Research Article

Alternative External Resource Allocation Method to Information Security in Smart Cities

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There are alternative external resources among smart cities; that is, no matter which cities are invaded illegally, other cities can avoid losses. Keeping this in mind, urban resource allocation is studied. To be specific, the external resource substitution rate model among smart cities is introduced when describing and modeling the problem, and then, the influences of the independent strategy and joint strategy are analyzed. Based on a comparison of the Nash equilibrium solutions for the above two strategies, a cooperative mechanism is proposed to deal with the problem of overallocation of resources in the process of independent strategy. Finally, the conclusions are verified by numerical experiments.

1. Introduction

A smart city is an open and sharing system with ecological circulation, and there are various exchanges and decisions between cities in terms of information security [1–3]. Through a city, illegal users can attack other cities linked thereto and easily transfer resources from a more secure city to a less secure one [4, 5]. In other words, in the smart city network, the information security level of cities can be determined by the volume of resource allocation [3, 6]. Moreover, the decision will affect other cities linked thereto, and the resource allocation has obvious externalities [7–9].

In addition to complementary external resources, there are alternative external resources between smart cities [10, 11]. After illegal users make their intrusion into a city, the incremental benefits of intruding into other cities are significantly lower than the costs, and then the resources of the city and cities linked thereto constitute alternative resources [12–14]. In this case, once illegal users intrude, they will acquire the information they need and will not intrude elsewhere [15–17]. On the contrary, if illegal users fail to intrude, they will intrude into other cities linked thereto [18–20]. The following two examples can illustrate the problem.

Example 1. After illegal users gain access to the system of a hospital in City A through intrusion, they can obtain much information about its patients. After that, illegal users benefit from selling unapproved drugs to patients based on the information, and they can get information about patients without the need to intrude into the same type of hospitals in City B.

Example 2. Large supermarkets and shopping malls exchange electronic data with suppliers through the network, so that suppliers can trace sales of goods in supermarkets or shopping malls at any time. Once out of stock, suppliers can supplement goods immediately. In order to steal information from a large supermarket and a shopping mall in City A and suppliers in City B, illegal users only need to attack any one of the two cities to acquire the information they need.

In the presence of alternative resources in information security, the information security level of a city will have a significant influence on the security of other urban agglomerations linked thereto [21–24]. In particular, the complex external environment, sudden intrusions by illegal users, and uncertainty in the process of resource allocation in cities make it more difficult to make decisions pertaining
to resource allocation to information security in urban agglomerations [25–27]. The substitutability of resources in the cities with alternative external resources will affect the probability of intrusion by illegal users and thus influence the expected costs of the city. Therefore, it is necessary to conduct in-depth analyses of resource allocation paradigms in cities.

Some scholars study this problem by introducing social planners. Their research found that if a city improves its level of information security, illegal users tend to attack other more vulnerable cities [28–31]. Other urban resource allocation approaches are based on distributed models [32], especially focusing on the communications infrastructure. Many solutions were discussed in the 19th International Conference on Ad Hoc Networks and Wireless, and its proceeding covers a wide spectrum of traditional networking topics ranging from routing to the application layer, to localization in various networking environments such as wireless sensor and ad hoc networks, and gives insights in a variety of application areas [32].

For alternative external resources, the substitutability of resources between cities will affect the probability of illegal users intruding into the city, thus affecting the expected costs associated with construction of information security systems. Thus, it is necessary to consider the substitutability into the model, as well as other factors, such as city size and the probability of intrusion by illegal users. The resource allocation methods under independent and joint strategies are separately discussed. By comparing their differences, collaborative mechanisms of information sharing are introduced further to optimize resource allocation to information security [32–36].

A model of the rate of substitution of external resources between smart cities is introduced to describe and model the problem according to characteristics of alternative external resources. Meanwhile, the influences of different factors, such as the size of the city and the probability of intrusion by illegal users, on resource allocation are considered, and independent and joint strategies are studied. Based on this, the influences of factors such as city size (in terms of population or area), probability of intrusion by illegal users, and substitution rate of resources between cities on the above two strategies are determined. In addition, Nash equilibrium solutions under the aforementioned two strategies are compared. Collaborative mechanisms are introduced to stimulate cities to implement the joint strategy, thus solving the problem of excessive resource allocation under the independent strategy. To verify the analysis and conclusions, numerical experiments are conducted.

