

## Research Article

# Feedback Control for Passivity of Memristor-Based Multiple Weighted Coupled Neural Networks

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This paper investigates the passivity of multiple weighted coupled memristive neural networks (MWCMMNs) based on the feedback control. Firstly, a kind of memristor-based coupled neural network model with multiple weights is presented for the first time. Furthermore, a novel passivity criterion for MWCMMNs is established by constructing an appropriate Lyapunov functional and developing a suitable feedback controller. In addition, with the assistance of some inequality techniques, sufficient conditions for ensuring the input strict passivity and output strict passivity of MWCMMNs are derived. Finally, the validity of the theoretical results is verified by a numerical example.

## 1. Introduction

Neural networks (NNs) have aroused widespread attention since they have been applied in numerous fields including machine learning, deep learning, and engineering data prediction [1–3]. As the fourth two-terminal circuit element, the memristor was predicted to exist by Chua in 1971, and the prototype of memristor was obtained by the research team of HP for the first time [4–6]. Memristor is considered to be an excellent candidate for imitating biological synapses in circuit implementation of NNs owing to its characteristics of nanometer size, high storage capacity, and low energy consumption [7]. Through replacing the resistors with memristors in NNs circuit implementation, a new type of NN called memristive NN (MNN) has been successfully introduced [8]. Recently, it is reported that MNNs have many potential applications in face detection, bio-engineering, pattern recognition, feature extraction, and associative memory [9–11]. To our knowledge, these applications for MNNs were to a great extent from the dynamical behaviors of MNNs. Particularly, the stability as a significant dynamical behavior for MNNs is one of the hot

research topics [12]. Zhang et al. [12] obtained several sufficient conditions to insure the stability for MNNs.

The theory of passivity is valid and robust when studying the stability for nonlinear systems since passivity properties of a system can keep the internal stability of the system [13–15]. Up to now, a lot of interesting results about passivity of MNNs have been reported [16–18]. In [16], Meng and Xiang conducted the passivity analysis for a kind of complex-valued MNNs. Xiao et al. [17] obtained a new passivity criterion by utilizing set-valued mapping as well as transforming MNNs into traditional NNs. Based on the Lyapunov–Krasovskii method, Wu and Zeng [18] acquired an exponential passivity criterion for MNNs with mixed time-varying delays.

Complex networks (CNs) have attracted more and more interests of researchers in recent years, and CNs are ubiquitous in our life, such as communication networks, metabolic system networks, and food networks. Coupled NNs (CNNs) are a special class of CNs, which are composed of many NNs through mutually coupling [19, 20]. Considering the fact that the passivity of CNNs has been broadly applied to many fields including chaos generators and brain science.

Consequently, it is interesting to research the passivity for coupled MNNs (CMNNs) [21]. In [21], Yue et al. investigated the passivity of delayed CMNNs with reaction-diffusion terms with the aid of two pinning control schemes and some inequality techniques.

It is well known that most of networks in the practical world are supposed to be described as multiple weighted CNs, such as human social networks and public transport networks. However, only a few researchers have discussed multiple weighted CNNs (MWCNNs) in recent years [22]. Chen et al. [22] dealt with the dissipativity problem of MWCNNs via dynamic event-triggered pinning control. It should be noted that the passivity of multiple weighted CMNNs has only been discussed by few researchers so far.

It is worth noticing that the passivity of MNNs usually cannot be achieved on their own [23]. In consequence, it is essential to make use of some control strategies to make MNNs passive [24, 25]. To ensure exponential synchronization for MNNs, Lin et al. [24] developed a nonlinear feedback controller. Zhang et al. [25] derived some sufficient conditions for achieving finite time synchronization based on the feedback control. To our knowledge, the problem of passivity for multiple weighted CMNNs (MWCNNs) under the feedback control has never been considered.

Motivated by the above analyses, this paper considers the passivity of MWCNNs via the feedback control. The primary contributions of this paper are displayed as follows:

- (1) A kind of memristor-based coupled neural network model with multiple weights is firstly proposed.
- (2) It is first time that the feedback control strategy is adopted to ensure the passivity, output strict passivity, and input strict passivity of MWCNNs.
- (3) Several new passivity criteria are established according to linear matrix inequalities that can be checked through utilizing standard numerical packages.

## 2. Preliminaries

Let  $\mathcal{N} = \{1, 2, \dots, N\}$ ,  $\mathbb{N} = \{1, 2, \dots\}$ ,  $\mathbb{R}^{m \times n}$  be the set of real matrices of order  $m \times n$ .  $\mathbb{R}^{n \times n} \ni \Gamma > 0$  ( $\mathbb{R}^{n \times n} \ni \Gamma < 0$ ) stands for that the matrix  $\Gamma$  is symmetric and positive (negative) definite.  $\mathbb{R}^{n \times n} \ni \Gamma \geq 0$  ( $\mathbb{R}^{n \times n} \ni \Gamma \leq 0$ ) stands for that the matrix  $\Gamma$  is symmetric and semipositive (seminegative) definite.  $\Gamma^T$  represents the transpose of matrix  $\Gamma$ . For any  $\chi(t) = (\chi_1(t), \chi_2(t), \dots, \chi_n(t)) \in \mathbb{R}^n$ ,  $\|\chi(t)\|_2 = \sqrt{\chi^T(t)\chi(t)}$ .  $\lambda_m(\Gamma)$  and  $\lambda_M(\Gamma)$  mean the minimal as well as the maximal eigenvalue of matrix  $\Gamma$ , respectively.

