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Research Article

Optimal Valuation of Retailer Equity Financing Based on Gambling Agreements in Centralized Supply Chain

Liang Wang and Guiru Wang

School of Economics and Management, Xi'an University of Technology, Xi'an 710048, China

Correspondence should be addressed to Liang Wang; wangliang@xaut.edu.cn

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This paper explores the issue of optimal valuation of retailer equity financing based on gambling agreements in a centralized supply chain. Firstly, the betting target settings are classified into three cases: high, medium, and low. Secondly, the optimal valuation models for retailer equity financing in a centralized supply chain without and with introducing a gambling agreement are constructed separately. Finally, this paper clarifies the relationships among factors such as the betting transfer share ratio, ordering price, and optimal valuation level through simulation analysis. The results illustrate the following: (i) There is always an only optimal valuation level and optimal effort level that makes the retailer and supplier might reach an equity financing agreement. (ii) When the betting target is set too high, the betting transfer share ratio varies in the same direction as the optimal valuation level and the opposite direction as the optimal effort level. (iii) When the betting target is set moderately or too low, the betting transfer share ratio has an inverse relationship with the optimal valuation level. However, the retailer will increase its effort level at this time. (iv) In addition, the optimal valuation level is affected by the retailer's firm growth and ordering price.

1. Introduction

In recent years, supply chain financing has become an effective way to deal with difficult and expensive finance for small and medium-sized enterprises. Supply chain financing methods mainly include debt financing and equity financing. Equity financing is a financing method in which the shareholders of a company give up part of their shareholdings and thus obtain funds. Different from debt financing, the capital raised by equity financing is mostly permanent, and dividends are usually based on the profitability of the company, without the financial risk of debt repayment, etc.

Equity financing is usually done by way of PE/VC in practice, but to improve the overall performance of the supply chain and strengthen cooperation, equity financing among supply chain members has become one of the main ways of supply chain financing. For example, China Modern Dairy Holdings Ltd. injected RMB 21.24 million into its forage supplier in 2011 and 2012, respectively, to address the former's financing needs. In 2020, HUAWEI began to focus on supplier equity financing projects due to the U.S. government crackdown, successively acquiring core supply chain partner vendors such as ORIENTAL SEMI, SKYVERSE, and ALLSEMI. In August 2020, JINGDONG Group acquired a controlling interest in KUAYUE-EXPRESS for a total consideration of 3 billion CNY. In September 2020, ALIBABA Group invested an additional 12% stake in YUANTONG for 6.6 billion CNY.

However, no matter what type of equity financing is adopted, it is most critical for both investors and financiers to reach a reasonable corporate valuation. In this process, both parties have to evaluate the value of the target company's assets and determine the percentage of shares to be offered by the financier and the size of the investor's investment. A reasonable corporate valuation can lead to an equity agreement between the investor and financier. But in practice, the parties may disagree on valuation issues due to factors, such as information asymmetry, future growth of the enterprise, differences in valuation methods, and moral hazard. A high valuation allows the financier to raise more money with fewer shares, while a low valuation may cause a larger loss to the financier. In addition, since supply chain equity financing needs to consider the influence of the market environment, price, ordering demand, revenue, and other uncertainties, supply chain equity financing is a typical stochastic optimization issue. For stochastic optimality, the existing literature has mainly researched in the fields of supply chain decision making [1–13], risk investment [14], and disease transmission [15–17] by applying stochastic process theory, generalized function analysis, and operations research methods. However, different from the existing studies, the valuation of financier enterprises by investors in the process of supply chain equity financing is inherently more complex and stochastic characteristic, which may result in larger valuation conflicts.

For the above issue, the gambling agreement as an effective "valuation adjustment mechanism" has received widespread attention from the practical and theoretical community. The gambling agreement is usually a predetermined performance target set by both parties to an agreement to guarantee the investor's rights and interests. If the financier completes the predetermined target as agreed, it will receive a corresponding equity award or additional investment from the investor. Otherwise, the financier will have to compensate the investor accordingly. Therefore, through the gambling agreement mechanism, both parties can later "adjust" or "remedy" the corporate valuation and equity ratio by the actual business performance to attract the investor and motivate the financier. For example, the famous Mengniu equity financing gambling agreement in 2003 stipulated that Mengniu would be required to transfer 70 million shares to Morgan Stanley and other investors if Mengniu's performance grew at a compound annual rate of less than 50% between 2003 and 2006. In 2007, Yolo Ltd. had signed a gambling agreement with Morgan Stanley and CDH Investments that the foreign shareholders would transfer 46,973,800 shares of Yolo Ltd. to its management if the company's 2007 net profit was higher than RMB 750 million. However, if the net profit was equal to or less than RMB 675 million, Yolo management would transfer the corresponding shares to the foreign shareholders. Xiaomi Ltd. signed a gambling agreement with Morningside Investment in 2011 that if Xiaomi met its performance targets from 2011 to 2015 and successfully listed in 2018, it would receive a bonus. 360 Technology Ltd. had also signed a gambling agreement when it was listed on the shell in 2017, the company was required to achieve net profits of RMB 2.2, 2.9, 3.8, and 4.15 billion from 2017 to 2020, respectively, or it would need to compensate the investors in shares and cash.

In summary, when supply chain members intend to solve the capital bottleneck problem through equity financing, to avoid the impact of optimality on valuation accuracy and better protect the interests of investors and motivate the financiers to make maximum efforts to improve performance, introducing a gambling agreement is a better choice. This paper will take the secondary centralized supply chain as the research object and constructs an optimal retailer valuation model by taking the expected value of the stochastic revenue function to explore the optimal valuation of retailer equity financing based on the gambling agreement. Furthermore, it will conduct a study on the following issues.

(i) How does the optimal valuation level change in a centralized supply chain with retailer equity financing based on the betting agreement when there are differences in the setting of betting targets?

(ii) What is the mechanism of influence of factors such as the gambling target setting level, the betting transfer share ratio, and the firm growth on the optimal valuation level and the retailer's optimal effort level?

2. Literature Review and Research Issues

2.1. Supply Chain Financing. For the study of supply chain financing, the existing literature has mainly focused on the mechanism of supply chain financing's role, financing efficiency, and the choice of supply chain equity financing methods. Supply chain financing can reduce information asymmetry and financing costs [18–25]. Moreover, the joint decision of the supply chain can improve the operational efficiency of retailers, suppliers, and banks. Product characteristics, technology, and service level are critical factors that determine the sustainability of supply chain financing performance [26–30]. In practice, accounts receivable financing by reverse factoring is an effective supply chain financing model. Fixed asset financing, inventory financing, and accounts receivable financing are valid methods to improve financing efficiency. Furthermore, the decision of a retailer to choose debt or equity financing is closely related to its market share, equity structure, financial costs, supply chain cooperation, firm growth, and optimal decisions of the bank. However, most retailers prefer equity financing for two reasons [31–36]. On the one hand, it is due to the characteristics of supply chain equity financing with low cost, high yield, and supporting technological innovation to reduce transportation costs. On the other hand, it is based on the consideration of capital structure and market competitiveness.

2.2. Gambling Agreements in Equity Financing. The research on the gambling agreement has been discussed in the literature mainly in terms of its role, risks, and mechanism design. The gambling agreement mechanism can alleviate conflicts in the investment and financing process, improve the efficiency of equity financing [37–40], reduce the uncertainty of valuation, diminish moral hazard, create incentive constraints, and even decrease the external financing cost. However, installment payment, uncertainty about the expected outcome, and inefficiencies and transaction costs due to government regulation may increase risks. The default risk by the financier can make it impossible for the investor to use the ordinary option method for the valuation of the gambling agreement [41, 42]. Meanwhile, the design of the gambling mechanism should follow the principle of reducing information asymmetry and mitigating the risk of adverse selection, and the setting of the betting target will form a different degree of incentive for the target enterprise. Furthermore, the gambling mechanism with the real options method can effectively avoid excessive dilution of corporate equity, and the structure design using repeated bets is better than the once bet.

2.3. Corporate Valuation Methods in Equity Financing. The valuation methods in corporate equity financing have been studied in the existing literature mainly from financial perspectives such as the price-to-earnings (P/E) ratio and price-to-book (P/B) ratio methods. Some scholars proposed that the P/B ratio valuation applies to listed companies in the Asia-Pacific region [43–45] and that this method is more accurate than the P/E ratio valuation. However, the lowest valuation of a company through the P/B ratio method is not the optimal valuation for investors. Valuation using the P/E ratio valuation method, forward earnings target, the book value of equity index, and other methods also requires consideration of factors such as the type and nature of the company. Zmeskal; Pringles, Olsina, and Garces; and Guo and Zmeskai discussed valuation methods of corporate equity financing from the perspective of real options.

2.4. The Impact of Firm Growth on Corporate Valuation. The established literature has mainly discussed the impact of firm growth on corporate valuation in terms of market share, intangible asset value, and firm network characteristics. The valuation of high-tech companies with high growth needs to take into account factors such as shareholder value, market share, and the life cycle of customers, thus reducing the issue of resource mismatch in investment. It is also necessary to consider the influence of the structural characteristics and network size of the enterprise's associated network nodes on the valuation result. Since companies with higher growth have more valuable intangible assets, the valuation process should pay attention to the value of such assets. The real option model through PFM (Private Firm Model) is more suitable for the valuation of growth companies, while the traditional discounted cash flow model makes it difficult to properly value them.

2.5. Research Issues and Structure of the Paper. The existing literature has mainly studied supply chain financing from the aspects of its role mechanism, efficiency, and equity financing model and explored the effect, risk, and mechanism design of the gambling agreement. Furthermore, the choice of valuation methods in company equity financing and the impact of firm growth on corporate valuation have also become the focus of academic attention, but current research needs to be further expanded in the following areas. Firstly, the existing literature mainly has focused on the mechanism of the role of supply chain financing, financing efficiency, and financing model selection but has not explored the issue of equity financing in a centralized supply chain based on the mechanism of gambling agreement. Secondly, the existing literature has not investigated the relationship between the heterogeneity of the gambling target setting and the optimal valuation and effort level when exploring the design of the gambling agreement mechanism. Thirdly, the existing studies have not clarified the mechanism of the impact of factors such as the share of gambling transfer and firm growth on corporate valuation, nor have they verified the validity of valuation methods in supply chain equity financing.

Therefore, this paper takes a secondary centralized supply chain consisting of a retailer and a supplier as the research object and focuses on the issue of optimal valuation of retailer equity financing based on a gambling agreement. First of all, through the analysis of the retailer equity financing process, the optimal valuation model for retailer equity financing without introducing a gambling agreement in a centralized supply chain is mathematically derived. Next, the betting target setting is divided into three scenarios and valued by the P/B ratio method, whereby it determines the optimal valuation level for equity financing and the optimal effort level for the retailer when the gambling agreement mechanism is introduced. Finally, through simulation analysis, this paper clarifies the influence mechanism between the betting transfer share ratio, ordering price, firm growth and optimal valuation level, and optimal effort level under the condition of heterogeneity of betting target setting.

The main contributions of this paper are the following four points. (i) Combining with the practical background of supply chain equity financing, it introduces the gambling agreement mechanism in centralized supply chain equity financing, which enriches the relevant research in the fields of company equity financing and supply chain finance. (ii) Following the Stackelberg game model, this paper constructs an optimal valuation model for retailer equity financing based on the gambling agreement and derives the optimal effort level, which provides a reference for decision-making to alleviate the valuation conflicts between retailer and supplier in a centralized supply chain. (iii) Through mathematical derivation and simulation analysis, it investigates the influence mechanism of factors such as the betting transfer share ratio, ordering price, and firm growth to the optimal valuation level and effort level. (iv) The comparative research finds that the parties of centralized supply chain equity financing should consider factors such as firm growth, ordering price, and future revenue expectations to determine the appropriate gambling target level and gambling compensation method.

The structure of this paper is composed as follows. Section 2 summarizes the existing literature and presents the research issue. Section 3 provides an analysis of the retailer's equity financing decision and assumptions. Section 4 establishes an optimal valuation model for retailer equity financing in a centralized supply chain without the introduction of a gambling agreement. In Section 5, the optimal valuation model with gambling agreement is constructed. Section 6 conducts inferences and Section 7 develops the simulation analysis. In the end, Section 8 presents the research conclusion.

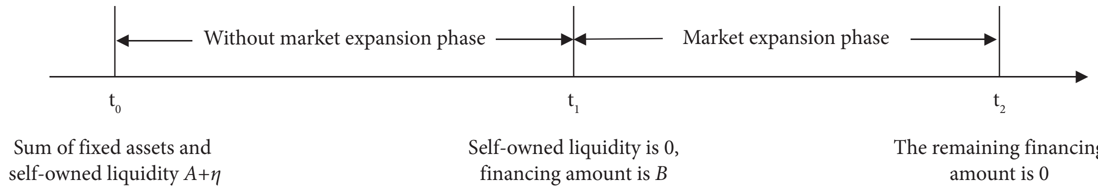

Figure 1: Time-series diagram of retailer equity financing decision.

3. Retailer Equity Financing Decision and Assumptions

3.1. Retailer Equity Financing Decision. This paper considers a secondary, centralized supply chain consisting of one supplier and one retailer. The retailer is an unlisted company in its growth stage and needs equity financing for market expansion. Since all parties in a centralized supply chain share the benefits and risks, this paper assumes that the retailer is willing to cede a portion of its equity to the supplier to obtain funding.

For further study, a time-series diagram of the retailer's equity financing process is presented here. As shown in Figure 1, the moments t_0 to t_2 are the sales cycle for the retailer. Assume that point t_0 is the beginning of the retailer's sales phase, at which time the retailer's total assets are $A + \eta$, where the amount of fixed assets is A and self-owned liquidity is η (Yu and Wang, 2018) [37]. t_1 is a point when the retailer gets the amount of equity financing, and it is presumed that both parties can quickly reach an agreement, and t_1 is also the starting moment of the retailer's market expansion. t_0 to t_1 is the premarket expansion stage, in which the retailer's liquidity η meets its basic ordering demand and operating expenses, and η is a constant, but the retailer's liquidity is used only for premarket expansion expenses. At the time t_1, the retailer's liquidity position is 0. Furthermore, the amount of equity financing received by the retailer is B, which will be used in its further market expansion, mainly including a series of costs for basic operations, procurement, sales, and t_1 to t_2 is the market expansion phase. Meanwhile, the time t_2 is the end moment of market expansion, and this paper assumes that the remaining financing amount at that point is 0. The retailer's behavior at these three moments collectively affects its corporate value and also determines the supplier's assessment of the retailer's corporate value.

