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Research Article

Choice and Influence of Return Policy and Remanufacture in a Dual-Channel Supply Chain

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To investigate the choice of return strategy and its impact on a dual-channel supply chain, a game model is constructed to analyze the equilibrium outcomes of five scenarios: no returns allowed, refunds without returns, returns allowed but no return shipping insurance, returns allowed and return shipping insurance purchased by the manufacturer, and returns allowed and return shipping insurance purchased by the consumer. The study found that manufacturers offering refund policies generate more sales in the direct online channel, while retailers choose to reduce the retail price of their products. It is important to note that price reductions by retailers have a very limited effect and do not lead to an increase in sales in the retail channel. Manufacturers offering refund policies will inevitably infringe on retailers' profits, and the variability of manufacturers' profits depends on the residual value of returned products. Manufacturers should decide whether to offer a refund policy in online direct sales channels based on the residual value of the returned product; otherwise, the action would be detrimental to themselves. The price of direct online sales is the same whether the manufacturer buys the return shipping insurance or the consumer buys the return shipping insurance, but when the return shipping insurance is bought by the consumer, sales are higher in the direct online channel and lower in the retail channel. When the return shipping cost reimbursement received after purchasing return shipping insurance is low, it should be purchased by the manufacturer, and when the return shipping cost reimbursement received after purchasing return shipping insurance is high, it will be better for the consumer to purchase return shipping insurance.

1. Introduction

With the continuous development of information technology and e-commerce, online shopping has become one of the consumers' main shopping methods. Statistics show that as of June 2021, the number of online shopping users in China reached 812 million [1]. This shows that the rapid growth of e-commerce is changing the traditional sales model and will have a huge impact on the physical retail sector. While enjoying the convenience of online shopping, it is also important to see the problems that exist. Compared to physical retail stores, online shopping is more prone to returns due to the lack of real-life experience. For this reason, the major online shopping platforms offer a variety of different return services to consumers. For merchants, there is a choice of whether to allow consumers to return goods and whether to provide return shipping insurance. It is optional for the consumer to choose whether to purchase the product and whether to take out return shipping insurance. While merchants' return policies undoubtedly enhance consumer satisfaction with online shopping, these policies can also lead to increased speculative spending by consumers and invariably increase the cost to merchants. To reduce the additional costs caused by returns, merchants will remanufacture returned products and resell them, thus making the subject of return shipping insurance and the remanufacturing of returned products a point of contention. Therefore, manufacturers selling products in online channels will compete with brick-and-mortar retailers.

The innovations of this research and the main objectives are as follows:

(1) Whether manufacturers should provide return services and return shipping insurance in the face of return claims from consumers in online channels.
(2) Which return policy should the manufacturers provide?
(3) Whether they should remanufacture returned products are the issues that this paper will study.

2. Literature Review

There are two main areas of literature relevant to the study of this paper: one is research related to the issue of returns, and the other is research on the issue of remanufacturing. Regarding research on returns, Oghazi et al. [2] showed that a lenient return policy can increase customer trust and thus customer willingness to buy. Jeng [3] studied the impact of return policies on consumer purchase intentions in terms of brand awareness. Yang and Li [4] showed that under different sales models, the money-back guarantee always leads to lower demand for the product and has an impact on optimal pricing. Further research by Jin et al. [5] found that money-back guarantees can have an impact on the pricing strategies of both sides of the brand differentiation supply chain but that manufacturers and online retailers are able to make more profit when certain conditions are met. Huang et al. [6] studied a secondary supply chain yield decision problem considering consumer returns, analyzing the impact of offering a money-back guarantee on the equilibrium outcome and the threshold for opening a direct sales channel. The study found that the equilibrium outcome of a manufacturer offering a money-back guarantee compared to a retailer offering a money-back guarantee was consistent, except for the higher wholesale price. Zhao et al. [7] study the impact of return guarantees on merchants' pricing and revenue from the perspective of consumers' opportunistic behavior. Hu et al. [8] studied the pricing strategies of dual-channel retailers when offering customers a return strategy and showed that proper return policies not only improve the flexibility of business operations but also enhance consumers' sense of security. Li et al. [9] studied the strategic effects of return strategies in a dual-channel supply chain and showed that customer return rates were the main factor influencing refund strategies. J. Chen and B. Chen [10] consider the impact of return policy tolerance on retailer pricing when there are multiple retailers coexisting in the market. Zhao and Hu [11] investigated the issue of return shipping cost coverage and showed that to maximize profits, online retailers should choose strategies that are appropriate to the characteristics of the proportion of defect-free returns of goods. Nie et al. [12] studied the impact of money-back guarantees on the opening of direct sales channels by manufacturers and the respective money-back guarantee strategies and market equilibrium of manufacturers and retailers under a dual-channel model. Huang et al. [13] studied the impact of money-back guarantees on product quality and service in the supply chain. Zhang and Jin [14] investigated the issue of optimal pricing for e-retailers and contract design for manufacturers under a money-back guarantee. Wu et al. [15] showed that factors such as product quality, market demand, and price have an impact on the amount of product returns and the manufacturer's optimal decision. Radhi and Zhang [16] studied the impact of customer preferences and customer recall rates on dual-channel pricing. The above literature has examined the impact of return guarantees and return policies on consumers and firms but has not addressed the issue of how returned products are subsequently handled.

The National Development and Reform Commission issued the "opinions on promoting the development of remanufacturing industry" and mentioned that accelerating the development of remanufacturing industry is an effective way to promote the development of manufacturing and modern service industry; remanufacturing is the organic combination of manufacturing and repair, recycling and utilization, and production and circulation. The study of remanufacturing issues is of great significance in enhancing the efficiency of enterprises and promoting the development of a circular economy. Lotfi et al. [17] proposed a resilient and sustainable supply chain network design considering renewable energy sources. Lotfi et al. [18] proposed a medical waste chain network design for medical waste generated during COVID-19 treatment. Waste segregation was proposed to locate the wastes to reduce them, recycle them, and send them to the waste purchase contractor. Lotfi et al. [19] proposed a viable closed-loop supply chain network that considers resiliency, sustainability, and agility. In order to solve the problem, they suggested a hybrid robust stochastic optimization by minimizing the weighted expected, maximum, and entropic value at risk (EVaR) of the cost function for this problem. Li et al. [20] studied the time value of returned products in the reverse supply chain. By analyzing retailers' optimal return prices, Li et al. [21] found that manufacturers are able to reconcile retailers' optimal return prices through buybacks and remanufacturing of returned products to achieve supply chain coordination. Radhi and Zhang [22] studied the impact of resalable returns on order volume across channels. Xing et al. [23] studied the impact of remanufacturing links on the cost of dual-source inventory in a closed-loop supply chain. Xia et al. [24] studied the impact of changes in return logistics costs on price volatility and supply chain system efficiency for remanufacturers and retailers. Hu et al. [25] analyzed the impact of return rates, remanufacturing ratios, and customer sensitivity factors to retail and return prices on return prices and profits and gave corresponding return price pricing strategies. Xu et al. [26] showed that the choice of a manufacturer's remanufacturing strategy depends on the production cost of the remanufactured product, and when the cost of the remanufactured product is low, the implementation of a remanufacturing strategy can increase the profits of both the manufacturer and the retailer, allowing the whole supply chain system to be optimized. Chen and Dong [27] analyzed the impact of own funds investment in emission reduction and carbon emission thresholds on remanufacturing decisions. Lin et al. [28] analyzed the impact of retailers on remanufacturing from the perspective of their complicity with remanufacturers. Cao and Zhu [29] analyzed the impact of firms' investment efficiency on the equilibrium decision to recycle and remanufacture. Xia and Zhu [30] investigated the impact of remanufacturing design on manufacturing/remanufacturing under authorized remanufacturing. Ullah et al. [31] investigated the optimal remanufacturing strategy for a single-/multi-retailer closed-loop supply chain under stochastic demand and return rates. Shekarian et al. [32] studied the impact of carbon emissions and remanufacturing on the dual channel of forward and reverse logistics. Bansal et al. [33] studied the role of remanufacturing in product development and associated profit estimation. In the above literature, scholars have focused on the issue of remanufacturing but have not considered the issue of the remanufacturing of returned products and its impact on supply chain return strategies.

In summary, the existing literature focuses on the impact of money-back guarantees on consumers and businesses, and research on the handling of returned products has focused on the impact on inventory costs and product prices, with little research on the impact of remanufactured returned products on supply chain members' return strategies under competitive channels. Based on the above analysis, this paper examines the choice of a manufacturer's return strategy based on a two-tier supply chain consisting of a single manufacturer and a single retailer, where the manufacturer sells its products both through traditional retailers and directly through online channels, to provide theoretical guidance on the choice of a manufacturer's remanufacturing pricing and return strategy. We selected some studies related to our work as a comparison to clearly illustrate the similarities and differences between the relevant literature and this paper in Table 1.

3. Problem Description and Notations

Consider a two-tier supply chain consisting of a single manufacturer and a single retailer, with the manufacturer selling products both through traditional retail channels and through direct online sales channels. The manufacturer is the dominant player in the supply chain, with the retailer as the follower. The manufacturer decides on the wholesale price of the product, the direct internet price, and the return strategy; the retailer decides on the retail price of the product. The manufacturer sells the product to the end consumer at the direct network price p_d and the retailer at the retail price p_r, and the production cost per unit of product is c. According to the Hotelling model, the total market is defined as 1, consumers are uniformly distributed in a unit linear market, and consumers value the product as v.

As consumers cannot physically experience the product when shopping online, it is reasonable to assume that if the product purchased online does not meet the consumer's expectations, the consumer will request a refund from the retailer. In commercial practice, merchants allow consumers to simply request a refund without returning the product if the product cannot be resold after the return or if the product is of low value. If the product itself has no quality problems or is repaired and does not affect secondary sales, the consumer will have to pay the return shipping costs s to return the product. Some merchants choose to spend t on return shipping insurance to reduce the negative impact of return shipping costs on consumers' willingness to buy. If the merchant does not offer return shipping insurance, consumers can also choose to purchase their own return shipping insurance at a cost of t to reduce the risk of returning the goods. When merchants give away return shipping insurance or consumers buy their return shipping insurance, consumers will receive a certain amount of compensation for return shipping costs once the goods are returned from online shopping r. Products returned by consumers are sorted by the manufacturer according to the residual value φ, and those of them without quality problems are put back on the market, while those of them with quality problems are remanufactured and resale. The cost used to remanufacture the returned product is c_i, and the price to resale the returned product after it has been remanufactured is ρ. Based on whether the manufacturer offers consumers a return policy and complimentary return shipping insurance in the online direct sales channel, whether the consumer purchases the product, and whether the consumer purchases return shipping insurance, the following five scenarios are considered:

(1) No returns allowed (*NN*): The manufacturer will not accept returns from consumers who are not satisfied with the products they receive after purchasing them through direct online sales channels.

(2) Refunds without returns (*RN*): Consumers who are not satisfied with the products they receive after purchasing them through direct online sales channels can apply for a refund without having to return the products.

(3) Return allowed but no return shipping insurance (*MN*): The manufacturer offers a return policy that allows the consumer to return unsatisfactory products, but the manufacturer does not give return shipping insurance and the consumer does not purchase return shipping insurance. In this case, the manufacturer allows the consumer to return the product, but the return shipping costs incurred are borne by the consumer.

(4) Allowing returns and having the manufacturer purchase return shipping insurance (*MR*): The manufacturer allows the consumer to return the product and gives the consumer return shipping insurance. This practice by the manufacturer can alleviate consumers' concerns and increase their willingness to buy.

(5) Allowed returns with return shipping insurance purchased by the consumer (*MC*): The manufacturer allows the consumer to return the product, but the return shipping insurance is purchased by the consumer. In this case, the consumer purchases his or her own return shipping insurance, which will reimburse him or her for the return shipping costs if

Table 1: Comparison among recent relevant studies.

Literature	Dual-channel	Return policy	Buyer's return insurance	Seller's return insurance	Remanufacturing	Remanufacturing of returned products	Impact on return strategies
Oghazi et al. [2]	—	✓	—	—	—	—	—
Hu et al. [8]	✓	✓	—	—	—	—	—
Li et al. [9]	✓	✓	—	✓	—	—	—
Zhao and Hu [11]	—	✓	✓	✓	—	—	—
Nie et al. [12]	✓	✓	—	—	—	—	—
Radhi and Zhang [16]	✓	✓	—	—	—	—	—
Radhi and Zhang [22]	✓	✓	—	—	—	✓	—
Xia et al. [24]	—	—	—	—	—	✓	—
Shekarian et al. [32]	✓	—	—	—	—	✓	—
This paper	✓	✓	✓	✓	✓	✓	✓

the consumer is not satisfied with the product after receiving it online.

To define the research question and to facilitate the construction of the subsequent model, the following research assumptions are made:

(1) In the process of the game, the manufacturer and the retailer are information symmetric, and they are risk neutral (similar to literature [34]).

(2) Whether a consumer purchases a product depends on the consumer's willingness to pay and utility. To illustrate the rate of consumer demand for a product, the total market is defined as 1.

(3) It is assumed that the consumer purchases only 1 unit of the product during the sales cycle and does not consider the case where the consumer purchases more than one product.

(4) The return rate of products purchased by consumers at offline retailers is low because consumers can experience the products firsthand when they purchase them at offline retailers. To simplify and highlight the research questions, this paper only considers returns that occur when consumers purchase products in online channels.

(5) Consumers are unable to experience the products they purchase online, and as a result, it is the case that online purchases do not meet expectations. Assume that the probability that a consumer's online purchase meets his or her expectations is $\theta (0 < \theta < 1)$.

(6) When a manufacturer offers a return policy in the online channel, if a consumer's online purchase does not meet expectations, the consumer will return the product and receive a full refund.

(7) In the process of the game, the manufacturer and the retailer are information symmetric, and they are risk neutral.

The notations used in this paper are described in Table 2.