*Definition 1* (see [26]). A system with supply rate  $\mathcal{W}(u, y)$  is dissipative if there is a nonnegative storage function  $\mathcal{V}: [0, +\infty) \rightarrow [0, +\infty)$ , such that

$$\int_{t_0}^{t_\varepsilon} \mathcal{W}(u(t), y(t)) dt \geq \mathcal{V}(t_\varepsilon) - \mathcal{V}(t_0), \quad (1)$$

for any  $t_0, t_\varepsilon \in [0, +\infty)$  and  $t_\varepsilon \geq t_0$ , where  $\mathbb{R}^w \ni y(t)$ ,  $\mathbb{R}^m \ni u(t)$  are the output and input of the system, respectively.

*Definition 2* (see [27]). If a system is dissipative and satisfying

$$\mathcal{W}(u(t), y(t)) = y^T(t)Zu(t), \quad (2)$$

in which  $Z \in \mathbb{R}^{w \times m}$  is a constant matrix, the system can achieve the passivity.

*Definition 3* (see [27]). If a system is dissipative and satisfying

$$\mathcal{W}(u(t), y(t)) = y^T(t)Zu(t) - y^T(t)\Phi_1 y(t) - u^T(t)\Phi_2 u(t), \quad (3)$$

in which  $Z \in \mathbb{R}^{w \times m}$ ,  $0 \leq \Phi_1 \in \mathbb{R}^{w \times w}$ ,  $0 \leq \Phi_2 \in \mathbb{R}^{m \times m}$ , and  $\lambda_m(\Phi_1) + \lambda_m(\Phi_2) > 0$ , then the system can achieve the strict passivity.

If  $0 \leq \Phi_1$ , the system is output-strictly passive, and if  $0 \leq \Phi_2$ , the system is input-strictly passive.

## 3. Passivity of MWCNNs

*3.1. Network Model.* The model of coupled neural network with multiple weights is given by

$$\begin{aligned} \dot{x}_i(t) = & -Kx_i(t) + Af(x_i(t)) + \sum_{r=1}^s \sum_{j=1}^N d_r G_{ij}^r H^r x_j(t) \\ & + J + Cu_i(t), \end{aligned} \quad (4)$$

where  $i = 1, 2, \dots, N$ ,  $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$  indicates the state vector of the  $i$ th node;  $0 < K = \text{diag}(k_1, k_2, \dots, k_n) \in \mathbb{R}^{n \times n}$ ,  $f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)), \dots, f_n(x_{in}(t)))^T \in \mathbb{R}^n$ , and  $f_\rho(\cdot)$  means the activation function of  $\rho$ th neuron;  $\mathbb{R}^{n \times n} \ni A$  denotes a constant matrix;  $J = (J_1, J_2, \dots, J_n)^T \in \mathbb{R}^n$  is the constant external input vector;  $C \in \mathbb{R}^{n \times m}$  is a constant matrix;  $u_i(t) = (u_{i1}(t), u_{i2}(t), \dots, u_{im}(t))^T \in \mathbb{R}^m$  denotes the external input vector;  $0 < d_r$  stands for the coupling strength of the  $r$ th coupling form;  $\mathbb{R}^{n \times n} \ni H^r = \text{diag}(h_1^r, h_2^r, \dots, h_n^r) > 0$  means the inner coupling matrix in the  $r$ th coupling form;  $G^r = (G_{ij}^r)_{N \times N}$  is the external coupling matrix for the  $r$ th coupling form, where  $G_{ij}^r$  satisfies the following conditions:

$$\begin{aligned} G_{ij}^r &= G_{ji}^r \geq 0, \quad i \neq j; \\ G_{ii}^r &= - \sum_{\substack{j=1 \\ j \neq i}}^N G_{ij}^r, \end{aligned} \quad (5)$$

if there is a link between node  $i$  and node  $j$ , then  $G_{ij}^r > 0$  or else  $G_{ij}^r = 0$ .

Consider the following multiple weighted coupled memristive neural network (MWCNN) consisting of  $N$  identical MNNs with multiple weights:

$$\begin{aligned} \dot{x}_i(t) = & -Kx_i(t) + A(x_i(t))f(x_i(t)) \\ & + B(x_i(t))g(x_i(t-\tau)) + Cu_i(t) \\ & + \sum_{r=1}^s \sum_{j=1}^N d_r G_{ij}^r H^r x_j(t) \\ & + J + \psi_i(t), \quad i = 1, 2, \dots, N, \end{aligned} \quad (6)$$

where  $K, J, x_i(t), d_r, G_{ij}^r, H^r, C, u_i(t), f(x_i(t))$  have the same meanings as in network (1);  $A(x_i(t)) = (a_{\eta\rho}(x_{i\eta}(t)))_{n \times n}$ ,  $B(x_i(t)) = (b_{\eta\rho}(x_{i\eta}(t)))_{n \times n}$ ,  $\eta = 1, 2, \dots, n, \rho = 1, 2, \dots, n$ ;  $x_{i\eta}(t) \in \mathbb{R}$  stands for the voltage for capacitor  $\mathcal{C}_{\eta}$ ;  $x_i(t-\tau) = (x_{i1}(t-\tau), x_{i2}(t-\tau), \dots, x_{in}(t-\tau))^T \in \mathbb{R}^n$ ;  $g(x_i(t-\tau)) = (g_1(x_{i1}(t-\tau)), g_2(x_{i2}(t-\tau)), \dots, g_n(x_{in}(t-\tau)))^T \in \mathbb{R}^n$ , and  $g_\rho(\cdot)$  means the activation function of  $\rho$ th neuron;  $\tau$  indicates the propagation delay;  $\psi_i(t) \in \mathbb{R}^n$  is the control input; and  $a_{\eta\rho}(x_{i\eta}(t))b_{\eta\rho}(x_{i\eta}(t))$  are described by

