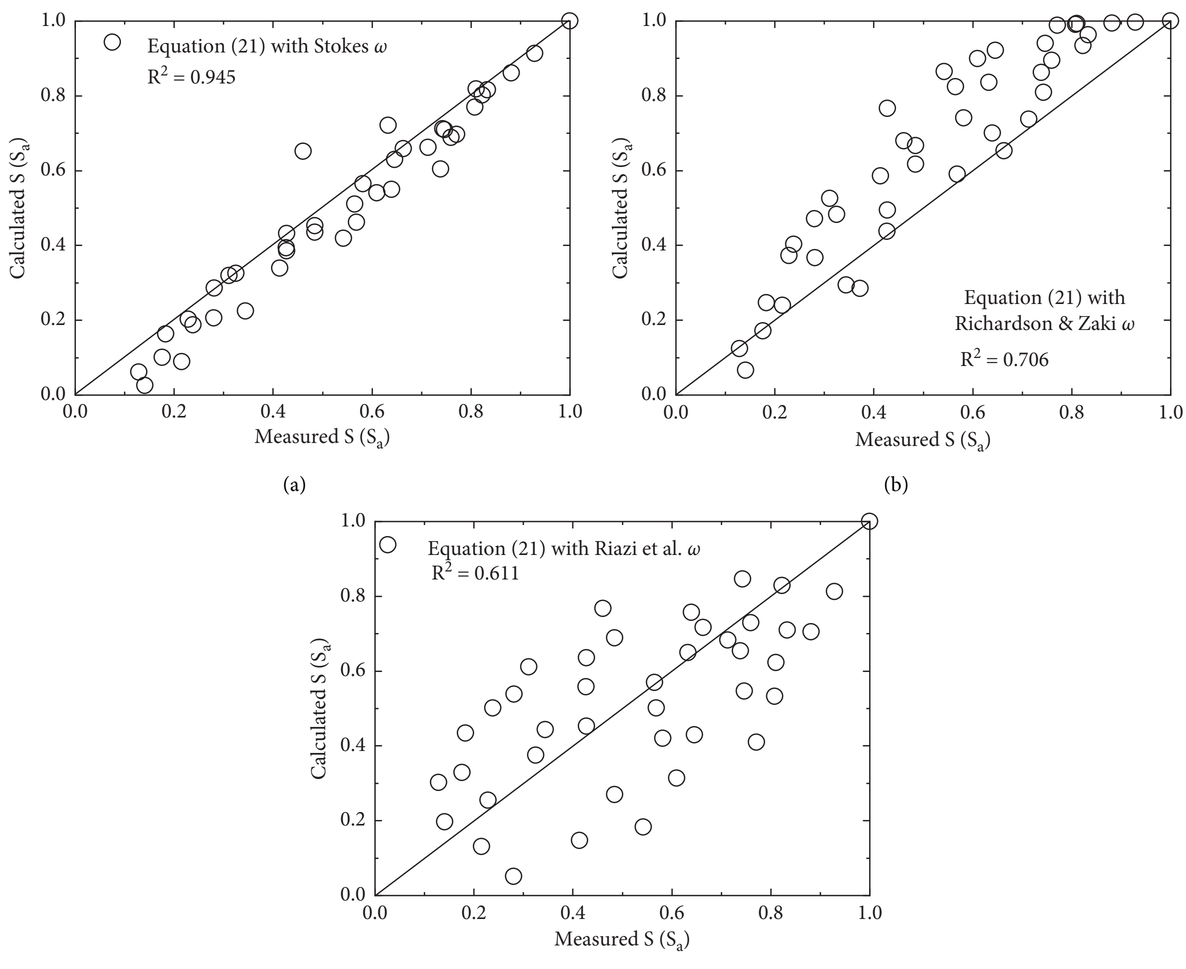

Figure 8: Verification of sediment concentration.

