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Research Article

A New Idea to Evaluate Networking Problem and MCGDM Problem in Parametric Interval Valued Pythagorean Arena

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In this article, the concept of parametric interval valued Pythagorean number (PIVPN) has been introduced, which is an extended version of Pythagorean number (PN). Here, a new score and accuracy function have been innovated in the PIVPN environment along with the De-Pythagorean value concept. The new tool and techniques have been fruitfully applied to two realistic problems, namely the networking critical path model (CPM) problem and the multicriteria group decision making problem (MCGDM) problem. In order to solve the MCGDM problem, we have prepared Parametric Interval valued Pythagorean Weighted Arithmetic Mean Operator (PIVPWAMO) and Parametric Interval valued Pythagorean Weighted Geometric Mean Operator (PIVPWGMO) operator in PIVPN environment. Finally, sensitivity analysis and industrious comprehensive numerical simulations have been performed to identify the reliability, efficiency, and usefulness of this novel work. In this article, we have shown that PIVPNs are a more well-organized representation to grip a real-life problem, and they can handle inconsistent conditions in a better compatible way in comparison to the other existing methods.

1. Introduction

Fuzzy set theory [1] plays a quintessential role in modern science, medical diagnoses, engineering, and technical problem, but there is a basic note of interrogation as to how we can relate or use the impreciseness concept in our computational mathematical modeling. Myriad researchers from the distinct field have defined many approaches to define it and have given numerous recommendations like triangular [2], trapezoidal [3], pentagonal [4] fuzzy number, and their perspectives to use the theory of uncertainty. Further, researchers developed the concept of Intuitionistic fuzzy number (IFN) [5], introducing the perception of both belongingness and nonbelongingness membership functions. After that, triangular [6] and trapezoidal [7] IFN's are formulated and applied in the distinct domain of the mathematical field. Further, the idea of interval-valued IFN [8] is manifested, which is the logical extension of IFN. After that, the concept of neutrosophic set (NS) [9] is established in the research domain which is the extension of IFN. In the case of NS theory, it actually deals with three components, namely, truth, false, and hesitation, whereas IFN contains two components, namely membership and nonmembership functions. Apart from this literature survey, researchers observed one important point. For instance, someone considered their liking towards any item is 0.7 whereas malaise is 0.4, and then the definition of intuitionistic fuzzy number has been violated. Thus, researchers developed the idea of Pythagorean number [10], which is the modified version of the intuitionistic fuzzy number considering the nonlinearity portions of the membership functions. Further, aggregation operators [11], generalized averaging aggregation operators [12], and confidence level-based aggregation operators [13] are developed by researchers day by day. Also, strategic decision-making approach [14], interval-valued Pythagorean number [15], some new results on Pythagorean set [16], decision-making approach on Pythagorean set [17], new novel score and accuracy function development [18], and linear programming method based on score value [19] have been developed by several researchers from different aspects. Apart from it, several researchers are worked in this field till now, and they proposed lots of concepts [20–25] related to multicriteria decision-making, score value, correlation coefficient, accuracy function of Pythagorean or intervalued Pythagorean number. There is plentiful literary evidence to classify some basic uncertain parameters. It should be adumbrated that there is no such unique representation of this uncertainty parameter. It can be optimized according to the choice of decision-makers for solving problems & can be varied as well as presented as different applications. Here, we present some information about the parameter of uncertainty showing how they vary from other concepts maneuvering the idea of uncertainty by utilizing some definitions, flowcharts, and diagrams. In this paper, we recommend that the researchers take the uncertainty parameter as PIVPN.

In this current era, the MCGDM problem is a predominant topic in the advance science and engineering field. The information of several parameters is precisely unspecified to create a model in real-life circumstances. Involving such cases, there is an occurrence of the notion of impreciseness. Uncertainty too forms the chief theme of the MCGDM problem, but the complication of such a method is how we can measure the uncertainty. Buyukozkan and Guleryuz [26] have adopted MCGDM approach to select better smart phones; Wang et al. [27] manifested intuitionistic linguistic operators on MCGDM problems; You et. al. [28] focused on an entropy based MCGDM method in interval-valued IFN; interval valued intuitionistic work in the MCGDM zone has been surveyed by Wang et al. [29]; Chakraborty A. [30–33] introduced MCGDM based problems in distinct domain, Chaio [34] derived MCGDM methodology based on type 2 fuzzy linguistic judgments; Wibowo [35] introduced MCGDM model in human resources management information projects; Chuu [36] established MCGDM model in manufacturing selection problem; Liu and Liu [37] proposed MCGDM model using Power Bonferroni Mean operator in IFN domain; Liu and Jiang [38] incorporated MCDM problem in interval valued intuitionistic set; Kumar and Garg [39] introduced TOPSIS in interval valued domain; Garg and Kumar [40] proposed advanced TOPSIS method in using SPA theory in IFS domain; Du et al. [41] focused on MADM problem in IVPN environment; Li et al. [42] described MADM model in IVPN environment. Apart from this literature survey, the concept of the Pythagorean set has also been fruitfully applied in several domains like operation research, graph theory, matrix evolution, etc. Luqman et al. [43] incorporated the Pythagorean concept in risk management problems using Digraph and matrix concept; Akram et al. [44] proposed LP model in the Pythagorean arena with equality constraints; Akram et al. [44, 45] manifested L-R type Pythagorean LP approaches with mixed and equality constraints; Enayattabar et al. [46] introduced the concept of shortest path problem in IVPN environment; Di Caprio et al. [47] focused on ant colony algorithm based on Pythagorean arena; Ebrahimnejad et al. [48] developed advanced bee colony algorithm in IVPN domain; Akram et al. [49] incorporated the concept of a planar graph in the Pythagorean environment which plays an essential role in graph theory problem.

From the available literature, it is observed that all the works which had been done earlier are mainly based on a general Pythagorean set or in an IVPN environment. Researchers have not explored the parametric representation of IVPN and its utility and usefulness in the domain of decision-making. In this article, we focused on the parametric representation of IVPN and constructed a few operators, de-Pythagorean skills in this environment that have been successfully and effectively applied to some real-life problems in the field of decision making. In this research article, we have developed two different operators, namely PIVPWAMO and PIVPWGMO, in an uncertain environment and introduced a new de-Pythagorean technique along with a new score and accuracy function for the crispification of PIVPN. The newly developed tools and techniques have been fruitfully applied to two realistic problems, namely the networking critical path model (CPM) problem and the multicriteria group decision-making (MCGDM) problem. Finally, sensitivity analysis and industrious comprehensive numerical simulations has been performed to identify the reliability, efficiency, and usefulness of this novel work.

1.1. Motivation. Although a good number of research works have already been toiled in the area of Pythagorean number that has been effectively and successfully applied to enrich MCDM/MADM/MCGDM techniques but only a few work is available in the area of IVPN. It is fascinating to note that many researchers have written Pythagorean numbers where they deploy the extension concept of FN and IFN. But why can we not structure the concept of the Pythagorean number using a simple parametric interval valued concept? Also, what will be the mathematical structure of PIVPN, and how can we relate it to real-life problems? Moreover, can we tackle the networking problem in PIVPN environment? If all the entities of decision matrices are considered as PIVPN, then how can we find out the best alternative! How can we perform a rigorous sensitivity and numerical analysis of an MCGDM model in a PIVPN environment? In this research article, an attempt is made to address all these issues and we proposed an efficient layout of MCGDM and networking when the decision-maker scored the feasible alternatives in the form of PIVPN. Different classifications of uncertain parameters and the conceptual workflow have been presents in Figure 1.

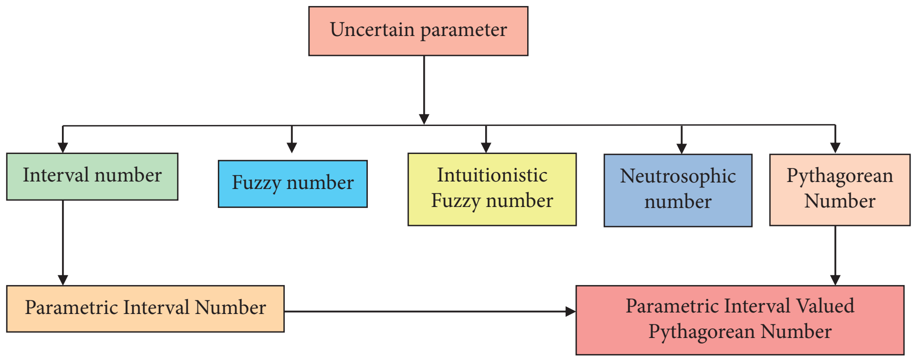

Figure 1: Classification of uncertain parameters and the workflow.

1.2. Novelties. The novelties of the work are described as follows:

(i) Formulation of PIVPN based on the concept of IVPN
(ii) Establishment of De-Pythagorean value using a noble technique to crispify a PIVPN into a real number.
(iii) Realistic application of PIVPN into MCGDM and Networking model.
(iv) Apply PIVPWAMO and PIVPWGMO in decision matrices to evaluate ranking.
(v) Well planned sensitivity analysis and comprehensive numerical simulations have been performed in order to understand the effect of weights on the PIVPN in the MCGDM problem.

2. Mathematical Preliminaries

Definition 1. Interval Number: An interval number X is denoted by $[X_L, X_R]$ and defined as $X = [X_L, X_R] = \{x: X_L \le x \le X_R,\ x \in R\}$, where R real number set and X_L and X_R generally denoted the left and right range of the interval, respectively.

Lemma 1. *The interval $[X_L, X_R]$ can also be represented as $P(\alpha) = (X_L)^{1-\alpha}(X_R)^{\alpha}$ for $\alpha \in [0,1]$.*

Definition 2. Fuzzy Set: A set $\widetilde{S}$, generally defined as $\widetilde{S} = \left\{(\alpha, \mu_{\widetilde{S}}(\alpha)):\ \alpha \in S,\ \mu_{\widetilde{S}}(\alpha) \in [0,1]\right\}$, denoted by the pair $(\alpha, \mu_{\widetilde{S}}(\alpha))$, where $\alpha \epsilon S$ and $\mu_{\widetilde{S}}(\alpha) \epsilon [0,1]$, then set $\widetilde{S}$ is called a fuzzy set.