$$\begin{aligned} a_{\eta\rho}(x_{i\eta}(t)) &= \frac{\mathcal{W}_{\eta\rho}}{\mathcal{E}_{\eta}} \times \text{sign}_{\eta\rho}, \\ b_{\eta\rho}(x_{i\eta}(t)) &= \frac{\mathcal{M}_{\eta\rho}}{\mathcal{E}_{\eta}} \times \text{sign}_{\eta\rho}, \\ \text{sign}_{\eta\rho} &= \begin{cases} 1, & \eta \neq \rho, \\ -1, & \eta = \rho, \end{cases} \end{aligned} \quad (7)$$

where  $\mathcal{W}_{\eta\rho}$  and  $\mathcal{M}_{\eta\rho}$  represent the memductances of memristors  $\mathcal{A}_{\eta\rho}$  and  $\mathcal{B}_{\eta\rho}$ , respectively.  $\mathcal{A}_{\eta\rho}$  indicates the memristor between  $x_{i\eta}(t)$  and the function  $f_\rho(x_{i\rho}(t))$ , and  $\mathcal{B}_{\eta\rho}$  indicates the memristor between  $x_{i\eta}(t)$  and the function  $g_\rho(x_{i\rho}(t-\tau))$ . In the light of the traits of voltage and current of memristor, we can obtain that

$$\begin{aligned} a_{\eta\rho}(x_{i\eta}(t)) &= \begin{cases} \hat{a}_{\eta\rho}, & |x_{i\eta}(t)| \leq \Psi_\eta, \\ \check{a}_{\eta\rho}, & |x_{i\eta}(t)| > \Psi_\eta, \end{cases} \\ b_{\eta\rho}(x_{i\eta}(t)) &= \begin{cases} \hat{b}_{\eta\rho}, & |x_{i\eta}(t)| \leq \Psi_\eta, \\ \check{b}_{\eta\rho}, & |x_{i\eta}(t)| > \Psi_\eta, \end{cases} \end{aligned} \quad (8)$$

$$\begin{aligned} i = 1, 2, \dots, N, z_i(t-\tau) &= (z_{i1}(t-\tau), z_{i2}(t-\tau), \dots, z_{in}(t-\tau))^T, \\ f(z_i(t)) &= f(x_i(t)) - f(x^*), g(z_i(t-\tau)) = g(x_i(t-\tau)) - g(x^*). \end{aligned} \quad (12)$$

According to the network (11), a feedback controller is developed as follows:

$$\psi_i(t) = -\text{sign}(z_i(t))(\bar{A}\bar{\lambda} + \bar{B}\bar{\mu}), \quad (13)$$

where the switching jumps  $\Psi_\eta > 0$ ;  $\check{a}_{\eta\rho}, \hat{a}_{\eta\rho}, \check{b}_{\eta\rho}, \hat{b}_{\eta\rho}$  are constants,  $\eta, \rho = 1, 2, \dots, n$ .

Define  $\bar{a}_{\eta\rho} = |\hat{a}_{\eta\rho} - \check{a}_{\eta\rho}|$ ,  $\bar{A} = (\bar{a}_{\eta\rho})_{n \times n}$ ,  $\bar{a}_{\eta\rho} = \max\{|\hat{a}_{\eta\rho}|, |\check{a}_{\eta\rho}|\}$ ,  $\bar{A} = \text{diag}(\sum_{\rho=1}^n \bar{a}_{1\rho}^2, \sum_{\rho=1}^n \bar{a}_{2\rho}^2, \dots, \sum_{\rho=1}^n \bar{a}_{n\rho}^2)$ ,  $\bar{b}_{\eta\rho} = |\hat{b}_{\eta\rho} - \check{b}_{\eta\rho}|$ ,  $\bar{B} = (\bar{b}_{\eta\rho})_{n \times n}$ ,  $\bar{b}_{\eta\rho} = \max\{|\hat{b}_{\eta\rho}|, |\check{b}_{\eta\rho}|\}$ ,  $\bar{B} = \text{diag}(\sum_{\rho=1}^n \bar{b}_{1\rho}^2, \sum_{\rho=1}^n \bar{b}_{2\rho}^2, \dots, \sum_{\rho=1}^n \bar{b}_{n\rho}^2)$ .

*Remark 1.* Considering the fact that plenty of networks in the actual world ought to be described by CNs with multiple weights, for instance, human social networks and public transport networks. Nevertheless, only some researchers have investigated the passivity of MWCNNs [22]. Therefore, it is interesting to discuss the passivity of MWCNNs. In this paper, a kind of memristor-based coupled neural network model with multiple weights is proposed.

Throughout this paper, we put forward the following assumption.

*Assumption 1.* If there are positive constants  $\lambda_\rho, \mu_\rho, \bar{\lambda}_\rho, \bar{\mu}_\rho, \rho = 1, 2, \dots, n$ , such that

$$\begin{aligned} |f_\rho(\delta_1) - f_\rho(\delta_2)| &\leq \lambda_\rho |\delta_1 - \delta_2|, & |f_\rho(\delta)| &\leq \bar{\lambda}_\rho, \\ |g_\rho(\delta_1) - g_\rho(\delta_2)| &\leq \mu_\rho |\delta_1 - \delta_2|, & |g_\rho(\delta)| &\leq \bar{\mu}_\rho, \end{aligned} \quad (9)$$

for any  $\delta, \delta_1, \delta_2 \in \mathbb{R}$ .