In addition, Riazi et al. [44, 45] put forward a setting velocity equation as in (27) by considering the influence of the particle shape, which improves the accuracy of sediment concentration calculation:

$$\omega = \frac{1}{18}\frac{\gamma_s - \gamma}{\gamma} g \frac{D^2}{\nu_f}, \tag{25}$$

$$\omega_s = \omega (1 - S)^{n_H}, \tag{26}$$

$$\omega_s^2 = \frac{4}{3}\left(\frac{\gamma_s}{\gamma - 1}\right) g \left(\frac{a_1 \times \nu_f}{d_n^{1.5} \times g^{0.5}} + a_2\right)^{-a_3} S_f^{\frac{2}{3}} d_n. \tag{27}$$

Among them, ω is the settling velocity, ω_s is the hindered settling velocity, D is the sediment particle diameter, ν_f is the fluid kinematic viscosity, n_H is the exponent of reduction in the settling velocity, γ_s and γ are the specific gravity of the sediment and water respectively, d_n is the sediment particle nominal diameter, and S_f is the Corey shape factor. a_1, a_2, and a_3 are the coefficients associated with S_f.

To analyze the influences of different settling velocity equations on the accuracy of sediment concentration calculation, equations (25)–(27) are substituted into (21) for sediment concentration verification. The sediment concentration correlation results are shown in Figure 8. The data used for correlation analysis are shown in Table 2, and the verification results of the vertical distribution of suspended sediment with different settling velocities against the data group h are shown in Figure 9.

The calculation results of (21) with different sediment settling velocity equations against field data given in Table 2