Definition 3. Intuitionistic Fuzzy Set (IFS): [2] $\widetilde{S}$ in the universal discourse X which is denoted generically by x is said to be a IFS if $\widetilde{S} = \{x; [\tau(x), \varphi(x)] \vdots x \in X\}$, where $\tau(x): X \longrightarrow [0,1]$ is called the truth membership function, $\varphi(x): X \longrightarrow [0,1]$ is called the indeterminacy membership function. $\tau(x), \varphi(x)$ exhibits the following relation:

$$0 \le \tau(x) + \varphi(x) \le 1. \tag{1}$$

The degree of hesitation is between membership function and nonmembership function is defined as $\pi(x) = 1 - \tau(x) - \varphi(x)$.

Definition 4. Pythagorean Fuzzy Set (PFS): [4] $\widetilde{P}$ in the universal discourse X which is denoted generically by x is said to be a Pythagorean fuzzy set if $\widetilde{P} = \{x; [\tau(x), \varphi(x)] \vdots x \in X\}$, where $\tau(x): X \longrightarrow [0,1]$ is called the truth membership function which represents the degree of confidence, $\varphi(x): X \longrightarrow [0,1]$ is called the indeterminacy membership function which represents the degree of falsity. Geometrical representation of PFN is shown in Figure 2. Here, $\tau(x), \varphi(x)$ exhibits the following relation:

$$0 \le (\tau(x))^2 + (\varphi(x))^2 \le 1. \tag{2}$$

The degree of hesitation is between the membership function and the nonmembership function is defined as $\pi(x) = \sqrt{1 - (\tau(x))^2 - (\varphi(x))^2}$. Zhang and Xu [6] defined the score function as $S(\check{P}) = (\tau(x))^2 - (\varphi(x))^2$, where $S(\check{P}) \in [-1, 1]$ for two Pythagorean fuzzy numbers $\check{P}_1$ and $\check{P}_2$ if

(i) $S(\check{P}_1) > S(\check{P}_2)$, then $\check{P}_1 > \check{P}_2$
(ii) $S(\check{P}_1) < S(\check{P}_2)$, then $\check{P}_1 < \check{P}_2$
(iii) $S(\check{P}_1) = S(\check{P}_2)$, then $\check{P}_1 \sim \check{P}_2$

and accuracy function is defined as $H(\check{P}) = (\tau(x))^2 + (\varphi(x))^2$, where $H(\check{P}) \in [0,1]$

Definition 5. Interval Valued Pythagorean fuzzy Set (IVPFS): A set "A" in X is defined as follows:

$$A = \left\{ < X; \left[\tau^L(x), \tau^U(x)\right], \left[\varphi^L(x), \varphi^U(x)\right] > | x \in X \right\}, \tag{3}$$

where $0 \le \tau^L(x) \le \tau^U(x) \le 1$ and $0 \le \varphi^L(x) \le \varphi^U(x) \le 1$

Also, $(\tau(x))^2 + (\varphi(x))^2 \le 1$.

For each element $x \in X$ its hesitation interval relative to A is defined as follows:

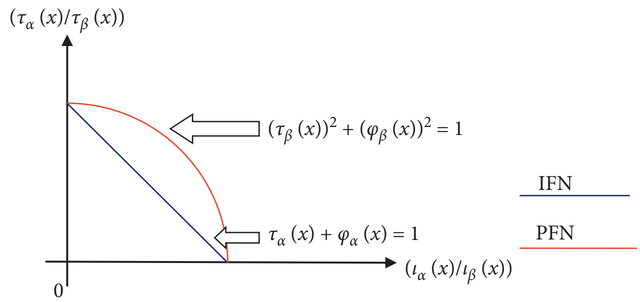

Figure 2: Graphical representation of IFN and PFN.

$$\pi_A(x) = \left[\pi_A^L(x), \pi_A^U(x)\right] = \left[\sqrt{1-\left(\tau^L(x)\right)^2-\left(\varphi^L(x)\right)^2}, \sqrt{1-\left(\tau^U(x)\right)^2-\left(\varphi^U(x)\right)^2}\right]. \tag{4}$$

Definition 6. Triangular Interval Valued Pythagorean fuzzy Set (TIVPFS): A TIVPFS is denoted as $\widetilde{A}_{invi} = [\{(a_1, b, c_1; \lambda), (a, b, c; \omega)\}, \{(d_1, b, e_1; \delta), (d, b, e; \mu)\}]$ where $0 < \omega \leq \lambda \leq 1$, $0 < \delta \leq \mu \leq 1$ and $a_1 < a < b < c < c_1$ also $0 < \omega^2 + \mu^2 \leq 1$ and $0 < \lambda^2 + \delta^2 \leq 1$. Here λ, ω are the maximum degree of the membership function of the triangular fuzzy number (a_1, b, c_1) and (a, b, c), respectively, and δ, μ are the minimum degree of the nonmembership function of the triangular fuzzy number (d_1, b, e_1) and (d, b, e), respectively.

The upper and lower membership function is defined by the following:

$$\mu_{\tilde{A}_{invi}^U}(x) = \begin{cases} \lambda\left(\dfrac{x-a_1}{b-a_1}\right), & a_1 \leq x \leq b, \\ \lambda\left(\dfrac{c_1-x}{c_1-b}\right), & b \leq x \leq c_1, \\ 0, & \text{otherwise}, \end{cases}$$
$$\mu_{\tilde{A}_{invi}^L}(x) = \begin{cases} \omega\left(\dfrac{x-a}{b-a}\right), & a \leq x \leq b, \\ \omega\left(\dfrac{c-x}{c-b}\right), & b \leq x \leq c, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

and non membership functions are defined as follows:

$$\vartheta_{\tilde{A}_{invi}^U}(x) = \begin{cases} \delta\left(\dfrac{b-x}{b-d_1}\right), & d_1 \leq x \leq b, \\ \delta\left(\dfrac{x-b}{e_1-b}\right), & b \leq x \leq e_1, \\ 1, & \text{otherwise}, \end{cases}$$
$$\vartheta_{\tilde{A}_{invi}^L}(x) = \begin{cases} \mu\left(\dfrac{b-x}{b-d}\right), & d \leq x \leq b, \\ \mu\left(\dfrac{x-b}{e-b}\right), & b \leq x \leq e, \\ 1, & \text{otherwise}. \end{cases} \tag{6}$$

3. Parametric Interval Valued Pythagorean Number and Its Properties

Definition 7. Parametric Interval Valued Pythagorean Number: PIVPN is defined as follows:

$$S_\alpha = \left\{\left\langle\left[\left(\tau^L(x)\right)^{\alpha_1}, \left(\tau^U(x)\right)^{1-\alpha_1}\right], \left[\left(\varphi^L(x)\right)^{\alpha_2}, \left(\varphi^U(x)\right)^{1-\alpha_2}\right]\right\rangle\right\}, \tag{7}$$

where $0 \le \tau^{L}(x) \le \tau^{U}(x) \le 1$; $0 \le \varphi^{L}(x) \le \varphi^{U}(x) \le 1$; $\alpha_1, \alpha_2 \in [0,1]$ and $0 \le \alpha_1 + \alpha_2 \le 1$.

Also, $(\tau(x))^2 + (\varphi(x))^2 \le 1$.

3.1. De-Pythagorean of Parametric Interval Valued Pythagorean Number. A PIVPN $A = \{\langle [(\tau^{L}(x))^{\alpha_1}, (\tau^{U}(x))^{1-\alpha_1}], [(\varphi^{L}(x))^{\alpha_2}, (\varphi^{U}(x))^{1-\alpha_2}]\rangle\}$. can be converted into a crisp number C as follows:

$$C = \frac{\rho\left[\left(\tau^{L}(x)\right)^{\alpha_1} + \left(\tau^{U}(x)\right)^{1-\alpha_1}\right]/2 + (1-\rho)\left[\left(\varphi^{L}(x)\right)^{\alpha_2} + \left(\varphi^{U}(x)\right)^{1-\alpha_2}\right]/2}{2}, \tag{8}$$

where ρ is an index parameter of the decision-maker and $\rho \in [0,1]$. That is, the crisp value of an IVPN fully depends on the parameter ρ.

Example 1. There is an IVPN written in the parametric form as $[(1)^{(\alpha_1)}(2.5)^{(1-\alpha_1)}], [(3.5)^{(\alpha_2)}(4.5)^{(1-\alpha_2)}]$. Then the value of the crisp number when index $\lambda = 0.5$ is 1.48.

Let A_1, A_2 be two IVPNs whose De-Pythagorean values are C_1 and C_2; then the relation between them is defined as follows:

$$\begin{aligned} &1. C_1 > C_2 \text{ then } A_1 > A_2,\\ &2. C_1 < C_2 \text{ then } A_1 < A_2,\\ &3. C_1 = C_2 \text{ then } A_1 = A_2. \end{aligned} \tag{9}$$

We have computed some ranking results using the De-Pythagorean value in Table 1 for different IVPNs.