To better motivate the retailer, as shown in Figure 1, this paper assumes that in the process of supplier-to-retailer equity financing, the two parties also sign a gambling agreement at the moment t_1 when the financing agreement is reached. Moreover, the gambling agreement takes the retailer's revenue as the subject of the betting at the time t_2, with the equity transfer as the measure of compensation, and the parties set the target revenue in the agreement. At t_2, the gambling agreement expires, and the side that lost the bet is required to transfer its equity to the other side. Furthermore, both sides will distribute the retailer's earnings at t_2 in proportion to their shareholdings. Let ε be the betting transfer share ratio and let π_0 be the target revenue set by the betting (referred to as the betting target in the following). If the retailer's revenue at t_2 exceeds π_0, it wins the betting, and the supplier is required to transfer ε of its shareholdings to the retailer. If the retailer's revenue at the time t_2 is lower than π_0, in this case, the supplier wins the betting, the retailer is obliged to transfer ε of its shareholdings to the supplier. Assume that both π_0 and ε are jointly negotiated by the supplier and the retailer, and once the gambling agreement is signed, π_0 and ε are established constants. Furthermore, this paper assumes that both parties are risk-averse.

3.2. Assumptions

3.2.1. Retailers' Revenue Components. Suppose there is only one retailer in the market and that retailer sells only one product, which has a retail price of p. Before market expansion through equity financing, the market demand is q. The relationship between demand and price is $q = \delta - \gamma p$, where $\delta, \gamma > 0$ are constants. Assuming that the retailer's ordering price is w, wq can be considered the variable cost to be paid by the retailer for selling q units of the product. The fixed cost of the retailer is d, which is a constant. Therefore, without considering other factors, the retailer's revenue with no market expansion is $\pi_m = (p - w)q - d$, Meanwhile, this paper assumes that the incremental demand resulting from retailer market expansion will not affect w and p.

3.2.2. Retailers' Effort Cost Function in Market Expansion. Let e denote the level of effort retailers put into market expansion [12], which is measured mainly in terms of increasing advertising, conducting technology research, and widening sales channels. $C(e)$ represents the effort cost function of retailers in market expansion, which conforms to the law of increasing marginal cost, and $C(0) = 0$. Following Laffont and Tirole [21], the effort cost function of a retailer is assumed to be $C(e) = 1/2se^2$; this function is a quadratic function with an image opening upward and U-shaped, where $s > 0$ is the effort cost coefficient.

3.2.3. Retailers' Revenue Function Based on Firm Growth. Let β denote the incremental market demand per unit level of effort for retailer market expansion; then, βe represents the retailer's market demand increment when the level of effort is e. The following sections refer to β as the firm growth factor, which indicates the growth of a firm and can generally be measured in terms of the firm's innovation capability, management capability, capital operation capability, and overall employee quality [32] (Yu and Wang, 2018). Therefore, the new market demand faced by the retailer during the market expansion phase is

$q + \beta e$. Without considering other factors, the revenue function of the retailer at the time t_2 is $\pi = (p - w)(q + \beta e) - d$.

3.2.4. Corporate Valuation Based on the Price-to-Book Ratio Method. There are common corporate valuation methods such as the price-to-earnings ratio, the price-to-sales ratio, and the price-to-book ratio method. Compared with the previous two methods, the price-to-book ratio method has the characteristics of simple calculation, easy access to data, and not being easy to be manipulated, so this paper assumes that both the retailer and supplier use this method for corporate valuation. The price-to-book ratio is the ratio of the share price per share to the net assets per share, and the total assets of the retailer before the equity financing are $A + \eta$. Therefore, let the price-to-book ratio be α; then, the value of the retailer's assets determined by both parties at the time of the equity financing is $V = \alpha(A + \eta)$, and in the later section, the valuation level is proposed to be denoted by α.

3.2.5. Amount of Equity Financing and Method of Revenue Distribution. To simplify the study, suppose that the equity financing amount B is a constant determined by mutual agreement between the parties and that the financing amount ensures future market expansion by the retailer. In addition, this paper assumes that during the period of the gambling agreement, the supplier and the retailer will share the revenue from the retailer's market expansion phase in proportion to their shareholdings.

3.3. Variables and Symbols Description

A: the number of fixed assets of the retailer before the equity financing

η: the amount of the retailer's own liquidity before the equity financing

d: fixed cost of the retailer

α: the retailer's price-to-book ratio

β: the growth factor of the retailer and $\beta > 0$

e: the effort level expended by the retailer for market expansion, and $e > 0$

θ: the excess revenue from market expansion by retailers through equity financing

B: the amount of the retailer's equity financing, with $B > 0$

V: the value of the retailer's corporate assets as determined by both parties at the point of the equity financing

π_0: the betting target set by both parties

ε: the betting transfer share ratio, $0 < \varepsilon < 1$

$\alpha^*, \alpha_1^*, \alpha_2^*, \alpha_3^*$: optimal valuation levels under different situations

4. Optimal Valuation Model for Retailer Equity Financing without Introducing Gambling Agreement in Centralized Supply Chain

To explore the possibility of reaching an equity financing agreement between retailer and supplier in a centralized supply chain, this paper intends to analyze the existence of the optimal valuation level through mathematical derivation, and this section examines the case when gambling agreements are not introduced. The effort level of the retailer's market expansion affects the future value of the firm and plays a critical role in the value creation of the entire supply chain, and the effort level is affected by the high or low valuation. Therefore, the optimal effort level of the retailer needs to be considered along with the determination of the optimal valuation level.

Proposition 1. *Under the condition of without introducing a gambling agreement in a centralized supply chain and value using the P/B method, there exists an only optimal effort level for the retailer's market expansion and optimal valuation level, and the retailer and supplier can reach an equity financing agreement.*

Proof. By the previous assumptions and Figure 1, it can be seen that the revenue function of the retailer at the moment t_2 is

$$\pi = (p - w)(q + \beta e) - d + \theta. \tag{1}$$

θ represents the retailer's excess revenue gained through market expansion after the equity financing. This excess revenue may be the result of market expansion that enhances the brand influence and service level and broadens sales channels, or it may be the result of economies of scale that result from market expansion or the revenue-sharing mechanism of the centralized supply chain that allows the retailer to earn excess revenue. Because of this, to simplify the study, θ is assumed to obey uniform distribution on the interval $[0, k]$, and k is the retailer's maximum possible excess revenue in the market expansion process. Following the definition of uniform distribution, the probability density function of excess revenue θ is known as $g(\theta) = 1/k$. In a centralized supply chain, let $E(\pi)$ denote the expected revenue function obtained by the retailer through selling the product at the moment t_2 without introducing a gambling agreement and then have

$$\begin{aligned} E(\pi) &= E[(p - w)(q + \beta e) - d + \theta] \\ &= (p - w)(q + \beta e) - d + E(\theta) \\ &= (p - w)(q + \beta e) - d + \int_0^k \theta g(\theta) \mathrm{d}\theta \\ &= (p - w)(q + \beta e) - d + \int_0^k \frac{\theta}{k} \mathrm{d}\theta \\ &= (p - w)(q + \beta e) - d + \frac{k}{2}. \end{aligned} \tag{2}$$

Furthermore, this paper discusses the expected revenue function of both the supplier and the retailer when they share the revenue in proportion to their shareholdings. By the previous assumptions, V represents the value of the retailer's corporate assets as determined by the parties at the equity financing, and B is the amount of the retailer's equity financing. At t_1, the retailer takes equity financing and acquires the financing amount, and its total assets are $V+B$. Therefore, $V/(V+B)$ and $B/(V+B)$), respectively, denote the shareholdings of the retailer and the supplier at that point.

In the case of no betting agreement introduced, the expected revenue received by the supplier at the moment t_2 in proportion to its shareholdings after the equity financing of the retailer is

$$\Phi(\alpha) = \frac{B}{V+B} E(\pi) - B. \tag{3}$$

Substituting equation (2) with $V = \alpha(A+\eta)$ into equation (3), we further obtain

$$\Phi(\alpha) = \frac{B}{\alpha(A+\eta)+B}\left[(p-w)(q+\beta e) - d + \frac{k}{2}\right] - B. \tag{4}$$

At the time t_2, the expected revenue received by the retailer in proportion to its shareholdings after equity financing is

$$\Psi(e) = \frac{V}{V+B} E(\pi) - \frac{1}{2} s e^2. \tag{5}$$

Substituting (2) into (5), we can obtain

$$\Psi(e) = \frac{\alpha(A+\eta)}{\alpha(A+\eta)+B}\left[(p-w)(q+\beta e) - d + \frac{k}{2}\right] - \frac{1}{2} s e^2. \tag{6}$$

In the equity financing process, the supplier who serves as the leader first gives the initial valuation level, with the retailer as the follower, and both parties play the Stackelberg game. Furthermore, the valuation level will affect the retailer's effort level, so the effort level e is a function of the valuation level α. Equation (6) takes the first-order derivative of the effort level e; that is,

$$\frac{d\Psi(e)}{de} = \frac{\alpha(A+\eta)}{\alpha(A+\eta)+B}(p-w)\beta - se. \tag{7}$$

Let $d\Psi(e)/de = 0$; the optimal effort level of the retailer is obtained as

$$e^* = \frac{\alpha(A+\eta)(p-w)\beta}{s[\alpha(A+\eta)+B]}. \tag{8}$$

It follows that the optimal effort level exists and is unique and that e^* is a function of α. Substituting (8) into (4) and taking the first-order derivative of the function $\Phi(\alpha)$ concerning α, we get

$$\begin{aligned} \frac{d\Phi(\alpha)}{d\alpha} &= \frac{\left\{2\Theta^2 B + [2(p-w)q - 2d + k]sB\right\} 2[\alpha(A+\eta)+B]}{4s[\alpha(A+\eta)+B]^3}(A+\eta) \\ &= \frac{4s\left\{2\Theta^2 B\alpha(A+\eta) + [2(p-w)q - 2d + k]B[\alpha(A+\eta)+B]\right\}}{4s[\alpha(A+\eta)+B]^3}(A+\eta), \end{aligned} \tag{9}$$

where $(p-w)\beta = \Theta$ and let $d\Phi(\alpha)/d\alpha = 0$; we further obtain

$$\begin{aligned} &\left\{2s\Theta^2 + s^2[2(p-w)q - 2\,d + k]\right\}\alpha^2(A+\eta)^2 - 2s\Theta^2 B^2 \\ &+ 2s^2[2(p-w)q - 2\,d + k]B\alpha(A+\eta) \\ &+ s^2[2(p-w)q - 2\,d + k]B^2 = 0. \end{aligned} \tag{10}$$

According to equation (10), the optimal valuation level of retailer equity financing in a centralized supply chain without introducing a gambling agreement is

$$\alpha^* = \frac{\left\{2\Theta^2 - s[2(p-w)q - 2\,d + k]\right\}B}{\left\{2\Theta^2 + s[2(p-w)q - 2\,d + k]\right\}(A+\eta)}. \tag{11}$$

Therefore, the optimal valuation level exists and is unique. Following the previous section, the retailer's revenue before financing should be greater than the fixed cost, that is, $(p-w)q > d$. Since the effort cost coefficient $s > 0$ and the maximum possible excess revenue $k > 0$ in the retailer's market expansion process, the denominator part of the right end of equation (11) is positive. Thus, to make $\alpha^* > 0$, the numerator part of the right end of equation (11) should be greater than 0. To ensure that the optimal valuation level is greater than 0, the parameters p, w, β, and so on need to satisfy $2(p-w)^2\beta^2 > s[2(p-w)q - 2\,d + k]$ between them. End of proof. □

5. Optimal Valuation Model for Retailer Equity Financing with Introducing Gambling Agreement in Centralized Supply Chain

Since this paper considers a centralized supply chain, the supplier and the retailer adopt a revenue-sharing mechanism, and they are both risk-averse. When the supplier takes equity financing for the retailer, introducing a gambling agreement can effectively guarantee investment returns for the supplier and maximize incentives for the retailer to explore the market while reducing the losses caused to both parties by overvaluation or undervaluation. Therefore, for centralized supply chains, the introduction of a gambling agreement is a preferable institutional arrangement when equity financing is undertaken.

From the study in Section 4, an optimal valuation model for retailer equity financing with introducing a gambling agreement will be developed here. The gambling agreement studied in this paper is by the retailer's revenue at the end moment of market expansion as the betting subject, with the transfer of equity as the compensation. If the retailer's revenue exceeds the betting target π_0 at the time t_2, the supplier transfers ε of shares to the retailer, and the retailer wins the betting. If the retailer's revenue at the time t_2 is lower than the betting target π_0, the retailer is required to transfer ε of shares to the supplier, who wins the betting at this moment.

From the above discussion, it is clear that if the betting target is set too high, the supplier is more likely to win the betting. Conversely, if the betting target is set too low, the retailer has a greater chance of winning. Therefore, this paper intends to explore the optimal valuation model for retailer equity financing from three cases of high, medium, and low betting target setting.

By equation (1), at the moment t_2, if θ takes the maximum value k, the retailer can obtain the maximum earnings from market expansion; that is,

$$\pi_1 = (p - w)(q + \beta e) - d + k. \tag{12}$$

If $\theta = 0$, the retailer can obtain the minimum revenue; that is,

$$\pi_2 = (p - w)(q + \beta e) - d. \tag{13}$$

For further research, the following definitions are available in this paper.

Definition 1. If the betting target is greater than the possible maximum gain for the retailer, that is, $\pi_1 < \pi_0$, then the target is set too high. If the betting target is between the minimum and maximum possible revenue for the retailer, that is, $\pi_2 \le \pi_0 \le \pi_1$, the target is set moderately. If the betting target is less than the minimum possible revenue for the retailer, that is, $\pi_0 < \pi_2$, the target is set too low.

However, in the three cases where the betting target is set at high, medium, or low, if the probability of the occurrence of the betting result under various circumstances is considered, it makes the process of determining the revenue function of both the investor and financier more complicated. To simplify the research, the latter part of the article assumes that the supplier (retailer) will win the betting when the revenue target is too high (too low), and it accordingly presents an expected revenue function by the shareholding ratio and discusses the optimal valuation level versus the optimal effort level. For the sake of research, this paper assumes that the corresponding optimal valuation level, respectively, is α_1^*, α_2^*, and α_3^* when the betting target is set too high, moderate, and too low.