2. Problem Modeling

For any game problem, it can be described as follows: \( GT = \{ P, St, Ut \} \). In terms of alternative external resources, cities are linked to each other and may be attacked by illegal users. After attacking any city, the information about all cities linked thereto can be acquired.

**Assumption 1.** Illegal users do not have any prior information relating to vulnerability in construction of information security in cities, and all cities share the same probability of illegal intrusion, recorded as \( \beta \).

**Assumption 2.** The rate of substitution of alternative resources between cities is the same and is recorded as \( \alpha \). After a city suffers an illegal intrusion, each city will bear the same loss, recorded as \( L_t \).

**Assumption 3.** When resources in information security in cities are not allocated, the probability of intrusion by illegal users is the same, namely, \( v \).

It is assumed that the number of cities forming alternative external resources is \( n \) and the probability of intrusion by illegal users after resource allocation to information security in the \( j(j = 1, 2, \ldots, n) \)th city is \( p_j \). The volume of resource allocated to information security is \( x_{jk} \), loss recovered per unit amount of money is \( E \), and the expected loss after resource allocation to information security in the city is \( C_{j} \). Based on this, the probability of intrusion by illegal users in the \( j \)th city is \( p_j \),

\[
\begin{align*}
  p_j & = \beta v^{Ex_j + 1}.
\end{align*}
\]

It is considered that there is a substitutability of resources between cities. In other words, illegal users can obtain any information they need after they intrude into one of cities and will not intrude into other cities; however, once they fail to intrude into a city, they will attack other cities. If illegal users want to maximize their profit, the probability of intrusion into City B is \( P_B \),

\[
\begin{align*}
  P_B & = p_1 + \sigma p_1 \prod_{j=2}^{n} (1-p_j),
\end{align*}
\]

where \( p_j \) represents the probability of intrusion by illegal users into City B, and \( \sigma p_1 \prod_{j=2}^{n} (1-p_j) \) denotes the probability that illegal users fail to intrude into other cities but turn to attack City B. The notations applied in this paper are summarized in Table 1.

3. Methods for Allocation of Complementary External Resources to Information Security

3.1. Resource Allocation to Information Security in Cities under the Independent Strategy. Here, resource allocation is analyzed under the independent strategy in smart cities for alternative external resources. According to formulae (1) and (2), the probability of intrusion by illegal users in the \( j(j = 1, 2, \ldots, n) \)th city is \( p_j + \sigma p_j \prod_{k=2}^{n} (1-p_k) \), so the minimum expected loss \( C_{j} \) of the city is taken as a loss function:

\[
\begin{align*}
  & \text{Min } C_{j} = \left[ p_j + \sigma p_j \prod_{k=2}^{n} (1-p_k) \right] L_t + x_j. \quad (3)
\end{align*}
\]

Substituting formula (1) into formula (3) gives

\[
\begin{align*}
  & \text{Min } C_{j} = \left[ \beta v^{Ex_j + 1} + \sigma \beta v^{Ex_j + 1} \prod_{k=2}^{n} (1-p_k) \right] L_t + x_j. \quad (4)
\end{align*}
\]
Based on Conclusion 1, users, and substitution rate of resources, the optimal volume of resource allocation to information security in cities reduces correspondingly; that is, $x_1^*$ is negatively correlated with $n$. When $n = \ln (\beta E v L_i \ln v - 1) - \ln (\sigma E v L_i \ln v) / \ln (1 - \beta v) + 1$, $x_1^*$ is 0.