3.2. Properties and Operators in Parametric Interval Valued Pythagorean Number. Let us consider two IVPNs $S_1 = <[(\tau^{L1}(x))^{\alpha_1}, (\tau^{U1}(x))^{1-\alpha_1}], [(\varphi^{L1}(x))^{\alpha_2}, (\varphi^{U1}(x))^{1-\alpha_2}]>$ and $S_2 = <[(\tau^{L2}(x))^{\alpha_1}, (\tau^{U2}(x))^{1-\alpha_1}], [(\varphi^{L2}(x))^{\alpha_2}, (\varphi^{U2}(x))^{1-\alpha_2}]>$ where $\alpha_1, \alpha_2 \in [0,1]$ and $0 \le \alpha_1 + \alpha_2 \le 1$ then, we have the following:

3.2.1. Addition

$$S = S_1 + S_2 = <\left[\left(\tau^{L1}(x) + \tau^{L2}(x)\right)^{\alpha_1}, \left(\tau^{U1}(x) + \tau^{U2}(x)\right)^{1-\alpha_1}\right], \left[\left(\varphi^{L1}(x) + \varphi^{L2}(x)\right)^{\alpha_1}, \left(\varphi^{U1}(x) + \varphi^{U2}(x)\right)^{1-\alpha_1}\right]>. \tag{10}$$

3.2.2. Subtraction

$$S = S_1 - S_2 - <\left[\left(\tau^{L1}(x) - \tau^{L2}(x)\right)^{\alpha_1}, \left(\tau^{U1}(x) - \tau^{U2}(x)\right)^{1-\alpha_1}\right], \left[\left(\varphi^{L1}(x) - \varphi^{L2}(x)\right)^{\alpha_1}, \left(\varphi^{U1}(x) - \varphi^{U2}(x)\right)^{1-\alpha_1}\right]>. \tag{11}$$

3.2.3. Multiplication

$$\begin{aligned} S = S_1 \times S_2 = &<\left[\left(\mathrm{Min}\left\{\tau^{L1}(x)\tau^{L2}(x), \tau^{U1}(x)\tau^{U2}(x), \tau^{L1}(x)\tau^{U2}(x), \tau^{L2}(x)\tau^{U1}(x)\right\}\right)^{\alpha_1},\right.\\ &\left.\left\{\left(\mathrm{Max}\tau^{L1}(x)\tau^{L2}(x), \tau^{U1}(x)\tau^{U2}(x), \tau^{L1}(x)\tau^{U2}(x), \tau^{L2}(x)\tau^{U1}(x)\right)\right\}^{1-\alpha_1}\right],\\ &\left[\left(\mathrm{Min}\left\{\varphi^{L1}(x)\varphi^{L2}(x), \varphi^{U1}(x)\varphi^{U2}(x), \varphi^{L1}(x)\varphi^{U2}(x), \varphi^{L2}(x)\varphi^{U1}(x)\right\}\right)^{\alpha_2},\right.\\ &\left.\left(\mathrm{Max}\left\{\varphi^{L1}(x)\varphi^{L2}(x), \varphi^{U1}(x)\varphi^{U2}(x), \varphi^{L1}(x)\varphi^{U2}(x), \varphi^{L2}(x)\varphi^{U1}(x)\right\}\right)^{1-\alpha_2}\right]>. \end{aligned} \tag{12}$$

3.2.4. Multiplication by a Constant

$$\begin{aligned} \mathrm{S} = \mathrm{k} \times \mathrm{S}_1 &= <\left[\left(k\tau^{L1}(x)\right)^{\alpha_1}, \left(k\tau^{U1}(x)\right)^{1-\alpha_1}\right], \left[\left(k\varphi^{L1}(x)\right)^{\alpha_2}, \left(k\varphi^{U1}(x)\right)^{1-\alpha_2}\right]> \text{ when } k > 0\\ &= <\left[\left(k\tau^{U1}(x)\right)^{\alpha_1}, \left(k\tau^{L1}(x)\right)^{1-\alpha_1}\right], \left[\left(k\varphi^{U1}(x)\right)^{\alpha_2}, \left(k\varphi^{L1}(x)\right)^{1-\alpha_2}\right]> \text{ when } k > 0. \end{aligned} \tag{13}$$

Table 1: Numerical example.

Interval valued Pythagorean number	Value of ρ	De-Pythagorean value	Ranking
A = < [1.5, 2.5], [2, 3] >	0.4	1.15	$B > A$
B = <[3.5, 5.5], [4, 6]>		2.4	
C = <[3.5, 6.5], [5, 7]>	0.3	2.85	$D > C$
D = <[7.5, 9.5], [8, 11]>		9.2	

3.2.5. Division

$$\begin{aligned} S = \frac{S_1}{S_2} &= \left\langle \left[\left(\mathrm{Min}\left\{ \frac{\tau^{L1}(x)}{\tau^{L2}(x)}, \frac{\tau^{U1}(x)}{\tau^{U2}(x)}, \frac{\tau^{L1}(x)}{\tau^{U2}(x)}, \frac{\tau^{L2}(x)}{\tau^{U1}(x)} \right\} \right)^{\alpha_1}, \left(\mathrm{Max}\left\{ \frac{\tau^{L1}(x)}{\tau^{L2}(x)}, \frac{\tau^{U1}(x)}{\tau^{U2}(x)}, \frac{\tau^{L1}(x)}{\tau^{U2}(x)}, \frac{\tau^{L2}(x)}{\tau^{U1}(x)} \right)^{1-\alpha_1} \right] \right. \\ &= \left. \left[\left(\mathrm{Min}\left\{ \frac{\varphi^{L1}(x)}{\varphi^{L2}(x)}, \frac{\varphi^{U1}(x)}{\varphi^{U2}(x)}, \frac{\varphi^{L1}(x)}{\varphi^{U2}(x)}, \frac{\varphi^{L2}(x)}{\varphi^{U1}(x)} \right\} \right)^{\alpha_2}, \left(\mathrm{Max}\left\{ \frac{\varphi^{L1}(x)}{\varphi^{L2}(x)}, \frac{\varphi^{U1}(x)}{\varphi^{U2}(x)}, \frac{\varphi^{L1}(x)}{\varphi^{U2}(x)}, \frac{\varphi^{L2}(x)}{\varphi^{U1}(x)} \right)^{1-\alpha_2} \right] \right\rangle. \end{aligned} \tag{14}$$

3.2.6. Inverse

$$\begin{aligned} [h(p)]^{(-1)} &= \frac{1}{\left\langle \left[\left(\tau^{L1}(x)\right)^{\alpha_1}, \left(\tau^{U1}(x)\right)^{1-\alpha_1} \right], \left[\left(\varphi^{L1}(x)\right)^{\alpha_2}, \left(\varphi^{U1}(x)\right)^{1-\alpha_2} \right] \right\rangle} \\ &= \left\langle \left[\left(\tau^{L1}(x)\right)^{-\alpha_1}, \left(\tau^{U1}(x)\right)^{\alpha_1-1} \right], \left[\left(\varphi^{L1}(x)\right)^{-\alpha_2}, \left(\varphi^{U1}(x)\right)^{\alpha_2-1} \right] \right\rangle. \end{aligned} \tag{15}$$

3.2.7. Example. If $I = [2.5^{\alpha_1} 6.3^{(1-\alpha_1)}], [1^{\alpha_2} 2^{(1-\alpha_2)}]$ and $J = [10^{\alpha_1} 20^{(1-\alpha_1)}], [5^{\alpha_2} 7.5^{(1-\alpha_2)}]$ for $\alpha_1, \alpha_2, \alpha_3 \in [0, 1]$ Find $I + J, I - J, IJ, kI, (I/J)$ for $k = 0.5, -0.2$.
Solution:

$$\begin{aligned} I + J &= \left\langle \left[12.5^{\alpha_1} 26.3^{(1-\alpha_1)} \right], \left[6^{\alpha_2} 9.5^{(1-\alpha_2)} \right] \right\rangle, \\ I - J &= \left\langle \left[(-17.5)^{\alpha_1} (-3.7)^{(1-\alpha_1)} \right], \left[(-6.5)^{\alpha_2} (-3)^{(1-\alpha_2)} \right] \right\rangle, \\ IJ &= \left\langle \left[25^{\alpha_1} 126^{(1-\alpha_1)} \right], \left[5^{\alpha_2} 15^{(1-\alpha_2)} \right] \right\rangle, \\ kI &= \begin{cases} \left\langle \left[1.25^{\alpha_1} 3.15^{(1-\alpha_1)} \right], \left[0.5^{\alpha_2} 1^{(1-\alpha_2)} \right] \right\rangle k = 0.5, \\ \left\langle \left[-1.26^{\alpha_1} - 0.5^{(1-\alpha_1)} \right], \left[-0.4^{\alpha_2} - 0.2^{(1-\alpha_2)} \right] \right\rangle k = -0.2, \end{cases} \\ \frac{I}{J} &= \left\langle \left[0.125^{\alpha_1} 0.63^{(1-\alpha_1)} \right], \left[0.133^{\alpha_2} 0.4^{(1-\alpha_2)} \right] \right\rangle. \end{aligned} \tag{16}$$

3.3. Operators. We consider some operators used for interval valued parametric Pythagorean fuzzy numbers.

3.3.1. Parametric Interval Valued Pythagorean Weighted Arithmetic Mean Operator (PIVPWAMO). Assuming $S_i = < [(\tau^{L1}(x))^{\alpha_1}, (\tau^{U1}(x))^{1-\alpha_1}], [(\varphi^{L1}(x))^{\alpha_2}, (\varphi^{U1}(x))^{1-\alpha_2}] >$, where $i = 1, \ldots n$ is family of PIVPN, λ is a set of PIVPN. A mapping PIVPWAMO such that $\lambda^n \longrightarrow \lambda$, then PIVPWAMO $(S_1, \ldots S_N) = < [\sum_{i=1}^n (w_i \tau_i^{L1})^{\alpha_1}, (w_i \tau_i^{U1})^{1-\alpha_1}], [\sum_{i=1}^n (w_i \varphi_i^{L1})^{\alpha_2}, (w_i \varphi_i^{U1})^{1-\alpha_2}]$. where w_i is the weight vector assigned with the PIVPN and $\sum_{i=1}^n w_i = 1$, also each $w_i \in [0, 1]$.