5.1. Optimal Valuation Model in Case the Betting Target Is Set Too High. Following the research above, this section explores the optimal valuation model when the betting target is set too high, that is, when the target π_0 is greater than the maximum possible revenue π_1 for the retailer. This paper intends to determine the optimal valuation level in the case that the betting target is set too high through mathematical derivation. Given that the effort level of the retailer can bring a large impact on the final revenue and gambling outcome, those are discussed next.

Proposition 2. *Under the condition of introducing a gambling agreement in the equity financing of a centralized supply chain, the supplier may win the betting when the betting target is set too high, $\pi_1 < \pi_0$. There exists an only optimal valuation level and an optimal effort level.*

Proof. Proposition 2 can be proved here by the expected revenue function of the supplier and the retailer. When $\pi_1 < \pi_0$, according to Equation (12), π_1 represents the retailer's maximum revenue, and the betting target π_0 is the constant determined by both parties when the agreement is reached. For the convenience of this research, it is assumed that the maximum possible revenue for the retailer is the revenue it could obtain if it received equity financing and put in maximum effort to develop the market. If the betting target π_0 is higher than this revenue, there is a high probability that the retailer may not win the betting. This case is because, in practice, the betting target agreed upon between the investor and financier is too high. The reasons are as follows. On the one hand, the retailer may be overconfident and blindly optimistic about the market and willing to accept overly high betting targets. On the other hand, the retailer may be forced to choose the way of equity financing due to the lack of capital. But at this point, the supplier wants to set a higher betting target to motivate the retailer and reduce its investment risk.

When the betting target is set too high, referring to the previous assumptions, the supplier will win the betting at the time t_2. By equation (3), the expected revenue of the supplier at that moment is

$$\Phi_1(\alpha) = \left[\frac{B}{\alpha(A + \eta) + B} + \varepsilon\right] E(\pi) - B, \tag{14}$$

where $B/(\alpha(A + \eta) + B)$ denotes the supplier's shareholdings at the time t_1, $E(\pi)$ represents the retailer's expected revenue function at t_2, and B is the amount of equity financing. If the supplier wins the betting, it will obtain the retailer's share of the betting transfer ε. Thus, $[B/(\alpha(A + \eta) + B) + \varepsilon]$ represents the supplier's entire shareholdings at the time t_2, when the gambling agreement expires and the betting is won.

Substituting equation (2) into (14), we further obtain

$$\Phi_1(\alpha) = \left[\frac{B}{\alpha(A + \eta) + B} + \varepsilon\right]\left[(p - w)(q + \beta e) - d + \frac{k}{2}\right] - B. \tag{15}$$

At the moment t_2, the retailer will probably not win the betting. Based on equation (5), the expected revenue of the retailer at this point according to its shareholdings is

$$\Psi_1(e) = \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} - \varepsilon\right] E(\pi) - \frac{1}{2} se^2, \tag{16}$$

where$\alpha(A+\eta)/(\alpha(A+\eta)+B)$ denotes the retailer's shareholdings at the moment t_1. Since the supplier wins the betting, the retailer needs to transfer ε of shares to the supplier, so$[\alpha(A+\eta)/(\alpha(A+\eta)+B)-\varepsilon]$ represents the retailer's entire shareholdings at t_2. $1/2se^2$ is the cost of effort for the retailer.

Substituting (2) into (16), we get

$$\Psi_1(e) = \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} - \varepsilon\right]\left[(p-w)(q+\beta e) - d + \frac{k}{2}\right] - \frac{1}{2} se^2. \tag{17}$$

Equation (17) takes the first-order derivative of the effort level e, and letting $d\Psi_1(e)/de = 0$, we obtain

$$\left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} - \varepsilon\right](p-w)\beta - se = 0. \tag{18}$$

Therefore, the optimal effort level of the retailer is

$$e_1^* = \frac{\alpha(A+\eta)(p-w)\beta}{[\alpha(A+\eta)+B]s} - \frac{\varepsilon(p-w)\beta}{s}. \tag{19}$$

It follows that the optimal effort level exists and is unique. Substituting (19) into (15), we have

$$\Phi_1(\alpha) = \left[\frac{B}{\alpha(A+\eta)+B} + \varepsilon\right]\left[(p-w)q - d + \frac{V(p-w)^2\beta^2}{(V+B)s} - \frac{\varepsilon(p-w)^2\beta^2}{s} + \frac{k}{2}\right] - B. \tag{20}$$

Equation (20) takes the first-order derivative of the valuation level α

$$\frac{d\Phi_1(\alpha)}{d\alpha} = \left\{\frac{4\Theta^2\varepsilon B - [2(p-w)q - 2\,d + k]sB}{2s[\alpha(A+\eta)+B]^2} + \frac{2\Theta^2B^2 - 2\Theta^2B\alpha(A+\eta)}{2s[\alpha(A+\eta)+B]^3}\right\}(A+\eta). \tag{21}$$

There have $(p-w)\beta = \Theta$. Let $d\Phi_1(\alpha)/d\alpha = 0$, so, we obtain the optimal valuation level of retailer equity financing with the high betting target in a centralized supply chain:

$$\alpha_1^* = \frac{\left\{4s\Theta^2\varepsilon - [2(p-w)q - 2d + k]s^2 + 2s\Theta^2\right\}B}{\left\{[2(p-w)q - 2d + k]s^2 + 2s\Theta^2 - 4s\Theta^2\varepsilon\right\}(A+\eta)}. \tag{22}$$

This shows that when the betting target π_0 is set too high, the optimal valuation level exists and is the only one under certain conditions, and the corresponding optimal effort level of the retailer can also be obtained. End of proof.

In practice, when the betting target is set too high, the supplier will, to a large extent, win the betting and thus obtain shares of the betting transfer, so its incentive to take equity financing is stronger. Meanwhile, due to financial constraints and the need for market expansion, the retailer may accept the risk of oversetting the betting target resulting in the transfer of equity, and both parties will eventually agree on the setting of the betting target. However, when the betting target is set too high, there is some uncertainty about whether the supplier can win the betting. Equity financing with an overly high betting target may provide some incentive for the retailer to increase the efficiency of its use of capital and enhance the effort level of market expansion. Moreover, if the retailer reaches the betting target through its efforts, then the supplier is required to transfer part of its equity to the retailer; in this case, it will receive excess benefits. Because of this, for the retailer, setting betting targets too high may enhance the operational performance to some extent. □

5.2. Optimal Valuation Model with Moderate Betting Target Setting. The optimal valuation model for the case in which the betting target is set too high has been constructed in the previous section, and this section explores the optimal valuation model when the betting target is set moderately, that is, $\pi_2 \le \pi_0 \le \pi_1$. In this case, the betting target lies between the minimum and maximum possible revenue for the retailer. In order to investigate whether the supplier and the retailer can reach an equity financing agreement and whether there is an optimal valuation level when the betting target is set moderately, this section will be further analyzed by constructing an expected revenue function and deriving a mathematical model.

Proposition 3. *Under the condition of introducing a gambling agreement in the equity financing of a centralized supply chain, when the betting target is set moderately, $\pi_2 \leq \pi_0 \leq \pi_1$, both the supplier and the retailer have the possibility of winning the bet. There exists an only optimal valuation level and optimal effort level.*

Proof. When the betting target setting is moderate, that is, $\pi_2 \leq \pi_0 \leq \pi_1$, by Equation. (12) and (13), π_1 and π_2, respectively, denote the maximum and minimum possible revenue for the retailer at the time t_2, and the betting target π_0 is a constant determined by both parties when the agreement is reached. When the retailer's final revenue at t_2 is between the minimum revenue π_2 and the betting target π_0, it is assumed for simplicity of the study that the retailer will not be able to win the betting. The retailer is more likely to win the betting when that revenue is between the betting target π_0 and the maximum revenue π_1. However, in practice, the retailer's expected revenue at the time t_2 is in the interval $[\pi_2, \pi_0]$ or $[\pi_0, \pi_1]$ is not determinable. Therefore, it can be deduced that when $\pi_2 \leq \pi_0 \leq \pi_1$, both the supplier and the retailer may win the betting. Since the betting target is generally determined by negotiation between the investor and financier, the betting target value set by both parties is usually at a reasonable level that is not inconsistent with the actual operating performance of the retailer and exceeds the retailer's own future ability to earn but is not so low that the retailer can easily meet the betting target. Therefore, by designing a moderate betting target agreement mechanism, it can reduce the investment risk of the supplier on the one side and enhance the retailer's market expansion efforts on the other.

If the betting target is set moderately, both the supplier and the retailer may win the betting at the moment t_2. Following the previous analysis and (5) and (12)-(13), the expected revenue of the retailer according to its shareholdings is

$$\Psi_2(e) = \int_{(p-w)(q+\beta e)-d}^{\pi_0} \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} - \varepsilon\right]\pi f(\pi)d\pi + \int_{\pi_0}^{(p-w)(q+\beta e)-d+k} \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} + \varepsilon\right]\pi f(\pi)d\pi - \frac{1}{2}se^2, \quad (23)$$

where $f(\pi)$ is the probability density function of the retailer's revenue π. The excess revenue θ in equation (1) obeys a uniform distribution over the interval $[0, k]$, and the probability density function of θ is $g(\theta) = 1/k$. Since the remaining variables in equation (1) are nonrandom variables, the probability density function of π is $f(\pi) = 1/k$. Therefore, equation (23) is further deformed as

$$\Psi_2(e) = \int_{(p-w)(q+\beta e)-d}^{\pi_0} \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} - \varepsilon\right]\frac{\pi}{k}d\pi + \int_{\pi_0}^{(p-w)(q+\beta e)-d+k} \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} + \varepsilon\right]\frac{\pi}{k}d\pi - \frac{1}{2}se^2. \quad (24)$$

Equally, by equation (3), the expected revenue of the supplier at the time t_2 according to the shareholding ratio can be obtained as

$$\Phi_2(\alpha) = \int_{(p-w)(q+\beta e)-d}^{\pi_0} \left[\frac{B}{\alpha(A+\eta)+B} + \varepsilon\right]\frac{\pi}{k}d\pi + \int_{\pi_0}^{(p-w)(q+\beta e)-d+k} \left[\frac{B}{\alpha(A+\eta)+B} - \varepsilon\right]\frac{\pi}{k}d\pi - B. \quad (25)$$

Equation (24) takes the first-order derivative of the effort level e and it obtains

$$\frac{d\Psi_2(e)}{de} = \frac{2\varepsilon(p-w)^2\beta^2 e}{k} + (p-w)\beta\left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} + \varepsilon\right] + \frac{2\varepsilon(p-w)\beta[(p-w)q-d]}{k} - se. \quad (26)$$

Let $(p-w)\beta=\Theta$ and $(p-w)q-d=m$. From this, we obtain the optimal effort level of the retailer when the betting target is set moderately as

$$e_2^*=\frac{k\Theta\left[\alpha(A+\eta)/(\alpha(A+\eta)+B)+\varepsilon\right]+2\varepsilon\Theta m}{ks-2\varepsilon\Theta^2}. \tag{27}$$

Therefore, the optimal effort level of the retailer exists and is only one. Substituting (27) into (25), we have

$$\begin{aligned}\Phi_2(\alpha)={}&\frac{\varepsilon\pi_0^2}{k}-\frac{1}{2k}\left\{m+\frac{2\varepsilon\Theta^2 m}{ks-2\varepsilon\Theta^2}+\frac{k\Theta^2\left(V/(V+B)+\varepsilon\right)}{ks-2\varepsilon\Theta^2}\right\}^2\\&\left[\frac{B}{\alpha(A+\eta)+B}+\varepsilon\right]+\frac{1}{2k}\left\{m+k+\frac{\left[2\varepsilon m+k\left(V/(V+B)+\varepsilon\right)\right]\Theta^2}{ks-2\varepsilon\Theta^2}\right\}^2\left[\frac{B}{\alpha(A+\eta)+B}-\varepsilon\right].\end{aligned} \tag{28}$$

Equation (28) takes the first-order derivative of the valuation level α and results in

$$\begin{aligned}\frac{d\Phi_2(\alpha)}{d\alpha}={}&\frac{B\left\{m\left[ks-2\varepsilon\Theta^2\right]+2\varepsilon\Theta^2 m+k\Theta^2\left[\alpha(A+\eta)/(\alpha(A+\eta)+B)+\varepsilon\right]\right\}^2(A+\eta)}{2k\left[\alpha(A+\eta)+B\right]^2\left[ks-2\varepsilon\Theta^2\right]^2}\\&-\frac{(A+\eta)B\left\{(m+k)\left[ks-2\varepsilon\Theta^2\right]+2\varepsilon\Theta^2 m+k\Theta^2\left[\alpha(A+\eta)/(\alpha(A+\eta)+B)+\varepsilon\right]\right\}^2}{2k\left[\alpha(A+\eta)+B\right]^2\left[ks-2\varepsilon\Theta^2\right]^2}\\&-\frac{(A+\eta)\left\{B+\varepsilon\left[\alpha(A+\eta)+B\right]\right\}B\Theta^2\left\{m\left[ks-2\varepsilon\Theta^2\right]+2\varepsilon\Theta^2 m+k\Theta^2\left[\alpha(A+\eta)/(\alpha(A+\eta)+B)+\varepsilon\right]\right\}}{\left[\alpha(A+\eta)+B\right]^3\left[ks-2\varepsilon\Theta^2\right]^2}\\&+\frac{(A+\eta)\left\{B-\varepsilon\left[\alpha(A+\eta)+B\right]\right\}B\Theta^2\left\{(m+k)\left[ks-2\varepsilon\Theta^2\right]+2\varepsilon\Theta^2 m+k\Theta^2\left[\alpha(A+\eta)/(\alpha(A+\eta)+B)+\varepsilon\right]\right\}}{\left[\alpha(A+\eta)+B\right]^3\left[ks-2\varepsilon\Theta^2\right]^2}.\end{aligned} \tag{29}$$

Letting $d\Phi_2(\alpha)/d\alpha=0$, we can obtain the optimal valuation level of retailer equity financing in a centralized supply chain when the betting target is set moderately:

$$\alpha_2^*=\frac{\left\{2mks^2+2\varepsilon ks\Theta^2+\left[2\varepsilon\Theta^2-ks\right]^2\right\}B-2\left[ks\Theta^2-2\varepsilon\Theta^4\right](B-\varepsilon B)}{\left\{4\varepsilon^2\Theta^4-2mks^2-2ks(1+2\varepsilon)\Theta^2-\left[2\varepsilon\Theta^2-ks\right]^2\right\}(A+\eta)}. \tag{30}$$

There has $(p-w)\beta=\Theta$ and $(p-w)q-d=m$.