Consider that  $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T \in \mathbb{R}^n$  is an equilibrium point for network (6), then

$$-Kx^* + A(x^*)f(x^*) + B(x^*)g(x^*) + J = 0. \quad (10)$$

Letting error vector  $z_i(t) = x_i(t) - x^*$ , we get

$$\begin{aligned} \dot{z}_i(t) = & -Kz_i(t) + A(x_i(t))f(z_i(t)) \\ & + [A(x_i(t)) - A(x^*)]f(x^*) \\ & + B(x_i(t))g(z_i(t-\tau)) \\ & + [B(x_i(t)) - B(x^*)]g(x^*) + Cu_i(t) \\ & + \sum_{r=1}^s \sum_{j=1}^N d_r G_{ij}^r H^r z_j(t) + \psi_i(t), \end{aligned} \quad (11)$$

where

where  $\bar{\lambda} = (\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_n)^T$ ,  $\bar{\mu} = (\bar{\mu}_1, \bar{\mu}_2, \dots, \bar{\mu}_n)^T$ ,  $\text{sign}(z_i(t)) = \text{diag}(\text{sign}(z_{i1}(t)), \text{sign}(z_{i2}(t)), \dots, \text{sign}(z_{in}(t)))$ ,  $i = 1, 2, \dots, N$ .

From (11) and (13), one has

$$\begin{aligned}
\dot{z}_i(t) &= -Kz_i(t) + A(x_i(t))f(z_i(t)) \\
&\quad + [A(x_i(t)) - A(x^*)]f(x^*) \\
&\quad + B(x_i(t))g(z_i(t-\tau)) \\
&\quad + [B(x_i(t)) - B(x^*)]g(x^*) + Cu_i(t) \\
&\quad + \sum_{r=1}^s \sum_{j=1}^N d_r G_{ij}^r H^r z_j(t) - \text{sign}(z_i(t))(\bar{A}\bar{\lambda} + \bar{B}\bar{\mu}).
\end{aligned} \tag{14}$$

The output vector  $y_i(t) \in \mathbb{R}^w$  of the network (14) is given by

$$y_i(t) = Q_1 z_i(t) + Q_2 u_i(t), \tag{15}$$

in which  $Q_1 \in \mathbb{R}^{w \times n}$  and  $Q_2 \in \mathbb{R}^{w \times m}$ .

For convenience, we denote

$$\begin{aligned}
\Lambda &= \text{diag}(\lambda_1^2, \lambda_2^2, \dots, \lambda_n^2), \\
M &= \text{diag}(\mu_1^2, \mu_2^2, \dots, \mu_n^2), \\
z(t) &= (z_1^T(t), z_2^T(t), \dots, z_N^T(t))^T, \\
u(t) &= (u_1^T(t), u_2^T(t), \dots, u_N^T(t))^T, \\
y(t) &= (y_1^T(t), y_2^T(t), \dots, y_N^T(t))^T.
\end{aligned} \tag{16}$$

### 3.2. Passivity Criteria

**Theorem 1.** *If there is a matrix  $F \in \mathbb{R}^{wN \times mN}$  satisfying*

$$\begin{pmatrix} \Xi_1 & & \Xi_2 \\ & & \\ \Xi_2^T & -\frac{1}{2}[(I_N \otimes Q_2^T)F + F^T(I_N \otimes Q_2)] & \end{pmatrix} \leq 0, \tag{17}$$

where  $\Xi_1 = I_N \otimes (-2K + \Lambda + \bar{A} + M + \bar{B}) + 2 \sum_{r=1}^s d_r G^r \otimes H^r$ ,  $\Xi_2 = I_N \otimes C - 1/2(I_N \otimes Q_1^T)F$ , then the network (14) is passive.

*Proof.* Select the following Lyapunov functional for the network (14):

$$V(t) = \sum_{i=1}^N z_i^T(t) z_i(t) + \sum_{i=1}^N \int_{t-\tau}^t z_i^T(s) M z_i(s) ds. \tag{18}$$

Then, one has

$$\begin{aligned}
D^+V(t) &= \sum_{i=1}^N z_i^T(t) M z_i(t) - \sum_{i=1}^N z_i^T(t-\tau) M z_i(t-\tau) \\
&\quad + 2 \sum_{i=1}^N z_i^T(t) \dot{z}_i(t) \\
&\leq 2 \sum_{i=1}^N z_i^T(t) \{-Kz_i(t) + A(x_i(t))f(z_i(t)) \\
&\quad + [A(x_i(t)) - A(x^*)]f(x^*) \\
&\quad + B(x_i(t))g(z_i(t-\tau)) \\
&\quad + [B(x_i(t)) - B(x^*)]g(x^*) + Cu_i(t) \\
&\quad + \sum_{r=1}^s \sum_{j=1}^N d_r G_{ij}^r H^r z_j(t) \\
&\quad - \text{sign}(z_i(t))(\bar{A}\bar{\lambda} + \bar{B}\bar{\mu})\} \\
&\quad + \sum_{i=1}^N z_i^T(t) M z_i(t) \\
&\quad - \sum_{i=1}^N z_i^T(t-\tau) M z_i(t-\tau).
\end{aligned} \tag{19}$$

Based on Assumption 1, one can obtain

$$\begin{aligned}
&2z_i^T(t)[A(x_i(t)) - A(x^*)]f(x^*) \\
&= 2 \sum_{\eta=1}^n \sum_{\rho=1}^n z_{i\eta}(t)(a_{\eta\rho}(x_{i\eta}(t)) - a_{\eta\rho}(x_{i\eta}^*))f_{\rho}(x_{\rho}^*) \\
&\leq 2 \sum_{\eta=1}^n \sum_{\rho=1}^n |z_{i\eta}(t)| |\check{a}_{\eta\rho} - \hat{a}_{\eta\rho}| \bar{\lambda}_{\rho} \\
&= 2 |z_i^T(t)| \bar{A}\bar{\lambda}.
\end{aligned} \tag{20}$$