3.3.2. Parametric Interval Valued Pythagorean Weighted Geometric Mean Operator (PIVPWGMO). Assuming $S_i = < [(\tau^{L1}(x))^{\alpha_1}, (\tau^{U1}(x))^{1-\alpha_1}], > [(\varphi^{L1}(x))^{\alpha_2}, (\varphi^{U1}(x))^{1-\alpha_2}], [(\tau^{L1}(x))^{\alpha_1}, (\tau^{U1}(x))^{1-\alpha_1}], [(\varphi^{L1}(x))^{\alpha_2}, (\varphi^{U1}(x))^{1-\alpha_2}] >$. where $i = 1, 2, 3, \ldots\ldots n$ is family of PIVPN and λ is a set of PIVPN. A mapping PIVPWGMO such that $\lambda^n \longrightarrow \lambda$, then

PIVPWGMO $(S_1, S_2, S_3, \ldots\ldots\ldots S_N) = < [\prod_{i=1}^n \{(\tau_i^{L1})^{\alpha_1}\}^{w_i}, \left\{(\tau_i^{U1})^{1-\alpha_1}\right\}^{w_i}], [\prod_{i=1}^n \{(\varphi_i^{L1})^{\alpha_1}\}^{w_i}, \left\{(\varphi_i^{U1})^{1-\alpha_1}\right\}^{w_i}] >$, where w_i is the weight vector assigned with the PIVPN and $\sum_{i=1}^n w_i = 1$, also each $w_i \in [0, 1]$.

4. Score and Accuracy Function IVPFN

A score function of IVPFS is mainly depends on hesitation indicator and score indicator. Let $\sigma = < [a, b], [c, d] >$ be an IVPFN, and a new score function $\check{S}(\sigma)$ depends on the following:

(i) $\sqrt{1 - b^2 - d^2}, \sqrt{1 - a^2 - c^2}$ are called hesitation indicators.

(ii) $a^2\sqrt{1 - a^2 - c^2}$, $b^2\sqrt{1 - b^2 - d^2}$ are called benefit degrees.

(iii) $c^2\sqrt{1 - a^2 - c^2}$, $d^2\sqrt{1 - b^2 - d^2}$ are called nonbeneficiary degrees.

So

$$\check{S}(\sigma) = \frac{\left(a^2 - c^2\right)\sqrt{1 - a^2 - c^2} + \left(b^2 - d^2\right)\sqrt{1 - b^2 - d^2}}{2}, \quad (17)$$

is the score function where $\check{S}(\sigma) \epsilon [-1, 1]$

If $\sigma = < [0, 0], [1, 1] >$ then $\check{S}(\sigma) = -1$ and if $\sigma = < [1, 1], [0, 0] >$ then $\check{S}(\sigma) = 1$

The new accuracy function is defined as follows:

$$\check{H}(\sigma) = \frac{\left(a^2 + c^2\right)\sqrt{1 - a^2 - c^2} + \left(b^2 + d^2\right)\sqrt{1 - b^2 - d^2}}{2}, \quad (18)$$

where $\check{H}(\sigma) \epsilon [0, 1]$.

Let γ, δ are two IVPFN then If

(1) $\check{s}(\gamma) > \check{s}(\delta)$ then $\gamma > \delta$,

(2) $\check{s}(\gamma) < \check{s}(\delta)$ then $\gamma < \delta$.

(3) $\check{s}(\gamma) = \check{s}(\delta)$ then.

(i) $\check{H}(\gamma) > \check{H}(\delta)$ then $\gamma > \delta$,

(ii) $\check{H}(\gamma) < \check{H}(\delta)$ then $\gamma < \delta$

(iii) $\check{H}(\gamma) = \check{H}(\delta)$ then $\gamma \sim \delta$

5. Critical Path Problem in Pythagorean Fuzzy Environment

The full form of CPM method is the critical path method which is a project management technique for progress planning that defines critical and noncritical tasks with the goal of preventing time frame problems. The critical path is defined as the (time) longest possible path from the start point to endpoint of the chart. Each project has at least one critical path. Each critical path consists of a list of activities on which the project manager should focus the most if he wants to ensure timely completion of the project. The date of the completion of the last task on the critical path is also the date of the completion of the project. For critical tasks to be applied, the total time reserve and thus free time reserve is equal to zero, i.e., that the start delay of this task or to extend its duration will affect the final date of the project. The critical path is reflected in the schedule and project management in all phases of the project lifecycle.

Activity: It is any portion of a project that has a definite beginning and ending and may use some resources such as time, labor, material, equipment, etc.

Event or Node: Beginning and ending points of activities denoted by circles are called nodes or events.

Critical Path: The sequence of activities in a network is called the critical path. It is the longest path in the network, from the starting event to the ending event, and defines the minimum time required to complete the project.

Forward pass method: Based on fixed occurrence time of the initial network event, the forward pass computation yields the earliest start and earliest finish times for each activity and, indirectly, the earliest expected occurrence time for each event.

(1) The computations begin from the start node and move to the end node. To accomplish this, the forward pass computations start with an assumed earliest occurrence time of zero for the initial project event.

(2) For any activity (i, j), let ES_i denote the earliest time of event i, then $S_j = ES_i + t_{ij}$.

(3) If more than one activity enters an event, the earliest start time for that event is computed as $ES_j = \max\left\{ES_i + t_{ij}\right\}$ for all activities emanating from node i entering into j.

Backward pass method: The latest occurrence event time specifies the time by which all activities entering into that event must be completed without delaying the total project. These are computed by reversing the method of calculation used for the earliest event times.

(1) To compute backward pass, starting from the final node, the computation proceeds from right to left, up to the initial event.
(2) For any activity (i, j), let LF_i denote the latest finished time of event i, then $F_i = LF_j - t_{ij}$.
(3) If more than one activity enters an event the latest finish time for that event is computed as $LF_i = \min \{LF_j - t_{ij}\}$ for all activities emanating from node j entering into i.

In our developed technique, we consider all the activities are IVPN to calculate the ES_i and LF_j; we utilize the concept of a new score function $\check{S}(\sigma) = ((a^2 - c^2)\sqrt{1 - a^2 - c^2} + (b^2 - d^2)\sqrt{1 - b^2 - d^2}/2)$, which can compare the finite number of distinct IVPFN to estimate the minimum and maximum value. If the score values are equal, then estimate the IVPFN using the new accuracy function defined as $\check{H}(\sigma) = ((a^2 + c^2)\sqrt{1 - a^2 - c^2} + (b^2 + d^2)\sqrt{1 - b^2 - d^2}/2)$.

5.1. Flowchart. Flowchart of critical path problem in Pythagorean fuzzy environment is presented in Figure 3.

5.2. Numerical Problem. Table 2 and Figure 4 present a numerical problem implemented in this study.

(1) Sketch the project network
(2) Evaluate the ES_i and LF_i
(3) Compute the critical path and total project duration.

Step-1

Step-2 After computation ES_i and LF_i we have a network as follows (Figure 5):

In step-1, we draw the network diagram of the given problem in Figure 3, and further after computation of forward pass and backward pass, we finally get Figure 4, which is the final diagram with node allocation. Hence, the expected project duration −0.879 and Critical Path- 1 $\longrightarrow$ 3 $\longrightarrow$ 7 $\longrightarrow$ 11

5.3. Comparative Analysis. In Table 3 we compare our work with other previous score function and accuracy function described by the authors [15, 18, 19] for finding the alternatives.

5.3.1. Discussion. We have compared the outcome of our proposed method with the outcome of other similar research work that has been presented in Table 2. Here, we observed that some score and accuracy functions fail to calculate the critical path as negativity arises in calculating ES_i and LF_i which is impossible for a networking problem. It is to be further noted that our proposed approach can find out a critical path that gives us a minimum project time duration in comparison to the approach of Garg [19]. Hence, we can conclude that it is a better and modified approach which can solve critical path problem in networking domain.

6. Proposed MCGDM Problem in Pythagorean Environment

In this MCGDM arena, we shall always focus on finding out the best feasible alternatives by disjunctive kind of factors, which is basically called the criteria. Though, it is not a very easy job to find out the attribute value in the format of crisp numbers due to the presence of impreciseness. All the information of the decision matrix is of the interval valued Pythagorean parametric form in nature. In this article, we actually consider an MCGDM, where there will be finite number of distinctive decision-makers available and according to their viewpoint we need to find out the best alternative. To do so, we construct an algorithm based on IVPWAMO and the normalization approach using the conception of IVPWGMO such that we can tackle the impreciseness problem very easily and using the conception of De-Pythagorean value, we shall choose the best alternative among all of them.

6.1. Illustration of the MCGDM Problem. We consider the problem as follows:

Let $\pi = \{\pi_1, \pi_2, \pi_3 \ldots \pi_m\}$. be "m" number of distinctive alternatives and $\sigma = \{\sigma_1, \sigma_2, \sigma_3 \ldots\ldots\ldots\ldots \sigma_n\}$. be "n" number of distinctive attribute values, respectively. Let $\delta = \{\delta e_1, \delta e_2, \delta e_3 \ldots\ldots\ldots\ldots \delta e_n\}$. be the weight set associated with the attributes σ where each $\delta e \geq 0$ and also satisfies the relation $\sum_{i=1}^{n} \delta e_i = 1$. We also consider the set of decision-maker $\mu = \{\mu_1, \mu_2, \mu_3 \ldots\ldots\ldots\ldots \mu_K\}$ associated with alternatives whose weight vector is defined as $\varphi = \{\varphi_1, \varphi_2, \varphi_3 \ldots\ldots\ldots\ldots \varphi_k\}$ where each $\varphi_i \geq 0$ and also satisfies the relation $\sum_{i=1}^{k} \varphi_i = 1$. This weight vector will be selected according to the decision-makers quality of judgment, knowledge, thinking power, etc. The flowchart of the proposed methodology is presented in Figure 6.