This is because, with the increase of $n$, $\prod_{k=2}^n (1 - \beta v^{E x_k^{v+1}})$ decreases, which increases $p_i = \beta v^{E x_k^{v+1}}$. Moreover, owing to $v \in [0,1]$, $x_1^*$ is bound to decrease therewith. This indicates that the volume of resource allocation in each city can decrease correspondingly with the increase in the size of cities with alternative resources under the independent strategy. Alternative resources can lead to excessive resource allocation under the independent strategy, so the increase of city size can mitigate this problem to some extent; however, it is not infinite to increase the city size, and there is a critical threshold. When the city size exceeds this threshold, resources will not be allocated in cities.

Under the independent strategy, by studying the relationship between probability of intrusion by illegal users and resource allocation to information security in cities, Conclusion 3 can be drawn as follows.

Conclusion 3. Under the independent strategy, for the probability $\beta \in [0,1]$ of intrusion by any illegal users, the optimal volume $x_1^*$ of resource allocation to information security in cities monotonically increases; that is, $\partial x_1^*/\partial \beta > 0$ is always valid.

Conclusion 3 shows that the volume of resource allocation to information security in cities rises with the increase in probability of intrusion by illegal users in the model of alternative external resource allocation in smart cities, which is consistent with the dictates of common sense. When the probability of intrusion by illegal users rises, cities will invest more to prevent illegal intrusion, thus increasing their level of information security.

Under the independent strategy, by analyzing the relationship between the substitution rate of resources between cities and resource allocation to information security in cities, Conclusion 4 can be drawn as follows.

Conclusion 4. Under the independent strategy, for the substitution rate $\sigma \in [0,1]$ between resources in any cities, the optimal volume of resource allocation to information security in cities monotonically decreases; that is, $\partial x_1^*/\partial \sigma > 0$ is always valid.

Conclusion 4 suggests that, under the independent strategy, with the increase in the substitution rate of resources between cities, the optimal volume of resource allocation to information security in cities increases correspondingly. This finding implies that, in the case of a higher substitution rate of resources between cities, investment in information security should be strengthened. When the rate of substitution is lower, the input should be reduced as appropriate to reduce expected costs.
3.2. Resource Allocation to Information Security in Cities under the Joint Strategy. Compared with the case under the independent strategy, there is a large difference in the case under the joint strategy. Under the joint strategy, it is assumed that the probability of intrusion by illegal users after resource allocation to information security in cities is \( p \) and the volume of resource allocation to information security in cities is \( x \). Therefore, the loss function of urban agglomerations in this case can be derived from modeling of the aforementioned problem and formula (2).

\[
\text{Min } C_J = \left[ p + \sigma p \prod_{k=2}^{n} (1 - p_k) \right] nL_1 + nx. \tag{8}
\]

Substituting formula (1) into formula (8) gives

\[
\text{Min } C_J = \left[ \beta v^{\text{Ext}+1} + a\beta v^{\text{Ext}+1} \prod_{k=2}^{n} (1 - p_k) \right] nL_1 + nx. \tag{9}
\]

Let \( \Omega = \prod_{k=2}^{n} (1 - p_k) \), and the following formula is obtained by partial differentiation of formula (9):

\[
\frac{\partial C_J}{\partial x} = \beta E n^2 L_1 (1 + \sigma \Omega_1) v^{\text{Ext}+1} \ln v + n. \tag{10}
\]

By further partial differentiation of formula (10), the second-order derivate of formula (9) can be obtained.

\[
\frac{\partial^2 C_J}{\partial x^2} = \beta E n^3 L_1 (1 + \sigma \Omega_1) v^{\text{Ext}+1} (\ln v)^2. \tag{11}
\]

According to formula (11), \( \partial^2 C_J/\partial x^2 \geq 0 \) is always valid, so the loss function \( C_J \) is minimized at \( \partial C_J/\partial x = 0 \), thus obtaining Conclusion 5.

Conclusion 5. Under the joint strategy between smart cities with alternative external resources, the Nash equilibrium solution can be attained through games when the optimal volume of resource allocation in cities is \( x^* \) and \( x^* \) is obviously found to be smaller than \( x \). Therefore, the information security level in cities under the independent strategy is lower than that under the joint strategy.

By further evaluating the influences of each parameter on the optimal volume of resource allocation under the joint strategy, Conclusions 6 to 8 are reached as follows.