3.1. No Returns Allowed. In cases where returns are not allowed (indicated by the superscript *NN*), the manufacturer will not accept returns if the consumer is not satisfied with the product received after purchasing it through the direct online sales channel. In this case, if the consumer's online purchase meets his or her expectations, then the utility gained by the consumer is $v - p_d$. If a consumer's online purchase does not meet expectations and since the manufacturer does not allow returns, the consumer receives a utility of $0 - p_d$. Therefore, in a scenario where returns are not allowed, the expected utility obtained by a consumer purchasing a product from a direct online sales channel is

$$U_d^{NN} = \theta (v - p_d) + (1 - \theta)(0 - p_d). \tag{1}$$

The expected utility received by the consumer from the retailer for purchasing the product is:

$$U_r^{NN} = v - p_r. \tag{2}$$

Consumers make purchase decisions based on the magnitude of expected utility, which is based on max $(U_d, U_r, 0)$. Defining the total market as 1, consumers' valuation of product v is heterogeneous, and v follows a uniform distribution of $[0, 1]$. Clearly, there are three critical states of consumer choice:

(1) $\theta (v - p_d) + (1 - \theta)(0 - p_d) = 0$, the consumer valuation at this point is defined as $V_d = p_d/\theta$

(2) $v - p_r = 0$, the consumer valuation at this point is defined as $V_r = p_r$

(3) $\theta (v - p_r) + (1 - \theta)(0 - p_d) = v - p_r$, the consumer valuation at this point is defined as $V_{dr} = p_r - p_d/1 - \theta$ Then the demand for both channels can be discussed in the following three scenarios:

(1) When $V_r \geq V_d$ and $V_{\mathrm{dr}} \leq 1$, that is, $p_d/\theta \leq p_r \leq 1 + p_d - \theta$. At this point, both channels

Table 2: Summary of notations.

Parameter	Definition
p_d	Manufacturer network channel direct sales prices
p_r	Retail price
w	Wholesale price
c	Unit production cost
v	Consumer valuation of products
θ	Probability that a consumer's online purchase will meet his or her expectations
φ	Returned product salvage rate
ρ	Price of returned products remanufactured for resale
c_i	Returned product remanufacturing costs
s	Return shipping costs
r	Reimbursement of return shipping costs following the purchase of return shipping insurance
t	Cost of purchasing return shipping insurance

have consumers buying the product, and the demand functions for the two channels are $D_d^{NN} = p_r - p_d/1 - \theta - p_d/\theta$ and $D_r^{NN} = 1 - p_r - p_d/1 - \theta$, respectively.

(2) When $V_r \geq V_d$ and $V_{\mathrm{dr}} \geq 1$, then $p_r \geq 1 + p_d - \theta$. At this point, the consumer's utility from purchasing the product from the direct online channel is greater than that from the retail channel, and the demand functions for the two channels are $D_d^{NN} = 1 - p_d/\theta, D_r^{NN} = 0$.

(3) When $V_r \leq V_d$, that is, $p_r \leq p_d/\theta$. At this point, the consumer obtains more utility from purchasing the product from the retail channel, and the demand functions for the two channels are $D_d^{NN} = 0, D_r^{NN} = 1 - p_r$, respectively.

As this paper examines the situation where both channels of demand exist simultaneously, the paper will be followed up with an analysis based on the following demand function:

$$\begin{aligned} D_d^{NN} &= \frac{p_r - p_d}{1 - \theta} - \frac{p_d}{\theta}, \\ D_r^{NN} &= 1 - \frac{p_r - p_d}{1 - \theta}. \end{aligned} \tag{3}$$

Based on the above demand function, the profit functions of the manufacturer and the retailer are further obtained as follows:

$$\begin{aligned} \pi_d^{NN} &= (w - c)D_r^{NN} + (p_d - c)D_d^{NN}, \\ \pi_r^{NN} &= (p_r - w)D_r^{NN}. \end{aligned} \tag{4}$$

Proposition 1. *The selling price, sales volume, and profit of the retailer and manufacturer in the no return scenario are as follows:* $p_r^{NN} = 3 + 2c - \theta/4$ *and* $p_d^{NN} = \theta + c/2$, $D_r^{NN} = 1/4$ *and* $D_d^{NN} = \theta - 2c/4\theta$, *and* $\pi_r^{NN} = 1 - \theta/16$ *and* $\pi_d^{NN} = 2c^2 - 4\theta c + \theta^2 + \theta/8\theta$.

The Proof of Proposition 1 and subsequent propositions and corollaries is given in the Appendix. Corollary 1 can be further obtained from Proposition 1.

Corollary 1. *The impact of the probability θ of online goods meeting expectations in the no return scenario on the sales price, sales volume, and profit of the retailer and manufacturer, respectively, is as follows:* $\partial p_r^{NN}/\partial\theta < 0$ *and* $\partial p_d^{NN}/\partial\theta > 0$, $\partial D_r^{NN}/\partial\theta = 0$ *and* $\partial D_d^{NN}/\partial\theta > 0$, *and* $\partial \pi_r^{NN}/\partial\theta < 0$ *and* $\partial \pi_d^{NN}/\partial\theta > 0$.

Corollary 1 suggests that in a no-return scenario, the greater the probability that a consumer's purchase from an online direct marketing channel will meet his or her expectations, the lower the risk of the consumer's online purchase, and the greater the consumer's willingness to purchase the product from the online direct marketing channel, and the corresponding increase in sales volume of the online direct marketing channel. At the same time, manufacturers will set higher prices for direct online sales, and the combination of these two effects leads to higher profits for manufacturers. As the probability of consumers purchasing products from online direct sales channels increases to meet their expectations, retailers will choose to reduce the retail price of their products to attract consumers to continue to purchase products from the retail channel; however, the effect of price reduction by retailers is very limited and does not lead to an increase in sales volume but only to maintain the original sales volume. As a result of the reduction in the retail price, the retailer's profitability ultimately decreases.

3.2. Refunds without Returns. In the case of a refund without returns (indicated by the superscript *RN*), the manufacturer allows the consumer to receive a refund and does not have to return the product if the product does not meet expectations and the returned product cannot be resold or is of low value. In this case, if the consumer's online purchase meets his or her expectations, then the utility gained by the consumer is $v - p_d$; if a consumer's online purchase does not meet expectations, the consumer receives a utility of φv, where φ is the residual value of the returned product, as the merchant refunds the full amount and does not require a return:

$$U_d^{RN} = \theta(v - p_d) + (1 - \theta)\varphi v. \tag{5}$$

The utility gained by the consumer from purchasing the product at the retailer is

$$U_r^{RN} = v - p_r. \tag{6}$$

In line with the previous section, only the case where both channels of demand exist simultaneously is considered, at which point the demand functions for the direct online channel and the retail channel are

$$\begin{aligned} D_d^{RN} &= \frac{p_r - \theta p_d}{(1-\theta)(1-\varphi)} - \frac{\theta p_d}{\theta + (1-\theta)\varphi}, \\ D_r^{RN} &= 1 - \frac{p_r - \theta p_d}{(1-\theta)(1-\varphi)}. \end{aligned} \tag{7}$$

Based on the above demand function, the profit functions of the manufacturer and the retailer are further obtained as follows:

$$\begin{aligned} \pi_d^{RN} &= (w-c)D_r^{RN} + \theta(p_d - c)D_d^{RN} - (1-\theta)cD_d^{RN}, \\ \pi_r^{RN} &= (p_r - w)D_r^{RN}. \end{aligned} \tag{8}$$

Proposition 2. *The selling price, sales volume, and profit of the retailer and manufacturer in the refund-no-return scenario are as follows:* $p_r^{RN} = (\theta-1)\varphi + 2c + 3 - \theta/4$ *and* $p_d^{RN} = \theta + c + \varphi - \theta\varphi/2\theta$, $D_r^{RN} = 1/4$ *and* $D_d^{RN} = (\theta-1)\varphi + 2c - \theta/(4\theta-4)\varphi - 4\theta$, *and* $\pi_r^{RN} = (\theta-1)(\varphi-1)/16$ *and* $\pi_d^{RN} = -(\theta-1)^2\varphi^2 - (4c-2\theta-1)(\theta-1)\ \varphi - \theta^2 + (4c-1)\theta - 2c^2/(8\theta-8)\varphi - 8\theta$.

Proposition 2 leads to Corollary 2, which further analyses the impact of the probability θ of online goods meeting expectations on the sales price, sales volume, and profits of retailers and manufacturers.

Corollary 2. *The impact of the probability* $\partial p_d^{RN}/\partial\theta < 0$ *of an online purchase meeting expectations in the refund-no-return scenario on the sales price, sales volume, and profits of retailers and manufacturers is as follows:* $\partial p_r^{RN}/\partial\theta < 0$ *and* $\partial p_d^{RN}/\partial\theta < 0$, $\partial D_r^{RN}/\partial\theta = 0$ *and* $\partial D_d^{RN}/\partial\theta > 0$, *and* $\partial\pi_r^{RN}/\partial\theta < 0$ *and* $\partial\pi_d^{RN}/\partial\theta > 0$.

Corollary 2 suggests that in a refund-no-return scenario, as the probability that consumers will meet their expectations by purchasing products from online direct sales channels increases, both manufacturers and retailers will reduce their selling prices to attract more consumers. At this time, sales volume in the direct online sales channel will increase due to the increased willingness of consumers to purchase products in the direct online sales channel and the lower prices of direct online sales. Although the price of direct internet sales is reduced, the act of reducing the price can lead to a rapid increase in sales volume, which ultimately leads to higher profits for the manufacturer. However, the effect of the retailers' price cuts was very limited, and only the original sales volume was maintained. For retailers, the reduction in retail prices while sales volumes remain the same ultimately leads to a reduction in retailer profits.

3.3. Returns Allowed but No Return Shipping Insurance. In the case of returns allowed but no return shipping insurance (indicated by the superscript *MN*), when a consumer's online purchase does not meet expectations, the manufacturer allows the consumer to return the goods but does not provide return shipping insurance, and the consumer is responsible for the return shipping costs. In this case, if the consumer's online purchase meets his or her expectations, then the utility gained by the consumer is $v - p_d$; if a consumer's online purchase does not meet expectations, the utility gained by the consumer at this point is $0 - s$, where s is the return shipping cost incurred by the consumer in returning the product. Therefore, in a scenario where the manufacturer allows returns but does not provide return shipping insurance, the expected utility obtained by the consumer from purchasing the product in the direct online channel is

$$U_d^{MN} = \theta(v - p_d) + (1-\theta)(0 - s). \tag{9}$$

The utility gained by the consumer from purchasing the product in the retail channel is

$$U_r^{MN} = v - p_r. \tag{10}$$

In line with the previous section, only the case where both channels of demand exist simultaneously is considered, at which point the demand functions for the direct online channel and the retail channel are

$$\begin{aligned} D_d^{MN} &= \frac{p_r - \theta p_d - (1-\theta)s}{1-\theta} - \frac{\theta p_d + (1-\theta)s}{\theta}, \\ D_r^{MN} &= 1 - \frac{p_r - \theta p_d - (1-\theta)s}{1-\theta}. \end{aligned} \tag{11}$$

In this case, the manufacturer will remanufacture and resell the returned product after evaluation, at which point the profit functions of the retailer and manufacturer are

$$\begin{aligned} \pi_r^{MN} &= (p_r - w)D_r^{MN}, \\ \pi_d^{MN} &= (w-c)D_r^{MN} + \theta(p_d - c)D_d^{MN} + (1-\theta)(\rho - c - c_i)D_d^{MN}. \end{aligned} \tag{12}$$

Proposition 3. *The selling price, sales volume, and profit for the retailer and manufacturer in the scenario where returns are allowed but there is no return shipping insurance are as follows:* $p_r^{MN} = \rho\theta - c_i\theta - s\theta - \rho + 2c + c_i + s - \theta + 3/4$ *and* $p_d^{MN} = \rho\theta - c_i\theta + s\theta - \rho + c + c_i - s + \theta/2\theta$, $D_r^{MN} = (1-\theta)(\rho - s - c_i - 1)/4\theta - 4$ *and* $D_d^{MN} = -(\rho - s - c_i - 1)\theta - 2\rho + 2c + 2c_i + 2s/4\theta$, *and* $\pi_r^{MN} = -(\theta-1)^2\ (\rho - s - c_i - 1)^2/16(\theta-1)$ *and* $\pi_d^{MN} = (\rho - s - c_i - 1)^2\theta^2 - (3\rho - 4c - 3s - 3c_i + 1)(\rho - s - c_i - 1)\theta + 2(\rho - c - s - c_i)^2/8\theta$.

Proposition 3 leads to Corollary 3, which further analyses the impact of the probability θ of online goods meeting expectations on the sales price, sales volume, and profits of retailers and manufacturers.

Corollary 3. *The probability of an online purchase meeting expectations in a scenario where returns are allowed but there is no return shipping insurance the impact of* θ *on the sales price, sales volume, and profits of retailers and manufacturers is as follows:* $\partial p_r^{MN}/\partial\theta < 0$ *and* $\partial p_d^{MN}/\partial\theta > 0$, $\partial D_r^{MN}/\partial\theta = 0$ *and* $\partial D_d^{MN}/\partial\theta > 0$, *and* $\partial\pi_r^{MN}/\partial\theta < 0$ *and* $\partial\pi_d^{MN}/\partial\theta > 0$.

Corollary 3 suggests that in a scenario where returns are allowed but there is no return shipping insurance, as the probability that consumers will meet their expectations by purchasing products from online direct sales channels increases, manufacturers will set higher online direct sales prices, while retailers will choose to reduce the retail price of their products to attract consumers to continue purchasing products from retail channels. As manufacturers offer consumers a return policy in the direct online sales channel, the risk of purchasing products from the direct online sales channel is reduced, and consumers' willingness to purchase products in the direct online sales channel increases; therefore, sales in the direct online sales channel will increase. The combination of the two effects leads to higher profits for manufacturers, as both online direct sales prices and sales volumes in the online direct sales channel are on the rise. However, the effect of price reductions by retailers is very limited and does not lead to an increase in sales but only to the maintenance of the same sales volume. As a result of lower retail prices, this ultimately leads to lower profits for retailers.

Next, the impact of the cost of remanufacturing returned products c_i on the selling prices, sales volumes, and profits of retailers and manufacturers is further analyzed.