Similarly, one has

$$\begin{aligned}
&2z_i^T(t)[B(x_i(t)) - B(x^*)]g(x^*) \\
&= 2 \sum_{\eta=1}^n \sum_{\rho=1}^n z_{i\eta}(t)(b_{\eta\rho}(x_{i\eta}(t)) - b_{\eta\rho}(x_{i\eta}^*))g_{\rho}(x_{\rho}^*) \\
&\leq 2 \sum_{\eta=1}^n \sum_{\rho=1}^n |z_{i\eta}(t)| |\check{b}_{\eta\rho} - \hat{b}_{\eta\rho}| \bar{\mu}_{\rho} \\
&= 2 |z_i^T(t)| \bar{B}\bar{\mu}.
\end{aligned} \tag{21}$$

It can be obtained from Lemma 2.1 in [28] and Assumption 1 that

$$\begin{aligned}
 & 2z_i^T(t)A(x_i(t))f(z_i(t)) \\
 & \leq z_i^T(t)A(x_i(t))A^T(x_i(t))z_i(t) + f^T(z_i(t))f(z_i(t)) \\
 & = \sum_{\eta=1}^n \sum_{\rho=1}^n z_{i\eta}^2(t)a_{\eta\rho}^2(x_{i\eta}(t)) + \sum_{\rho=1}^n f_\rho^2(z_{i\rho}(t)) \\
 & \leq \sum_{\eta=1}^n \sum_{\rho=1}^n z_{i\eta}^2(t)\bar{a}_{\eta\rho}^2 + \sum_{\rho=1}^n \lambda_\rho^2 z_{i\rho}^2(t) \\
 & = z_i^T(t)\tilde{A}z_i(t) + z_i^T(t)\wedge z_i(t).
 \end{aligned} \tag{22}$$

Similarly, we get

$$\begin{aligned}
 & 2z_i^T(t)B(x_i(t))g(z_i(t-\tau)) \\
 & \leq z_i^T(t)B(x_i(t))B^T(x_i(t))z_i(t) \\
 & \quad + g^T(z_i(t-\tau))g(z_i(t-\tau)) \\
 & = \sum_{\eta=1}^n \sum_{\rho=1}^n z_{i\eta}^2(t)b_{\eta\rho}^2(x_{i\eta}(t)) + \sum_{\rho=1}^n g_\rho^2(z_{i\rho}(t-\tau)) \\
 & \leq \sum_{\eta=1}^n \sum_{\rho=1}^n z_{i\eta}^2(t)\bar{b}_{\eta\rho}^2 + \sum_{\rho=1}^n \mu_\rho^2 z_{i\rho}^2(t-\tau) \\
 & = z_i^T(t)\tilde{B}z_i(t) + z_i^T(t-\tau)Mz_i(t-\tau).
 \end{aligned} \tag{23}$$

According to (19)–(23), one gets

$$\begin{aligned}
 D^+V(t) & \leq \sum_{i=1}^N z_i^T(t)(-2K + \wedge + \tilde{A} + M + \tilde{B})z_i(t) \\
 & \quad + 2 \sum_{i=1}^N z_i^T(t)Cu_i(t) \\
 & \quad + 2 \sum_{r=1}^s \sum_{i=1}^N \sum_{j=1}^N d_r G_{ij}^r z_i^T(t)H^r z_j(t) \\
 & = \sum_{i=1}^N z_i^T(t)(-2K + \wedge + \tilde{A} + M + \tilde{B})z_i(t) \\
 & \quad + 2 \sum_{i=1}^N z_i^T(t)Cu_i(t) \\
 & \quad + 2 \sum_{r=1}^s d_r z^T(t)(G^r \otimes H^r)z(t) \\
 & = z^T(t)[I_N \otimes (-2K + \wedge + \tilde{A} + M + \tilde{B}) \\
 & \quad + 2 \sum_{r=1}^s d_r G^r \otimes H^r]z(t) \\
 & \quad + 2z^T(t)(I_N \otimes C)u(t).
 \end{aligned} \tag{24}$$

Further, one obtains

$$\begin{aligned}
 & D^+V(t) - y^T(t)Fu(t) \\
 & \leq z^T(t)[I_N \otimes (-2K + \wedge + \tilde{A} + M + \tilde{B}) \\
 & \quad + 2 \sum_{r=1}^s d_r G^r \otimes H^r]z(t) \\
 & \quad + z^T(t)[2I_N \otimes C - (I_N \otimes Q_1^T)F]u(t) \\
 & \quad - u^T(t)\left\{\frac{1}{2}[(I_N \otimes Q_2^T)F + F^T(I_N \otimes Q_2)]\right\}u(t) \\
 & = \vartheta^T(t) \begin{pmatrix} \Xi_1 & \Xi_2 \\ \Xi_2^T & -\frac{1}{2}[(I_N \otimes Q_2^T)F + F^T(I_N \otimes Q_2)] \end{pmatrix} \vartheta(t),
 \end{aligned} \tag{25}$$

where  $\vartheta(t) = (z^T(t), u^T(t))^T$ .

From (17), one can derive

$$y^T(t)Fu(t) \geq D^+V(t). \tag{26}$$

By (26), one has

$$\int_{t_0}^{t_\varepsilon} y^T(s)Fu(s)ds \geq V(t_\varepsilon) - V(t_0), \tag{27}$$

for any  $t_0, t_\varepsilon \in [0, +\infty)$  and  $t_\varepsilon \geq t_0$ .  $\square$

**Theorem 2.** *If there exist matrices  $F \in \mathbb{R}^{wN \times mN}$  and  $0 < L_1 \in \mathbb{R}^{mN \times mN}$  satisfying*

$$\begin{pmatrix} \Xi_1 & \Xi_2 \\ \Xi_2^T & -\frac{1}{2}[(I_N \otimes Q_2^T)F + F^T(I_N \otimes Q_2)] + L_1 \end{pmatrix} \leq 0, \tag{28}$$

where  $\Xi_1 = I_N \otimes (-2K + \wedge + \tilde{A} + M + \tilde{B}) + 2 \sum_{r=1}^s d_r G^r \otimes H^r$ ,  $\Xi_2 = I_N \otimes C - 1/2(I_N \otimes Q_1^T)F$ , then the network (14) is input-strictly passive.