Conclusion 6. Under the joint strategy, when \( L_1 > -1/p' + \sigma p' \Omega_1 \), with the increase of the size of linked cities with complementary external resources in information security, the optimal volume \( x^*_2 \) of resource allocation to information security in cities decreases correspondingly; that is, \( x^*_2 \) has a negative correlation with \( n \) and \( \partial x^*_2/\partial n < 0 \).

Conclusion 6 implies that, under the joint strategy, if the loss \( L_1 \) in a city exceeds a certain threshold, the volume of resource allocation in the city decreases with the increase in the size of the urban agglomerations. Meanwhile, Conclusions 2 and 6 imply that, for the cities with alternative external resources in information security, the optimal volume of resource allocation reduces with the increase in the size of urban agglomerations under either the independent strategy or the joint strategy.

Conclusion 7. Under the joint strategy, for the probability \( \beta \in [0,1] \) of intrusion by any illegal users, the optimal volume \( x^*_2 \) of resource allocation to information security in cities monotonically increases with \( \beta \); that is, \( \partial x^*_2/\partial \beta > 0 \) is always valid.

Conclusion 7 indicates that, under the joint strategy, the optimal volume of resource allocation rises with the increase in probability of intrusion by illegal users, which is the same as the trend under the independent strategy in Conclusion 3. To be specific, under either the independent strategy or the joint strategy, the optimal volumes of resource allocation in cities always increase with the probability of intrusion. Meanwhile, as \( x^*_1 < x^*_2 \), the increase in probability of intrusion under the independent strategy suggests that cities need to bear higher risks to their information security: the joint strategy can, however, reduce such risks to some extent. Therefore, for the alternative external resources, the joint strategy can ensure a higher level of information security and alleviate excessive resource allocation under the independent strategy.

Conclusion 8. Under the joint strategy, for the substitution rate \( \sigma \in [0,1] \) of resources between any cities, the optimal volume \( x^*_2 \) of resource allocation to information security in cities monotonically rises with \( \sigma \); that is, \( \partial x^*_2/\partial \sigma > 0 \) is always valid.

Conclusion 8 implies that, under the joint strategy, the optimal volume of resource allocation can increase with the rate of substitution of resources in cities, which is the same as the trend under the independent strategy in Conclusion 4. In other words, under either the independent strategy or the joint strategy, the optimal volumes of resource allocation in cities always increase with the rate of substitution of resources between cities. This finding indicates that the rate of substitution of information resources in cities can determine resource allocation to information security to a large extent.

In conclusion, the volume of resource allocation in cities with alternative external resources in information security changes with the size of urban agglomerations, the probability of intrusion by illegal users, and the rate of substitution of resources in cities. These trends are consistent under both independent and joint strategies.

4. Introduction of Collaborative Mechanisms

The volumes \( x^*_1 \) and \( x^*_2 \) of resource allocation and expected costs are compared by discussing the independent and joint strategies as the result indicates that the optimal volume of resource allocation in cities under the independent strategy is higher than that under the joint strategy, thus leading to Conclusion 9.

Conclusion 9. When \( L_1 > -1/p' + \sigma p' \Omega_1 \), then \( \partial C_J/\partial x > 0 \), namely, the optimal volume \( x^*_2 \) of resource allocation in cities obtained under the independent strategy is smaller.
than that $x^*_1$ under the joint strategy. Therefore, the expected cost $C_j$ under the independent strategy is lower than $C_j$ under the joint strategy.

Conclusion 9 implies that the optimal investment volume and expected costs in urban agglomerations with alternative external resources under the independent strategy are lower than those under the joint strategy. Under the independent strategy in cities, if any city in an urban agglomeration with alternative external resources increases the resources allocated to information security, it will reduce the probability of illegal-user intrusion into the city. As a result, this will increase the probability of illegal users turning to attack other cities, promoting resource allocation to information security in other cities, thus raising the overall level of information security in the urban agglomeration. Conclusion 9 also suggests that there is a hidden competitive relationship between cities with alternative external resources, which may lead to excessive resource allocation to information security in urban agglomerations.