Corollary 4. *The impact of the cost of remanufacturing returned products $\partial D_r^{MN}/\partial c_i > 0$ on the selling prices, sales volumes, and profits of retailers and manufacturers in a scenario where returns are allowed but there is no return freight insurance is as follows: $\partial p_r^{MN}/\partial c_i > 0$ and $\partial p_d^{MN}/\partial c_i > 0$, $\partial D_r^{MN}/\partial c_i > 0$ and $\partial D_d^{MN}/\partial c_i < 0$, and $\partial \pi_r^{MN}/\partial c_i > 0$ and $\partial \pi_d^{MN}/\partial c_i > 0$.*

Corollary 4 suggests that in a scenario where returns are allowed but there is no return shipping insurance, as the cost of remanufacturing the returned product rises, the manufacturer will set a higher direct internet sales price to cover its remanufacturing costs. As the price of direct online sales increases, sales volume in the direct online sales channel will consequently decrease, and more consumers will choose to buy products in the retail channel, with retailers taking the opportunity to increase the retail price of their products to make more profit. Due to the higher prices set by the manufacturer for direct online sales, the volume of sales in the direct online sales channel is declining, but it is not causing a decline in the manufacturer's profit.

3.4. Returns Are Permitted and Are Covered by the Manufacturer's Return Shipping Insurance. Where returns are permitted and return shipping insurance is taken out by the manufacturer (indicated by the superscript *MR*), when a consumer's online purchase does not meet expectations, the manufacturer allows the consumer to return the product and provides return shipping insurance, with the insurance company reimbursing the consumer for the cost of return shipping r. In this case, if the consumer's online purchase meets his or her expectations, then the utility gained by the consumer is $v - p_d$; if the consumer's online purchase does not meet expectations, the utility gained by the consumer at this point is $r - s$. Therefore, in a situation where the manufacturer allows returns and provides return shipping insurance, the expected utility gained by a consumer purchasing a product in a direct online sales channel is

$$U_d^{MR} = \theta(v - p_d) + (1 - \theta)(r - s). \quad (13)$$

The utility gained by the consumer from purchasing the product in the retail channel is

$$U_r^{MR} = v - p_r. \quad (14)$$

Consistent with the previous section, consider only the case where demand exists for both channels when the demand functions for the direct online channel and the retail channel are

$$D_d^{MR} = \frac{p_r - \theta p_d + (1 - \theta)(r - s)}{1 - \theta} - \frac{\theta p_d - (1 - \theta)(r - s)}{\theta},$$
$$D_r^{MR} = 1 - \frac{p_r - \theta p_d + (1 - \theta)(r - s)}{1 - \theta}. \quad (15)$$

In this case, the manufacturer will remanufacture and resell the returned product after evaluation, where the profit functions of the retailer and manufacturer are

$$\pi_r^{MR} = (p_r - w)D_r^{MR},$$
$$\pi_d^{MR} = (w - c)D_r^{MR} + \theta(p_d - c)D_d^{MR} + (1 - \theta)(\rho - c - c_i)D_d^{MR} - tD_d^{MR}. \quad (16)$$

Proposition 4. *The selling price, sales volume, and profit of the retailer and manufacturer in the case where returns are allowed and the manufacturer buys return shipping insurance are $p_r^{MR} = M\theta - N + c + t + 3/4$ and $p_d^{MR} = (M + 2)\theta - N + 2r - 2s + t/2\theta$, $D_r^{MR} = M - t - M\theta/4\theta - 4$ and $D_d^{MR} = -M\theta^2 + (3M - 2c - t + 2)\theta - 2N + 2t/4\theta(\theta - 1)$, and $\pi_r^{MR} = -(M\theta - M + t)^2/16\theta - 16$ and $\pi_d^{MR} = F + (E + z_1 c_i + z_2\rho + z_3 r + z_4 s + z_6)\theta/8\theta(\theta - 1)$, where $M = \rho + r - s - c_i - 1$, $N = \rho - c + r - s - c_i$, $F = M^2\theta^3 - 4M(N + t/2)\theta^2 - 2(N + t)^2$, $z_1 = -10\rho + 8c - 10r + 10s + 6t + 2$, $z_2 = -8c + 10r - 10s - 6t - 2$, $z_3 = -8c - 10s - 6t - 2$, $z_4 = 8c + 6t + 2$, $z_5 = -4t + 4$, $z_6 = t^2 + 2t - 1$, and $E = 5c_i^2 + 5\rho^2 + 5r^2 + 5s^2 + 2c^2 + z_5 c$.*

Proposition 4 leads to Corollary 5, which further analyses the impact of the probability θ of an online purchase meeting expectations on the sales price, sales volume, and profit of the retailer and manufacturer in a scenario where returns are allowed and the manufacturer purchases return shipping insurance.

Corollary 5. *The impact of the probability of an online purchase meeting expectations on the retailer's and manufacturer's sales price, sales volume, and profit in a scenario where returns are allowed and the manufacturer purchases return shipping insurance is as follows: $\partial p_r^{MR}/\partial\theta < 0$ and $\partial p_d^{MR}/\partial\theta > 0$, $\partial D_r^{MR}/\partial\theta > 0$ and $\partial D_d^{MR}/\partial\theta > 0$, and $\partial \pi_r^{MR}/\partial\theta < 0$ and $\partial \pi_d^{MR}/\partial\theta > 0$.*

Corollary 5 suggests that in a scenario where returns are allowed and the manufacturer purchases return shipping insurance, as the probability that consumers will meet their expectations by purchasing products from the direct online channel increases, the manufacturer will set a higher direct online price, and the retailer will choose to reduce the retail price of the product to entice consumers to continue purchasing the product from the retail channel. As manufacturers offer consumers a return policy in the direct online sales channel, the risk of purchasing products from the direct online sales channel is reduced, and consumers' willingness to purchase products in the direct online sales channel increases; therefore, sales in the direct online sales channel will increase. The combination of the two effects leads to higher profits for manufacturers, as both online direct sales prices and sales volumes in the online direct sales channel are on the rise. Unlike the previous three scenarios (where the retailer does not increase sales by reducing prices), in this scenario, the retailer can increase sales by reducing prices. Although sales volumes increased in the retail channel, it was difficult to recover from the decline in profits.

The impact of the cost of remanufacturing returned products c_i on retailers' and manufacturers' selling prices, sales volumes, and profits is then further analyzed, leading to Corollary 6.

Corollary 6. *The impact of the cost of remanufacturing the returned product* $\partial D_d^{MR}/\partial c_i < 0$ *on the selling price, sales volume, and profit of the retailer and manufacturer in a scenario where returns are allowed and the manufacturer purchases return shipping insurance is as follows:* $\partial p_r^{MR}/\partial c_i > 0$ *and* $\partial p_d^{MR}/\partial c_i > 0$, $\partial D_r^{MR}/\partial c_i > 0$ *and* $\partial D_d^{MR}/\partial c_i > 0$, *and* $\partial \pi_r^{MR}/\partial c_i > 0$ *and* $\partial \pi_d^{MR}/\partial c_i < 0$.

Corollary 6 suggests that in a scenario where returns are allowed and the manufacturer purchases return shipping insurance, as the cost of remanufacturing the returned product rises, the manufacturer will set a higher direct internet sales price to cover its remanufacturing costs. As a result of the increase in online direct sales prices, the sales volume in the online direct sales channel will consequently decrease; more consumers will choose to buy the products in the retail channel; and retailers will take the opportunity to increase the retail price of the products to gain more profit. As manufacturers set higher prices for direct online sales, sales volumes in the direct online sales channel fall sharply, ultimately leading to lower profits for manufacturers.

Next, the impact of the cost of purchasing return shipping insurance t and the return shipping reimbursement r received after purchasing return shipping insurance on retailers' and manufacturers' sales prices, sales volumes, and profits is analyzed, leading to Corollary 7.

Corollary 7. *The effect of parameters t and r on the selling price, sales volume, and profit of the retailer and manufacturer in the case where returns are allowed and the manufacturer purchases return shipping insurance is as follows: (1)* $\partial p_r^{MR}/\partial t > 0$ *and* $\partial p_d^{MR}/\partial t > 0$, $\partial D_r^{MR}/\partial t > 0$ *and* $\partial D_d^{MR}/\partial t < 0$, *and* $\partial \pi_r^{MR}/\partial t > 0$ *and* $\partial \pi_d^{MR}/\partial t < 0$. *(2)* $\partial p_r^{MR}/\partial r < 0$ *and* $\partial p_d^{MR}/\partial r > 0$, $\partial D_r^{MR}/\partial r < 0$ *and* $\partial D_d^{MR}/\partial r > 0$, *and* $\partial \pi_r^{MR}/\partial r < 0$ *and* $\partial \pi_d^{MR}/\partial r > 0$.

Corollary 7 suggests that the higher the reimbursement of return shipping costs provided by return shipping insurance, the lower the risk to consumers of purchasing products in online direct sales channels, where returns are allowed and return shipping insurance is purchased by the manufacturer. Consumers' willingness to buy products in the direct online sales channel increases, and therefore, the sales volume in the direct online sales channel will increase. Manufacturers offering return shipping insurance to consumers make consumers more willing to buy products in online direct sales channels, so manufacturers can set higher online direct sales prices to make more profit. With the increase in return shipping reimbursement, consumers will be more willing to purchase products in the online direct sales channel, and retailers will choose to reduce the retail price of their products to attract consumers to continue to purchase products in the retail channel. However, the effect of the price cuts by retailers has been very limited, and sales in the retail channel are still down compared to before. For retailers, lower retail prices and declining sales volumes ultimately lead to lower profits for retailers.

Corollary 7 also shows that as the manufacturer's cost of purchasing return shipping insurance rises, the manufacturer will set a higher direct online price to cover its cost of purchasing return shipping insurance. As the price of direct online sales increases, sales in the direct online sales channel will consequently decline, and more consumers will turn to the retail channel to buy products, with retailers taking the opportunity to increase the retail price of their products to make more profit. As manufacturers set higher prices for direct online sales, this leads to a sharp drop in sales in the direct online sales channel, which ultimately leads to a drop in manufacturer profits.

3.5. Returns Allowed and Return Shipping Insurance Purchased by the Consumer. Where returns are permitted and the consumer has taken out return shipping insurance (indicated by the superscript *MC*) when a consumer's online purchase does not meet expectations, the manufacturer allows the consumer to return the goods, but the return shipping insurance is purchased by the consumer. In this scenario, if the consumer's online purchase does not meet expectations and needs to be returned, the insurance company reimburses the consumer for the return shipping cost $D_d^{MC} = p_r - \theta p_d + (1-\theta)(r-s) + t/1 - \theta - \theta p_d - (1-\theta)(r-s) - t/\theta$,, and the consumer receives a utility of $r - s - t$. If the consumer's online purchase meets his or her expectations, then the consumer obtains utility $v - p_d - t$. Therefore, in a scenario where returns are allowed and consumer purchases return shipping insurance, the expected utility gained by the consumer from purchasing the product in the direct online channel is

$$U_d^{MC} = \theta(v - p_d - t) + (1-\theta)(r - s - t). \tag{17}$$

The utility gained by the consumer from purchasing the product in the retail channel is

$$U_r^{MC} = v - p_r. \tag{18}$$

In line with the previous section, considering only the case where both channels are in demand, the demand functions for the direct online channel and the retail channel are

$$\begin{aligned} D_d^{MC} &= \frac{p_r - \theta p_d + (1-\theta)(r-s) + t}{1-\theta} - \frac{\theta p_d - (1-\theta)(r-s) - t}{\theta},\\ D_r^{MC} &= 1 - \frac{p_r - \theta p_d + (1-\theta)(r-s) + t}{1-\theta}. \end{aligned} \tag{19}$$

In this case, the manufacturer will remanufacture and resell the returned product after evaluation, where the profit functions of the retailer and manufacturer are

$$\begin{aligned} \pi_r^{MC} &= (p_r - w)D_r^{MC},\\ \pi_d^{MC} &= (w-c)D_r^{MC} + \theta(p_d - c)D_d^{MC} + (1-\theta)(\rho - c - c_i)D_d^{MC}. \end{aligned} \tag{20}$$

Proposition 5. *The selling price, sales volume, and profit for the retailer and manufacturer in the scenario where returns are allowed and return shipping insurance is taken out by the consumer are as follows:* $p_r^{MC} = M\theta - N + c - t + 3/4$ *and* $p_d^{MC} = (\rho - r + s - c_i + 1)\theta - \rho + c + r - s + t + c_i/2\theta$, $D_r^{MC} = M + t - M\theta/4\theta - 4$ *and* $D_d^{MC} = (3N + c - 1)\ \theta - M\theta^2 - 2N - 2t/4\theta(\theta - 1)$, *and* $\pi_r^{MC} = -(M\theta - M - t)^2/16\theta - 16$ *and* $\pi_d^{MC} = F + (E + \alpha_1 c_i + \alpha_2\rho + \alpha_3 r + \alpha_4 s + \alpha_5)\theta/8\theta(\theta - 1)$*, where* $M = \rho + r - s - c_i - 1$, $N = \rho - c + r - s - c_i$, $\alpha_1 = -10\rho + 8c - 10r + 10s - 6t + 2$, $\alpha_2 = -8c + 10r - 10s + 6t - 2$, $\alpha_3 = -8c - 10s + 6t - 2$, $\alpha_4 = 8c - 6t + 2$, $z_5 = -4t + 4$, $\alpha_5 = t^2 - 2t - 1$, $F = M^2\theta^3 - 2M(2N + t)\theta^2 - 2(N + t)^2$, *and* $E = 5c_i^2 + 5\rho^2 + 5r^2 + 5s^2 + 2c^2 + z_5 c$.

Proposition 5 leads to Corollary 8, which further analyses the impact of the probability $\partial\pi_r^{MC}/\partial\theta < 0$ of an online purchase meeting expectations on the sales price, sales volume, and profits of retailers and manufacturers in a scenario where returns are allowed and consumers purchase return shipping insurance.

Corollary 8. *The probability of an online purchase meeting expectations in a scenario where returns are allowed and return shipping insurance is purchased by the consumer. The effect of θ on the sales price, sales volume, and profit of retailers and manufacturers is as follows:* $\partial p_r^{MC}/\partial\theta < 0$ *and* $\partial p_d^{MC}/\partial\theta > 0$, $\partial D_r^{MC}/\partial\theta < 0$ *and* $\partial D_d^{MC}/\partial\theta > 0$, *and* $\partial\pi_r^{MC}/\partial\theta < 0$ *and* $\partial\pi_d^{MC}/\partial\theta > 0$.