As a result, it is proved that the optimal valuation level α_2^* exists and is unique in this case, and the corresponding only optimal effort level e_2^* is obtained. End of proof.

In practice, setting a moderate betting target will have a motivating effect on the investor and financier. On the one hand, for the supplier, a moderate betting target setting makes it easier for the supplier to win the betting and thus obtain shares of the betting transfer than in the previous case where the betting target is set low. On the other hand, for the retailer, it is easier to meet the betting target when the target is set moderately, so the retailer will exert a higher level of effort to win the betting. It can be seen that there has a strong incentive for both parties to cooperate in equity financing in this situation. The retailer and the supplier can find the optimal valuation level that is in the interest of both parties, which also fits with the revenue-sharing mechanism in a centralized supply chain. □

5.3. Optimal Valuation Model in Case of Underset Betting Target. The optimal valuation models with the target set high and moderate are constructed in the previous. Furthermore, there are cases where the betting target is set too low, that is, $\pi_0 < \pi_2$, in the gambling agreement signed between the investor and financier. This section explores the optimal valuation level and effort level in this case.

Proposition 4. *Under the condition of introducing a gambling agreement in the equity financing of a centralized supply chain, the retailer may win the betting when the betting target is set too low, $\pi_0 < \pi_2$. There exists an only optimal valuation level and optimal effort level.*

Proof. By equation. (13), π_2 represents the minimum revenue available to the retailer at the moment t_2, and $\pi_0 < \pi_2$ indicates that the betting target π_0 is less than the minimum revenue available to the retailer. The betting target is determined at the time of signing the agreement between the investor and financier, and it is a constant. Since the minimum revenue available to the retailer is the revenue earned by the retailer from selling $q + \beta e$ units of a product at unit price p, but the excess revenue is 0, therefore, if the betting target is lower than the minimum revenue, to simplify the research, this paper assumes that the retailer will win the betting at this point. Moreover, combined with equation. (13), it is known that $\pi_0 \leq (p-w)(q+\beta e) - d$, which further leads to $e \in [(\pi_0 + d - (p-w)q)/\beta, +\infty)$, so the retailer's effort level may theoretically reach infinity in this case. The results also suggest that a lower betting target setting creates a greater incentive for the retailer to work extremely hard to win the betting before the target is met. □

In practice, the retailer prefers to reach a lower betting target with the supplier to win the betting and gain more control to achieve its absolute discourse power in a centralized supply chain. The supplier, on the contrary, is more likely to accept the higher betting target. However, there are two possible reasons why the two sides finally reached a lower betting target. On the one hand, it may be due to the lack of working capital of the retailer; if they do not have enough funds for market expansion, the performance of the supplier in a centralized supply chain will also be affected. Moreover, the revenue-risk sharing mechanism in a centralized supply chain will force the supplier to make equity investments to the retailer and ultimately accept the lower betting target. On the other hand, the pessimistic expectations of both sides about the market outlook will also lower the expectation of the retailer's future earnings, which may result in the agreed betting targets of both parties being low.

When the betting target is set too low, assume that the supplier lost the betting at the time t_2. By equation (3), the expected revenue of the supplier at that moment according to its shareholdings is

$$\Phi_3(\alpha) = \left[\frac{B}{\alpha(A+\eta)+B} - \varepsilon\right]E(\pi) - B. \quad (31)$$

Since, in this case, the supplier has to transfer ε share of equity to the retailer, $[B/(\alpha(A+\eta)+B) - \varepsilon]$ is the shareholdings of the supplier at t_2. Substituting equation (2) into (31), the following can be derived:

$$\Phi_3(\alpha) = \left[\frac{B}{\alpha(A+\eta)+B} - \varepsilon\right]\left[(p-w)(q+\beta e) - d + \frac{k}{2}\right] - B. \quad (32)$$

Meanwhile, given that the betting target is set too low, the retailer will win the betting at the time t_2. By equation (5), the expected revenue function of the retailer is

$$\Psi_3(e) = \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} + \varepsilon\right]E(\pi) - \frac{1}{2}se^2. \quad (33)$$

Substituting equation (2) into (33), the following can be derived:

$$\Psi_3(e) = \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} + \varepsilon\right]\left[(p-w)(q+\beta e) - d + \frac{k}{2}\right] - \frac{1}{2}se^2. \quad (34)$$

Equation (34) takes the first-order derivative of the effort level e and we get

$$\frac{d\Psi_3(e)}{de} = \left[\frac{\alpha(A+\eta)}{\alpha(A+\eta)+B} + \varepsilon\right](p-w)\beta - se. \quad (35)$$

Letting $d\Psi_3(e)/de = 0$, the only optimal effort level is obtained as

$$e_3^* = \frac{\alpha(A+\eta)(p-w)\beta}{[\alpha(A+\eta)+B]s} + \frac{\varepsilon(p-w)\beta}{s}. \quad (36)$$

Substituting equation (36) into (32), we have

$$\Phi_3(\alpha) = \left[\frac{B}{\alpha(A+\eta)+B} - \varepsilon\right]\left[(p-w)q - d + \frac{\alpha(A+\eta)\Theta^2}{[\alpha(A+\eta)+B]s} + \frac{\varepsilon\Theta^2}{s} + \frac{k}{2}\right] - B, \quad (37)$$

where $(p-w)\beta = \Theta$, and equation (37) for the first-order derivative of the valuation level α has

$$\frac{d\Phi_3(\alpha)}{d\alpha} = \frac{\Theta^2B^2 - \Theta^2B\alpha(A+\eta)^2}{s[\alpha(A+\eta)+B]^3} - \frac{4\Theta^2\varepsilon B + [2(p-w)q - 2d + k]sB(A+\eta)}{2s[\alpha(A+\eta)+B]^2}. \quad (38)$$

Letting $d\Phi_3(\alpha)/d\alpha = 0$, the optimal valuation level of retailer equity financing in a centralized supply chain when the betting target is set too low can be obtained as

$$\alpha_3^* = \frac{\left\{4s\Theta^2\varepsilon + [2(p-w)q - 2d + k]s^2 - 2s\Theta^2\right\}B}{-\left\{[2(p-w)q - 2d + k]s^2 + 2s\Theta^2 + 4s\Theta^2\varepsilon\right\}(A+\eta)}. \quad (39)$$

Therefore, at this situation, the optimal valuation level α_3^* and the corresponding optimal effort level e_3^* of the retailer exist and are unique. Thus, the valuation level that maximizes the revenue of both parties exists at this moment and the equity financing agreement can be reached. End of proof.

When the betting target is set too low, the retailer is more likely to meet the betting target and may win the betting to gain more shares, which will provide further incentive to exert a higher level of effort to achieve excess revenue. In this case, even if the supplier does not win the betting, the excess revenue increase for the retailer will improve its returns on investment. Meanwhile, the retailer's efforts will expand market demand, and under the mechanism of the centralized supply chain effect, the supplier will thus gain greater benefits. The gambling agreement itself can alleviate the information asymmetry between the two parties and reduce the investment risk of the supplier. Therefore, when the betting target is set too low, the parties may still reach an equity financing partnership agreement even though the retailer may win the betting. However, since the supplier, in this case, can receive the equity share corresponding to the financing amount B at the beginning of the investment, the supplier will lose ε of shares when the agreement expires if the retailer wins the betting. Thus, if the betting target is set too low, the supplier will weigh the loss from a failed betting against the returns from an increased level of effort by the retailer to determine whether to pursue an equity financing partnership.

Up to now, the optimal valuation level and the optimal effort level of the retailer under the three cases of betting target setting are obtained, and it is proved that after introducing the gambling agreement, the investor and financier can always find an optimal valuation level for reaching the equity financing agreement. Propositions 1 to 4 show that, despite the conflict between the two parties in the valuation process, the valuation level that allows both sides to reach a win-win situation can be found in practice, and the optimal valuation level and the optimal effort level still exist when both parties introduce a gambling agreement in the contract. Therefore, there is a possibility of equity financing cooperation between centralized supply chain entities and the introduction of gambling agreements, which can alleviate the investment risks caused by information asymmetry and improper valuation, and provide maximum incentives for the retailer to explore the market. In practice, the valuation level is influenced by various factors, such as the betting transfer share ratio, the ordering price, and the firm growth, which will be further discussed below.

6. The Influence of the Betting Transfer Share Ratio, Ordering Price, and Firm Growth on the Optimal Valuation Level

Section 5 constructs an optimal valuation model for retailer equity financing with gambling agreement in a centralized supply chain. Meanwhile, it separately obtains the optimal valuation level and the optimal retailer effort level under different betting target settings. Following (11), (22), (30), and (39), the optimal valuation level α^* is a multivariate function of variables regarding the betting transfer share ratio ε, ordering price w, and firm growth β. ε is a critical variable for both parties to raise equity financing and reach a gambling agreement. w reflects the negotiating ability of both parties in a centralized supply chain, and β is an important factor to measure the incremental demand following market expansion. Therefore, the discussion will later focus on the impact of these three variables on the optimal valuation level or the optimal effort level.

6.1. The Effect of the Betting Transfer Share Ratio on the Optimal Valuation Level. The betting transfer share ratio ε determines the share of equity to be given by the side that does not win the betting, which directly affects the effort level of the retailer and the returns on investment of the supplier and thus indirectly affects the valuation level. From the study in Section 5, it is clear that there is variability in the impact of the betting transfer share ratio on the optimal valuation level in the three cases where the betting target is set too high, medium, and too low, for which there is further inference as follows.

Corollary 1. *When the betting target is set too high, $\pi_1 < \pi_0$, the greater the betting transfer share ratio ε, the higher the optimal valuation level α^*. Conversely, α^* will be lower. The parties show a relationship of change in the identical direction.*

Proof. Following Definition 1 and Proposition 2, when the betting target is set too high, that is, $\pi_1 < \pi_0$, the betting target value is higher than the maximum revenue that the retailer can obtain. In this situation, the optimal valuation level α_1^* is obtained from equation (22), whose first-order partial derivative about ε is

$$\begin{aligned}\frac{\partial\alpha_1^*}{\partial\varepsilon} &= \frac{4s(p-w)^2\beta^2 B\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\ &\quad+\frac{4s(p-w)^2\beta^2 B\left\{4s(p-w)^2\beta^2\varepsilon-[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2\right\}}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\ &= \frac{\left(16s^2(p-w)^4\beta^4 B\right)}{\left(\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)\right)}.\end{aligned}\tag{40}$$

If the fixed cost of the retailer is higher than its revenue, it is bound to suffer losses, so for the retailer, the benefits before financing should be greater than the fixed cost, that is, $(p-w)q>d$. It is known from the previous assumptions that the effort cost coefficient $s>0$, the maximum possible excess revenue $k>0$ in the retailer's market expansion process, and the financing amount $B>0$, so $\partial\alpha_1^*/\partial\varepsilon>0$ is obtained.

Therefore, when the betting target is set too high, the optimal valuation level α_1^* increases monotonically concerning the betting transfer share ratio ε; that is, α_1^* and ε vary in the same direction. At this moment, if the retailer fails in the betting, the supplier will receive the betting transfer share ratio set in the agreement. Accordingly, from the supplier's view, the betting transfer share ratio ε can be considered as an additional "incentive" for the supplier, the more shares the supplier can receive, the greater the degree of incentive for the supplier, and the stronger motivation for the supplier to cooperate in equity financing. Moreover, in practice, the high valuation is more favorable for the retailer. As a result, the supplier may accept a higher valuation level to reach an equity financing agreement. From the perspective of the retailer, the size of the betting transfer share ratio represents the "cost" of its bet failure. If this "cost" is high, the retailer may make more efforts to expand the market or improve operational efficiency to the revenue at the time t_2, exceed the betting target, and eventually win the betting. However, even if the retailer still does not win the betting, its upfront efforts will improve the overall performance of the centralized supply chain. At this point, the supplier is also able to receive more excess returns from its shareholdings. Hence, when the betting target is set too high, the revenue-sharing mechanism is enhanced by equity financing and the gambling agreement. Because of this, the supplier will tend to a higher valuation level. End of proof. □

Corollary 2. *When the betting target is set moderately or too low, $\pi_2\le\pi_0\le\pi_1$ or $\pi_0<\pi_2$, the larger the betting transfer share ratio ε, the lower the optimal valuation level α^*. Conversely, α^* will be higher. The parties show a relationship of change in the reverse direction.*

Proof. Following Definition 1 and Proposition 3, when the betting target is set moderately, that is, $\pi_2\le\pi_0\le\pi_1$, the betting target lies between the retailer's minimum and maximum revenue. The optimal valuation level α_2^* can be obtained from equation (30), and the first-order partial derivative of equation (30) on ε is

$$\frac{\partial\alpha_2^*}{\partial\varepsilon}=\frac{-4(p-w)^4\beta^4B\left[2mks^2+k^2s^2+2ks(p-w)^2\beta^2\right]}{\left\{4\varepsilon^2(p-w)^4\beta^4-2mks^2-2ks(1+2\varepsilon)(p-w)^2\beta^2-\left[2\varepsilon(p-w)^2\beta^2-ks\right]^2\right\}^2(A+\eta)}. \tag{41}$$

Since $B>0$, $m>0$, $k>0$, and $s>0$, $\partial\alpha_2^*/\partial\varepsilon<0$ can be derived.

Equally, when the betting target is set too low, that is $\pi_0<\pi_2$, the betting target is smaller than the minimum revenue that the retailer can obtain. By equation (39), the optimal valuation level α_3^* can be obtained; in this case, the first-order partial derivative of α_3^* on ε is

$$\frac{\partial\alpha_3^*}{\partial\varepsilon}=\frac{-16s^2(p-w)^4\beta^4B}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)}<0. \tag{42}$$

Therefore, when the betting target is set moderately or too low, α_2^* and α_3^* are both monotonically decreasing concerning ε. Thus, the optimal valuation level and the betting transfer share ratio vary in the opposite direction. When the betting target is set moderately, both the investor and financier may win the betting. In this situation, if the supplier wins the betting, it will get shares of the bet transfer, and if it does not win, it may lose part of the equity and increase the investment risk. As a result, the risk-averse supplier may face greater investment risk in the event of uncertain betting outcomes. Furthermore, according to the previous section, the total assets of the retailer are its shareholdings, and the financing amount represents the shareholdings of the supplier. Because of this, the higher the valuation level, the proportion of equity held by the retailer$V/(V+B)$ is relatively greater, while the proportion held by the supplier$B/(V+B)$ is lower. If the betting transfer share is high at this time, the equity it holds will be further reduced once the supplier fails to win the betting. Hence, for risk-averse reasons, the supplier may depress the retailer's valuation, and even if the supplier fails in its betting and needs to transfer part of its equity, the lower valuation will keep its shareholdings from being significantly reduced.