Step-1 Creation of Decision Matrices

First, we formulate the decision matrices for each decision maker's choice related to alternatives versus attribute functions. We consider the elements of the matrices in IVPN in parametric form, so all a_{ij}*'s* are members of the Pythagorean fuzzy number set. The related matrix is described as follows:

$$C^K = \begin{pmatrix} . & \sigma_1 & \sigma_2 & \sigma_3 & . & . & . & \sigma_n \\ \pi_1 & a_{11}^k & a_{12}^k & a_{13}^k & . & . & . & a_{1n}^k \\ \pi_2 & a_{21}^k & a_{22}^k & a_{23}^k & . & . & . & a_{2n}^k \\ \pi_3 & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \pi_m & a_{m1}^k & a_{m2}^k & a_{m3}^k & . & . & . & a_{mn}^k \end{pmatrix}. \quad (19)$$

Step-2 Creation of Single decision matrix

To create the single group decision matrix, we utilize the operator PIVPWAMO for each decision matrix C^i . Thus, we get the modified matrix as follows:

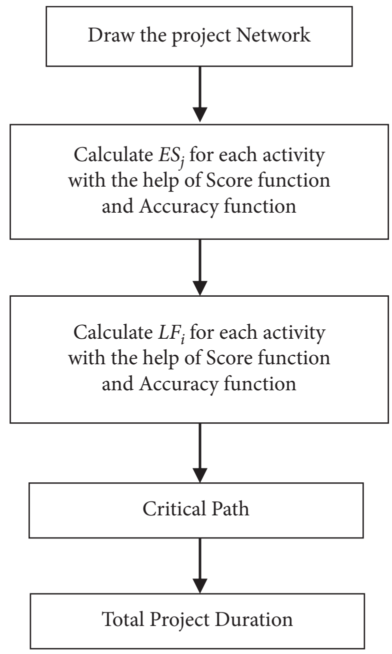

Figure 3: Critical path problem in pythagorean fuzzy environment.

Table 2: Numerical example.

Nodes	Description	Predecessors	Activity
1	Choice of vehicles	—	a = [(0.4, 0.7); (0.3, 0.5)]
2	Choice of destination	—	b = [(0.3, 0.8); (0.2, 0.6)]
3	Choice of driver	1	e = [(0.3, 0.7); (0.2, 0.6)]
4	Final plan and blueprint	2	n = [(0.6, 0.8); (0.3, 0.5)]
5	Route map creation	2	O = [(0.7, 0.9); (0.2, 0.3)]
6	Fuel consumption for different places	1	d = [(0.6, 0.8); (0.5, 0.6)]
7	Instruction and training	3	h = [(0.4, 0.7); (0.3, 0.6)]
8	Initial startup time	4	m = [(0.4, 0.7); (0.2, 0.6)]
9	Vehicle testing timing	1	c = [(0.4, 0.6); (0.2, 0.4)]
10	Run the system	5, 7, 8	p = [(0.8, 0.9); (0.1, 0.2)]k = [(0.4, 0.7); (0.3, 0.5)]l = [(0.5, 0.7); (0.3, 0.6)]
11	Final destination	6, 9, 10	g = [(0.3, 0.6); (0.2, 0.5)] f = [(0.1, 0.8); (0.05, 0.3)] j = [(0.4, 0.8); (0.3, 0.6)]

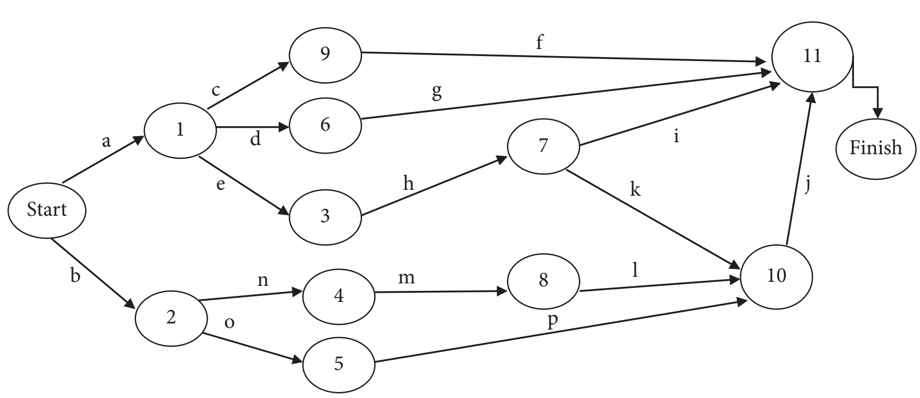

Figure 4: Network diagram.

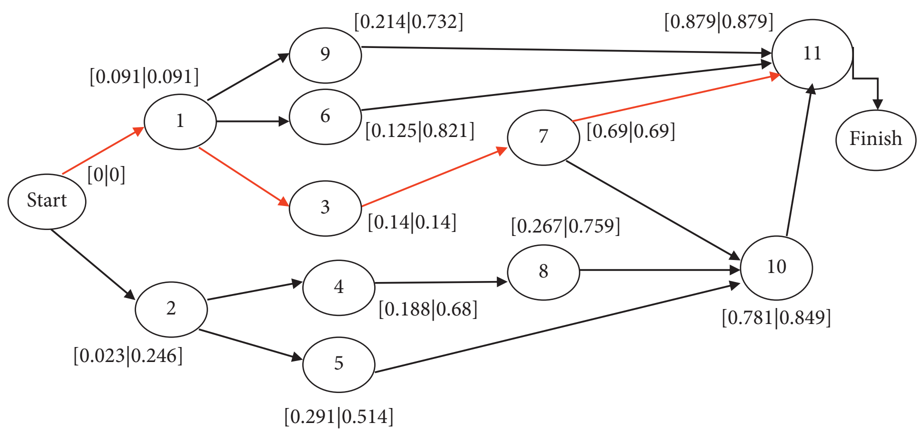

FIGURE 5: Final computational network diagram. ⟶ line denote the critical path.

TABLE 3: Comparison of work.

Score function	Critical path	Project duration
Garg [18]	Can't be determined since negativity arise at the earliest and latest time counting	—
Garg [19]	2 ⟶5 ⟶ 10 ⟶ 11	2.281
Zhang [15]	Can't determined since negativity arise in earliest and latest time counting	—
Our proposed	1 ⟶ 3 ⟶ 7 ⟶ 11	0.879

$$C = \begin{array}{c} \\ \pi_1 \\ \pi_2 \\ \pi_3 \\ . \\ \pi_m \end{array} \begin{pmatrix} \sigma_1 & \sigma_2 & \sigma_3 & . & . & . & \sigma_n \\ a'_{11} & a'_{12} & a'_{13} & . & . & . & a'_{1n} \\ a'_{21} & a'_{22} & a'_{23} & . & . & . & a'_{2n} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ a'_{m1} & a'_{m2} & a'_{m3} & . & . & . & a'_{mn} \end{pmatrix}. \quad (20)$$

Step-3 Creation of Final matrix according to weight Priority

To obtain a single Column decision matrix, we all use the operator PIVPWGMO for each Column, and hence we get the decision matrix as follows:

$$C = \begin{array}{c} \\ \pi_1 \\ \pi_2 \\ . \\ . \\ \pi_m \end{array} \begin{pmatrix} \sigma_1 \\ a''_{11} \\ a''_{21} \\ . \\ . \\ a''_{m1} \end{pmatrix}. \quad (21)$$

Step-4 Ranking

Now, we consider the De-Pythagorean value and convert the matrix (3) into a crisp one such that we can evaluate the best alternative corresponding to the best attributes.

6.2. Illustrative Example. Let us consider a problem associated with four distinctive products and their three different attributes. Generally, lots of commodities are available in the market, having different types of components with different quality and facilities. Now in this problem, there are four different kinds of decision-makers available. We choose the problem as follows:

C_1 = Comodity 1, C_2 = Comodity 2, C_3 = Comodity 3, C_4 = Comodity 4 are the different kinds of alternatives. S_1 = Quality, S_2 = longibity, S_3 = Price, S_4 = Service are the four different attributes. There are three distinct types of decision-makers classified according to their ages D_1 = Young aged people, D_2 = Middle-aged people, D_3 = Old aged people having corresponding weight set $D = \{0.30, 0.33, 0.37\}$ and we also consider the weight function related to the attribute function $\sigma = \{0.35, 0.33, 0.32\}$. We consider a verbal matrix for the designer to assist the decision-maker in the creation of his decision matrix. The verbal phrases are listed in Table 4 and the verbal matrix of this MCGDM problem is shown in Table 5.

Step-1 All the members of the matrices are PIVPN in nature. So, the decision matrices are as follows:

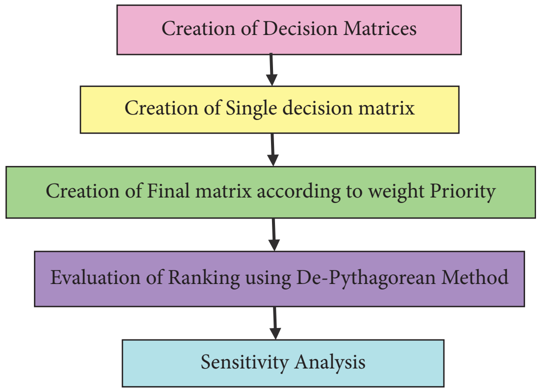

Figure 6: Flowchart of the MCGDM methodology.

Table 4: List of verbal phrase.

Sl no.	Attribute	Verbal phrase
		Quantitative attributes
1	Quantity of the commodity	Very high (VH), high (L), intermediate (I), small (S), very small (VS)
2	Legibility of the commodity	Very high (VH), high (H), mid (M), low (L), very low (VL)
3	Price rate of the commodity	Very high (VH), high (H), mid (M), low (l), very low (VL)
4	Service life of the commodity	Very high (VH), high (H), mid (M), low (l), very low (VL)

Table 5: The verbal matrix.