*Proof.* For the network (14), choosing the identical Lyapunov functional as (18), then one can obtain

$$\begin{aligned}
 & D^+V(t) - y^T(t)Fu(t) + u^T(t)L_1u(t) \\
 & \leq z^T(t)[I_N \otimes (-2K + \wedge + \tilde{A} + M + \tilde{B}) \\
 & \quad + 2 \sum_{r=1}^s d_r G^r \otimes H^r]z(t) \\
 & \quad + z^T(t)[2I_N \otimes C - (I_N \otimes Q_1^T)F]u(t) \\
 & \quad - u^T(t)\left\{\frac{1}{2}[(I_N \otimes Q_2^T)F + F^T(I_N \otimes Q_2)] - L_1\right\}u(t) \\
 & = \vartheta^T(t) \begin{pmatrix} \Xi_1 & \Xi_2 \\ \Xi_2^T & -\frac{1}{2}[(I_N \otimes Q_2^T)F + F^T(I_N \otimes Q_2)] + L_1 \end{pmatrix} \vartheta(t),
 \end{aligned} \tag{29}$$

where  $\vartheta(t) = (z^T(t), u^T(t))^T$ .

From (28), we can get

$$y^T(t)Fu(t) - u^T(t)L_1u(t) \geq D^+V(t). \quad (30)$$

By (30), one has

$$\int_{t_0}^{t_\varepsilon} (y^T(s)Fu(s) - u^T(s)L_1u(s))ds \geq V(t_\varepsilon) - V(t_0), \quad (31) \quad \text{where}$$

for any  $t_0, t_\varepsilon \in [0, +\infty)$  and  $t_\varepsilon \geq t_0$ .  $\square$

**Theorem 3.** *If there exist matrices  $F \in \mathbb{R}^{wN \times mN}$  and  $0 < L_2 \in \mathbb{R}^{wN \times wN}$  satisfying*

$$\begin{pmatrix} \Xi_3 & \Xi_4 \\ \Xi_4^T & \Xi_5 \end{pmatrix} \leq 0, \quad (32)$$

$$\Xi_3 = I_N \otimes (-2K + \Lambda + \tilde{A} + M + \tilde{B}) + 2 \sum_{r=1}^s d_r G^r \otimes H^r + (I_N \otimes Q_1^T) L_2 (I_N \otimes Q_1),$$

$$\Xi_4 = I_N \otimes C - \frac{1}{2} (I_N \otimes Q_1^T) F + (I_N \otimes Q_1^T) L_2 (I_N \otimes Q_2), \quad (33)$$

$$\Xi_5 = -\frac{1}{2} [(I_N \otimes Q_2^T) F + F^T (I_N \otimes Q_2)] + (I_N \otimes Q_2^T) L_2 (I_N \otimes Q_2),$$

then the network (14) is output-strictly passive.

*Proof.* For the network (14), choosing the identical Lyapunov functional as (18), then we can derive

$$\begin{aligned} & D^+V(t) - y^T(t)Fu(t) + y^T(t)L_2y(t) \\ & \leq z^T(t) \left[ I_N \otimes (-2K + \Lambda + \tilde{A} + M + \tilde{B}) \right. \\ & \quad \left. + 2 \sum_{r=1}^s d_r G^r \otimes H^r + (I_N \otimes Q_1^T) L_2 (I_N \otimes Q_1) \right] z(t) \\ & \quad + z^T(t) \left[ 2I_N \otimes C - (I_N \otimes Q_1^T) F \right. \\ & \quad \left. + 2(I_N \otimes Q_1^T) L_2 (I_N \otimes Q_2) \right] u(t) \\ & \quad - u^T(t) \left\{ \frac{1}{2} [(I_N \otimes Q_2^T) F + F^T (I_N \otimes Q_2)] \right. \\ & \quad \left. - (I_N \otimes Q_2^T) L_2 (I_N \otimes Q_2) \right\} u(t) \\ & = \vartheta^T(t) \begin{pmatrix} \Xi_3 & \Xi_4 \\ \Xi_4^T & \Xi_5 \end{pmatrix} \vartheta(t), \end{aligned} \quad (34)$$

where  $\vartheta(t) = (z^T(t), u^T(t))^T$ .

From (32), one can get

$$y^T(t)Fu(t) - y^T(t)L_2y(t) \geq D^+V(t). \quad (35)$$

By (35), one has

$$\int_{t_0}^{t_\varepsilon} (y^T(s)Fu(s) - y^T(s)L_2y(s))ds \geq V(t_\varepsilon) - V(t_0), \quad (36)$$

for any  $t_0, t_\varepsilon \in [0, +\infty)$  and  $t_\varepsilon \geq t_0$ .  $\square$

*Remark 2.* It is a key issue that NNs are unable to achieve the passivity by themselves in some circumstances [23]. As a consequence, it is necessary to utilize an appropriate control method to make NNs passive. For all we know, the passivity problem of MWCMNNs via feedback control has not been researched. In the above discussion, with the help of a developed feedback controller, several criteria are established to ensure that the proposed network is passive, output-strictly passive, and input-strictly passive, respectively.