Conclusion 9 shows that if the independent strategy is implemented in cities, excessive resource allocation may arise; therefore, it is necessary to consider the introduction of collaborative mechanisms, so that the overall optimal volume of resource allocation in urban agglomerations reaches an optimal state. It is considered to use information sharing to realize collaboration of alternative external resources, thus solving the problem of excessive resource allocation.

Assumption 4. Cities ignore information leakage when sharing information.

Assumption 5. Any one city can obtain information from other cities with alternative resources; that is, the city can make decisions on resource allocation to information security based on the volume of resource allocation in other cities. Supposing that $\delta \in [0, 1]$ indicates the information sharing rate of city $j$ with other cities, the volume of resource allocation to information security obtained by city $j$ from other cities is $\delta \sum_{k=1, k \neq j}^n x_k$.

If Assumption 5 is valid, the loss function $C_j$ of city $j$ is given by

$$\text{Min } C_{c_j} = \left[p_j + \sigma p_j \sum_{k=1, k \neq j}^n (1 - p_k) \right] L_j + x_j. \quad (12)$$

Let $\Gamma_1 = \prod_{k=1, k \neq j}^n (1 - p_k)$, and substituting formula (1) into formula (12) gives

$$\text{Min } C_{c_j} = \left[p_j + \sigma p_j \sum_{k=1, k \neq j}^n \Gamma_{k} \right] L_j + x_j. \quad (13)$$

The optimal volume of resource allocation can be calculated using formula (13).

$$x^*_3 = \frac{\ln[-\beta L_j E(1 + \sigma \Gamma_1) \ln v]}{E \ln v} \delta \sum_{k=1, k \neq j}^n x_k. \quad (14)$$

By comparing $x^*_1$ with the optimal volumes $x^*_1$ and $x^*_2$ of resource allocation under the independent and joint strategies, $x^*_1 < x^*_2 < x^*_2$ indicates that the optimal volume of resource allocation in urban agglomerations under information sharing is larger than that under the independent strategy but is smaller than that under the joint strategy. Therefore, such mechanisms can cope with the problem of excessive resource allocation. Meanwhile, by comparing the expected costs, if a city bears loss $L > -1/p^1 + p^1(1 + \sigma \Gamma_1)$ after intrusion by illegal users, $\partial C_{c_j}/\partial x_j < 0$ and the expected cost $C_{c_j}(x^*_1)$ of the city in this case is lower than $C_j(x^*_1)$ under the independent strategy. Therefore, under information sharing mechanisms, urban agglomerations can not only reduce the volume of resource allocation, but also decrease the expected costs. Therefore, Conclusion 10 can be reached.

Conclusion 10. For urban agglomerations with alternative external resources, under information sharing, if the loss borne by the city after being illegal intrusion is $L > -1/p^1 + p^1(1 + \sigma \Gamma_1)$, the optimal volume $x^*_3$ of resource allocation is greater than $x^*_1$ under the independent strategy, but smaller than $x^*_2$ under the joint strategy. Moreover, its expected cost $C_{c_j}$ is lower than $C_j$ under the independent strategy.

Conclusion 10 shows that if the loss exceeds a certain threshold, information sharing in cities can not only solve the problem of excessive resource allocation, but also reduce the expected costs compared with the independent strategy.

The relationship between the optimal volume of resource allocation and the substitution rate of resources in cities is further expounded in this case. Owing to $\partial x^*_3/\partial \sigma = -\Gamma_1/E^2 \beta L_j (1 + \sigma \Gamma_1) (\ln v)^2 > 0$, the optimal volume of resource allocation in a city in urban agglomerations rises with the increase in the rate of substitution of resources across cities. Furthermore, because $\partial x^*_3/\partial \delta = -\sum_{k=1, k \neq j}^n x_k < 0$, the optimal volume of alternative resource allocation reduces with the increase in the rate of substitution under information sharing, thus leading to the following conclusion.