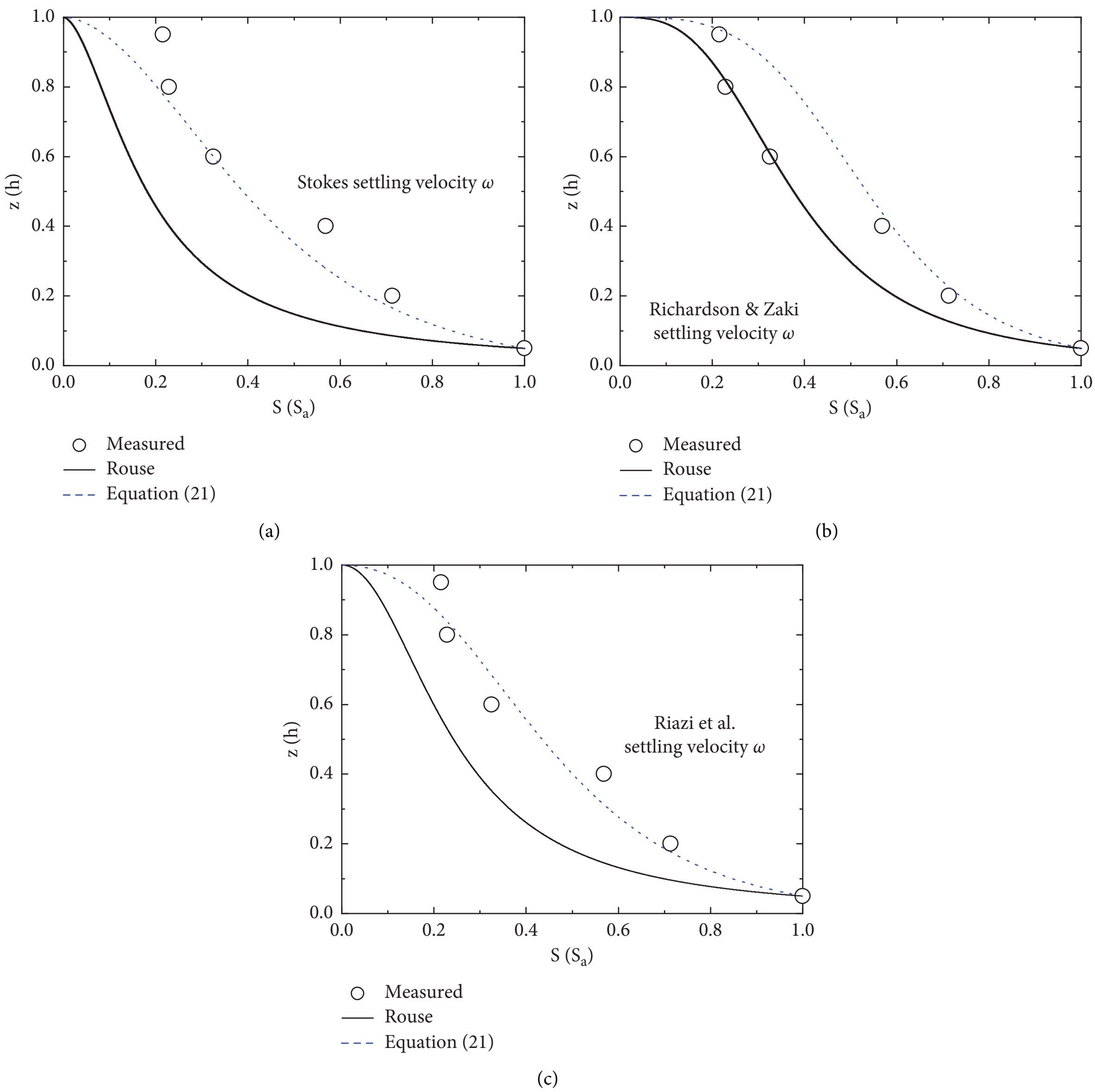

Figure 9: Suspended sediment concentration profile.

are shown in Figures 8(a)–8(c). It is obvious that the settling velocity has a significant impact on the suspended sediment distribution described in (21). Overall, (21) with the settling velocity in clear water yields higher calculation accuracy than it does with the hindered settling velocity. The coefficient of determination of (21) with the Stokes settling velocity is 0.945 and that of the equation with the Riazi settling velocity is 0.611, which is the lowest among the three cases. The error in the case of the Riazi settling velocity may be due to missing data on the sediment particle shape, so we treated sediment particles as spheres in calculation.

Using the Richarson and Zaki settling velocity equation in the Rouse equation could revise the underestimation of the Rouse equation to some degree (Figure 9(b)), which is consistent with the research results by Kumbhakar et al. [43]. In contrast, the sediment concentration calculated by equation (21) using the Richarson and Zaki settling velocity equation is overestimated (Figure 8(b)). Because in the derivation of equation (21), the effect of sediment concentration on the flow turbulence had already been taken into account. Therefore, the utilization of the Richarson and Zaki settling velocity in which the effect of the sediment concentration is considered in equation (21) leads that the effect of sediment concentration on the flow turbulence is considered twice, which may be the main reason for the overestimation of the equation (21).

Actually, the differential equation obtained by substituting the Richarson and Zaki settling velocity equation into the Schmidt diffusion equation has the same form with the differential equation that takes account of the effect of the vertical flow velocity, which is

$$\varepsilon_z \frac{dS}{dz} + S\omega\left(1 - mS\right)^{n_H} = 0. \tag{28}$$

When the exponent $n_H = 1$, (28) becomes the sediment diffusion equation considering the influence of the vertical flow velocity, and when the coefficient $m = 1$, it becomes the sediment diffusion equation using the Richardson and Zaki settling velocity equation. Therefore, (21) should be used with the settling velocity in the clear water instead of a hindered settling velocity equation related to sediment concentration.

6. Conclusions

A suspended sediment concentration profile considering the influence of the vertical velocity is derived, which avoids the underestimation of the Rouse equation in practical application. The coefficient m in the derived equation is a parameter that characterizes the effect of the vertical velocity on the vertical distribution of sediment concentration, and it is related to water depth. In addition, the derived equation should be used with the settling velocity in clear water rather than the hindered settling velocity related to sediment concentration.