Equally, when the betting target is set too low, following Definition 1, the retailer will win the betting, so the supplier needs to transfer ε of equity, thus reducing its shareholdings. In this case, the supplier's investment incentive will be weakened. The same principle as the betting target is set moderately, if the betting transfer share ratio is larger, the supplier will be more inclined to a lower valuation level to avoid the investment risk associated with a lower final shareholding ratio. □

6.2. Effect of the Betting Transfer Share Ratio on the Optimal Effort Level. The betting transfer share ratio is a key element reflecting the mechanism of the gambling agreement, which determines the shareholdings of the investor and financier after the expiration of the agreement. Section 6.1 examines the relationship between the betting transfer share ratio and the optimal valuation level and focuses on the analysis of the former's impact on supplier behavior. This section intends to analyze the extent to which the betting transfer share ratio affects the retailer's optimal effort level and focuses on the ratio's impact on its behavior.

Corollary 3. *When the betting target is set too high, $\pi_1 < \pi_0$, the larger the betting transfer share ratio ε, the lower the retailer's optimal effort level e^*. Conversely, e^* will be higher. The parties show a relationship of change in the reverse direction.*

Proof. When the betting target is set too high, the optimal effort level e_1^*, in this case, is obtained by equation (19), and its first-order partial derivative about ε is found, and the following can be derived:

$$\frac{\partial e_1^*}{\partial \varepsilon} = -\frac{(p-w)\beta}{s}, \tag{43}$$

where $p-w>0$, $\beta>0$, $s>0$, so $\partial e_1^*/\partial\varepsilon<0$, which means that e_1^* is a decreasing function about ε, and the parties change in opposite directions.

When the betting target is set too high, the retailer cannot win the betting at the end of market expansion, and it needs to transfer ε of the shares held before the expiration of the gambling agreement to the supplier. The larger the ε, the more the retailer may lose absolute control of the company. Therefore, during the market expansion phase, the pessimistic expectation of the retailer will reduce its motivation for expansion, thus making its effort level decrease. Conversely, when ε is low, even if the supplier wins the betting, the impact on the retailer's control is modest, so it faces a relatively low loss from a failed betting. Furthermore, the greater the level of retailer effort will increase the overall performance of the supply chain, so increasing revenue. Hence, the retailer's effort level is may larger when ε is lower. End of proof. □

Corollary 4. *When the betting target is set moderately or too low, that is, $\pi_2 \le \pi_0 \le \pi_1$ or $\pi_0 < \pi_2$, the greater the betting transfer share ratio ε, the higher the retailer's optimal effort level e^*. Conversely, e^* will be lower. The parties show a relationship of change in an identical direction.*

Proof. When the betting target is set moderately, that is, $\pi_2 \le \pi_0 \le \pi_1$, the optimal effort levele_2^* is obtained by equation (27), and it takes first-order partial derivative regarding ε:

$$\frac{\partial e_2^*}{\partial \varepsilon} = \frac{\left\{k^2(p-w)\beta s + 2ks(p-w)\beta[(p-w)q-d]\right\}[\alpha(A+\eta)+B] + 2\alpha k(A+\eta)(p-w)^3\beta^3}{[\alpha(A+\eta)+B]\left[ks - 2\varepsilon(p-w)^2\beta^2\right]^2}. \tag{44}$$

Since $p-w>0$, $(p-w)q>d$, $\beta>0$, $s>0$, $k>0$, and $\alpha>0$, $\partial e_2^*/\partial\varepsilon>0$, e_2^* is an increasing function with respect to ε. Therefore, at this time, the optimal effort level is in the same direction as the betting transfer share ratio.

In the same way, when the betting target is set too low, that is $\pi_0 < \pi_2$, the optimal effort level e_3^* can be obtained through equation (36), and further it has $\partial e_3^*/\partial\varepsilon = (p-w)\beta/s$, and $\partial e_3^*/\partial\varepsilon>0$ can be derived, that is, e_3^* and ε show a relationship of change in the same direction. End of proof.

The retailer is more likely to win the betting when the betting target is set moderately or too low, as opposed to when the target is set too high. At this moment, if both parties set a larger betting transfer share ratio, the retailer will face a double incentive to win the betting and get more shareholdings, and a larger betting transfer share ratio represents a higher opportunity cost for the retailer to lose the betting. As a result, its motivation to expand markets to win the betting is necessarily stronger, resulting in a correspondingly higher effort level. When the other party sets a lower betting transfer share ratio, the share of equity that the retailer receives for winning the bet will be lower; in this case, the retailer faces insufficient incentives and a correspondingly lower effort level.

During equity financing, the investor often takes advantage of the financier's eagerness to obtain funds for corporate expansion and increases the target revenue value to incentivize the financier to achieve better performance so that both parties can achieve a win-win situation. However, compared with Corollary 3, it can be seen that a higher betting target does not represent a greater degree of incentive generated for the retailer. Only when the betting target is set to match the retailer's strength, or the retailer can achieve the target with its existing resources, the appropriate increase in the betting target may make the retailer work harder on market expansion to improve its performance. Thus, the betting target should not be set too high. □

6.3. Effect of Ordering Price on Optimal Valuation Level. Ordering price affects the market sales of products by influencing the ordering demand of the retailer, which ultimately affects the performance of the supply chain and the profit of the participants. Therefore, ordering price is the main factor that needs to be focused on in supply chain equity financing. As a result, this section will focus on the effect of the ordering price on the optimal valuation level.

Corollary 5. *When the betting target is set too high or too low, $\pi_1 < \pi_0$ or $\pi_0 < \pi_2$, there exists an only ordering price w^* that maximizes the optimal valuation level α^*. And when the retailer's ordering price w is lower than the optimal ordering price w^*, α^* changes in the identical direction as w. When w is higher than w^*, α^* changes in the opposite direction as w.*

Proof. :

(1) When the betting target is set too high, that is, $\pi_1 < \pi_0$, the optimal valuation level α_1^* is obtained by equation. (22), and its first-order partial derivative concerning w is found, and the following can be derived:

$$
\begin{aligned}
\frac{\partial \alpha_1^*}{\partial w} &= \frac{-s^3\beta^2 B(p-w)(8\varepsilon+4)[2(p-w)q-2d+k](A+\eta)-s^2\beta^4 B(p-w)^3(8\varepsilon+4)(2-4\varepsilon)}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\
&+\frac{4s^2(p-w)^3\beta^4 B(1-2\varepsilon)(4\varepsilon+2)(A+\eta)-4s^3B\beta^2(p-w)(1-2\varepsilon)[2(p-w)q-2d+k]}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\
&+\frac{2qs^4B[2(p-w)q-2d+k](A+\eta)+2qs^3B(p-w)^2\beta^2(2-4\varepsilon)}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\
&+\frac{2qs^3B(p-w)^2\beta^2(4\varepsilon+2)(A+\eta)-2qs^4B[2(p-w)q-2d+k]}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\
&=\frac{\left\{-8s^3\beta^2B(p-w)[2(p-w)q-2d+k]+8s^3\beta^2B(p-w)^2q\right\}}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\
&=\frac{\left(8s^3\beta^2B(p-w)[-q(p-w)+2d-k]\right)}{\left(\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)\right)}.
\end{aligned}
\tag{45}
$$

Let $\partial\alpha_1^*/\partial w = 0$, and since $p > w$, $s > 0$, $\beta > 0$, and $B > 0$, it follows that $-q(p-w)+2d-k=0$. Moreover, it is known from the previous assumption that $q=\delta-\gamma p$, so there have $(\gamma p-\delta)(p-w)+2d-k=0$. Then, the second-order derivative of the ordering price w has $\partial^2\alpha_1^*/\partial w^2 < 0$, so the valuation level is a concave function of the ordering price w. Therefore, there exists a unique stationary point of the function, the stationary point is$w^* = ((\delta-\gamma p)p+k-2d)/(\delta-\gamma p)$, and it can be proved that when $w > w^*$, there is $\partial\alpha_1^*/\partial w < 0$. When $w < w^*$, there is $\partial\alpha_1^*/\partial w > 0$.

(2) When the betting target is set too low, that is $\pi_0 < \pi_2$, similarly, the optimal valuation level α_3^* is obtained by equation. (39), and its first-order partial derivative regarding w is obtained as

$$
\begin{aligned}
\frac{\partial \alpha_3^*}{\partial w} &= \frac{\left[-s\beta^2B(p-w)(-8\varepsilon+4)-2qs^2B\right]\left\{[2(p-w)q-2d+k]s^2+s(p-w)^2\beta^2(2+4\varepsilon)\right\}}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2+4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\
&+\frac{\left[-2qs^2-4s\beta^2(p-w)(1+2\varepsilon)\right]\left\{s(p-w)^2\beta^2(4\varepsilon-2)+[2(p-w)q-2d+k]s^2\right\}B}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)} \\
&=\frac{8s^3\beta^2B(p-w)[-q(p-w)+2d-k]}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)}.
\end{aligned}
\tag{46}
$$

This result is consistent with the result when the betting target is set too high, the stationary point $w^* = ((\delta - \gamma p)p + k - 2\,d)/(\delta - \gamma p)$is obtained, and it can be proved that when $w > w^*$, there is $\partial\alpha_3^*/\partial w < 0$. When $w < w^*$, there is $\partial\alpha_3^*/\partial w > 0$.

From the above results, there exists an only ordering price w^* that maximizes the optimal valuation level. Since the retailer's shareholding ratio is $V/(V+B)$, and $V = \alpha(A+\eta)$, so the higher the valuation level α, the greater the shareholding ratio; thus, this ordering price w^* is the retailer's optimal ordering price. When $w^* = ((\delta - \gamma p)p + k - 2\,d)/(\delta - \gamma p)$, the optimal valuation level reaches the highest. When $w > w^*$, the higher the ordering price, the lower the optimal valuation level of the supplier to the retailer. When $w < w^*$, the higher the ordering price, the higher the optimal valuation level. End of proof.

In summary, if the ordering price is in a higher range, that is $w > w^*$, the optimal valuation level has an inverse relationship with the ordering price. This is mainly because higher-ordering prices push up retailers' selling prices, causing them to lose their competitive price advantage and thereby weakening their market share and ultimately hitting firm value. Hence, when $w > w^*$, the supplier tends to give a lower valuation to the retailer.

However, when the ordering price is in the lower range, that is, $w < w^*$, the optimal valuation level varies in the same direction as the ordering price. In this range, the lower ordering price of the retailer will weaken the profit of the supplier. At this time, the supplier may depress the retailer's valuation to compensate for the loss of such a case and thus increase its stake to obtain more shareholding returns. Therefore, the lower the ordering price, the cheaper the retail price, which reduces the retailer's revenue and decreases the overall performance of the centralized supply chain, thus leading lower valuation level. Thus, the optimal valuation level varies in the same direction as the ordering price when the ordering price is in the lower range.

Furthermore, when the betting target is set moderately, which means $\pi_2 \le \pi_0 \le \pi_1$, the first-order partial derivative of α_2^* concerning w is obtained to be

$$\begin{aligned}\frac{\partial\alpha_2^*}{\partial w} &= \frac{\left\{4ksB(p-w)\beta^2 - 16\varepsilon B(p-w)^3\beta^4\right\}\left\{-2mks^2 - 2ks(p-w)^2\beta^2 - k^2s^2\right\}}{\left\{4\varepsilon^2(p-w)^4\beta^4 - 2mks^2 - 2ks(1+2\varepsilon)(p-w)^2\beta^2 - \left[2\varepsilon(p-w)^2\beta^2 - ks\right]^2\right\}^2(A+\eta)}\\ &\quad - \frac{4ks\beta^2(p-w)\left\{2mks^2B - k^2s^2B - 2ks(p-w)^2\beta^2B + 4\varepsilon(p-w)^4\beta^4B\right\}}{\left\{4\varepsilon^2(p-w)^4\beta^4 - 2mks^2 - 2ks(1+2\varepsilon)(p-w)^2\beta^2 - \left[2\varepsilon(p-w)^2\beta^2 - ks\right]^2\right\}^2(A+\eta)}\\ &= \frac{-16k^2s^3Bm\beta^2(p-w) + 16\varepsilon B(p-w)^5\beta^6ks + 16\varepsilon B(p-w)^3\beta^4ks^2(4m+k)}{\left\{4\varepsilon^2(p-w)^4\beta^4 - 2mks^2 - 2ks(1+2\varepsilon)(p-w)^2\beta^2 - \left[2\varepsilon(p-w)^2\beta^2 - ks\right]^2\right\}^2(A+\eta)}.\end{aligned} \tag{47}$$

Letting $\partial\alpha_2^*/\partial w = 0$, we get $\varepsilon(p-w)^4\beta^4 + \varepsilon(p-w)^2\beta^2s(4m+k) - ks^2m = 0$. Since it is too complicated to solve this unary quartic equation and discuss the functional characteristics of $\partial\alpha_2^*/\partial w$, this paper will not discuss it for the sake of space. □

6.4. Impact of Firm Growth on Optimal Valuation Level. In general, when the firm growth is higher, it has more impact on the overall performance of the centralized supply chain. Following Yu and Wang (2018), if both the investor and financier adopt the price-to-net ratio method for corporate valuation, the optimal valuation level will be influenced by the firm growth of the retailer. As the firm growth β increases, the optimal valuation level α^* will keep rising. From the study in Section 5, it is clear that in centralized supply chain equity financing, the optimal valuation level remains the function of the firm growth after introducing the gambling agreement, so that α^* is still affected by β.