### 4. Numerical Example

Example 1. The MWCMNN is considered as follows:

$$\begin{aligned} \dot{x}_i(t) = & -Kx_i(t) + A(x_i(t))f(x_i(t)) \\ & + B(x_i(t))g(x_i(t - \tau)) + Cu_i(t) \\ & + \sum_{r=1}^3 \sum_{j=1}^6 d_r G_{ij}^r H^r x_j(t) + J + \psi_i(t), \end{aligned} \tag{37}$$

in which  $i = 1, 2, \dots, 6, f_\rho(\delta) = g_\rho(\delta) = 1/8(|\delta + 1| - |\delta - 1|), \rho = 1, 2, 3, K = \text{diag}(1.6, 2.1, 2.6), \tau = 1, J = (0, 0, 0)^T, d_1 = 0.1, d_2 = 0.2, d_3 = 0.3, H^1 = \text{diag}(0.3, 0.6, 0.4), H^2 = \text{diag}(0.5, 0.6, 0.2), H^3 = \text{diag}(0.6, 0.5, 0.3),$  and the matrices  $G^1, G^2, G^3, C, A(x_i(t)), B(x_i(t))$  are chosen as follows:

$$\begin{aligned} G^1 = & \begin{pmatrix} -0.2 & 0 & 0.1 & 0 & 0.1 & 0 \\ 0 & -0.4 & 0 & 0.2 & 0.1 & 0.1 \\ 0.1 & 0 & -0.5 & 0 & 0.2 & 0.2 \\ 0 & 0.2 & 0 & -0.5 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0.1 & -0.7 & 0.2 \\ 0 & 0.1 & 0.2 & 0.2 & 0.2 & -0.7 \end{pmatrix}, \\ G^2 = & \begin{pmatrix} -0.3 & 0 & 0.1 & 0 & 0.1 & 0.1 \\ 0 & -0.5 & 0 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0 & -0.4 & 0 & 0.3 & 0 \\ 0 & 0.2 & 0 & -0.6 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.3 & 0.2 & -0.9 & 0.2 \\ 0.1 & 0.2 & 0 & 0.2 & 0.2 & -0.7 \end{pmatrix}, \\ G^3 = & \begin{pmatrix} -0.4 & 0.1 & 0.1 & 0 & 0.1 & 0.1 \\ 0.1 & -0.4 & 0 & 0.1 & 0.2 & 0 \\ 0.1 & 0 & -0.7 & 0.2 & 0.3 & 0.1 \\ 0 & 0.1 & 0.2 & -0.5 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.3 & 0.1 & -1.0 & 0.3 \\ 0.1 & 0 & 0.1 & 0.1 & 0.3 & -0.6 \end{pmatrix}, \\ C = & \begin{pmatrix} 0.3 & 0.6 \\ 0.3 & 0.1 \\ 0.2 & 0.4 \end{pmatrix}, \end{aligned} \tag{38}$$

$$\begin{aligned} a_{11}(x_{i1}(t)) = & \begin{cases} -0.20, |x_{i1}(t)| \leq 1, \\ -0.36, |x_{i1}(t)| > 1, \end{cases} \\ a_{12}(x_{i1}(t)) = & \begin{cases} 0.43, |x_{i1}(t)| \leq 1, \\ -0.20, |x_{i1}(t)| > 1, \end{cases} \\ a_{13}(x_{i1}(t)) = & \begin{cases} -0.20, |x_{i1}(t)| \leq 1, \\ 0.32, |x_{i1}(t)| > 1, \end{cases} \\ a_{21}(x_{i2}(t)) = & \begin{cases} 0.47, |x_{i2}(t)| \leq 1, \\ -0.30, |x_{i2}(t)| > 1, \end{cases} \\ a_{22}(x_{i2}(t)) = & \begin{cases} -0.53, |x_{i2}(t)| \leq 1, \\ 0.40, |x_{i2}(t)| > 1, \end{cases} \\ a_{23}(x_{i2}(t)) = & \begin{cases} 0.33, |x_{i2}(t)| \leq 1, \\ -0.66, |x_{i2}(t)| > 1, \end{cases} \\ a_{31}(x_{i3}(t)) = & \begin{cases} 0.56, |x_{i3}(t)| \leq 1, \\ 0.40, |x_{i3}(t)| > 1, \end{cases} \\ a_{32}(x_{i3}(t)) = & \begin{cases} 0.20, |x_{i3}(t)| \leq 1, \\ -0.42, |x_{i3}(t)| > 1, \end{cases} \\ a_{33}(x_{i3}(t)) = & \begin{cases} -0.45, |x_{i3}(t)| \leq 1, \\ 0.48, |x_{i3}(t)| > 1, \end{cases} \\ b_{11}(x_{i1}(t)) = & \begin{cases} 0.58, |x_{i1}(t)| \leq 1, \\ -0.49, |x_{i1}(t)| > 1, \end{cases} \\ b_{12}(x_{i1}(t)) = & \begin{cases} -0.66, |x_{i1}(t)| \leq 1, \\ -0.22, |x_{i1}(t)| > 1, \end{cases} \\ b_{13}(x_{i1}(t)) = & \begin{cases} 0.38, |x_{i1}(t)| \leq 1, \\ 0.53, |x_{i1}(t)| > 1, \end{cases} \\ b_{21}(x_{i2}(t)) = & \begin{cases} -0.32, |x_{i2}(t)| \leq 1, \\ 0.28, |x_{i2}(t)| > 1, \end{cases} \\ b_{22}(x_{i2}(t)) = & \begin{cases} 0.40, |x_{i2}(t)| \leq 1, \\ 0.32, |x_{i2}(t)| > 1, \end{cases} \\ b_{23}(x_{i2}(t)) = & \begin{cases} -0.56, |x_{i2}(t)| \leq 1, \\ -0.30, |x_{i2}(t)| > 1, \end{cases} \\ b_{31}(x_{i3}(t)) = & \begin{cases} 0.24, |x_{i3}(t)| \leq 1, \\ -0.36, |x_{i3}(t)| > 1, \end{cases} \\ b_{32}(x_{i3}(t)) = & \begin{cases} 0.34, |x_{i3}(t)| \leq 1, \\ 0.18, |x_{i3}(t)| > 1, \end{cases} \\ b_{33}(x_{i3}(t)) = & \begin{cases} 0.46, |x_{i3}(t)| \leq 1, \\ -0.55, |x_{i3}(t)| > 1. \end{cases} \end{aligned} \tag{39}$$