Appendix

A Solution of the Two-Dimensional Sediment Diffusion Equation

The sediment diffusion equation including the vertical velocity is as follows:

$$\beta kzU_* \frac{h-z}{h}\frac{dS}{dz} + S\omega(1-mS) = 0. \tag{A.1}$$

Among them, S is the concentration of sediment, z denotes the vertical coordinate, ω indicates the sediment settling velocity, U_* denotes the shear velocity, h indicates the water depth, m is the vertical time average velocity coefficient, and β and k are the proportional constant and the Karman constant, respectively.

Separating variables, equation (A.1) is rearranged as

$$\frac{dS}{S(1-mS)} = -\frac{\omega}{\beta kU_*}\cdot\frac{h\,dz}{z(h-z)}. \tag{A.2}$$

Integrating equation (A.2), we obtain

$$\int\left(\frac{1}{S}+\frac{m}{1-mS}\right)\mathrm{d}S = -\frac{\omega}{\beta kU_*}\int\left(\frac{1}{z}+\frac{1}{h-z}\right)\mathrm{d}z. \tag{A.3}$$

According to the integral rule, equation (A.3) can be solved as

$$\ln\frac{S}{1-mS} = -\frac{\omega}{\beta kU_*}\ln\frac{C_1 z}{h-z}, \tag{A.4}$$

where C_1 indicates the integration constant.

According to Vanoni's research results, the parameter term $\omega/\beta kU_*$ is the modified Rouse number, and then, equation (A.4) can be expressed as follows:

$$\frac{S}{1-mS} = C_2\left(\frac{h-z}{z}\right)^{\omega/\beta kU_*}, \tag{A.5}$$

where C_2 is the integral constant related to C_1.

Combining similar terms, equation (A.5) can be written as

$$S\left[1+mC_2\left(\frac{h-z}{z}\right)^{\omega/\beta kU_*}\right] = C_2\left(\frac{h-z}{z}\right)^{\omega/\beta kU_*}. \tag{A.6}$$

Considering the sediment concentration S_a at the reference level $z = a$, we get

$$S_a\left[1+mC_2\left(\frac{h-a}{a}\right)^{\omega/\beta kU_*}\right] = C_2\left(\frac{h-a}{a}\right)^{\omega/\beta kU_*}. \tag{A.7}$$

Combining equations (A.6) and (A.7), we obtain

$$\frac{S}{S_a} = \frac{1+mC_2(h-a/a)^{\omega/\beta kU_*}}{1+mC_2(h-z/z)^{\omega/\beta kU_*}}\left(\frac{h-z}{z}\frac{a}{h-a}\right)^{\omega/\beta kU_*}. \tag{A.8}$$

The vertical time-averaged velocity coefficient m and the integration constant C_2 are both coefficients to be determined. Assuming that the product of m and C_2 is still represented by m, the vertical distribution of sediment concentration is as follows:

$$\frac{S}{S_a} = \frac{1+m(h-a/a)^{\omega/\beta kU_*}}{1+m(h-z/z)^{\omega/\beta kU_*}}\left(\frac{h-z}{z}\frac{a}{h-a}\right)^{\omega/\beta kU_*}. \tag{A.9}$$

Equation (A.9) is the solution of the sediment diffusion equation incorporating the vertical velocity.

B Notation

The following symbols are used in this paper:

$\overline{V}$ is the time-averaged velocity in the z-axis direction

ω is the sediment settling velocity

$\overline{S}$ is the mean value of suspended sediment concentration

l is the thickness of the water layer attached to the sediment surface

d_{50} is the medium diameter of sediment

ρ_s is the sediment density

ρ_f is the fluid density

m is the vertical time average velocity coefficient

S is the instantaneous concentration of sediment

u, v are the instantaneous velocity components in the x -axis direction and z-axis direction, respectively

x, z are horizontal and vertical physical coordinates, respectively.

$\overline{u}, \overline{v}$ are the time-averaged values of horizontal and vertical velocity components, respectively.

$u^{'}, v^{'}$ are the pulsation values of horizontal and vertical velocity components, respectively.