Corollary 6. *When the betting target is set too high or too low, that is, $\pi_1 < \pi_0$ or $\pi_0 < \pi_2$, the greater the retailer's firm growth β, the higher the optimal valuation level α^*. Conversely, α^* will be lower. The parties show a relationship of change in an identical direction.*

Proof. When the betting target is set too high, that is $\pi_1 < \pi_0$, the optimal valuation level α_1^* is obtained by equation. (22), and the first-order partial derivative of it concerning β can be found. The following can be derived:

$$\begin{aligned}\frac{\partial \alpha_1^*}{\partial \beta} &= \frac{8\varepsilon B(p-w)\beta[2(p-w)q-2d+k]s^3 + 4B(p-w)\beta[2(p-w)q-2d+k]s^3}{\left(ks^2+2s\beta^2-4s\beta^2\varepsilon\right)^2(A+\eta)} \\ &\quad - \frac{32\varepsilon^2 B(p-w)^3\beta^3 s^2 + 16\varepsilon B(p-w)^3\beta^3 s^2 + 16\varepsilon B(p-w)^3\beta^3 s^2 - 32\varepsilon^2 B(p-w)^3\beta^3 s^2}{\left(ks^2+2s\beta^2-4s\beta^2\varepsilon\right)^2(A+\eta)} \\ &\quad + \frac{4B(p-w)\beta ks^3 - 8\varepsilon B(p-w)\beta ks^3 + 16\varepsilon B(p-w)^3\beta^3 s^2 + 16\varepsilon B(p-w)^3\beta^3 s^2}{\left(ks^2+2s\beta^2-4s\beta^2\varepsilon\right)^2(A+\eta)} \\ &= \frac{8s^3(p-w)^2\beta B[2(p-w)q-2d+k]}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2-4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)}.\end{aligned} \tag{48}$$

Since $s>0, k>0, B>0, A>0, \eta>0, p>w, (p-w)q>d$, it follows that $\partial\alpha_1^*/\partial\beta>0$.

Equally, when the betting target is set too low, that is, $\pi_0<\pi_2$, α_3^* is obtained by equation (39), and the first-order partial derivative of it concerning β is obtained:

$$\frac{\partial \alpha_3^*}{\partial \beta} = \frac{8s^3(p-w)^2\beta B[2(p-w)q-2d+k]}{\left\{[2(p-w)q-2d+k]s^2+2s(p-w)^2\beta^2+4s(p-w)^2\beta^2\varepsilon\right\}^2(A+\eta)}. \tag{49}$$

The derivative result is similar to those when the betting target is set too high, so it also has $\partial\alpha_3^*/\partial\beta>0$. Therefore, when the betting target is set too high or too low, there exists a positive relationship between the retailer's firm growth and the optimal valuation level. End of proof.

In general, the higher the firm growth is, the greater it is potential for market expansion, and the higher the revenue to the various actors in the centralized supply chain. As a result, in practice, equity investors prefer high-growth companies, such as those in emerging industries like biotechnology, the Internet of Things, and 4D technology. When the betting target is low, even though the supplier might not win the betting, it can gain more from either the original shares or from product sales because the high-growth retailer is likely to be more profitable in the future. The higher firm growth represents a greater possibility of future value creation for the firm, and the investor will not only assess the value of the financier's current assets but will also consider its future value. As a result, the supplier is willing to give a higher corporate valuation to high-growth retailers even in cases where it will lose part of its equity due to a failed betting. In this case, the supplier does not aim to gain control of the retailer but rather to strengthen the revenue-sharing mechanism and create more value in the whole centralized supply chain by improving the retailer's performance.

When the betting target is set too high, the supplier may win the betting and gain the equity transferred by the retailer, thereby increasing its control over the retailer. Meanwhile, the higher firm growth retailer is possible to achieve better performance in future market expansion. This control of the retailer by the supplier may result in excess revenue for the supplier in the centralized supply chain. Such a case may result in the supplier entering into equity financing agreements with the higher firm growth retailer and giving its higher valuation level. □

Corollary 7. *When the betting target is set moderately, that is, $\pi_2\le\pi_0\le\pi_1$, the optimal valuation level α^* varies in the identical direction with the firm growth β if the retailer's firm growth $\beta\in(0,\sqrt{ks}/((p-w)\sqrt{2\varepsilon}))$. α^* varies in the opposite direction with β if $\beta\in(\sqrt{ks}/((p-w)\sqrt{2\varepsilon}),+\infty)$.*

Proof. When the betting target is set moderately, that is, $\pi_2\le\pi_0\le\pi_1$, the optimal valuation level α_2^* is obtained by equation. (30), and the first-order partial derivative of it concerning β is found, and the following can be derived:

$$\frac{\partial \alpha_2^*}{\partial \beta} = \frac{8k\beta s^2 B(2m+k)(p-w)^2\left[ks-2\varepsilon(p-w)^2\beta^2\right]}{\left\{4\varepsilon^2(p-w)^4\beta^4-2mks^2-2ks(1+2\varepsilon)(p-w)^2\beta^2-\left[2\varepsilon(p-w)^2\beta^2-ks\right]^2\right\}^2(A+\eta)}. \tag{50}$$

Letting $\partial\alpha_2^*/\partial\beta=0$, we get $\beta=0$ or $ks-2\varepsilon(p-w)^2\beta^2=0$; this yields three stationary points, namely, $\beta^*=0$, $\sqrt{ks}/((p-w)\sqrt{2\varepsilon})$, and $-\sqrt{ks}/((p-w)\sqrt{2\varepsilon})$. The firm growth factor $\beta>0$ in the market expansion process, so only the stationing point $\beta^* = \sqrt{ks}/(p-w)\sqrt{2\varepsilon}$ is considered here. It is further known that α_2^* is a concave function about β when $\beta \in (0, +\infty)$. When $0 < \beta < \sqrt{ks}/(p-w)\sqrt{2\varepsilon}$, there is $\partial\alpha_2^*/\partial\beta > 0$, so the greater the firm growth, the higher the optimal valuation level. When $\beta > \sqrt{ks}/(p-w)\sqrt{2\varepsilon}$, $\partial\alpha_2^*/\partial\beta < 0$, so at this point when the firm's growth becomes higher, the optimal valuation level will be reduced instead; when the firm's growth reaches $\beta = \sqrt{ks}/(p-w)\sqrt{2\varepsilon}$, the optimal valuation level reaches the maximum.

Similar to the previous analysis, in general, the greater the firm's growth, the higher its optimal valuation level. However, the optimal valuation level cannot be infinitely higher. If the firm growth factor $\beta > \sqrt{ks}/((p-w)\sqrt{2\varepsilon})$, the valuation level will no longer increase monotonically but will tend to decrease. This is because when the betting target is set moderately, both the retailer and the supplier may win the betting. If the firm growth of the retailer is too high and exceeds a specific range, the possibility of the retailer winning the bet will increase. As a result, the supplier will lower the valuation to avoid losing the betting and reducing its equity share. Therefore, with a moderate betting target setting, the valuation level does not increase with the firm growth of the retailer, and the two variables vary in opposite directions when the valuation level reaches a certain threshold. □

7. Numerical Simulation

The earlier part of the article constructs an optimal valuation model for retailer equity financing based on the gambling agreement in a centralized supply chain and explores the effects of critical variables on valuation levels and effort levels. This section proposes using MATLAB to perform numerical simulations and explore the following questions: (i) the relationship between the betting transfer share ratio, valuation level, and effort level; (ii) the relationship between firm growth, ordering price, and valuation level; (iii) the relationship between firm growth, ordering price, and effort level.

7.1. The Betting Transfer Share Ratio, Valuation Level, and Effort Level. Letting the retailer's fixed assets amounts $A = 3000$ before equity financing, the retailer's own liquidity amounts $\eta = 1000$ before equity financing, the equity financing amounts $B = 8000$, the retail price $p = 200$, the market demand $q = 2050–10p$, the ordering price $w = 180$, and the retailer's fixed cost $d = 50$. This section verifies Corollary 1 and Corollary 2 of the previous part by using a two-dimensional relationship diagram between the betting transfer share ratio and valuation level (the relationship diagram is omitted due to the limitation of space), whereby the relationship between the former two variables and effort level is further explored.

In addition, to obtain Figure 2, we set retailer growth factor $\beta = 30$ and effort cost coefficient $s = 5$. Figure 2 reflects the relationship between the betting transfer share ratio ε, the valuation level α, and the effort level e when $\pi_1 < \pi_0$, that is, when the betting target is too high. It can be deduced that if the betting target is set too high, the retailer may not win the betting; in this situation, the effort level decreases accordingly as the betting transfer share ratio increases. This is because the higher the betting transfer share ratio set in the agreement, the greater the equity share of the retailer may lose at the expiration of the agreement, thus reducing the retailer's incentive to explore the market, which is consistent with Corollary 3. Therefore, even if the supplier has an advantageous position in the game of equity financing, it should consider the incentive effect of the betting target setting on the retailer. A too-large betting transfer share may, in turn, reduce the retailer's efforts and the overall performance of the centralized supply chain, resulting in the supplier failing to achieve expected revenue. It is also clear from Figure 2 that for a given betting transfer share ratio, the effort level increases with the valuation level. It is because the higher the valuation level, the greater the incentive for the retailer, and the bigger the level of effort.

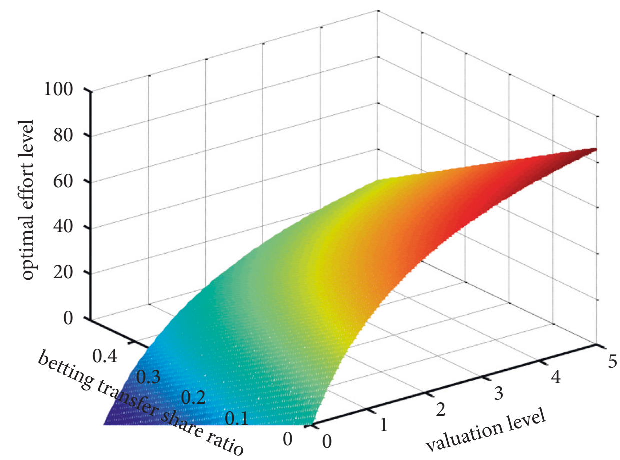

Figure 2: Relationship between ε, α, and e when the betting target is set too high.

Figure 3 presents the relationship between ε, α, and e when $\pi_2 \le \pi_0 \le \pi_1$, that is, when the betting target is set moderately. To obtain Figure 3, with the assigned values of A, η, B, p, w, and d unchanged from the previous section, let the effort cost coefficient $s = 50$, the retailer's growth factor $\beta = 4$, and the maximum possible excess revenue $k = 360$ in the retailer market expansion process. As shown in Figure 3, the level of effort increases with the rise in the betting transfer share ratio ε, which is consistent with the results of Corollary 4. When the valuation level is given, the effort level is a concave function regarding the betting transfer share ratio. When the betting target is set moderately, the retailer may win the betting, but if the betting transfer share ratio is low, the degree of incentive for the retailer will remain weak. Therefore, from Figure 3, it can be seen that the level of retailer effort increases relatively slowly with the betting transfer share ratio when this proportion is low. When the share of betting transfer is higher, the level of the retailer's effort rises with the former rate. By increasing the level of effort, the retailer can, on the one hand, avoid the need to transfer more shares if the gambling fails. On the other hand,

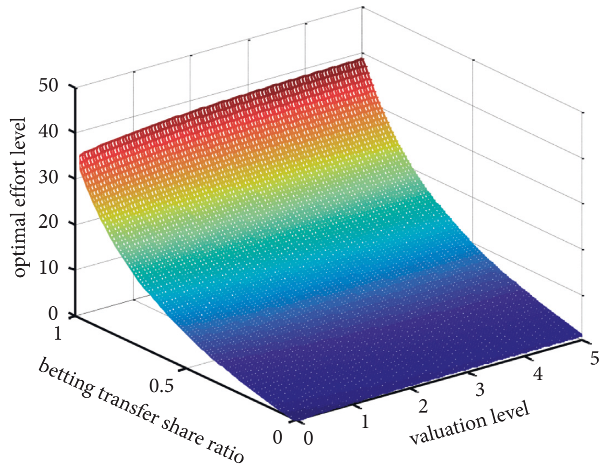

Figure 3: Relationship between ε, α, and e when the betting target is set moderately.

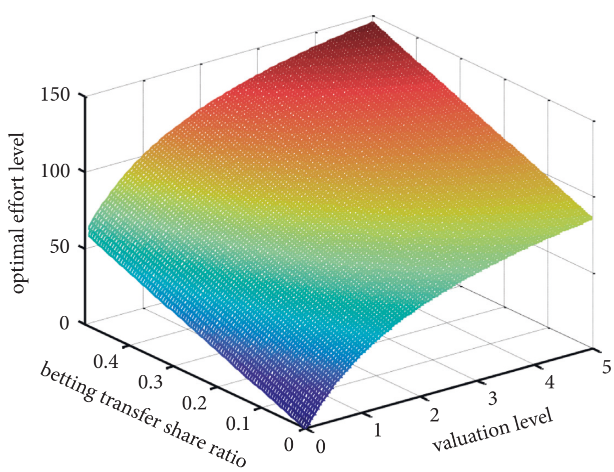

Figure 4: Relationship between ε, α, and e when the betting target is set too low.

it can improve the overall performance of the centralized supply chain. Consistent with the findings in Figure 2, with a given betting transfer share ratio, the level of retailer effort in Figure 3 still increases with the valuation level. The incentive effect of different valuation levels on the retailer varies so that a higher valuation level can affect the effort level that the retailer puts into market expansion.

Figure 4 reflects the relationship between ε, α, and e when $\pi_0 < \pi_2$, that is the betting target is too low. It can be seen that the effort level e rises correspondingly with the increase in the betting transfer share ratio ε, which is consistent with the conclusion of Corollary 4.

When the target value set in the gambling agreement is at a low level, the retailer may win the betting. A larger betting transfer share ratio will make the retailer receive a larger share of the equity upon the expiration of the agreement. However, a higher share of the betting transfer also represents the greater gambling risk and investment risk for the supplier, who may not receive the expected investment returns and thus tends to give a lower valuation to the retailer. Nevertheless, the too low valuation level will not be conducive to an equity financing agreement, and the retailer may seek alternative investment routes to avoid excessive equity dilution. Therefore, it should focus on several factors when setting the betting transfer share ratio, such as the behavior of the retailer and supplier, the possibility of equity financing realization, and the overall performance of the supply chain. The effort level varies in the same direction as the valuation level α, and it is a convex function about the valuation level, which is consistent with the findings obtained in Figures 2 and 3.

7.2. The Firm Growth, Ordering Price, and Valuation Level. From the previous analysis, it is clear that firm growth directly affects the incremental demand of the retailer through market expansion, and it is also a critical factor to be considered when valuing. Therefore, this section intends to explore the relationship between firm growth and valuation level through simulation analysis, and accordingly, then investigate the effect of ordering price on both.