	S^1	S^2	S^3	S^4
C^1	L	I	L	VH
C^2	VL	M	H	VH
C^3	L	I	I	H
C^4	VL	L	M	H

$$M^1 = \left(\begin{array}{cccc} . & S_1 & S_2 & S_3 \\ C_1 & \left\langle \left[5.5^{\alpha_1}6.5^{(1-\alpha_1)}\right], \left[1.3^{\alpha_2}2.8^{(1-\alpha_2)}\right]\right\rangle & \left\langle \left[5.3^{\alpha_1}6.5^{(1-\alpha_1)}\right], \left[6.7^{\alpha_2}8.6^{(1-\alpha_2)}\right]\right\rangle & \left\langle \left[5.8^{\alpha_1}6.3^{(1-\alpha_1)}\right], \left[0.9^{\alpha_2}2.5^{(1-\alpha_2)}\right]\right\rangle \\ C_2 & \left\langle \left[6.3^{\alpha_1}7.2^{(1-\alpha_1)}\right], \left[1.5^{\alpha_2}2.3^{(1-\alpha_2)}\right]\right\rangle & \left\langle \left[5.0^{\alpha_1}6.0^{(1-\alpha_1)}\right], \left[1.9^{\alpha_2}2.5^{(1-\alpha_2)}\right]\right\rangle & \left\langle \left[4.2^{\alpha_1}6.0^{(1-\alpha_1)}\right], \left[1.5^{\alpha_2}2.2^{(1-\alpha_2)}\right]\right\rangle \\ C_3 & \left\langle \left[1.2^{\alpha_1}2.4^{(1-\alpha_1)}\right], \left[5.9^{\alpha_2}6.4^{(1-\alpha_2)}\right]\right\rangle & \left\langle \left[1.9^{\alpha_1}2.5^{(1-\alpha_1)}\right], \left[9.0^{\alpha_2}9.9^{(1-\alpha_2)}\right]\right\rangle & \left\langle \left[0.9^{\alpha_1}2.5^{(1-\alpha_1)}\right], \left[7.2^{\alpha_2}8.3^{(1-\alpha_2)}\right]\right\rangle \\ C_4 & \left\langle \left[4.0^{\alpha_1}5.3^{(1-\alpha_1)}\right], \left[5.5^{\alpha_2}6.5^{(1-\alpha_2)}\right]\right\rangle & \left\langle \left[2.5^{\alpha_1}3.0^{(1-\alpha_1)}\right], \left[5.2^{\alpha_2}6.5^{(1-\alpha_2)}\right]\right\rangle & \left\langle \left[3.2^{\alpha_1}4.6^{(1-\alpha_1)}\right], \left[2.5^{\alpha_2}3.0^{(1-\alpha_2)}\right]\right\rangle \end{array}\right). \tag{22}$$

For decision maker D_1

$$M^2 = \begin{array}{c} \\ \\ \\ \\ \end{array}\left(\begin{array}{cccc} \cdot & S_1 & S_2 & S_3 \\ C_1 & \left\langle \left[4.5^{\alpha_1} 7.5^{(1-\alpha_1)}\right], \left[1.3^{\alpha_2} 2.8^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[5.3^{\alpha_1} 6.5^{(1-\alpha_1)}\right], \left[6.7^{\alpha_2} 8.6^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[5.8^{\alpha_1} 6.3^{(1-\alpha_1)}\right], \left[0.9^{\alpha_2} 2.5^{(1-\alpha_2)}\right] \right\rangle \\ C_2 & \left\langle \left[5.3^{\alpha_1} 7.2^{(1-\alpha_1)}\right], \left[1.2^{\alpha_2} 2.8^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[5.0^{\alpha_1} 6.0^{(1-\alpha_1)}\right], \left[1.9^{\alpha_2} 2.5^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[4.2^{\alpha_1} 6.0^{(1-\alpha_1)}\right], \left[1.5^{\alpha_2} 2.2^{(1-\alpha_2)}\right] \right\rangle \\ C_3 & \left\langle \left[2.2^{\alpha_2} 5.4^{(1-\alpha_2)}\right], \left[7.9^{\alpha_3} 8.4^{(1-\alpha_3)}\right] \right\rangle & \left\langle \left[1.9^{\alpha_2} 2.5^{(1-\alpha_2)}\right], \left[9.0^{\alpha_3} 9.9^{(1-\alpha_3)}\right] \right\rangle & \left\langle \left[0.9^{\alpha_2} 2.5^{(1-\alpha_2)}\right], \left[7.2^{\alpha_3} 8.3^{(1-\alpha_3)}\right] \right\rangle \\ C_4 & \left\langle \left[3.0^{\alpha_2} 5.8^{(1-\alpha_2)}\right], \left[4.5^{\alpha_3} 6.8^{(1-\alpha_3)}\right] \right\rangle & \left\langle \left[2.5^{\alpha_2} 3.0^{(1-\alpha_2)}\right], \left[5.2^{\alpha_3} 6.5^{(1-\alpha_3)}\right] \right\rangle & \left\langle \left[3.2^{\alpha_1} 4.6^{(1-\alpha_1)}\right], \left[2.5^{\alpha_2} 3.0^{(1-\alpha_2)}\right] \right\rangle \end{array}\right). \tag{23}$$

For decision maker D_2

$$M^3 = \left(\begin{array}{cccc} \cdot & S_1 & S_2 & S_3 \\ C_1 & \left\langle \left[2.5^{\alpha_1} 4.5^{(1-\alpha_1)}\right], \left[2.3^{\alpha_2} 3.3^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[4.3^{\alpha_1} 6.8^{(1-\alpha_1)}\right], \left[3.7^{\alpha_2} 5.6^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[5.2^{\alpha_1} 7.3^{(1-\alpha_1)}\right], \left[1.9^{\alpha_2} 4.5^{(1-\alpha_2)}\right] \right\rangle \\ C_2 & \left\langle \left[6.0^{\alpha_1} 8.0^{(1-\alpha_1)}\right], \left[2.5^{\alpha_2} 6.3^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[5.2^{\alpha_1} 6.8^{(1-\alpha_1)}\right], \left[2.9^{\alpha_2} 4.5^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[3.2^{\alpha_1} 6.5^{(1-\alpha_1)}\right], \left[2.5^{\alpha_2} 4.2^{(1-\alpha_2)}\right] \right\rangle \\ C_3 & \left\langle \left[7.2^{\alpha_1} 9.4^{(1-\alpha_1)}\right], \left[2.9^{\alpha_2} 5.4^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[3.9^{\alpha_1} 4.5^{(1-\alpha_1)}\right], \left[7.0^{\alpha_2} 8.9^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[2.9^{\alpha_1} 5.5^{(1-\alpha_1)}\right], \left[4.2^{\alpha_2} 6.3^{(1-\alpha_2)}\right] \right\rangle \\ C_4 & \left\langle \left[2.0^{\alpha_1} 6.3^{(1-\alpha_1)}\right], \left[4.5^{\alpha_2} 7.5^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[1.5^{\alpha_1} 4.0^{(1-\alpha_1)}\right], \left[3.2^{\alpha_2} 4.5^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[1.2^{\alpha_1} 3.6^{(1-\alpha_1)}\right], \left[1.5^{\alpha_2} 3.5^{(1-\alpha_2)}\right] \right\rangle \end{array}\right). \tag{24}$$

For decision maker D_3

Step-2Now, we utilized our parametric interval valued Pythagorean weighted arithmetic mean operator (PIVPWAMO)to calculate all individual decision matrices and we get the final weighted normalized matrix as follows:

$$M = \left(\begin{array}{cccc} \cdot & S_1 & S_2 & S_3 \\ C_1 & \left\langle \left[4.06^{\alpha_1} 6.09^{(1-\alpha_1)}\right], \left[1.67^{\alpha_2} 2.99^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[4.93^{\alpha_1} 6.61^{(1-\alpha_1)}\right], \left[5.59^{\alpha_2} 7.49^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[5.58^{\alpha_1} 6.67^{(1-\alpha_1)}\right], \left[1.27^{\alpha_2} 3.24^{(1-\alpha_2)}\right] \right\rangle \\ C_2 & \left\langle \left[5.86^{\alpha_1} 7.50^{(1-\alpha_1)}\right], \left[1.77^{\alpha_2} 3.95^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[5.08^{\alpha_1} 6.30^{(1-\alpha_1)}\right], \left[2.27^{\alpha_2} 3.24^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[3.83^{\alpha_1} 6.19^{(1-\alpha_1)}\right], \left[1.87^{\alpha_2} 2.94^{(1-\alpha_2)}\right] \right\rangle \\ C_3 & \left\langle \left[3.75^{\alpha_1} 5.98^{(1-\alpha_1)}\right], \left[5.45^{\alpha_2} 6.69^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[2.64^{\alpha_1} 3.24^{(1-\alpha_1)}\right], \left[8.26^{\alpha_2} 9.53^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[1.64^{\alpha_1} 3.61^{(1-\alpha_1)}\right], \left[6.09^{\alpha_2} 7.56^{(1-\alpha_2)}\right] \right\rangle \\ C_4 & \left\langle \left[3.75^{\alpha_1} 5.84^{(1-\alpha_1)}\right], \left[4.80^{\alpha_2} 6.97^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[2.13^{\alpha_1} 3.37^{(1-\alpha_1)}\right], \left[4.46^{\alpha_2 \cdot} 5.76^{(1-\alpha_2)}\right] \right\rangle & \left\langle \left[2.46^{\alpha_1} 4.23^{(1-\alpha_1)}\right], \left[2.13^{\alpha_2} 3.19^{(1-\alpha_2)}\right] \right\rangle \end{array}\right). \tag{25}$$

Step-3 Now, we utilized our parametric interval valued Pythagorean weighted geometric mean operator (PIVPWGMO) considering the weight factors of attributes to calculate the matrix M in step 2, and we get the following:

$$\mathbf{M} = \left(\begin{array}{cc} \cdot & \mathbf{S} \\ C_1 & \left\langle \left[4.9346^{\alpha_1} 7.4917^{(1-\alpha_1)}\right], \left[0.8581^{\alpha_2} 2.9546^{(1-\alpha_2)}\right] \right\rangle \\ C_1 & \left\langle \left[3.8891^{\alpha_1} 7.3514^{(1-\alpha_1)}\right], \left[0.9580^{\alpha_2} 2.2418^{(1-\alpha_2)}\right] \right\rangle \\ C_1 & \left\langle \left[1.1482^{\alpha_1} 3.1841^{(1-\alpha_1)}\right], \left[7.0812^{\alpha_2} 9.9008^{(1-\alpha_2)}\right] \right\rangle \\ C_1 & \left\langle \left[1.4718^{\alpha_1} 3.7473^{(1-\alpha_1)}\right], \left[1.9331^{\alpha_2} 3.5835^{(1-\alpha_2)}\right] \right\rangle \end{array}\right). \tag{26}$$

Step-4 Now, we apply our developed De-Pythagorean value to convert the fuzzy number into a crisp one considering = 0.5, $\alpha_1 = 0.5, \alpha_2 = 0.5$, and then we have the final decision matrix M as follows:

Table 6: Comparison of work.