Obviously,  $f_\rho(\cdot)$  as well as  $g_\rho(\cdot)$  satisfy Assumption 1 with  $\lambda_\rho = \bar{\lambda}_\rho = 0.25$  and  $\mu_\rho = \bar{\mu}_\rho = 0.25$ . Besides,  $\mathbb{R}^3 \ni x^* = (0, 0, 0)^T$  is an equilibrium point of the isolated node for MWCMNN (37).

The output vector  $y_i(t) \in \mathbb{R}^3$  is chosen as follows:

$$y_i(t) = Q_1 z_i(t) + Q_2 u_i(t), \quad i = 1, 2, \dots, 6, \tag{40}$$

(38) in which

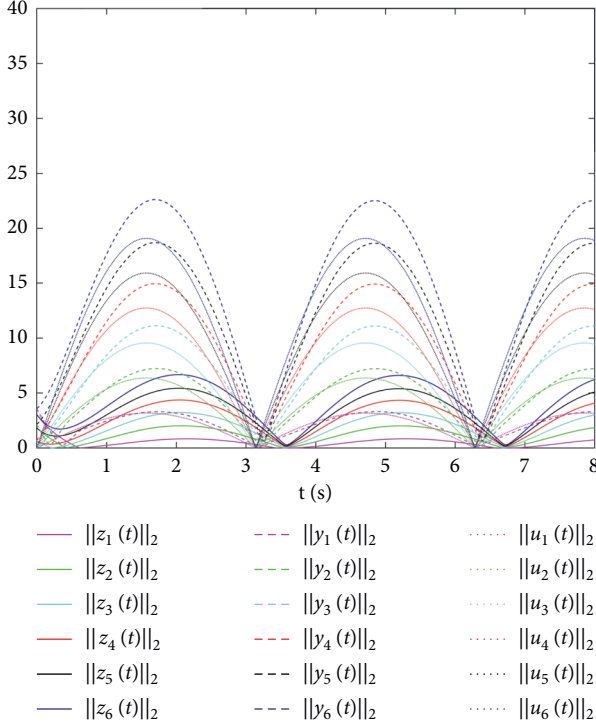


FIGURE 1:  $\|z_i(t)\|_2, \|y_i(t)\|_2, \|u_i(t)\|_2, i = 1, 2, \dots, 6$ .

$$Q_1 = \begin{pmatrix} 0.4 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.3 \\ 0.6 & 0.4 & 0.6 \end{pmatrix}, \quad (41)$$

$$Q_2 = \begin{pmatrix} 0.3 & 0.4 \\ 0.2 & 0.5 \\ 0.6 & 0.1 \end{pmatrix}.$$

*Case 1.* The following matrix  $F$  which satisfies (17) can be derived with the assistance of the MATLAB YALMIP Toolbox:

$$F = I_6 \otimes \begin{pmatrix} -0.6168 & 8.0766 \\ -1.6951 & 0.2803 \\ 4.1206 & -3.0097 \end{pmatrix}. \quad (42)$$

In accordance with Theorem 1, the MWCMNN (37) is passive via the controller (13).

*Case 2.* The following matrices  $F$  and  $L_1$  which satisfy (28) can be derived with the assistance of the MATLAB YALMIP Toolbox:

$$F = I_6 \otimes \begin{pmatrix} -0.2914 & 12.7012 \\ -2.1342 & 1.2881 \\ 4.8077 & -5.9296 \end{pmatrix}, \quad (43)$$

$$L_1 = I_6 \otimes \begin{pmatrix} 1.1852 & -0.0482 \\ -0.0482 & 2.5658 \end{pmatrix}.$$

By Theorem 2, the MWCMNN (37) is input-strictly passive via the controller (13).

*Case 3.* The following matrices  $F$  and  $L_2$  which satisfy (32) can be derived with the assistance of the MATLAB YALMIP Toolbox:

$$F = I_6 \otimes \begin{pmatrix} -1.4129 & 6.4117 \\ -0.9625 & 1.8288 \\ 5.2249 & -1.8118 \end{pmatrix}, \quad (44)$$

$$L_2 = I_6 \otimes \begin{pmatrix} 2.2956 & -0.2844 & -0.6467 \\ -0.2844 & 2.4627 & -0.4603 \\ -0.6467 & -0.4603 & 1.8159 \end{pmatrix}.$$

On account of Theorem 3, the MWCMNN (37) is output-strictly passive via the controller (13). Figure 1 exhibits the simulation results.

## 5. Conclusion

It is the first time that the passivity of MWCMNNs has been discussed in this paper. Based on the Lyapunov stability theory, feedback control theory, and functional differential equations, several novel criteria have been set up to guarantee that the considered network is passive, output-strictly passive, and input-strictly passive. Finally, one numerical simulation example has been presented to demonstrate the effectiveness of the theoretical results. In the future work, the investigation of the synchronization and passivity for multiple weighted CMNNs with time-varying delays via adaptive control will be considered.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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