Let the parameters A, η, B, p, and d take the same values as in the previous Section 7.1 and take the possible maximum excess revenue for the retailer $k = 1000$ and the effort cost factor $s = 4$. Figure 5 presents the relationship between firm growth β, ordering price w, and valuation level α when $\pi_1 < \pi_0$, that is, the betting target is set too high. As shown in Figure 5, the valuation level of the retailer tends to increase monotonically as the growth of its firm rises, which is consistent with the conclusion obtained in Corollary 6. For this reason, when the betting target is set too high, the supplier may win the betting to the large extent and receive shares of the betting transfer, which will create an incentive for the supplier. Furthermore, the retailer with higher firm growth is likely to be more profitable in the future, with the supplier receiving more returns from either its shareholdings or products sales under the revenue-sharing mechanism of a centralized supply chain. Therefore, in the case of greater retailer's firm growth, the valuation level by the supplier increases with the level of retailer effort due to the consideration of obtaining maximum revenue. As shown in Figure 5, under the condition that the ordering price w =130, the valuation level is $\alpha_1^* = 2.8$ when the firm growth $\beta = 5.2$. When $\beta = 6.4$, there is $\alpha_1^* = 4.9$, and when $\beta = 7.4$, there is $\alpha_1^* = 7.2$. It can be seen that the valuation level increases accordingly with the firm growth, and the relationship between them is characterized by a concave function in the interval shown in the figure, which means that the curve is flatter when the firm growth is lower. The valuation level increases relatively faster and the curve is steeper when companies have higher growth.

From Figure 5, it can also be deduced that when the betting target is set too high, the ordering price will impact the relationship between firm growth and valuation level. When the firm growth is invariant, if the ordering price is higher, the valuation level is correspondingly lower. Conversely, the valuation level is relatively greater. This is mainly because higher-ordering prices can push up retailer's selling prices and cause it to lose its competitive price advantage,

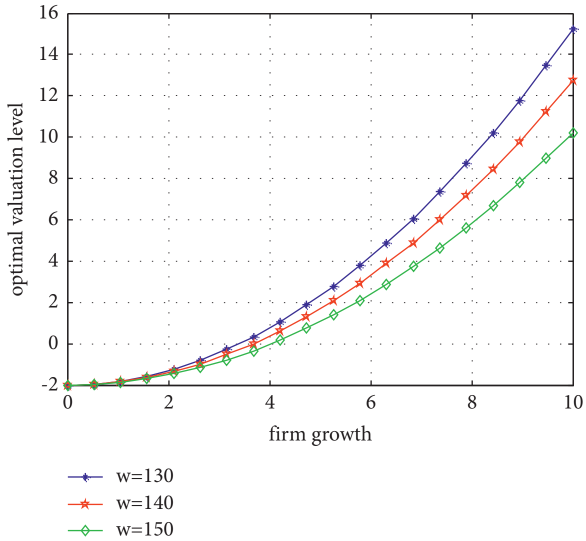

FIGURE 5: Relationship between β and α when the betting target is set too high.

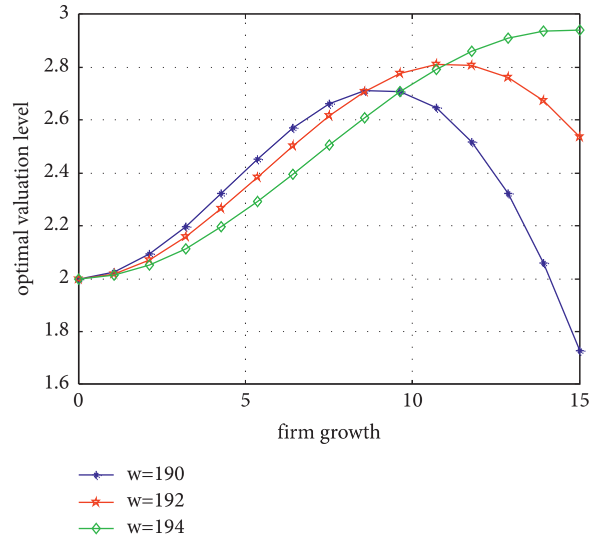

FIGURE 6: Relationship between β and α when the betting target is set moderately.

which may weaken its market share and ultimately impact the value of the firm. Such shocks may also cause losses to the supplier under the revenue-sharing mechanism in centralized supply chain. As a result, the supplier may give the retailer a lower valuation to hedge its risk.

The values of parameters A, η, B, p, d, and s are the same as before, and let the retailer's maximum possible excess revenue $k = 1500$. Figure 6 reflects the relationship between firm growth β, ordering price w and valuation level α for $\pi_2 \leq \pi_0 \leq \pi_1$, that is when the betting target is moderate. This situation illustrates the uncertainty of the gambling outcome when the betting target is between the minimum and maximum revenue available to the retailer. As can be seen from the figure, the valuation level exhibits monotonically increasing as the firm growth β rises and it then becomes monotonically decreasing after reaching the extreme value point, a case consistent with Corollary 7. This is because greater firm growth means more room for future growth, and thus a greater possibility of creating high value. When firm growth is at the low level, the retailer's earnings increase with higher firm growth, so the supplier's potential returns on investment in the future also rises accordingly. The likelihood of a retailer winning the betting increases when the firm growth is at a high level, which increases the probability of reduced shareholdings by the supplier, and excessive valuations will further aggravate this case.

Then we consider the effect of ordering price on both. From Figure 6, it can be seen the curve of the relationship between firm growth and valuation level is flatter when the ordering price is higher. As the ordering price increases, the amount of change in the valuation level that each unit of firm growth can bring decreases. This is because higher-ordering prices can push up the retailer's selling prices, reduce its market competitiveness, and affect the overall performance of the centralized supply chain. In this case, even though the firm growth of the retailer may be high, the supplier will depress the valuation level of the company to reduce the risk of investment due to the failure of the betting.

Figure 7 presents the relationship between firm growth β, ordering price w, and valuation level α when $\pi_0 < \pi_2$, that is, the betting target is set too low. Let the parameters A, η, B, p, d, and s take the same values as before and consider the possible maximum excess revenue $k = 100$ for the retailer. In this case, the valuation level of the retailer increases monotonically as the firm growth rises, which is consistent with the conclusion of Corollary 6. As shown in Figure 7, when the ordering price $w = 140$, the valuation level is $\alpha_3^* = 1.54$ if the firm growth $\beta = 30$. If $\beta = 35.3$, then there is $\alpha_3^* = 1.568$, and if $\beta = 40$, there is $\alpha_3^* = 1.582$. Although the supplier will probably lose part of equity after the agreement expires, higher firm growth may bring more incremental market demand for the retailer. Therefore, as the firm growth increases, the supplier may receive more returns on investment, resulting in a win-win situation for all parties in centralized supply chain. As a result, the supplier's valuation to the retailer increases with its firm growth. It can be seen from Figure 7 that when the betting target is set too low, the relationship between them is characterized by a convex function in the interval shown in the figure. When firm growth is at a high level, the incremental increase in valuation level caused by the improvement in firm growth is relatively low, which is contrary to the conclusion obtained in Figure 5. Meanwhile, when the betting target is set too low, the effect of ordering price on the relationship between firm growth and valuation level is the same as in Figure 5. It can be deduced that when the firm growth is a constant if the ordering price is higher, the valuation level is lower. Conversely, the valuation level is greater.

7.3. Firm Growth, Ordering Price, and Effort Level. In general, provided that the retailer's growth remains constant, the higher the level of effort it puts in, the greater the potential for its value creation that the supplier perceives and thus raises the valuation level. Furthermore, this section intends

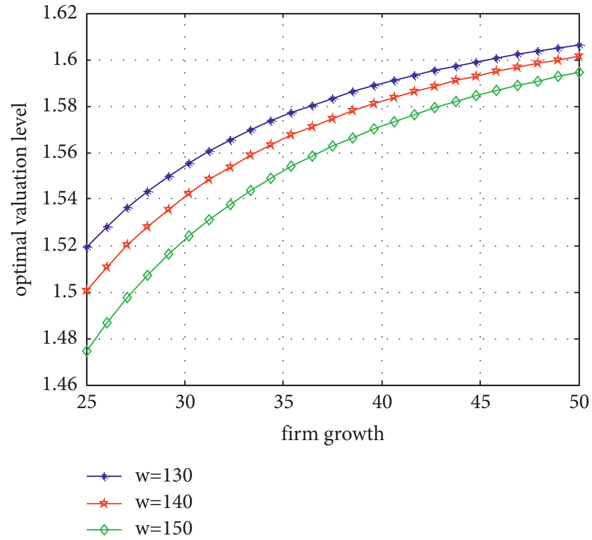

Figure 7: Relationship between β and α when the betting target is set too low.

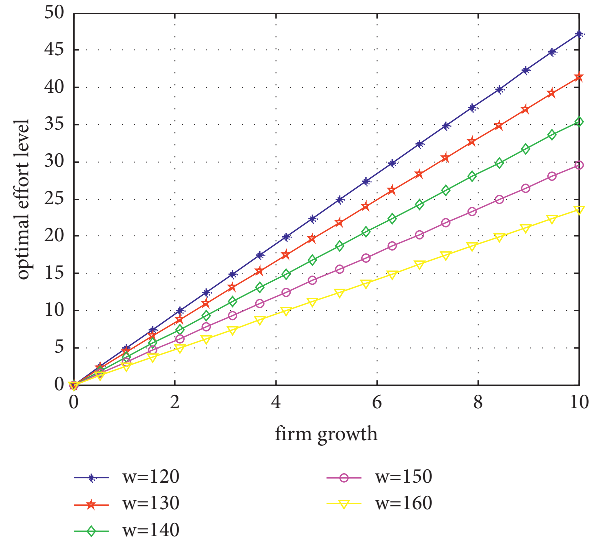

Figure 8: Relationship between β and e when the betting target is set too high.

to explore the relationship between firm growth, ordering price, and effort level through simulation analysis.

Let the parameters A, η, B, p, and s take the same values as in the previous sections 7.1 and 7.2, and take the valuation level as $\alpha = 3.5$. Figure 8 reflects the relationship between firm growth β, ordering price w, and effort level e when $\pi_1 < \pi_0$, that is when the betting target is set too high. From Figure 8, it can be seen that with a constant ordering price w, as the firm growth β of the retailer firm increases, its effort level of market expansion also increases accordingly, and the parties have the same proportional change relationship. Therefore, the effort level of the retailer is not only influenced by the betting transfer share ratio and valuation level but also depends on its growth. In general, the higher the firm growth, the more profitability it is, in this case, the retailer may increase its efforts in market expansion.

Furthermore, the ordering price influences the relationship between effort level and firm growth, and five different ordering prices are taken separately in Figure 8 for analysis. The steepest curve of the link between effort level and firm growth is observed when $w = 120$. The flattest curve is obtained when $w = 160$. In general, the higher the ordering price, the greater the ordering cost for the retailer, which may reduce its competitive price advantage and market share. When the betting target is set too high, the likelihood of retailer betting failure will increase, which reduces the incentive for the retailer to improve its effort level and reduces the overall performance of the supply chain. Thus, as shown in Figure 8, as the ordering price rises, the incremental level of effort from each unit increases in firm growth decreases.

Let the parameters A, η, B, p, s, and α take the same values as in the previous Sections 7.1 and 7.2, and take $k = 1500$. Figure 9 reflects the relationship between firm growth β, ordering price w, and effort level e for $\pi_2 \leq \pi_0 \leq \pi_1$, that is when the betting target is set moderately. This shows that the retailer's firm growth varies in the same direction as

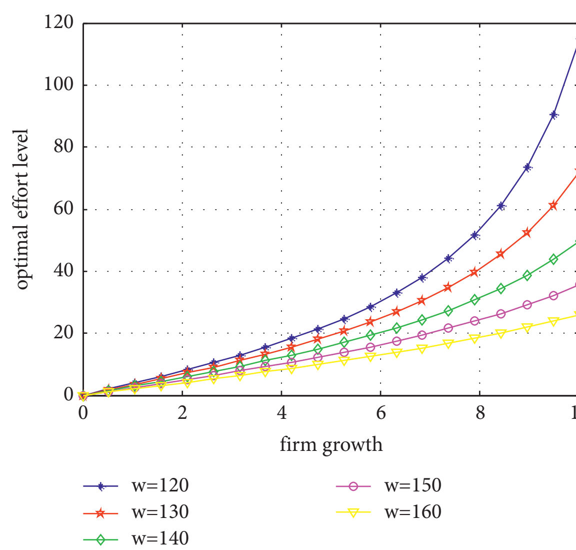

Figure 9: Relationship between β and e when the betting target is set moderately.

its effort level with constant ordering price, which is consistent with the findings in Figure 8. Higher firm growth represents the potential for greater future profitability for the retailer, which is more inclined to put in higher effort level. Figure 9 also selects five different ordering prices to explore the effect of ordering price on the relationship between firm growth and effort level. It can be inferred that the relationship between firm growth and effort level tends to flatten as the ordering price increases. In other words, if the firm's growth remains constant, the higher the ordering price, the weaker the retailer's cost advantage, thus the retailer's effort level decreases as the ordering price rises.

Let the parameters A, η, B, p, s, and α take the same values as the previous section. Figure 10 presents the relationship between firm growth β, ordering price w, and effort

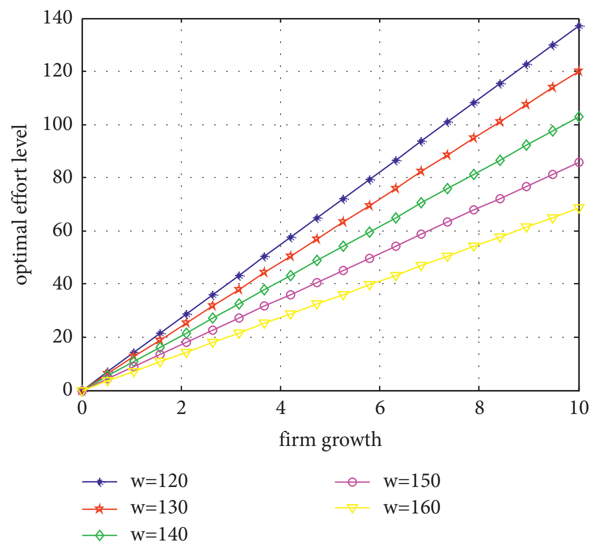

Figure 10: Relationship between β and e when the betting target is set too low.

level e when $\pi_0 < \pi_1$, that is, when the betting target is too low. As with the findings in Figures 8 and 9, all other conditions being equal, the retailer's firm growth varies in the same direction as the level of effort. As shown in Figure 10, for example, when the ordering price $w = 140$, the relationship between firm growth and effort level is reflected by the green curve in the figure. When the firm growth is $\beta = 1.6$, the effort level $e_3^* = 15$, $\beta = 2.1$, $e_3^* = 20$, and $\beta = 3.1$, $e_3^* = 31$. It can further be deduced that retailer who is willing to put more effort into market development as its growth increases and who has a higher effort level face less risk of low valuation in equity financing. In addition, when the firm growth remains constant, the effort level decreases with the increase in the ordering price.