Approach	Ranking
Ye [7]	$C_3 > C_1 > C_2 > C_4$
Garg [18]	$C_3 > C_1 > C_2 > C_4$
Li et al. [42]	$C_3 > C_1 > C_4 > C_2$
Our proposed model	$C_3 > C_1 > C_2 > C_4$

Table 7: Sensitivity analysis.

Value of α_1	Value of α_2	Value of ρ	Final decision	Matrix-ranking
		0.1	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.8163\rangle \\ C_2 & \langle 0.7300\rangle \\ C_3 & \langle 1.5915\rangle \\ C_4 & \langle 0.8979\rangle \end{pmatrix}$	$C_3 > C_4 > C_1 > C_2$
		0.2	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.8602\rangle \\ C_2 & \langle 0.7734\rangle \\ C_3 & \langle 1.4890\rangle \\ C_4 & \langle 0.8803\rangle \end{pmatrix}$	$C_3 > C_4 > C_1 > C_2$
		0.3	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.9040\rangle \\ C_2 & \langle 0.8167\rangle \\ C_3 & \langle 1.3865\rangle \\ C_4 & \langle 0.8627\rangle \end{pmatrix}$	$C_3 > C_1 > C_4 > C_2$
		0.4	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.94479\rangle \\ C_2 & \langle 0.8600\rangle \\ C_3 & \langle 1.2840\rangle \\ C_4 & \langle 0.8450\rangle \end{pmatrix}$	$C_3 > C_1 > C_2 > C_4$
0.6	0.3	0.5	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.9917\rangle \\ C_2 & \langle 0.9033\rangle \\ C_3 & \langle 1.1815\rangle \\ C_4 & \langle 0.8274\rangle \end{pmatrix}$	$C_3 > C_1 > C_2 > C_4$
		0.6	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.0355\rangle \\ C_2 & \langle 0.9467\rangle \\ C_3 & \langle 1.0790\rangle \\ C_4 & \langle 0.8098\rangle \end{pmatrix}$	$C_3 > C_1 > C_2 > C_4$
		0.7	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.0794\rangle \\ C_2 & \langle 0.9900\rangle \\ C_3 & \langle 0.9765\rangle \\ C_4 & \langle 0.7922\rangle \end{pmatrix}$	$C_1 > C_2 > C_3 > C_4$
		0.8	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.1232\rangle \\ C_2 & \langle 1.0333\rangle \\ C_3 & \langle 0.8739\rangle \\ C_4 & \langle 0.7746\rangle \end{pmatrix}$	$C_1 > C_2 > C_3 > C_4$
		0.9	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.1671\rangle \\ C_2 & \langle 1.0767\rangle \\ C_3 & \langle 0.7714\rangle \\ C_4 & \langle 0.7569\rangle \end{pmatrix}$	$C_1 > C_2 > C_4 > C_3$

Table 7: Continued.

Value of α_1	Value of α_2	Value of ρ	Final decision	Matrix-ranking
0.5	0.5	0.1	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.7191 \rangle \\ C_2 & \langle 0.6742 \rangle \\ C_3 & \langle 1.3781 \rangle \\ C_4 & \langle 0.8175 \rangle \end{pmatrix}$	$C_3 > C_4 > C_1 > C_2$
		0.2	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.7770 \rangle \\ C_2 & \langle 0.7294 \rangle \\ C_3 & \langle 1.3043 \rangle \\ C_4 & \langle 0.8141 \rangle \end{pmatrix}$	$C_3 > C_4 > C_1 > C_2$
		0.3	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.8348 \rangle \\ C_2 & \langle 0.7846 \rangle \\ C_3 & \langle 1.2305 \rangle \\ C_4 & \langle 0.8108 \rangle \end{pmatrix}$	$C_3 > C_1 > C_4 > C_2$
		0.4	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.8926 \rangle \\ C_2 & \langle 0.8397 \rangle \\ C_3 & \langle 1.1567 \rangle \\ C_4 & \langle 0.8074 \rangle \end{pmatrix}$	$C_3 > C_1 > C_2 > C_4$
		0.5	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.9505 \rangle \\ C_2 & \langle 0.8949 \rangle \\ C_3 & \langle 1.0829 \rangle \\ C_4 & \langle 0.8040 \rangle \end{pmatrix}$	$C_3 > C_1 > C_2 > C_4$
		0.6	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.0083 \rangle \\ C_2 & \langle 0.9501 \rangle \\ C_3 & \langle 1.0092 \rangle \\ C_4 & \langle 0.8007 \rangle \end{pmatrix}$	$C_3 > C_1 > C_2 > C_4$
		0.7	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.0661 \rangle \\ C_2 & \langle 1.0053 \rangle \\ C_3 & \langle 0.9354 \rangle \\ C_4 & \langle 0.7973 \rangle \end{pmatrix}$	$C_1 > C_2 > C_3 > C_4$
		0.8	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.1240 \rangle \\ C_2 & \langle 1.0605 \rangle \\ C_3 & \langle 0.8616 \rangle \\ C_4 & \langle 0.7940 \rangle \end{pmatrix}$	$C_1 > C_2 > C_3 > C_4$
		0.9	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.1818 \rangle \\ C_2 & \langle 1.1157 \rangle \\ C_3 & \langle 0.7878 \rangle \\ C_4 & \langle 0.7906 \rangle \end{pmatrix}$	$C_1 > C_2 > C_3 > C_4$

Table 7: Continued.

Value of α_1	Value of α_2	Value of ρ	Final decision	Matrix-ranking
0.3	0.4	0.1	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.7854 \rangle \\ C_2 & \langle 0.7250 \rangle \\ C_3 & \langle 1.4650 \rangle \\ C_4 & \langle 0.8679 \rangle \end{pmatrix}$	$C_3 > C_4 > C_1 > C_2$
		0.2	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.8567 \rangle \\ C_2 & \langle 0.7984 \rangle \\ C_3 & \langle 1.3937 \rangle \\ C_4 & \langle 0.8727 \rangle \end{pmatrix}$	$C_3 > C_4 > C_1 > C_2$
		0.3	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.9280 \rangle \\ C_2 & \langle 0.8718 \rangle \\ C_3 & \langle 1.3223 \rangle \\ C_4 & \langle 0.8775 \rangle \end{pmatrix}$	$C_3 > C_1 > C_4 > C_2$
		0.4	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.9993 \rangle \\ C_2 & \langle 0.9453 \rangle \\ C_3 & \langle 1.2510 \rangle \\ C_4 & \langle 0.8823 \rangle \end{pmatrix}$	$C_3 > C_1 > C_2 > C_4$
		0.5	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.0706 \rangle \\ C_2 & \langle 1.0187 \rangle \\ C_3 & \langle 1.1796 \rangle \\ C_4 & \langle 0.8871 \rangle \end{pmatrix}$	$C_3 > C_1 > C_2 > C_4$
		0.6	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.1419 \rangle \\ C_2 & \langle 1.0922 \rangle \\ C_3 & \langle 1.1083 \rangle \\ C_4 & \langle 0.8919 \rangle \end{pmatrix}$	$C_1 > C_3 > C_2 > C_4$
		0.7	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.2133 \rangle \\ C_2 & \langle 1.1656 \rangle \\ C_3 & \langle 1.0370 \rangle \\ C_4 & \langle 0.8967 \rangle \end{pmatrix}$	$C_1 > C_2 > C_3 > C_4$
		0.8	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.2846 \rangle \\ C_2 & \langle 1.2390 \rangle \\ C_3 & \langle 0.9656 \rangle \\ C_4 & \langle 0.9014 \rangle \end{pmatrix}$	$C_1 > C_2 > C_3 > C_4$
		0.9	$\begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 1.3559 \rangle \\ C_2 & \langle 1.3125 \rangle \\ C_3 & \langle 0.8943 \rangle \\ C_4 & \langle 0.9062 \rangle \end{pmatrix}$	$C_1 > C_2 > C_4 > C_3$

$$\mathbf{M} = \begin{pmatrix} . & \mathbf{S} \\ C_1 & \langle 0.9505 \rangle \\ C_2 & \langle 0.8949 \rangle \\ C_3 & \langle 1.0829 \rangle \\ C_4 & \langle 0.8040 \rangle \end{pmatrix}. \tag{27}$$

From the final decision matrix we can say that the ranking will be $C_3 > C_1 > C_2 > C_4$.

6.3. Comparison of Work. We compare our work with other previously established work described by the authors 31,8,32, for finding the best alternatives whenever $\rho = 0.5, \alpha_1 = 0.5, \alpha_2 = 0.5$. The comparison result has been presented in Table 6.