Conclusion 11. For the urban agglomerations with alternative external resources under information sharing, when $\forall \sigma \in [0, 1]$, the optimal volume $x^*_3$ of resource allocation in urban agglomerations increases monotonically. As for $\forall \delta \in [0, 1]$, the optimal volume $x^*_3$ of resource allocation in urban agglomerations decreases monotonically.
Conclusion 11 shows that the optimal volume of resource allocation in each city in urban agglomerations increases with the increase in the rate of substitution of resources in cities and decreases with the increase in the rate of information sharing between cities. In addition, after introducing collaborative mechanisms, when $\delta = 0$, then $x^*_3 = x^*_3$; when $\delta = 1$, then $x^*_1 = x^*_1$. In other words, when collaborative mechanisms are not introduced between cities, it will degenerate into the independent strategy. When collaborative mechanisms are introduced in all cities, they will evolve to form the joint strategy.

In conclusion, after introducing collaborative mechanisms for information sharing, the optimal volume of

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>8.3118</td>
<td>18.3118</td>
</tr>
<tr>
<td>10.3279</td>
<td>20.3279</td>
</tr>
<tr>
<td>11.2457</td>
<td>21.2457</td>
</tr>
<tr>
<td>12.1116</td>
<td>22.1116</td>
</tr>
<tr>
<td>12.9310</td>
<td>22.9310</td>
</tr>
<tr>
<td>13.7082</td>
<td>23.7082</td>
</tr>
</tbody>
</table>

**Table 3:** Influences of $\sigma$ and $\beta$ on the expected loss under the independent strategy.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>8.3118</td>
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<td>22.9310</td>
</tr>
<tr>
<td>13.7082</td>
<td>23.7082</td>
</tr>
</tbody>
</table>

**Table 4:** Partial results: the volume of resource allocation and expected loss when $n = 4$ under the independent strategy.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>Volume of resource allocation $e^*_1$</th>
<th>Expected loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>5.9609</td>
<td>20.3879</td>
</tr>
<tr>
<td>0.2</td>
<td>7.1277</td>
<td>21.5546</td>
</tr>
<tr>
<td>0.3</td>
<td>8.2202</td>
<td>22.6472</td>
</tr>
<tr>
<td>0.4</td>
<td>9.2458</td>
<td>23.6728</td>
</tr>
<tr>
<td>0.5</td>
<td>10.2110</td>
<td>24.6379</td>
</tr>
<tr>
<td>0.6</td>
<td>11.1215</td>
<td>25.5485</td>
</tr>
<tr>
<td>0.7</td>
<td>11.9827</td>
<td>26.4096</td>
</tr>
<tr>
<td>0.8</td>
<td>12.7990</td>
<td>27.2260</td>
</tr>
<tr>
<td>0.9</td>
<td>13.5746</td>
<td>28.0016</td>
</tr>
</tbody>
</table>
resource allocation to information security obtained in cities with alternative resources is greater than that under the independent strategy. Meanwhile, its expected cost is lower than that under the independent strategy. The optimal volume of resource allocation in each city in urban agglomerations can decrease with the increase in the extent of information sharing between cities. When cities share information sufficiently, the optimal volume of resource allocation to information security in cities can reach the optimal allocation level for the whole agglomeration.

Therefore, introducing collaborative mechanisms can stimulate reasonable resource allocation in cities.

5. Experimental Results and Analysis

Here, two problems are discussed through use of a simulation experiment:

(1) The optimal volumes of resource allocation and expected costs under the independent and joint strategies are compared based on the numerical simulation. The influence trends of city size, the probability \( \beta \) of intrusion by illegal users, and the rate of substitution \( \sigma \) between cities on the optimal volume of resource allocation and expected costs under numerical conditions are analyzed.

(2) The influences of the rate of sharing of information security \( \delta \) in cities on the optimal volume of resource allocation and expected costs are elucidated; that is, numerical analysis after introducing collaborative mechanisms is undertaken.

According to the actual conditions, the sizes of cities linked with complementary external resources are set to \( n = 3 \) and \( n = 4 \). Owing to it being impossible and unnecessary to consider all experimental parameters, herein, several representative values are only considered. It is assumed that \( L = 400, v = 0.5 \), and \( E = 0.1 \).