Corollary 8 suggests that the probability of a product meeting a consumer's expectations increases as the consumer purchases it from a direct online sales channel when returns are allowed and the consumer purchases return shipping insurance. Consumers' willingness to buy products in the direct online sales channel increases, and therefore, the sales volume in the direct online sales channel will increase. At this point, the manufacturer will set a higher price for direct online sales to make more profit, while the retailer will choose to lower the retail price of the product to attract consumers to continue to buy the product in the retail channel. However, the effect of retailers' price cuts was very limited and did not undo the decline in sales in the retail channel. As sales volumes and prices fall in the retail channel, this has led to a rapid decline in retailers' profits.

The impact of the cost of remanufacturing the returned product c_i on the retailer's and manufacturer's selling prices, sales volumes, and profits is then analyzed, leading to Corollary 9.

Corollary 9. The impact of the cost of remanufacturing the returned product c_i on the retailer's and manufacturer's selling price, sales volume, and profit if returns are allowed and the consumer purchases return shipping insurance is as follows: $\partial p_r^{MC}/\partial c_i > 0$ and $\partial p_d^{MC}/\partial c_i > 0$, $\partial D_r^{MC}/\partial c_i > 0$ and $\partial D_d^{MC}/\partial c_i < 0$, and $\partial\pi_r^{MC}/\partial c_i < 0$ and $\partial\pi_d^{MC}/\partial c_i < 0$.

Corollary 9 suggests that in a scenario where returns are allowed and return shipping insurance is purchased by the consumer, as the cost of remanufacturing the returned product rises, the manufacturer will set a higher online direct selling price to cover its remanufacturing costs. As the price of direct online sales increases, sales in the direct online sales channel will consequently decrease, and more consumers will choose to purchase products in the retail channel. As manufacturers set higher prices for direct online sales, sales volumes in the direct online sales channel fall sharply, ultimately leading to lower profits for manufacturers.

Next, the impact of the cost t for consumers to purchase return shipping insurance and the return shipping reimbursement r received after purchasing return shipping insurance on retailers' and manufacturers' sales prices, sales volumes, and profits is analyzed, leading to Corollary 10.

Corollary 10. The effect of parameters r and t on the selling prices, sales volumes, and profits of retailers and manufacturers in a situation where returns are allowed and consumers purchase return shipping insurance is as follows: (1) $\partial p_r^{MC}/\partial r < 0$ and $\partial p_d^{MC}/\partial r > 0$, $\partial D_r^{MC}/\partial r < 0$ and $\partial D_d^{MC}/\partial r > 0$, and $\partial\pi_r^{MC}/\partial r < 0$ and $\partial\pi_d^{MC}/\partial r > 0$. (2) $\partial p_r^{MC}/\partial t < 0$ and $\partial p_d^{MC}/\partial t > 0$, $\partial D_r^{MC}/\partial t < 0$ and $\partial D_d^{MC}/\partial t > 0$, and $\partial\pi_r^{MC}/\partial t < 0$ and $\partial\pi_d^{MC}/\partial t > 0$.

Corollary 10 suggests that in a scenario where returns are allowed and consumers purchase return shipping insurance, the higher the return shipping reimbursement provided by the purchase of return shipping insurance, the lower the risk of consumers purchasing products in the direct online sales channel, the greater the willingness of consumers to purchase products in the direct online sales channel, and therefore, the greater the sales volume in the direct online sales channel. Higher return shipping reimbursement makes consumers more willing to buy products in online direct sales channels, so manufacturers can set higher online direct sales prices to make more profit, while retailers will choose to lower the retail price of their products to attract consumers to continue to buy products in retail channels. However, the effect of the retailers' price cuts was very limited and did not undo the decline in sales in the retail channel. For retailers, lower retail prices and declining sales volumes ultimately lead to lower profits for retailers.

4. Results

4.1. Discussion

4.1.1. Refund Strategy Selection. To analyze whether a manufacturer should offer a refund policy in the online direct sales channel, this section will provide some management insights by comparing the changes in the manufacturer's sales price, sales volume, and profit in a no return allowed (*NN*) scenario and a refund not return (*RN*) scenario.

Proposition 6. *The impact of refund policies on retailers' and manufacturers' selling prices, sales volumes, and profits is expressed as follows:*

(1) $p_r^{RN} < p_r^{NN}$, $p_d^{RN} > p_d^{NN}$, $D_r^{RN} = D_r^{NN}$, $D_d^{RN} > D_d^{NN}$, *and* $\pi_r^{RN} < \pi_r^{NN}$

(2) *When* $\varphi > \theta^2 - 2c^2/\theta(\theta - 1)$, $\pi_d^{RN} > \pi_d^{NN}$; *when* $0 < \varphi < \theta^2 - 2c^2/\theta(\theta - 1)$, $\pi_d^{RN} < \pi_d^{NN}$

Proposition 6 suggests that manufacturers set higher direct internet sales prices when they offer a refund policy in the direct internet sales channel. Manufacturers offering refund policies in the online direct sales channel can bring more sales to the online direct sales channel. Retailers will choose to reduce the retail price of their products to attract consumers to continue to purchase products in the retail channel. However, it is important to note that price reductions by retailers have a very limited effect and do not lead to an increase in sales in the retail channel, only to maintain the same sales volume. As a result, manufacturers offering refund policies in online direct sales channels will inevitably infringe on retailers' profits. However, the variation in the manufacturer's own profitability depends specifically on the residual value of the returned product. By offering a refund policy in the online direct sales channel when the residual value of returned products is low, the manufacturer will infringe on the retailer's profitability while at the same time suffering a loss of its own. Only if the residual value of the returned product is high can the manufacturer make itself more profitable by offering a refund policy in the online direct sales channel. This suggests that manufacturers should decide whether to offer a refund policy in the online direct sales channel based on the residual value of the returned product; otherwise, the action is likely to be detrimental to others.

4.1.2. Return Strategy Selection. To analyze the choice of return strategy for a manufacturer's online direct sales channel, this section will provide some management insights by comparing the changes in selling prices, sales volumes, and profits of retailers and manufacturers in the no returns allowed (*NN*) scenario and the no return shipping insurance (*MN*) scenario.

Proposition 7. *The impact of return policies on the selling prices, sales volumes, and profits of retailers and manufacturers is demonstrated by the following:*

(1) *When* $s \geq \rho - c_i$, $p_r^{MN} \geq p_r^{NN}$, $D_r^{MN} \geq D_r^{NN}$, $D_d^{MN} \leq D_d^{NN}$, $\pi_r^{MN} \geq \pi_r^{NN}$, *and* $\pi_d^{MN} \geq \pi_d^{NN}$; *when* $s \leq \rho - c_i$, $p_r^{MN} \leq p_r^{NN}$, $D_r^{MN} \leq D_r^{NN}$, $D_d^{MN} \geq D_d^{NN}$, *and* $\pi_d^{MN} \leq \pi_d^{NN}$

(2) *When* $s \leq c + c_i + \theta - \rho$, $p_d^{MN} \geq p_d^{NN}$; *when* $s \geq c + c_i + \theta - \rho$, $p_d^{MN} \leq p_d^{NN}$

(3) *When* $s \geq \rho - c_i$ *or* $s \leq \rho - c_i - 2$, $\pi_r^{MN} \geq \pi_r^{NN}$; *when* $\rho - c_i - 2 \leq s \leq \rho - c_i$, $\pi_r^{MN} \leq \pi_r^{NN}$

Proposition 7 suggests that when return shipping costs are low, that is, when $s \leq c + c_i + \theta - \rho$, *the manufacturer sets a higher selling price when offering a return policy in the online direct sales channel than when returns are not allowed. Conversely, when return shipping costs are high, manufacturers set lower selling prices when offering return policies in online direct sales channels than when returns are not allowed. When the return shipping cost is lower than the profit from remanufacturing and reselling the returned product, that is,* $s \leq \rho - c_i$, *the manufacturer sells more when the return policy is offered in the online direct sales channel than when returns are not allowed, but the profit is lower instead. When return shipping costs are higher than the profit from remanufacturing and reselling the returned product, that is,* $s \geq \rho - c_i$, *the manufacturer's selling price, sales volume, and profit are higher when the return policy is offered in the online direct sales channel than when returns are not allowed. Therefore, manufacturers need to determine whether to offer a return policy based on the cost of return shipping and the size of the marginal profit from remanufacturing and reselling the returned product.*

4.1.3. Return Shipping Insurance Strategy Selection. To analyze the strategic choice of whether to purchase return shipping insurance and whether it is purchased by the manufacturer or by the consumer, this section will provide some management insights by comparing the equilibrium results of three scenarios: allowing returns but no return shipping insurance (*MN*), allowing returns and having the manufacturer purchase return shipping insurance (*MR*), and allowing returns and having the consumer purchase return shipping insurance (*MC*).

Proposition 8. *The effect of the manufacturer's purchase of return shipping insurance on the equilibrium outcome is expressed as follows:*

(1) $p_d^{MR} \geq p_d^{MN}$

(2) *When* $0 < r \leq t/1 - \theta$, $p_r^{MR} \geq p_r^{MN}$, $D_r^{MR} \geq D_r^{MN}$, $D_d^{MR} \geq D_d^{MN}$, $\pi_r^{MR} \geq \pi_r^{MN}$, *and* $\pi_d^{MR} \leq \pi_d^{MN}$

(3) *When* $r \geq t/1 - \theta$, $p_r^{MR} \leq p_r^{MN}$, $D_r^{MR} \leq D_r^{MN}$, $D_d^{MR} \geq D_d^{MN}$, $\pi_r^{MR} \leq \pi_r^{MN}$, *and* $\pi_d^{MR} \geq \pi_d^{MN}$

Proposition 8 suggests that manufacturers will set higher prices for direct online sales when purchasing return shipping insurance compared to the no return shipping insurance scenario. This is because it will undoubtedly increase the manufacturer's costs of offering free return shipping insurance to consumers, so the manufacturer will inevitably set a higher direct internet price to cover the cost of purchasing return *shipping insurance. It is also clear from Proposition 8 that when the return shipping reimbursement received after purchasing return shipping insurance is low, the manufacturer's provision of free return shipping insurance to consumers while generating more sales for the online direct sales channel can hurt its own profits. Manufacturers offering free return shipping insurance to consumers can both generate more sales for the online direct sales channel and bring in more profit for themselves only if the return shipping reimbursement received after purchasing return shipping insurance is high. It follows that whether a manufacturer offers free return shipping insurance to consumers depends on the size of the reimbursement of return shipping costs received after purchasing return shipping insurance.*

Proposition 9. *The effect of consumer purchase of return shipping insurance on the equilibrium outcome is expressed as follows:* $p_r^{MC} \leq p_r^{MN}$, $p_d^{MC} \geq p_d^{MN}$, $D_r^{MC} \leq D_r^{MN}$, $D_d^{MC} \geq D_d^{MN}$, $\pi_r^{MC} \leq \pi_r^{MN}$, *and* $\pi_d^{MC} \geq \pi_d^{MN}$.

Proposition 9 suggests that the risk of return of products purchased in online direct sales channels is also lower when consumers purchase return shipping insurance compared to the no return shipping insurance scenario, and consumers' willingness to purchase products in online direct sales channels increases. As a result, sales volume in the direct online channel will increase, and manufacturers can take advantage of the opportunity to set higher prices for direct online sales to make more profit. As consumers become more willing to purchase products in online direct sales channels, retailers will choose to reduce the retail price of their products to attract consumers to continue purchasing products in retail channels. However, the effect of the retailers' price cuts has been very limited, and sales in the retail channel are still down compared to before. For retailers, lower retail prices and declining sales volumes ultimately lead to lower profits for retailers.

Proposition 10. *The effect of the purchase of return shipping insurance by different subjects on the equilibrium outcome is shown by the following:*

(1) $p_r^{MC} \leq p_r^{MR}$, $p_d^{MC} = p_d^{MR}$, $D_r^{MC} \leq D_r^{MR}$, *and* $D_d^{MC} \geq D_d^{MR}$

(2) *When* $r \geq s + c_i + 1 - \rho$, $\pi_r^{MC} \geq \pi_r^{MR}$, *and* $\pi_d^{MC} \geq \pi_d^{MR}$

(3) *When* $0 < r \leq s + c_i + 1 - \rho$, $\pi_r^{MC} \leq \pi_r^{MR}$, *and* $\pi_d^{MC} \leq \pi_d^{MR}$

Proposition 10 suggests that the price set by the manufacturer for direct online sales is the same whether the manufacturer purchases the return shipping insurance or the consumer purchases the return shipping insurance. However, when it is up to the consumer to purchase return shipping insurance, sales are higher in the direct online channel and lower in the retail channel in terms of both retail price and sales volume. Proposition 10 also shows that return shipping insurance should be provided free of charge by the manufacturer to the consumer when the return shipping reimbursement received after the purchase of return shipping insurance is low, at which point both the manufacturer and the retailer make a higher profit. Conversely, it will be better for consumers to purchase their own return shipping insurance when they receive higher reimbursement for return shipping costs after purchasing return shipping insurance. The findings of this study can provide good management insights into the choice of return shipping insurance purchase strategies in commercial practice.

4.1.4. Remanufacturing Strategy Selection. To explore whether manufacturers should remanufacture returned products and analyze the impact of remanufacturing costs, this section provides some management insights by comparing changes in manufacturers' selling prices, sales volumes, and profits in the no-return-allowed (*NN*) and refund-no-return (*RN*) scenarios.