$\varepsilon_x, \varepsilon_z$ are the sediment diffusivity along the flow direction and perpendicular to the flow direction, respectively.

k is the Karman constant

U_* is the shear velocity

τ is the shear force, $\tau = \tau_0 (1 - z/h)$, and τ_0 is the shear force on the bed surface

S_a is the near-bottom sediment concentration corresponding to the reference height $z = a$.

Z_* is the Rouse index, $Z_* = \omega/kU_*$

α_a is the relative reference height, $\alpha_a = a/h$

α is the relative height of sediment, $\alpha = z/h$.

δ is the influence coefficient of the vertical velocity on the vertical relative distribution of sediment concentration

ε_m is the momentum exchange coefficient

β is the proportional constant

SSE is the sum of squares due to error

RMSE is the root mean squared error

R^2 is the coefficient of determination

ρ is the Pearson correlation coefficient

S_i is the field measured sediment concentration

$\widehat{S}_i$ is the sediment concentration calculated by the formula

w_i is the weight

$Cov(S_i, \widehat{S}_i)$ is the covariance between the field-measured sediment concentration and the formula-calculated sediment concentration

$Var(S_i)$ is the statistical variance of the field-measured sediment concentration

$Var(\widehat{S}_i)$ is the statistical variance of the formula-calculated sediment concentration

S_f is the Corey shape factor

n_H is the exponent of reduction in the settling velocity

d_n is the sediment particle nominal diameter.

Data Availability

The data presented in this study are available on reasonable request from the corresponding author.

Consent

Consent is not applicable to this study.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

Xiufeng Quan was responsible for writing of the original draft, reviewing and editing of the manuscript, funding acquisition, project administration, data curation, and supervision; Ruijie Li was responsible for writing of the original draft, reviewing and editing of the manuscript, and data curation; Yuting Li, Feng Luo, Xiaoyan Fu, and He Gou were responsible for reviewing and editing of the manuscript and data curation. All the authors read and agreed to the published version of the manuscript.

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References

[1] H. Rouse, "Modern conceptions of the mechanics of fluid turbulence," *Transactions of the American Society of Civil Engineers*, vol. 102, no. 1, pp. 463–505, 1937.

[2] V. A. Vanoni, "Transportation of suspended sediment by water," *Transactions of the American Society of Civil Engineers*, vol. 111, no. 1, pp. 67–102, 1946.

[3] H. M. Ismail, "Turbulent transfer mechanism and suspended sediment in closed channels," *Transactions of the American Society of Civil Engineers*, vol. 117, no. 1, pp. 409–434, 1952.

[4] J. N. Hunt, "The turbulent transport of suspended sediment in open channels," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, vol. 224, no. 1158, pp. 322–335, 1954.

[5] R. A. Bagnold, *An Approach to the Sediment Transport Problem from General Physics*, pp. 1–43, United States Government Printing Office, Washington, 1966.

[6] R. Soulsby, *Dynamics of Marine Sands*, Thomas Telford, London, 1997.

[7] L. C. v. Rijn, "Sediment transport, Part II: suspended load transport," *Journal of Hydraulic Engineering*, vol. 110, no. 11, pp. 1613–1641, 1984.

[8] M. Kumbhakar, K. Ghoshal, and V. P. Singh, "Derivation of Rouse equation for sediment concentration using Shannon entropy," *Physica A: Statistical Mechanics and Its Applications*, vol. 465, pp. 494–499, 2017.

[9] S. Q. Nie, H. G. Sun, Y. Zhang et al., "Vertical distribution of suspended sediment under steady flow: existing theories and fractional derivative model," *Discrete Dynamics in Nature and Society*, vol. 2017, no. 2, Article ID 5481531, 11 pages, 2017.

[10] S. Kundu, "Derivation of different suspension equations in sediment-laden flow from Shannon entropy," *Stochastic Environmental Research and Risk Assessment*, vol. 32, no. 2, pp. 563–576, 2018.