5.1. Resource Allocation under the Independent Strategy

When \( n = 3 \), the rate of substitution \( \sigma \) between cities and the probability \( \beta \) of cities suffering intrusion by illegal users are set between 0.1 and 0.9, to estimate the influences of \( \sigma \) and \( \beta \) on resource allocation. The volume of resource allocation and expected loss are listed in Tables 2 and 3. By further analyzing the data in Tables 2 and 3, when \( \sigma \) is set to 0.1 and \( \beta \) is \([0.1, 0.9]\) and when \( \beta \) is set to 0.1 and \( \sigma \) is \([0.1, 0.9]\), the results are obtained, as seen in Figures 1 and 2.

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According to the actual conditions, the sizes of cities linked with complementary external resources are set to \( n = 3 \) and \( n = 4 \). Owing to it being impossible and unnecessary to consider all experimental parameters, herein, several representative values are only considered. It is assumed that \( L = 400, v = 0.5 \), and \( E = 0.1 \).

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5.2. Resource Allocation under the Joint Strategy. When $n \geq 3$, the rate of substitution $\sigma$ of resources between cities and probability $\beta$ of intrusion by illegal users is set between 0.1 and 0.9, in increments of 0.1, when studying the influences of $\sigma$ and $\beta$ on resource allocation. The volume of resource allocation and expected loss are summarized in Tables 4 and 5.

By further analyzing the data in Tables 5 and 6, when $\sigma$ is set to 0.1 and $\beta$ is [0.1, 0.9] and when $\beta$ is set to 0.1 and $\sigma$ is [0.1, 0.9], the results in Figures 3 and 4 can be obtained.

It is seen that as $\beta$ increases, the volume $x^*_2$ of resource allocation continuously increases, which verifies the correctness of Conclusion 7. With the continuous increase of $\sigma$, the volume $x^*_2$ of resource allocation increases constantly, which confirms the correctness of Conclusion 8. When $n = 4$, the rate of substitution $\sigma$ of resources between cities is from 0.1 to 0.9, in increments of 0.1, and the probability of intrusion by illegal users is 0.1. The obtained volume of resource allocation and expected loss are listed in Table 7.

By comparing the results in Table 7 with those in Tables 5 and 6, with the increase of $n$, the volume $x^*_2$ of resource allocation decreases, while the expected loss increases, proving the correctness of Conclusion 6.

5.3. Numerical Analysis after Introducing Collaborative Mechanisms. When $\sigma = 0.1$ and $\beta = 0.1$, the rate of sharing $\delta$ of information between cities is set between 0.1 and 0.9, in increments of 0.1. On this basis, the influences of $\delta$ on resource allocation can be evaluated, and the volume of resource allocation and expected loss are listed in Table 8.

As shown in Table 8, with the constant increase of $\delta$, the volume $x^*_3$ of resource allocation continuously decreases. As $\delta$ increases, the expected loss increases, which verifies the correctness of Conclusion 11.

6. Conclusion

This study introduces the model of the substitution rate of external resources between smart cities to describe and model the problem of resource allocation in cities. An in-depth analysis and discussion of resource allocation to information security in cities under the independent and joint strategies are undertaken, and Nash equilibrium solutions in the two cases are deduced and compared. In addition, collaborative mechanisms are introduced to solve the problem of excessive resource allocation incurred by the independent strategy. Finally, through simulation, the trends in the influence of city size, probability of intrusion by illegal users, rate of substitution between cities, and rate of sharing of information in cities on the optimal volume of resource allocation and expected cost are analyzed, thus verifying conclusions based on a theoretical derivation.
Data Availability
All the data used in this paper can be obtained from the corresponding author (e-mail: xinglining@gmail.com).

Conflicts of Interest
The authors declare no conflicts of interest.

Authors’ Contributions
Li, Zou, and Xing jointly proposed the overall research plan. Li established the proposed method, implemented case study, and analyzed the data under the supervision of Zou and Xing. The manuscript was drafted by Li and revised and proofread by Zou and Xing. All authors have read and agreed to the published version of the manuscript.

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