Proposition 11. *The impact of returned product remanufacturing costs on retailers' and manufacturers' selling prices, sales volumes, and profitability performance is as follows:*

(1) *When* $c_i \geq \rho - s$, $p_r^{MN} \geq p_r^{NN}$, $D_r^{MN} \geq D_r^{NN}$, $D_d^{MN} \leq D_d^{NN}$, $\pi_r^{MN} \geq \pi_r^{NN}$, *and* $\pi_d^{MN} \geq \pi_d^{NN}$; *when* $c_i \leq \rho - s$, $p_r^{MN} \leq p_r^{NN}$, $D_r^{MN} \leq D_r^{NN}$, $D_d^{MN} \geq D_d^{NN}$, *and* $\pi_d^{MN} \leq \pi_d^{NN}$

(2) *When* $c_i \geq s - c - \theta + \rho$, $p_d^{MN} \geq p_d^{NN}$; *when* $c_i \leq s - c - \theta + \rho$, $p_d^{MN} \leq p_d^{NN}$

(3) *When* $c_i \geq \rho - s$ *or* $c_i \leq \rho - s - 2$, $\pi_r^{MN} \geq \pi_r^{NN}$; *when* $\rho - s - 2 \leq c_i \leq \rho - s$, $\pi_r^{MN} \leq \pi_r^{NN}$

Proposition 11 shows that when the cost of remanufacturing the returned product is higher, that is, $c_i \geq s - c - \theta + \rho$, *the manufacturer will set a higher direct network sales price when implementing a remanufacturing strategy. Conversely, when the cost of remanufacturing the returned product is low, that is,* $c_i \leq s - c - \theta + \rho$, *the manufacturer will set a lower direct network price when implementing a remanufacturing strategy. When* $c_i \geq \rho - s$, *the manufacturer's remanufacturing strategy does not result in more sales for the online direct sales channel; it does result in more profit for both the manufacturer and the retailer. Therefore, manufacturers need to decide whether to remanufacture returned products based on the cost of remanufacturing the returned product.*

4.2. Sensitivity Analysis. To verify the above theoretical model conclusions more intuitively, the model will be verified numerically by setting parameters that match reality. Depending on the conditions of the theoretical model and ensuring that the parameters are in the valid range, it may be useful to set the following selection of parameters: $c = 0.4$, $r = 0.06$, $t = 0.03$, $\theta = 0.8$, $s = 0.08$, $\varphi = 0.85$, $c_i = 0.2$, and $\rho = 0.85$. The following section analyzes the impact of the probability of a platform purchase meeting consumer demand, the cost per unit of product returned, the proportion of salvage value per unit of the returned product, and the cost of remanufacturing the returned product on the optimal decision.

4.2.1. Impact of the Probability of Consumer Demand Satisfaction. Figures 1 and 2 show the effect of the probability of the product satisfying the consumer on the optimal price and the optimal profit, respectively. As shown in Figure 1, offline retailers have seen prices fall as the probability of satisfying consumers by purchasing products online has increased. This is due to increased consumer satisfaction with online purchases and increased demand online, with offline retailers having no choice but to reduce the price at which they sell their products to attract customers. Online prices, however, vary under different return strategies, with prices rising under the *NN* strategy, *MN* strategy, *MR* strategy, and *MC* strategy and falling under the *RN* strategy. This is because the manufacturer will only allow refunds and not returns under the *RN* strategy, which will affect the consumer's online shopping experience, and the manufacturer will have no choice but to sell at a reduced price to retain customers. From Figure 1, it can be seen that manufacturers priced highest under the *MR* and *MC* strategies, followed by the *MN*, *NN*, and *RN* strategies. The reason for this is that under the *MR* strategy, the cost of return insurance is borne by the manufacturer, who raises the price of the product to cover this cost. Under the *MC* strategy, although the return insurance is paid for by the consumer, the demand from the manufacturer increases, and the manufacturer raises the price of the product to make more profit. In contrast, manufacturers in the *MN*, *NN*, and *RN* strategies do not offer return insurance or do not allow returns, which increases the risk that consumers will buy online and have to adopt a low-price strategy to attract consumers. For offline retailers, the product price is highest under the *NN* strategy with the lowest probability of meeting demand, that is, when θ is smallest, followed by the *MN* strategy, *MR* strategy, *MC* strategy, and *RN* strategy. This suggests that not allowing returns will significantly affect the manufacturer's demand, allowing the manufacturer to increase the price at which the product is sold. Under the *MN* strategy, the manufacturer does not provide free return insurance but allows returns, at which point the offline retailer can only lower the price of the product to attract consumers. Under the *MR* and *MC* strategies, manufacturers offer free return insurance to consumers or allow consumers to purchase their own insurance, greatly reducing the risk of online purchases, and offline retailers can only sell their products at lower prices to capture the customer base. Under the *RN* strategy, although the manufacturer does not provide return insurance and does not allow consumers to purchase their own return insurance, consumers can leave unsatisfactory products behind and receive a full refund from the merchant. The offline retailer sells its products at the lowest price under the *NN* strategy when the product satisfaction rate is high, that is, when θ is large. This is because when online shopping can satisfy consumers' needs to a greater extent, the convenience of online shopping will drive consumers to buy online although the merchant does not provide free return insurance, and the offline retailer can only sell its products at a reduced price. By comparing the prices of manufacturers and retailers under different strategies, it can be seen that when θ is low, the prices of offline retailers are higher than those of manufacturers, and as θ increases, the prices of manufacturers will gradually be higher than those of offline retailers. When the increase in consumer satisfaction with online purchases will attract more consumers, offline retailers can only occupy the market through a low-price strategy.

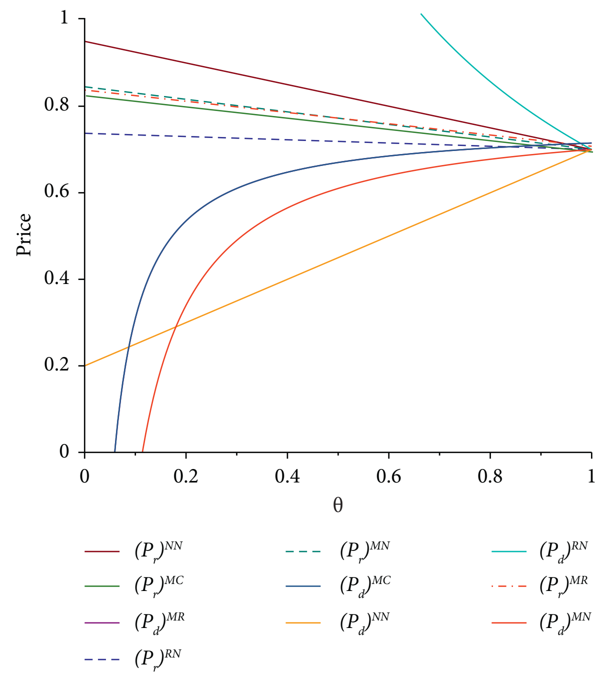

Figure 1: Effect of θ on optimal price.

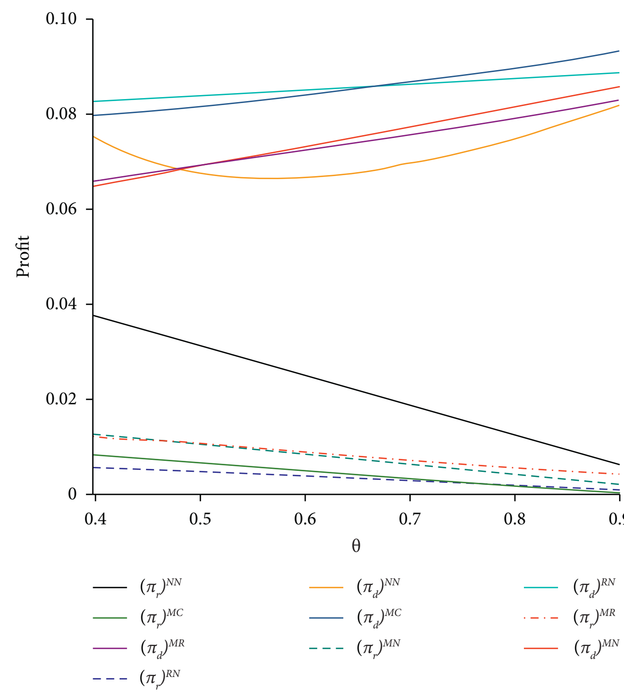

Figure 2: Effect of θ on optimal profit.

As shown in Figure 2, as θ increases, the profits of the retailers all decrease, while the profits of the manufacturers all

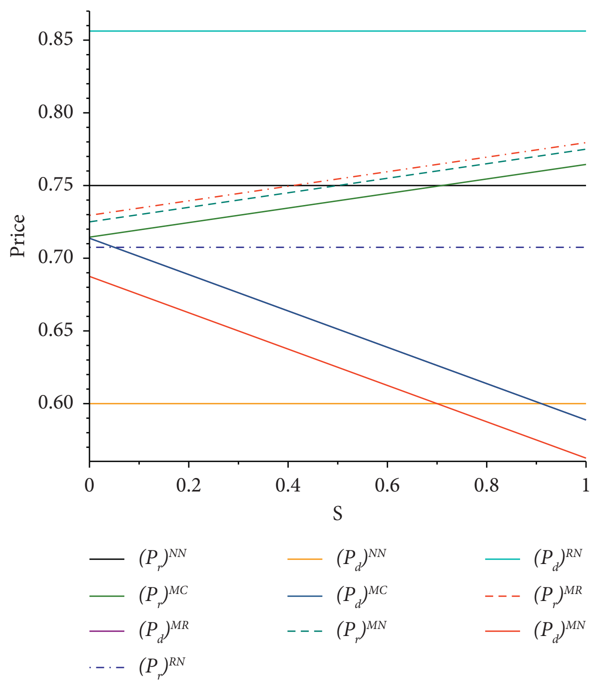

FIGURE 3: Effect of s on optimal price.

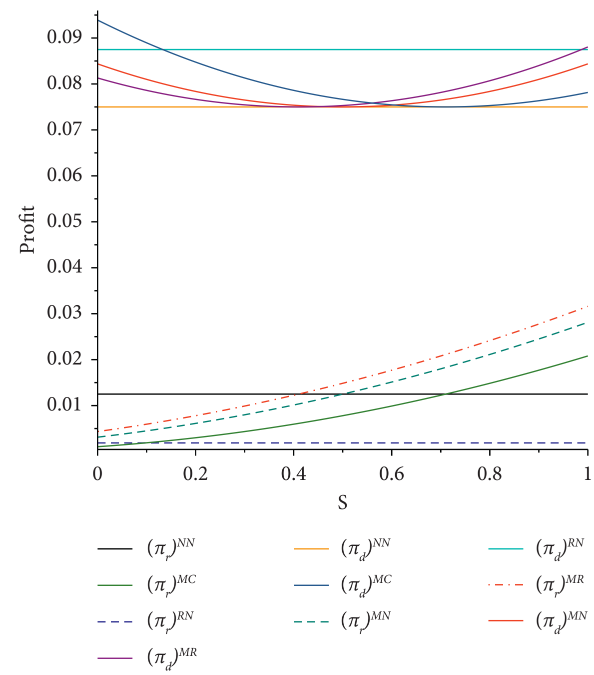

FIGURE 4: Effect of s on optimal profit.

increase. The reason for this is obvious: when the probability of online shopping meeting consumer demand is high, both demand and prices rise for manufacturers and fall for retailers, which leads to an upward and downward trend in profits for both manufacturers and retailers. For manufacturers, profits are highest under the *RN* strategy when θ is small because neither the manufacturer nor the consumer has to bear return costs under the *RN* strategy, and thus, demand increases, and therefore, profits are highest. Profits are highest under the *MC* strategy when θ is large. Therefore, when online shopping satisfaction is low, manufacturers should be more likely to opt for *RN* strategies, that is, offering refunds only without the need to return goods. The *MC* strategy should be chosen more often when online purchase satisfaction is high. For offline retailers, profits are highest under the *NN* strategy when θ is low, as consumers are less satisfied with their online purchases and will naturally choose to buy offline more often, increasing the profits of offline retailers accordingly. In concrete practice, therefore, both manufacturers and retailers should pay close attention to the probability that a platform purchase satisfies consumer demand, that is, θ. Offline retailers can price their goods accurately according to θ, and manufacturers should improve θ in many ways, such as the quality of goods and services, to ensure that they can have higher prices and profits.

4.2.2. Impact of Unit Product Return Costs. Figure 3 shows the impact of consumer return costs per unit of product on manufacturer and retailer sales prices in different scenarios. The graph shows that when the return cost per unit of product is low, the offline retailer has the highest selling price under the manufacturer's no returns allowed, that is, the *NN* strategy, and as the return cost per unit of product increases, the offline retailer has the highest selling price under the *MR* strategy. For manufacturers, regardless of the size of the return cost per unit of product, their selling price is highest at the time of the *RN* strategy, that is, refunds without returns, and is always higher than the selling price of offline retailers. Under the *RN* strategy, consumers can offer refunds for unsatisfactory goods but do not need to return the goods, so no return costs are incurred, and the goods left behind still have some use, hence the high demand and high selling prices of the products on the online platform under the *RN* strategy.

Figure 4 shows the impact of consumer return costs per product on manufacturer and retailer profits in different scenarios. As seen from the graph, the manufacturer's profit is always greater than the retailer's profit, regardless of the size of the return cost per unit of product and whether the manufacturer offers a return policy or return insurance. Specifically, offline retailers are most profitable when the return cost per unit of product is small and when the *NN* strategy, that is, the manufacturer does not offer a return strategy, is in place. Because the online sales channel does not allow consumers to return unsatisfactory products at this time, all losses will be borne by the consumer in the event of a failed purchase, and consumers prefer to shop offline to avoid this risk. As the cost per unit of product returned increases, the offline retailer's profits are greatest when the *MR* strategy, that is, the manufacturer, allows returns and provides return insurance. For manufacturers, the highest profit is made when the return cost per unit of product is small and when the *MC*, that is, the consumer, purchases his own return insurance. As the cost per unit of product returned increases, manufacturers are most profitable under the *RN*, refund not return strategy, and the *MR*, manufacturer-provided return insurance strategy, as neither strategy requires the consumer to bear the loss caused by the return of the product.