[11] J. T. Wallwork, J. H. Pu, S. Kundu et al., "Review of suspended sediment transport mathematical modelling studies," *Fluid*, vol. 7, no. 1, pp. 23–39, 2022.

[12] M. A. Velikanov, *Fluvial Process of Rivers*, Physics Mathematics Literature Press, Moscow, 1958.

[13] R. J. Zhang, "Discussion of gravitation theory and mechanism of sediment suspension," *Journal of Hydraulic Engineering*, vol. 3, pp. 11–23, 1963.

[14] N. Chien and Z. H. Wan, *Mechanics of Sediment Transport*, pp. 412–440, American Soc of Civil Engineers, Virginia, 1999.

[15] H. W. Zhang, “Vertical distribution of velocity in sediment-laden flow,” *Journal of Sedimentary Research*, vol. 2, pp. 1–10, 1995.

[16] J. Zheng, R. J. Li, Q. Feng, and S. s. Lu, “Vertical profiles of fluid velocity and suspended sediment concentration in nearshore,” *International Journal of Sediment Research*, vol. 28, no. 3, pp. 406–412, 2013.

[17] Y. T. Li, Z. Y. Song, G. Peng et al., “Modeling hydro-dynamics in a harbor area in the Daishan island, China,” *Water*, vol. 11, no. 2, pp. 192–214, 2019.

[18] B. P. Boudreau and P. S. Hill, “Rouse revisited: the bottom boundary condition for suspended sediment profiles,” *Marine Geology*, vol. 419, Article ID 106066, 2020.

[19] J. J. Baronas, E. I. Stevenson, C. R. Hackney et al., “Integrating suspended sediment flux in large alluvial river channels: application of a Synoptic Rouse-Based model to the Irrawaddy and Salween rivers,” *Journal of geophysical research Earth surface*, vol. 125, no. 9, pp. 1–21, 2020.

[20] P. Jain, M. Kumbhakar, and K. Ghoshal, “A mathematical model on depth-averaged β-factor in open-channel turbulent flow,” *Environmental Earth Sciences*, vol. 77, no. 6, pp. 1–10, 2018.

[21] R. Absi, “Rebuttal on “A mathematical model on depth-averaged β-factor in open-channel turbulent flow” Environmental Earth Sciences 77:253 (2018),” *Environmental Earth Sciences*, vol. 79, no. 5, pp. 1–5, 2020.

[22] C. H. Huang, C. W. Tsai, and S. M. Mousavi, “Quantification of probabilistic concentrations for mixed-size sediment particles in open channel flow,” *Stochastic Environmental Research and Risk Assessment*, vol. 35, no. 2, pp. 419–435, 2021.

[23] J. H. Pu, J. T. Wallwork, M. A. Khan et al., “Flood suspended sediment transport: combined modelling from dilute to hyper-concentrated flow,” *Water*, vol. 13, no. 3, p. 379, 2021.

[24] Z. L. Sun, H. L. Zheng, D. Xu, C. Hu, and C. Zhang, “Vertical concentration profile of nonuniform sediment,” *International Journal of Sediment Research*, vol. 36, no. 1, pp. 120–126, 2021.

[25] J. B. Keller, “Shallow-water theory for arbitrary slopes of the bottom,” *Journal of Fluid Mechanics*, vol. 489, Article ID S0022112003005342, 348 pages, 2003.

[26] N. Kobayashi and S. N. Seo, “Fluid and sediment interaction over a plane bed,” *Journal of Hydraulic Engineering*, vol. 111, no. 6, pp. 903–921, 1985.

[27] X. F. Zhang and Z. X. Chen, “On the distribution of sediment concentration,” *Journal of Hydraulic Engineering*, vol. 10, pp. 41–48, 1990.

[28] Z. X. Cao, L. Y. Wei, and J. H. Xie, “Sediment-laden flow in open channels from two-phase flow viewpoint,” *Journal of Hydraulic Engineering*, vol. 121, no. 10, pp. 725–735, 1995.