8. Conclusion

This paper studies the optimal valuation issue of retailer equity financing based on gambling agreement in centralized supply chain. It introduces a gambling agreement mechanism in centralized supply chain and classifies the gambling targets as high, medium, or low according to the revenue gained by the retailer's market expansion. The optimal valuation model for retailer equity financing is constructed by mathematical derivation using the price-to-book ratio method for corporate valuation and determining the optimal effort level of the retailer. Furthermore, it conducts simulation analysis to clarify the relationships of the betting transfer share ratio, ordering price, firm growth, and optimal valuation level and effort level. The main conclusions of the research are as follows.[2], [7, 28, 43].

(i) Based on the expressions for the retailer's optimal effort level e^*, the supplier's optimal valuation level to the retailer α^* obtained in Section 4, and the associated property analysis, it can be concluded that in the centralized supply chain, for equity financing with no gambling agreement introduced, only by price-to-book ratio method valuation, there exists only an optimal valuation level and correspondingly optimal effort level of the retailer. In this case, the supplier (investor) and the retailer (financier) may reach an equity financing agreement.

(ii) According to the expressions for different optimal effort levels and optimal valuation levels e_1^*, e_2^*, e_3^* , α_1^*, α_2^*, α_3^* obtained in Section 5, and the correlation analysis, it is clear that for the equity financing based on gambling agreements in a centralized supply chain, there exists a valuation level and an effort level that maximizes revenue for both the retailer and the supplier, regardless of the set of gambling targets. Moreover, the level of retailer effort increases with the valuation level. According to Section 5.1, Section 5.2, Section 5.3, when the betting target is set too high, moderate, and too low, the supplier [46], both parties, and the retailer will probably win the betting, respectively.

(iii) From the research results in Section 6.1, the betting transfer share ratio will affect the optimal valuation level of the equity financing. As evidenced by the $\partial\alpha_1^*/\partial\varepsilon > 0$ in the proof of Corollary 1, when the betting target is set too high, the betting transfer share ratio and the optimal valuation level are in the same direction of change. According to $\partial\alpha_2^*/\partial\varepsilon < 0$ and $\partial\alpha_3^*/\partial\varepsilon < 0$ in Corollary 2, when the betting target is moderate or too low, the supplier may depress the valuation for the retailer due to the increase in the betting transfer share ratio to hedge the risk of equity transfer from a failed betting.

(iv) As can be seen from the research in Section 6.2, the level at which the betting transfer share ratio is set can affect the retailer's optimal effort level. Based on $\partial e_1^*/\partial\varepsilon < 0$ of Corollary 3, it follows that when the betting target is set too high, the betting transfer share ratio varies in the opposite direction of the optimal effort level, and this may improve the retailer's operational performance to some extent. As evidenced by $\partial e_2^*/\partial\varepsilon > 0$ and $\partial e_3^*/\partial\varepsilon > 0$ obtained by Corollary 4, when the betting target is moderate or too low, the retailer is more likely to win the betting, and thus it will increase its level of effort.

(v) From the conclusion of Corollary 5 in Section 6.3, if the betting target is set too high or too low, there exists an only optimal ordering price w^* that maximizes the optimal valuation level. However, when ordering prices are in the lower range, the supplier may depress corporate valuation to the retailer to compensate for the loss of low profits.

(vi) As shown by the numerical simulation results in Figures 8–10 in Section 7.3, the retailer's optimal effort level for market expansion is higher when its firm growth becomes greater. In this case, from Corollary 6 of Section 6.4, it is known that there is $\partial\alpha_1^*/\partial\beta > 0$ with $\partial\alpha_3^*/\partial\beta > 0$. Therefore, the supplier will give the retailer a higher valuation to obtain more excess revenue, regardless of the betting target is set too high or too low. As evidenced by Corollary 7 in Section 6.4, when the betting target is moderate, both parties have the potential to win the betting. Nevertheless, if the retailer's firm growth is too high, it is more likely to win the betting, and thus the supplier may lower the valuation.

This paper also has the following research limitations.

(i) This paper only takes the secondary centralized supply chain as the research object, and does not extend to more complex supply chain forms, meanwhile does not consider dynamic inventory, storage cost, transaction costs, operational efficiency, overall supply chain performance, etc.

(ii) This research only studies the equity financing among supply chain members, and only considers the valuation of retailers from the perspective of suppliers. It does not consider the case when PE, VC, and other venture capital firms carry out equity financing in the supply chain, nor does it address the dynamic change of corporate value.

(iii) This paper also does not take into account the impact of factors, such as phased payment methods, risk premiums, and optimal timing of equity financing when introducing the gambling agreement mechanism.

Data Availability

The data used in this manuscript was obtained through MATLAB simulation, and the programming code is detailed in the Appendix section.

Disclosure

No potential conflicts of interest was reported by the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] M. Abdel-Basset, R. Mohamed, K. Sallam, and E. Mohamed, "A novel decision-making model for sustainable supply chain finance under uncertainty environment," *Journal of Cleaner Production*, vol. 269, no. 5, pp. 1–11, 2020.

[2] A. W. Alford, "The effect of the set of comparable firms on the accuracy of the price-earnings valuation method," *Journal of Accounting Research*, vol. 30, no. 1, pp. 94–108, 1992.

[3] T. W. Bates, J. B. Neyland, and Y. Y. Wang, "Financing acquisitions with earnouts," *Journal of Accounting and Economics*, vol. 66, no. 2-3, pp. 374–395, 2018.

[4] H. H. Bauer and M. Hammerschmidt, "Customer," *Management Decision*, vol. 43, no. 3, pp. 331–348, 2005.

[5] J. N. Beka, B. Nguema, G. Bi, and Z. Ali, "Exploring the factors influencing the adoption of supply chain finance in supply chain effectiveness: evidence from manufacturing firms," *Journal of Business & Industrial Marketing*, vol. 36, no. 5, pp. 706–716, 2021.

[6] J. A. Buzacott and R. Q. Zhang, "Inventory management with asset-based financing," *Management Science*, vol. 50, no. 9, pp. 1274–1292, 2004.

[7] S. Caselli, S. Gatti, and M. Visconti, "Managing M&A risk with collars, earn-outs, and CVRs," *The Journal of Applied Corporate Finance*, vol. 18, no. 4, pp. 91–104, 2006.

[8] S. Chakuu, D. Masi, and J. Godsell, "Exploring the relationship between mechanisms, actors and instruments in supply chain finance: a systematic literature review," *International Journal of Production Economics*, vol. 216, no. 10, pp. 35–53, 2019.

[9] B. S. Daniel and H. Rau, "A stochastic model for developing speculation-postponement strategies and modularization concepts in the global supply chain with demand uncertainty," *Computers & Industrial Engineering*, vol. 158, no. 8, p. 107392, 2021.

[10] A. Din and Y. Li, "Stationary distribution extinction and optimal control for the stochastic hepatitis B epidemic model with partial immunity," *Physica Scripta*, vol. 96, no. 7, p. 074005, 2021.

[11] A. Din, Y. Li, and A. Yusuf, "Delayed hepatitis B epidemic model with stochastic analysis," *Chaos, Solitons & Fractals*, vol. 146, no. 1, p. 110839, 2021.

[12] L. Fang, Y. Xia, and Y. M. Yang, "Research on equilibrium of supply chain finance decision with retailer's sale effort level," *Systems Engineering Theory Practice*, vol. 38, no. 1, pp. 135–144, 2018.

[13] H. Fu, G. Y. Ke, Z. Lian, and L. Zhang, "3PL firm's equity financing for technology innovation in a platform supply chain," *Transportation Research Part E Logistics and Transportation Review*, vol. 147, no. 4, p. 102239, 2021.

[14] M. Gneiser, J. Heidemann, M. Klier, A. Landherr, and F. Probst, "Valuation of online social networks taking into account users' interconnectedness," *Information Systems and E-Business Management*, vol. 10, no. 1, pp. 61–84, 2012.

[15] J. Guo and Z. Zmeškal, "Valuation of the China internet company under a real option approach," *Perspectives in Science*, vol. 7, no. 3, pp. 65–73, 2016.

[16] R. Hammami, Y. Frein, I. Nouira, and A. S. Albana, "On the interplay between local lead times, overall lead time, prices, and profits in decentralized supply chains," *International Journal of Production Economics*, vol. 243, p. 108324, 2022.

[17] L. Jing and N. J. Thomas, "Equity valuation using multiples," *Journal of Accounting Research*, vol. 40, no. 1, pp. 135–172, 2002.

[18] S. Khavul and D. Deeds, "The evolution of initial Co-investment syndications in an emerging venture capital market," *Journal of International Management*, vol. 22, no. 3, pp. 280–293, 2016.

[19] N. Kohers and J. Ang, "Earnouts in mergers: agreeing to disagree and agreeing to stay," *Journal of Business*, vol. 73, no. 3, pp. 445–476, 2000.

[20] P. Kouvelis and W. Zhao, "Financing the newsvendor: supplier vs. Bank, and the structure of optimal trade credit

contracts," *Operations Research*, vol. 60, no. 3, pp. 566–580, 2012.

[21] J. J. Laffont and J. Tirole, *A Theory of Incentives in Procurement and Regulation*, MIT Press, Cambridge, MA, USA, 1993.

[22] D. P. J. Leisen, "Staged venture capital contracting with ratchets and liquidation rights," *Review of Financial Economics*, vol. 21, no. 1, pp. 21–30, 2012.

[23] S. D. Lekkakos and A. Serrano, "Supply chain finance for small and medium sized enterprises: the case of reverse factoring," *International Journal of Physical Distribution & Logistics Management*, vol. 46, no. 4, pp. 367–392, 2016.

[24] H. Mitani, "Capital structure and competitive position in product market," *International Review of Economics & Finance*, vol. 29, no. 1, pp. 358–371, 2014.

[25] D. Nissim, "Relative valuation of U.S. insurance companies," *Review of Accounting Studies*, vol. 18, no. 2, pp. 324–359, 2013.

[26] N. R. S. Raghavan and V. K. Mishra, "Short-term financing in a cash-constrained supply chain," *International Journal of Production Economics*, vol. 134, no. 2, pp. 407–412, 2011.

[27] R. Ragozzino and J. J. Reuer, "Contingent earnouts in acquisitions of privately held targets," *Journal of Management*, vol. 35, no. 4, pp. 857–879, 2009.

[28] S. H. Penman, "Combining earnings and book value in equity valuation," *Contemporary Accounting Research*, vol. 15, no. 3, pp. 291–324, 1998.

[29] R. Pringles, F. Olsina, and F. Garces, "Real option valuation of power transmission investments by stochastic simulation," *Energy Economics*, vol. 47, no. 1, pp. 215–226, 2015.

[30] S. Sehgal and A. Pandey, "Equity valuation using price multiples: a comparative study for bricks," *Asian Journal of Finance & Accounting*, vol. 2, no. 1, pp. 68–70, 2010.

[31] D. Srikant, F. Richard, and W. Mark, "Earnouts: the effects of adverse selection and agency costs on acquisition techniques," *Journal of Law, Economics, and Organization*, vol. 17, no. 1, pp. 201–238, 2001.

[32] T. A. Taylor, "Supply chain coordination under channel rebates with sales effort effects," *Management Science*, vol. 48, no. 8, pp. 992–1007, 2002.

[33] C. Toll and J. P. Rolinck, "Earn-outs to bridge gap between negotiation parties – curse or blessing?" *Managerial Economics*, vol. 18, no. 1, pp. 103–106, 2017.

[34] M. Ueda, "Banks versus venture capital: project evaluation, screening, and expropriation," *The Journal of Finance*, vol. 59, no. 2, pp. 601–621, 2010.

[35] L. Viarengo, S. Gatti, and A. Prencipe, "Enforcement quality and the use of earnouts in M&A transactions: i," *Journal of Business Finance & Accounting*, vol. 45, no. 3-4, pp. 437–481, 2018.

[36] Q. Wang, D. M. Kilgour, and K. W. Hipel, "Facilitating risky project negotiation: an integrated approach using fuzzy real options, multicriteria analysis, and conflict analysis," *Information Sciences*, vol. 295, no. 1, pp. 544–557, 2015.

[37] Y. Wang, J. Zhai, and J. Deng, "Study on equity financing model of dominant retailer on the perspective of supply chain," *Journal of Systems Science and Mathematical Sciences*, vol. 41, no. 2, pp. 466–480, 2021.

[38] H. R. Xiang, J. J. Li, and X. Liu, "A study of gambling contracts based on incentive perspective," *Shanghai Journal of Economics*, no. 3, pp. 92–98, 2009.

[39] L. Xu, L. Zhang, and D. Yao, "Optimal investment and reinsurance for an insurer under Markov-modulated financial market," *Insurance: Mathematics and Economics*, vol. 74, no. 5, pp. 7–19, 2017.

[40] H. Yang, W. Zhuo, and L. Shao, "Equilibrium evolution in a two-echelon supply chain with financially constrained retailers: the impact of equity financing," *International Journal of Production Economics*, vol. 185, no. 1, pp. 139–149, 2017.

[41] A. Battauz, S. Gatti, and A. Prencipe, "Earnouts: the real value of disagreement in mergers and acquisitions," *European Financial Management*, vol. 27, no. 5, pp. 981–1024, 2021.

[42] H. Yu and Y. Wang, "Growth enterprise financing under supply chain management: debt financing versus equity financing," *Chinese Journal of Management Science*, vol. 26, no. 3, pp. 74–85, 2018.

[43] S. A. Yang and J. R. Birge, "Trade credit, risk sharing, and inventory financing portfolios," *Management Science*, vol. 64, no. 8, pp. 3667–3689, 2018.

[44] R. Yitshaki, "Venture capitalist-entrepreneur conflicts: an exploratory study of determinants and possible resolutions," *International Journal of Conflict Management*, vol. 19, no. 3, pp. 262–292, 2008.

[45] H. Yu and X. Li, "Optimal valuation model of retailer equity financing in supply chains," *Journal of Management Sciences in China*, vol. 21, no. 9, pp. 91–104, 2018.

[46] Z. Zmeskal, "Application of the fuzzy–stochastic methodology to appraising the firm value as a European call option," *European Journal of Operational Research*, vol. 135, no. 2, pp. 303–310, 2001.