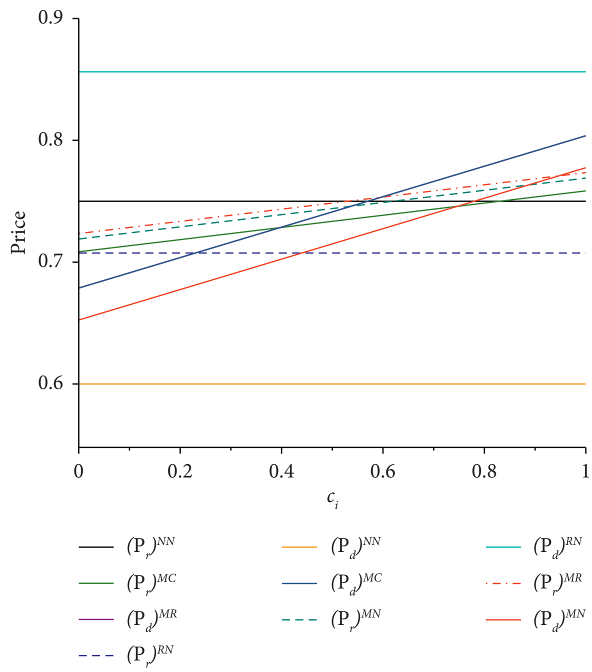

Figure 5: Effect of c_i on optimal price.

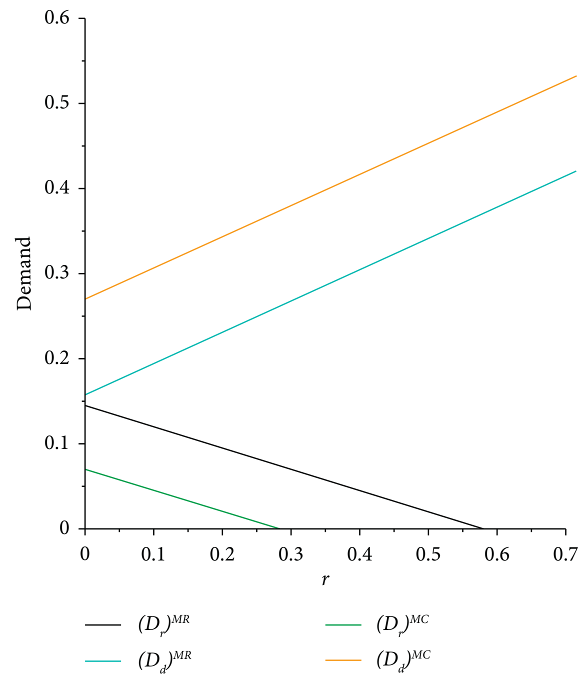

Figure 7: Effect of r on optimal demand.

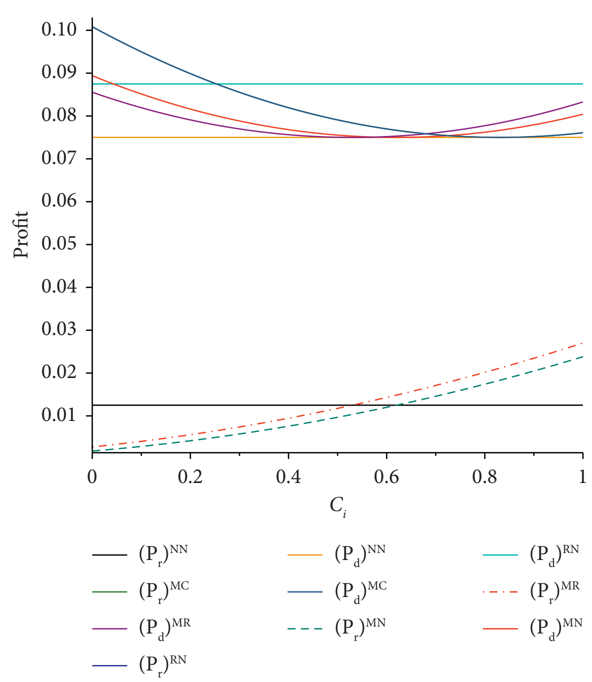

Figure 6: Effect of c_i on optimal profit.

4.2.3. Impact of Remanufacturing Costs per Unit of Returned Product. Figure 5 shows the impact of the remanufacturing cost per unit of returned product on the sales prices of manufacturers and retailers under different scenarios. As shown in the graph, when the cost of remanufacturing the returned product is small, the offline retailer's price is greatest when the *NN* strategy, that is, the manufacturer, does not allow returns. As the cost of remanufacturing returned products increases, the selling price for offline retailers is highest under the *MR* strategy when the manufacturer provides return insurance. For manufacturers, the selling price of their products is always the highest in the *RN* strategy, that is, when the manufacturer allows the consumer a refund but not a return. This is because, under this strategy, the manufacturer may lose some of the product due to consumer dissatisfaction and therefore set a higher selling price, whereas under the *NN* strategy, that is, when the manufacturer does not allow returns, the manufacturer's selling price is always the lowest.

Figure 6 shows the impact of the remanufacturing cost per unit of returned product on the profits of manufacturers and retailers in different scenarios. The graph shows that retailers' profits are highest under the *MC* strategy when the remanufacturing cost per unit of the returned product is low, that is, when consumers purchase their own return insurance, and that retailers' profits are always highest under the *MC* strategy as the remanufacturing cost per unit of returned product increases and as the return policy changes. For the manufacturer, the manufacturer's profit is highest under the *MC* strategy when the remanufacturing cost per returned product is low, and as the remanufacturing cost of the returned product increases, the manufacturer's profit is greatest under the *RN* strategy.

4.2.4. Impact and Reimbursement Return Shipping Costs after Purchase of Return Shipping Insurance and Return Shipping Costs. Figure 7 shows the impact of return shipping reimbursement on consumer demand following the purchase of return shipping insurance under the *MR* and *MC*

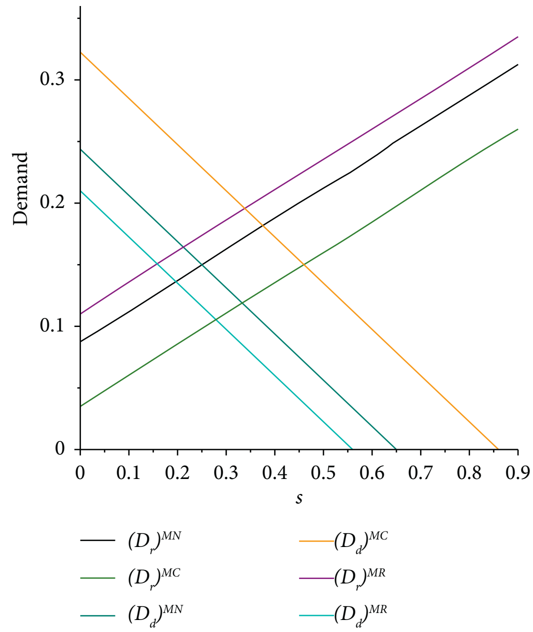

Figure 8: Effect of s on optimal demand.

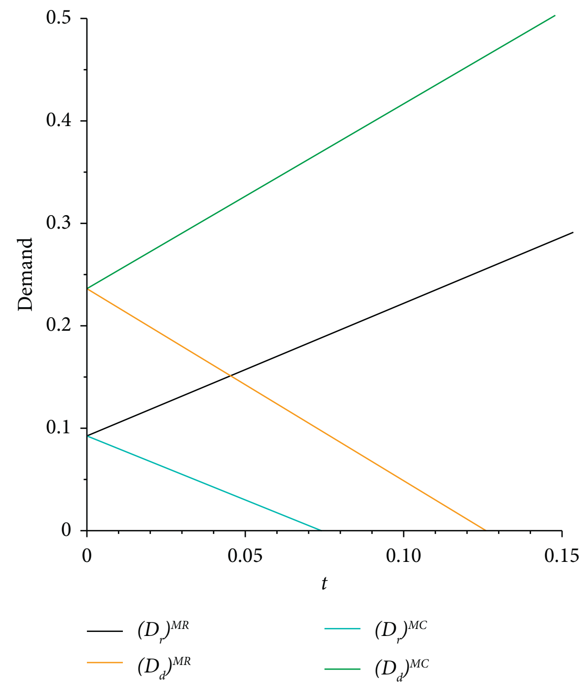

Figure 9: Effect of t on optimal demand.

strategies. As shown in the graph, under *MR* and *MC* strategies, the demand for the retailer's product decreases at the same rate as the amount of return compensation per unit of product increases, while the demand for the manufacturer's product increases monotonically and at the same rate as the amount of return compensation increases. In addition, product demand is always at a high level for manufacturers under both strategies and is higher under the *MC* strategy than the *MR* strategy. This suggests that the increasingly sophisticated return service for online sales has made consumers more inclined to choose the convenient online shopping platform when shopping, and that brick-and-mortar retail is bound to take a hit in this scenario. As a result, manufacturers can gain a larger market share by increasing the amount of compensation for product returns, and return strategies can be more profitable by allowing consumers to purchase their own return shipping insurance. For offline retailers, there is a need to ensure product quality and improve service levels to reduce the impact of e-commerce platforms.

Figure 8 shows the impact of return shipping costs on consumer demand under the *MN*, *MR*, and *MC* strategies. The graph shows that retailers' product demand and return shipping costs are positively related under the three strategies, while manufacturers' product demand and return shipping costs are negatively related. When return shipping costs are low, demand for manufacturers' products, although trending downwards, is still higher than that of retailers. With the rise in return shipping costs, manufacturers are gradually at a disadvantage in terms of sales. The obvious reason is that the high cost of return shipping increases the risk of online shopping for consumers who will turn to the more secure option of offline shopping. The graph shows that the retailer's product demand is highest under the *MR* strategy when the manufacturer will offer consumers return insurance and, accordingly, higher product prices. As a result, the demand for products under the *MR* strategy is lowest for manufacturers. Consumers are always looking for good quality products for less money, especially price-sensitive consumers. Therefore, manufacturers may choose to allow consumers to purchase their own shipping insurance to reduce the risk of online shopping, rather than offering them free shipping insurance by increasing the price of their products.

4.2.5. Effect of the Cost of Purchasing Return Shipping Insurance and the Price of Remanufacturing the Returned Product for Resale. Figure 9 shows the impact of the cost of purchasing return shipping insurance on consumer demand under the *MR* and *MC* strategies. The graph shows that under the *MR* strategy, the retailer's demand for the product increases monotonically with the cost of freight insurance; the manufacturer's demand for the product decreases monotonically with the cost of freight insurance. Under the *MC* strategy, the retailer's product decreases monotonically with the cost of freight insurance, while the manufacturer's demand increases monotonically. In terms of the magnitude of change, the change in demand for manufacturers was greater than for retailers under both strategies. This suggests that the cost of purchasing return shipping insurance has a greater impact on the manufacturer. In addition, the demand for the manufacturer's products under the *MC* strategy is always at the highest level, while the retailer's demand is at the lowest level. The reason for this is that by not providing return insurance, the manufacturer can reduce costs and therefore the selling price of the product. By capturing consumers' preferences for low prices, you can capture a favorable market. It follows that the adverse impact

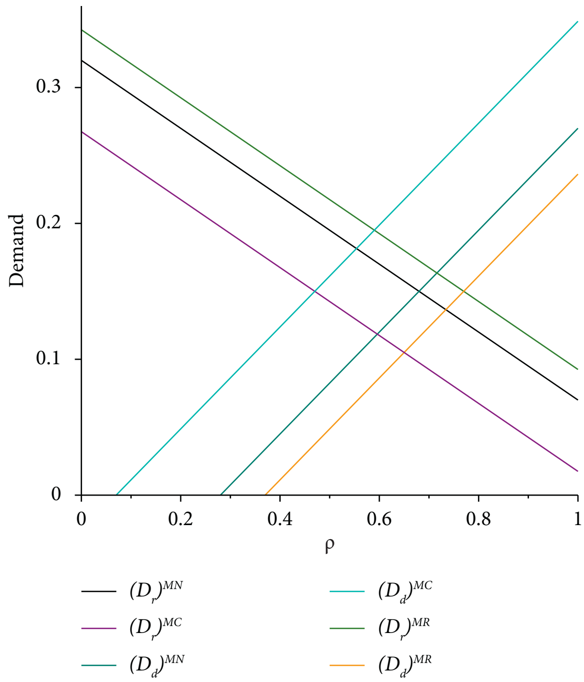

FIGURE 10: Effect of ρ on optimal demand.

of return shipping insurance on manufacturers can be reduced by making reasonable return policies.

Figure 10 shows the impact of the price of remanufacturing the returned product for resale under the *MN*, *MR*, and *MC* strategies on consumer demand. As shown in the graph, the retailer's product demand is negatively related to the price at which the returned product is remanufactured for resale, and conversely, the manufacturer's product demand is positively related to the price at which it is resold. Retailers have a higher demand for their products than manufacturers when the price of resale is lower. With higher sales prices, demand from manufacturers outpaced retailers and was at its highest under the *MC* strategy. This suggests that higher resale prices are more favorable to manufacturers because of the complete return service of the online sales channel. Therefore, it is more advantageous to choose online channels for the sale of returned and remanufactured products.

5. Managerial Insights and Practical Implications

Remanufacturing refers to the use of used products as blanks, using special technology and techniques to carry out new manufacturing based on the original product, the remanufactured product is no less than the new product in terms of quality and performance. Remanufacturing technology not only extends the life of products but also allows for near-zero waste of energy resources. At a time when resources and energy are relatively scarce, the advantages of remanufacturing technology cannot be ignored. In 2021, the State Council issued a guiding opinion on "accelerating the establishment of a sound green low-carbon cycle development economic system," proposing to promote industrial green upgrading, vigorously develop the remanufacturing industry, and strengthen the certification and promotion and application of remanufactured products. In the context of "double carbon," it is important to support remanufacturing in key industries to develop a green circular economy.

With the rapid growth in e-commerce transactions, the number of returns has exploded. Forecast data published by the NRF show that e-commerce returns spending will increase to US$604 billion, which will create huge pressure on warehousing. As a result, the question of how to deal with returns has become a real issue for merchants to consider. To relieve the pressure on warehousing, Amazon will advise sellers to dispose of their products for free through a donation scheme, which is undoubtedly extremely wasteful. For its part, Pangu De Ho has launched the De Ho after-sales service platform to solve overseas after-sales and returns problems for Chinese sellers. In the face of a large number of returns due to COVID-19 and other reasons, the platform has introduced four core services: return sorting, labeling and label exchange, quality inspection and refurbishment, and product repair. These services will effectively reduce the negative impact of returns on businesses and reduce environmental pressure. This paper studies the impact of returned product remanufacturing on the return strategy of supply chain members during the online sales process and provides theoretical guidance for online merchants in dealing with returns and the remanufacturing of returned products.