[29] S. Q. Yang, “Formula for sediment transport subject to vertical flows,” *Journal of Hydraulic Engineering*, vol. 145, no. 5, Article ID 04019013, 2019.

[30] S. Q. Yang, S. K. Tan, and S. Y. Lim, “Velocity and sediment concentration profiles in sediment-laden flows,” *China Ocean Engineering*, vol. 18, no. 2, pp. 229–244, 2004.

[31] J. X. Zhang and H. Liu, “A vertical 2-D numerical simulation of suspended sediment transport,” *Journal of Hydrodynamics*, vol. 19, no. 2, pp. 217–224, 2007.

[32] C. Cheng, Z. Y. Song, Y. G. Wang, and J. S. Zhang, “Parameterized expressions for an improved Rouse equation,” *International Journal of Sediment Research*, vol. 28, no. 4, pp. 523–534, 2013.

[33] J. Zheng, R. Li, Y. Yu, and A. Suo, “Influence of wave and current flow on sediment-carrying capacity and sediment flux at the water-sediment interface,” *Water Science and Technology*, vol. 70, no. 6, pp. 1090–1098, 2014.

[34] Z. H. Mao, D. L. Pan, C. L. Tang et al., “A dynamic sediment model based on satellite-measured concentration of the surface suspended matter in the East China Sea,” *Journal of geophysical research. Oceans*, vol. 121, no. 4, pp. 2755–2768, 2016.

[35] S. Wang, “Research on microstructure of the muddy-limited sediments,” Master’s thesis of China, Northwest A&F University, Xianyang, China, 2013.

[36] D. Chen, H. G. Sun, and Y. Zhang, “Fractional dispersion equation for sediment suspension,” *Journal of Hydrology*, vol. 491, pp. 13–22, 2013.

[37] M. H. Garcia, *Sediment Transport and Morphodynamics*, vol. 54, pp. 107–120, American Society of Civil Engineers, Reston,Virginia, 2008.

[38] R. J. Li, S. S. Lu, and J. Zheng, “Transport simulation of sorptive contaminants considering sediment-associated processes,” *Chinese Journal of Oceanology and Limnology*, vol. 30, pp. 668–674, 2012.

[39] R. J. Li, F. Luo, and W. J. Zhu, “The suspended sediment transport equation and its near-bed sediment flux,” *Science in China - Series E: Technological Sciences*, vol. 52, no. 2, pp. 387–391, 2009.

[40] F. Luo and R. J. Li, “3D water environment simulation for north Jiangsu offshore sea based on EFDC,” *Journal of Water Resource and Protection*, vol. 01, no. 01, pp. 41–47, 2009.

[41] J. F. Richardson and W. N. Zaki, “Sedimentation and fluidisation, Part 1,” *Institution of Chemical Engineers*, vol. 32, pp. 35–53, 1954.

[42] S. Mohan, M. Kumbhakar, K. Ghoshal, and J. Kumar, “Semi-analytical solution for one-dimensional unsteady sediment transport model in open channel with concentration-dependent settling velocity,” *Physica Scripta*, vol. 95, no. 5, Article ID 055204, 2020.

[43] M. Kumbhakar, S. Mohan, K. Ghoshal, J. Kumar, and V. P. Singh, “Semianalytical solution for nonequilibrium suspended sediment transport in open channels with concentration-dependent settling velocity,” *Journal of Hydrologic Engineering*, vol. 27, no. 2, Article ID 04021048, 2022.

[44] A. Riazi and U. Türker, “The drag coefficient and settling velocity of natural sediment particles,” *Computational Particle Mechanics*, vol. 6, no. 3, pp. 427–437, 2019.

[45] A. Riazi, A. Vila-Concejo, T. Salles, and U. Turker, “Improved drag coefficient and settling velocity for carbonate sands,” *Scientific Reports*, vol. 10, no. 1, pp. 9465–9469, 2020.