6. Conclusions

This paper investigates the optimal pricing decision problem for four scenarios in which the manufacturer does not offer a return strategy (*NN* strategy), the manufacturer offers a refund strategy but does not offer a return strategy (*RN* strategy), the manufacturer offers a return strategy but does not offer return insurance (*MN* strategy), the manufacturer offers a return strategy and offers return insurance (*MR* strategy), and the manufacturer offers a return strategy but the consumer purchases return insurance (*MC* strategy) under a competitive channel between the manufacturer and the retailer. The comparative study also answers the five questions of whether to offer a refund strategy, whether to offer a return strategy, whether to offer return insurance, who covers the return insurance, and what is the impact of returning recycled products on the return strategy of the supply chain members. The results of the study not only provide theoretical support for the study of the mechanism of combining return insurance and return strategy but also provide guidance for companies to develop appropriate return strategies.

Therefore, the results are as follows:

(1) Regarding whether to offer a refund strategy, by comparing the *NN* and *RN* strategies, we found that the manufacturer's selling price and product sales were always higher when offering a refund strategy, while the retailer's selling price and profits were lower. Therefore, it is more advantageous for the manufacturer to offer a refund strategy. Further

analysis of the impact of the residual value of the returned product on the manufacturer's profitability shows that when the residual value of the returned product is large, the manufacturer can make more profit by offering a refund strategy; when the residual value of the returned product is small, it is more beneficial not to offer a refund strategy.

(2) Regarding whether to offer a return strategy, a comparison of *NN* and *MN* strategies revealed that manufacturers offering return strategies sell their products at higher prices, in higher volumes, and at higher profits when the cost of return to consumers is lower. However, when the cost of return to the consumer is higher, the manufacturer's profit is reduced instead. Manufacturers can therefore choose the *MN* strategy when return shipping costs are low and the *NN* strategy when return shipping costs are high.

(3) About whether to provide return insurance, a comparison of the *MN*, *MR*, and *MC* strategies shows that manufacturers should set a higher selling price under the *MR* strategy when return compensation is greater, at which point the manufacturer should choose to provide consumers with return insurance to make more profit. When return shipping costs are high, manufacturers should choose to allow consumers to purchase their own return insurance, given the increase in price that would affect product sales if merchants offered return insurance. Further comparison of the *MR* and *MC* strategies reveals that the manufacturer's optimal selling price is the same under both strategies but that profits are higher under the *MC* strategy when the return compensation is greater when it is more advantageous for the manufacturer to have the return insurance borne by the consumer.

(4) Regarding the impact of returning remanufactured products on the return strategy of supply chain members, a comparison of *MN*, *MR*, and *MC* strategies revealed that although the increase in the cost of remanufacturing causes the manufacturer's profit to go through a process from decreasing to increasing, the manufacturer's profit is always higher than that of the retailer. When remanufacturing costs are low, the manufacturer may choose the *MC* strategy; when remanufacturing costs are high, it is more advantageous for the manufacturer to choose the *MR* strategy. Further analysis of the impact of the price of remanufactured resales of returned products on demand found that manufacturer product sales were highest under the *MC* strategy when the resale price was higher. Therefore, it is more advantageous for manufacturers to choose to allow consumers to purchase their own return insurance when selling remanufactured products in terms of the price at which the product is sold.

While this paper examines issues around whether to provide a return strategy, whether to provide return insurance, who provides return insurance, and the impact of returning recycled products on the return strategy of supply chain members and informs manufacturers' return decisions, it does not consider enough the behavior of consumers and recyclers. For example, studying consumers' willingness to accept remanufactured products, taking into account the psychological cost to consumers and the functional quality of the product,exploring the decision-making behavior of recyclers in the remanufacturing process based on carbon reduction and government subsidy policies,the relationship between consumers' willingness to buy and return services for both regular and remanufactured products, all of which will be worthy of future research.

Appendix

Proof of Proposition 1. The second-order derivatives of the profit functions of the manufacturer and the retailer with respect to p_d and p_r, respectively, are obtained: $\partial^2 \pi_d^{NN}/\partial (p_d^{NN})^2 = -2/\theta(1-\theta) < 0$ and $\partial^2 \pi_r^{NN}/\partial (p_r^{NN})^2 = -2/1-\theta < 0$. This gives the Hessian matrix $\begin{vmatrix} -2/1-\theta & 1/1-\theta \\ 1/1-\theta & -2/\theta(1-\theta) \end{vmatrix} = 4-\theta/\theta(1-\theta)^2 > 0$, that is, there is a unique optimal solution. From the first-order condition, $\partial \pi_r^{NN}/\partial p_r^{NN} = 0$; $\partial \pi_d^{NN}/\partial p_d^{NN} = 0$; and $\partial \pi_d^{NN}/\partial w = 0$. The authors have $p_r^{NN} = 3+2c-\theta/4$, $p_d^{NN} = \theta + c/2$, and $w = c + 1/2$. Substituting p_r^{NN}, p_d^{NN}, and w into the demand and profit functions gives the optimal demand and profit for retailers and manufacturers.

Proof of Corollary 1. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to θ to give $\partial p_r^{NN}/\partial\theta = -1/4 < 0$, $\partial p_d^{NN}/\partial\theta = 1/2 > 0$, $\partial D_r^{NN}/\partial\theta = 0$, $\partial D_d^{NN}/\partial\theta = c/2\theta^2 > 0$, $\partial \pi_r^{NN}/\partial\theta = -1/16 < 0$, and $\partial \pi_d^{NN}/\partial\theta = \theta^2 - 2c^2/8\theta^2 > 0$.

Proof of Proposition 2. The second-order derivatives of the profit functions of the manufacturer and the retailer with respect to p_d and p_r, respectively, are obtained: $\partial^2 \pi_d^{RN}/\partial (p_d^{RN})^2 = 2\theta^2/(\varphi-1)(\theta-1)(\theta\varphi-\theta-\varphi) < 0$ and $\partial^2 \pi_r^{RN}/\partial (p_r^{RN})^2 = -2/(1-\varphi)(1-\theta) < 0$. This gives the Hessian matrix- $\begin{vmatrix} \% -2/(1-\theta)(1-\varphi) & \theta/(1-\theta)(1-\varphi) \\ \theta/(1-\theta)(1-\varphi) & 2\theta^2/(\varphi-1)(\theta-1)(\theta\varphi-\theta-\varphi) \end{vmatrix} = -\theta^2(\theta\varphi-\theta-\varphi+4)/(\varphi-1)^2(\theta-1)^2(\theta\varphi-\theta-\varphi) > 0$, that is, there is a unique optimal solution. From the first-order condition, $\partial \pi_r^{RN}/\partial p_r^{RN} = 0$; $\partial \pi_d^{RN}/\partial p_d^{RN} = 0$; and $\partial \pi_d^{RN}/\partial w = 0$. The authors have $p_r^{RN} = (\theta-1)\varphi + 2c + 3 - \theta/4$, $p_d^{RN} = \theta + c + \varphi - \theta\varphi/2\theta$, and $w = c + 1/2$. Substituting p_r^{RN}, p_d^{RN}, and w into the demand and profit functions gives the optimal demand and profit for retailers and manufacturers.

Proof of Corollary 2. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to θ to give $\partial p_r^{RN}/\partial\theta =$

$\varphi - 1/4 < 0$ and $\partial p_d^{RN}/\partial\theta = -c - \varphi/2\theta^2 < 0$; $\partial D_r^{RN}/\partial\theta = 0$ and $\partial D_d^{RN}/\partial\theta = -(\varphi - 1)c/2(\theta\varphi - \varphi - \theta)^2 > 0$; and $\partial\pi_r^{RN}/\partial\theta = \varphi - 1/16 < 0$ and $\partial\pi_d^{RN}/\partial\theta = (2\varphi(\theta^2 - \theta) - (\theta - 1)^2\varphi^2 + 2c^2 - \theta^2)(\varphi - 1)/8(\theta\varphi - \varphi - \theta)^2 > 0$.

Proof of Proposition 3. The second-order derivatives of the profit functions of the manufacturer and the retailer with respect to p_d and p_r, respectively, are obtained: $\partial^2\pi_d^{MN}/\partial(p_d^{MN})^2 = 2\theta/\theta - 1 < 0$ and $\partial^2\pi_r^{MN}/\partial(p_r^{MN})^2 = -2/1 - \theta < 0$. This gives the Hessian matrix $\begin{vmatrix} -2/1-\theta & \theta/1-\theta \\ \theta/1-\theta & 2\theta/\theta-1 \end{vmatrix} = -\theta(\theta - 4)/(\theta - 1)^2 > 0$, that is, there is a unique optimal solution. From the first-order condition, $\partial\pi_r^{MN}/\partial p_r^{MN} = 0$; $\partial\pi_d^{MN}/\partial p_d^{MN} = 0$; and $\partial\pi_d^{MN}/\partial w = 0$. The authors have $p_r^{MN} = \rho\theta - c_i\theta - s\theta - \rho + 2c + c_i + s - \theta + 3/4$, $p_d^{MN} = \rho\theta - c_i\theta + s\theta - \rho + c + c_i - s + \theta/2\theta$, and $w = c + 1/2$. Substituting p_r^{MN}, p_d^{MN}, and w into the demand and profit functions gives the optimal demand and profit for retailers and manufacturers.

Proof of Corollary 3. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to θ to give $\partial p_r^{MN}/\partial\theta = \rho - c_i - s - 1/4 < 0$ and $\partial p_d^{MN}/\partial\theta = \rho - c_i - c + s/2\theta^2 > 0$, $\partial D_r^{MN}/\partial\theta = 0$ and $\partial D_d^{MN}/\partial\theta = c + c_i + s - \rho/2\theta^2 > 0$, and $\partial\pi_r^{MN}/\partial\theta = -(\rho - c_i - s - 1)^2/16 < 0$ and $\partial\pi_d^{MN}/\partial\theta = (\rho - c_i - s - 1)^2\theta^2 - 2(\rho - c - c_i - s)^2/8\theta^2 > 0$. □

Proof of Corollary 4. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to c_i to give $\partial p_r^{MN}/\partial c_i = 1 - \theta/4 > 0$ and $\partial p_d^{MN}/\partial c_i = 1 - \theta/2\theta > 0$, $\partial D_r^{MN}/\partial c_i = 1/4 > 0$ and $\partial D_d^{MN}/\partial c_i = \theta - 2/4\theta < 0$, and $\partial\pi_r^{MN}/\partial c_i = (\rho - s - c_i - 1)(\theta - 1)/8 > 0$ and $\partial\pi_d^{MN}/\partial c_i = (1 - \theta)((\rho - s - c_i - 1)\theta - 2(\rho - c - c_i - s))/4\theta > 0$.

Proof of Proposition 4. The second-order derivatives of the profit functions of the manufacturer and the retailer with respect to p_d and p_r, respectively, are obtained: $\partial^2\pi_d^{MR}/\partial(p_d^{MR})^2 = 2\theta/\theta - 1 < 0$ and $\partial^2\pi_r^{MR}/\partial(p_r^{MR})^2 = -2/1 - \theta < 0$. This gives the Hessian matrix $\begin{vmatrix} -2/1-\theta & \theta/1-\theta \\ \theta/1-\theta & 2\theta/\theta-1 \end{vmatrix} = -\theta(\theta - 4)/(\theta - 1)^2 > 0$, that is, there is a unique optimal solution. From the first-order condition, $\partial\pi_r^{MR}/\partial p_r^{MR} = 0$; $\partial\pi_d^{MR}/\partial p_d^{MR} = 0$; and $\partial\pi_d^{MR}/\partial w = 0$. The authors have $p_r^{MR} = (\rho + r - s - c_i - 1)\theta - \rho + 2c - r + s + t + c_i + 3/4$, $p_d^{MR} = (\rho - r + s - c_i + 1)\theta - \rho + c + r - s + t + c_i/2\theta$, and $w = c + 1/2$. Substituting p_r^{MR}, p_d^{MR}, and w into the demand and profit functions gives the optimal demand and profit for retailers and manufacturers.

Proof of Corollary 5. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to θ to give $\partial p_r^{MR}/\partial\theta = \rho + r - c_i - s - 1/4 < 0$ and $\partial p_d^{MR}/\partial\theta = \rho - c - r - c_i - t + s/2\theta^2 > 0$, $\partial D_r^{MR}/\partial\theta = t/4(\theta - 1)^2 > 0$ and $\partial D_d^{MR}/\partial\theta = (-2\rho + 2c - 2r + 2c_i + t + 2s)\theta^2 + (\rho - c + r - s - t - c_i)(4\theta - 2)/4\theta^2(\theta - 1)^2 > 0$, and $\partial\pi_r^{MR}/\partial\theta = -((\rho + r - s - c_i - 1)\theta - \rho - r + s - t + c_i + 1)\ ((\rho + r - s - c_i - 1)\theta - \rho - r + s + t + c_i + 1)/\ 16(\theta - 1)^2 < 0$; to simplify the formula, let $x = \rho + r - s - c_i - 1$, $y = \rho - c + r - s - c_i$, $\beta_1 = 2\rho - 4c + 2r - 2s - 4t + 2$, $\beta_2 = 4c - 2r + 2s + 4t - 2$, $\beta_3 = 4c + 2s + 4t - 2$, $\beta_4 = -4c - 4t + 2$, and $\beta_5 = -2c^2 - 4ct - t^2 + 1$; the derivative of the manufacturer's optimum profit with respect to θ is then simplified as $\partial\pi_d^{MR}/\partial\theta = x^2\theta^4 - 2x^2\theta^3 + (-c_i^2 + \beta_1 c_i - \rho^2 + \beta_2\rho - r^2 + \beta_3 r - s^2 + \beta_4 s + \beta_5)\theta^2 + (4\theta - 2)(y - t)^2/8\theta^2(\theta - 1)^2 > 0$.

Proof of Corollary 6. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to c_i to give $\partial p_r^{MR}/\partial c_i = > 0$ and $\partial p_d^{MR}/\partial c_i = 1 - \theta/2\theta > 0$, $\partial D_r^{MR}/\partial c_i = 1/4 > 0$ and $\partial D_d^{MR}/\partial c_i = \theta - 2/4\theta < 0$, and $\partial p_d^{MR}/\partial c_i = (\rho + r - c_i - s - 1)\theta - (\rho + r - s - t - c_i - 1)/8 > 0$ and $\partial\pi_d^{MR}/\partial c_i = (s + c_i - r - \rho + 1)\theta^2 + (3\rho - 2c + 3r - 3s - t - 3c_i - 1)\theta - 2(\rho - c + r - s - t - c_i)/4\theta < 0$.

Proof of Corollary 7. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to r to give $\partial p_r^{MR}/\partial r = \theta - 1/4 < 0$ and $\partial p_d^{MR}/\partial r = 1 - \theta/2\theta > 0$, $\partial D_r^{MR}/\partial r = -1/4 < 0$ and $\partial D_d^{MR}/\partial r = 2 - \theta/4\theta > 0$, and $\partial\pi_r^{MR}/\partial r = (s - \rho - r + c_i + 1)\theta + (\rho + r - s - t - c_i - 1)/8 < 0$ and $\partial\pi_d^{MR}/\partial r = (\rho + r - s - c_i - 1)\theta^2 + (2c - 3\rho - 3r + 3s + t + 3c_i + 1)\theta + 2(\rho - c + r - s - t - c_i)/4\theta > 0$.

The optimal selling price, the optimal demand, and the optimal profit for the manufacturer and the retailer are each derived with respect to t to give: $\partial p_r^{MR}/\partial t = 1/4 > 0$ and $\partial p_d^{MR}/\partial t = 1/2\theta > 0$, $\partial D_r^{MR}/\partial t = -1/4(\theta - 1) > 0$ and $\partial D_d^{MR}/\partial t = 2 - \theta/4\theta(\theta - 1) < 0$, and $\partial\pi_r^{MR}/\partial t = (s - \rho - r + c_i + 1)\theta + (\rho + r - s - t - c_i - 1)/8(\theta - 1) > 0$ and $\partial\pi_d^{MR}/\partial t = (\rho + r - s - c_i - 1)\theta^2 + (2c - 3\rho - 3r + 3s + t + 3c_i + 1)\theta + 2(\rho - c + r - s - t - c_i)/4\theta(\theta - 1) < 0$.

Proof of Proposition 5. The second-order derivatives of the profit functions of the manufacturer and the retailer with respect to p_d and p_r, respectively, are obtained: $\partial^2\pi_d^{MC}/\partial(p_d^{MC})^2 = 2\theta/\theta - 1 < 0$ and $\partial^2\pi_r^{MC}/\partial(p_r^{MC})^2 = -2/1 - \theta < 0$. This gives the Hessian matrix $\begin{vmatrix} -2/1-\theta & \theta/1-\theta \\ \theta/1-\theta & 2\theta/\theta-1 \end{vmatrix} = -\theta(\theta - 4)/(\theta - 1)^2 > 0$, that is, there is a unique optimal solution. From the first-order condition, $\partial\pi_r^{MC}/\partial p_r^{MC} = 0$; $\partial\pi_d^{MC}/\partial p_d^{MC} = 0$; and $\partial\pi_d^{MC}/\partial w = 0$. The authors have $p_r^{MC} = (\rho + r - s - c_i - 1)\ \theta - \rho + 2c - r + s - t + c_i + 3/4$, $p_d^{MC} = (\rho - r + s - c_i + 1)\ \theta - \rho + c + r - s + t + c_i/2\theta$, and $w = c + 1/2$.Substituting p_r^{MC}, p_d^{MC}, and w into the demand and profit functions gives the optimal demand and profit for retailers and manufacturers.

Proof of Corollary 8. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to θ to give $\partial p_r^{MC}/\partial\theta = \rho + r - c_i - s - 1/4 < 0$ and $\partial p_d^{MC}/\partial\theta = \rho - c - c_i - r$

$-t + s/2\theta^2 > 0$, $\partial D_r^{MC}/\partial\theta = -t/4\ (\theta-1)^2 < 0$ and $\partial D_d^{MR}/\partial\theta = (-2\rho + 2c - 2r + 2c_i - t + 2s)\theta^2 + (\rho - c + r - s + t - c_i)(4\theta - 2)/4\theta^2(\theta-1)^2 > 0$, and $\partial\pi_r^{MC}/\partial\theta = -((\rho + r - s - c_i - 1)\theta - \rho - r + s + t + c_i + 1)((\rho + r - s - c_i - 1)\theta - \rho - r + s - t + c_i + 1)/16(\theta-1)^2 < 0$; to simplify the formula, let $x = \rho + r - s - c_i - 1$, $y = \rho - c + r - s - c_i$, $\lambda_1 = 2\rho - 4c + 2r - 2s + 4t + 2$, $\lambda_2 = 4c - 2r + 2s - 4t - 2$, $\lambda_3 = 4c + 2s - 4t - 2$, $\lambda_4 = -4c + 4t + 2$, and $\lambda_5 = -2c^2 + 4ct - t^2 + 1$; the derivative of the manufacturer's optimum profit with respect to θ is then simplified as $\partial\pi_d^{MC}/\partial\theta = x^2\theta^4 - 2x^2\theta^3 + (-c_i^2 + \lambda_1 c_i - \rho^2 + \lambda_2\rho - r^2 + \lambda_3 r - s^2 + \lambda_4 s + \lambda_5)\theta^2 + (4\theta - 2)(y+t)^2/8\theta^2(\theta-1)^2 > 0$.

Proof of Corollary 9. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to c_i to give $\partial p_r^{MC}/\partial c_i = 1 - \theta/4 > 0$ and $\partial p_d^{MC}/\partial c_i = 1 - \theta/2\theta > 0$, $\partial D_r^{MC}/\partial c_i = 1/4 > 0$ and $\partial D_d^{MC}/\partial c_i = \theta - 2/4\theta < 0$, and $\partial\pi_r^{MC}/\partial c_i = (\rho - s + r - c_i - 1)\theta - (\rho + r - s + t - c_i - 1)/8 < 0$ and $\partial\pi_d^{MC}/\partial c_i = (s + c_i - r - \rho + 1)\theta^2 + (3\rho - 2c + 3r - 3s - 3c_i + t - 1)\theta - 2\ (\rho - c + r - s + t - c_i)/4\theta < 0$.

Proof of Corollary 10. The optimal selling price, the optimal demand, and the optimal profit of the manufacturer and the retailer are derived with respect to r to give $\partial p_r^{MC}/\partial r = \theta - 1/4 < 0$ and $\partial p_d^{MC}/\partial r = 1 - \theta/2\theta > 0$, $\partial D_r^{MC}/\partial r = -1/4 < 0$, $\partial D_d^{MC}/\partial r = 2 - \theta/4\theta > 0$, and $\partial\pi_r^{MC}/\partial r = (s - \rho - r + c_i + 1)\theta + (\rho + r - s + t - c_i - 1)/8 < 0$ and $\partial\pi_d^{MC}/\partial r = (\rho + r - s - c_i - 1)\theta^2 + (2c - 3\rho - 3r + 3s + 3c_i - t + 1)\theta + 2(\rho - c + r - s + t - c_i)/4\theta > 0$.

The optimal selling price, the optimal demand, and the optimal profit for the manufacturer and the retailer are each derived with respect to t to give $\partial p_r^{MC}/\partial t = -1/4 < 0$ and $\partial p_d^{MC}/\partial t = 1/2\theta > 0$, $\partial D_r^{MC}/\partial t = 1/4(\theta-1) < 0$ and $\partial D_d^{MC}/\partial t = \theta - 2/4\theta(\theta-1) > 0$, and $\partial\pi_r^{MC}/\partial t = (\rho + r - s - c_i - 1)\theta - (\rho + r - s + t - c_i - 1)/8(\theta-1) < 0$ and $\partial\pi_d^{MC}/\partial t = (s - \rho - r + c_i + 1)\theta^2 + (3\rho - 2c + 3r - 3s - 3c_i + t - 1)\theta + 2(\rho - c + r - s + t - c_i)/4\theta(\theta-1) > 0$.

Proof of Proposition 6. The sales prices, market shares, and profit differentials between manufacturers and retailers under the *NN* and *RN* strategies are as follows: $p_r^{RN} - p_r^{NN} = (\theta-1)\varphi/4$ and $p_d^{RN} - p_d^{NN} = -(\theta-1)(\theta + c + \varphi)/2\theta$, $D_r^{RN} - D_r^{NN} = 0$ and $D_d^{RN} - D_d^{NN} = c\varphi(\theta-1)/2\theta((\varphi-1)\theta - \varphi)$, and $\pi_r^{RN} - \pi_r^{NN} = \varphi(\theta-1)/16$ and $\pi_d^{RN} - \pi_d^{NN} = -\varphi(\theta-1)(\varphi\theta^2 + 2c^2 - \theta^2 - \theta\varphi)/8\theta((\varphi-1)\theta - \varphi)$.

Since $0 < \varphi < 1$, $0 < \varphi < 1$, therefore, $P_r^{RN} < P_r^{NN}$, $P_d^{RN} > P_d^{NN}$, $D_r^{RN} = D_r^{NN}$, $D_d^{RN} > D_d^{NN}$, and $\pi_r^{RN} < \pi_r^{NN}$.

When $\varphi > \theta^2 - 2c^2/\theta(\theta-1)$, $\pi_d^{RN} > \pi_d^{NN}$; when $0 < \varphi < \theta^2 - 2c^2/\theta(\theta-1)$, $\pi_d^{RN} < \pi_d^{NN}$.

Proof of Proposition 7. The sales prices, market shares, and profit differentials between manufacturers and retailers under the *NN* and *MN* strategies are as follows: $p_r^{MN} - p_r^{NN} = (\theta-1)(\rho - c_i - s)/4$ and $p_d^{MN} - p_d^{NN} = (\theta-1)(\rho - \theta - c - c_i + s)/2\theta$, $D_r^{MN} - D_r^{NN} = s + c_i - \rho/4$ and $D_d^{MN} - D_d^{NN} = -(\theta-2)(\rho - c_i - s)/4\theta$, and $\pi_r^{MN} - \pi_r^{NN} = -(\theta-1)(\rho - c_i - s)(\rho - s - c_i - 2)/16$ and $\pi_d^{MN} - \pi_d^{NN} = (\theta - 1)((\rho - s - c_i - 2)\theta - 2\rho + 4c + 2s + 2c_i)(\rho - s - c_i)/8\theta$.

Proof of Proposition 8. The sales prices, market shares, and profit differences between manufacturers and retailers under the *MR* and *MN* strategies are as follows: $p_r^{MR} - p_r^{MN} = (\theta-1)r + t/4$ and $p_d^{MR} - p_d^{MN} = (1-\theta)r + t/2\theta$, $D_r^{MR} - D_r^{MN} = (1-\theta)r - t/4(\theta-1)$ and $D_d^{MR} - D_d^{MN} = -(\theta-2)(\theta r - r + t)/4\theta(\theta-1)$, and $\pi_r^{MR} - \pi_r^{MN} = -((2\rho + r - 2s - 2c_i - 2)\theta - 2\rho - r + 2s + t + 2c_i + 2)(\theta r - r + t)/16(\theta-1)$ and $\pi_d^{MR} - \pi_d^{MN} = ((\rho + r/2 - s - c_i - 1)\theta^2 + (2c - 3\rho - 3r/2 + 3s + t/2 + 3c_i + 1)\theta + 2(\rho - c - s - c_i) + r - t)(\theta r - r + t)/4\theta(\theta-1)$.

Proof of Proposition 9. The sales prices, market shares, and profit differentials between manufacturers and retailers under the *MC* and *MN* strategies are as follows: $p_r^{MC} - p_r^{MN} = (\theta-1)r - t/4$ and $p_d^{MC} - p_d^{MN} = (1-\theta)r + t/2\theta$, $D_r^{MC} - D_r^{MN} = (1-\theta)r + t/4(\theta-1)$ and $D_d^{MC} - D_d^{MN} = -(\theta-2)(r\theta - r - t)/4\theta(\theta-1)$, and $\pi_r^{MC} - \pi_r^{MN} = -((2\rho + r - 2s - 2c_i - 2)\theta - 2\rho - r + 2s - t + 2c_i + 2)(\theta r - r - t)/16(\theta-1)$ and $\pi_d^{MC} - \pi_d^{MN} = (\theta r - r - t)\ ((\rho + r/2 - s - c_i - 1)\theta^2 + (2c - 3\rho - 3r/2 + 3s - t/2 + 3c_i + 1)\theta + 2(\rho - c - s - c_i) + r - t)/4\theta(\theta-1)$.

Proof of Proposition 10. The difference in selling prices, market shares, and profits between manufacturers and retailers under the *MC* and *MR* strategies are as follows: $p_r^{MC} - p_r^{MR} = -t/2$ and $p_d^{MC} - p_d^{MR} = 0$, $D_r^{MC} - D_r^{MR} = t/2(\theta-1)$ and $D_d^{MC} - D_d^{MR} = t(\theta-2)/2\theta(\theta-1)$, and $\pi_r^{MC} - \pi_r^{MR} = (\rho + r - s - c_i - 1)t/4$ and $\pi_d^{MC} - \pi_d^{MR} = t((\rho + r - s - c_i)(2-\theta) + 2c - \theta)/2\theta$.

Proof of Proposition 11. The sales prices, market shares, and profit differentials between manufacturers and retailers under the *NN* and *MN* strategies are as follows: $p_r^{MN} - p_r^{NN} = (\theta-1)(\rho - c_i - s)/4$ and $p_d^{MN} - p_d^{NN} = (\theta-1)(\rho - \theta - c - c_i + s)/2\theta$, $D_r^{MN} - D_r^{NN} = s + c_i - \rho/4$ and $D_d^{MN} - D_d^{NN} = -(\theta-2)(\rho - c_i - s)/4\theta$, and $\pi_r^{MN} - \pi_r^{NN} = -(\theta-1)(\rho - c_i - s)(\rho - s - c_i - 2)/16$ and $\pi_d^{MN} - \pi_d^{NN} = (\theta-1)((\rho - s - c_i - 2)\theta - 2\rho + 4c + 2s + 2c_i)\ (\rho - s - c_i)/8\theta$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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