From Table 5, we notice that C_3 has been declared the best alternative by our model, which matches with the outcomes of other researchers, namely Ye [7], Garg [18], Li et al. [42], although we have taken the parametric interval valued form of Pythagorean number which is

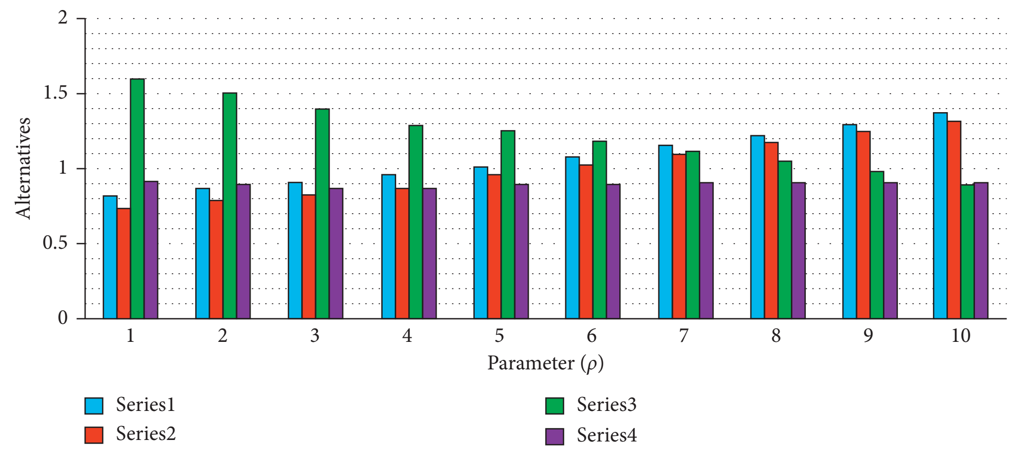

FIGURE 7: Chart for comparison of ranking when $\alpha_1 = 0.6$, $\alpha_2 = 0.3$, row (ρ) varies from 0.1 to 0.9.

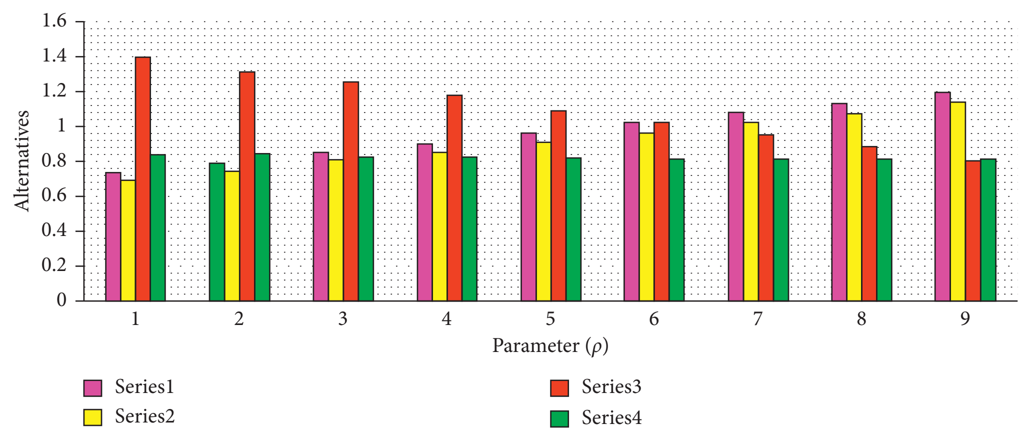

FIGURE 8: Chart for comparison of ranking when $\alpha_1 = 0.3$, $\alpha_2 = 0.4$, row (ρ) varies from 0.1 to 0.9.

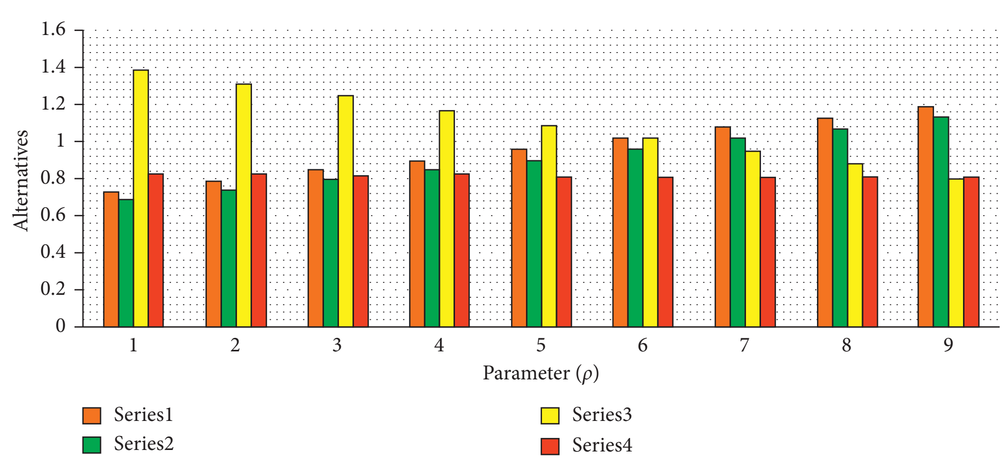

FIGURE 9: Chart for comparison of ranking when $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, row (ρ) varies from 0.1 to 0.9.

more generalized from in compare to the established forms. Thus, we may conclude that PIVPN is a more robust structure that is reliable in the decision-making process.

6.4. Sensitivity Analysis. The main target of performing the sensitivity analysis is to check the efficiency, effectiveness, and stability of the proposed work. Here, we performed a well-organized sensitivity analysis by exchanging the weights of the

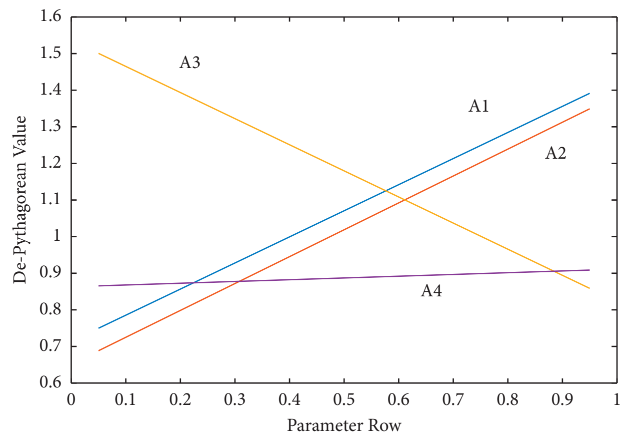

Figure 10: Graph of de-Pythagorean value of different alternatives for $\alpha_1 = 0.3$, $\alpha_2 = 0.4$.

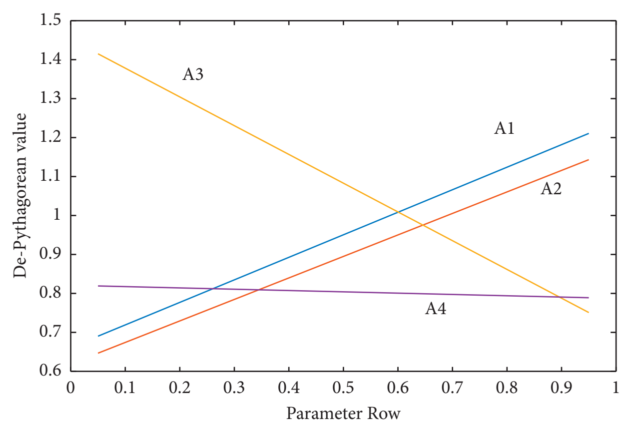

Figure 11: Graph of de-Pythagorean value of different alternatives for $\alpha_1 = 0.5$, $\alpha_2 = 0.5$.

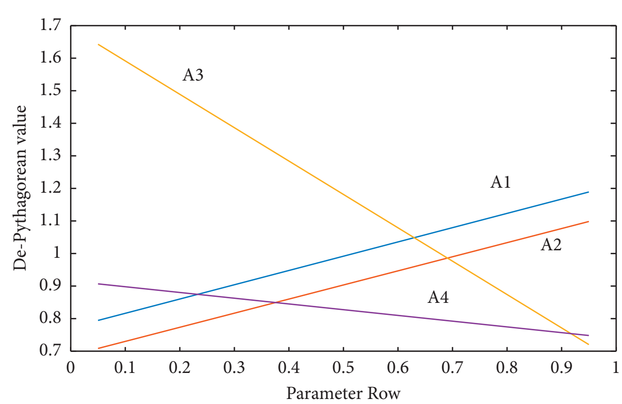

Figure 12: Graph of de-Pythagorean value of different alternatives for $\alpha_1 = 0.6$, $\alpha_2 = 0.3$.

attributes and values of the parameter ρ keeping values of other terms unchanged. For different values of α_1, α_2, ρ the change of ranking have been shown in Table 7.

6.4.1. Graphical Chart. Here, we computed the graphical chart of comparison of work in Figures 7–9 which shows the ranking of the alternatives for different values of α_1, α_2 and

ρ.Here, Series-1, Series-2, Series-3, and Series-4 denote the alternatives Commodity 1, Commodity 2, Commodity 3, and Commodity 4, respectively.

7. Numerical Analysis

In this section, we consider our sensitivity results and we did a numerical analysis whenever the value of Row (ρ) actually varies from 0.05 to 0.95. For different three sets of values of α_1, α_2, we got the following figures numerically (Figures 10–12),

7.1. Discussion. After the rigorous sensitivity analysis, numerical simulations, and graphical analysis, we have observed that in Figure 8, alternatives 3, 1 change their position in a range of (0.55, 0.6) and become the best one. Now, if we observe Figure 9, then we can conclude that alternatives 3, 1 change their position in a range of (0.6, 0.62) and become the best one. Lastly, if we observe Figure 10, then we can conclude that the alternatives 3, 1 change their position in a range of (0.6, 0.68) and becomes the best one. Finally, from the sensitivity analysis, we can conclude that the ranking of the alternatives depends on the range of the parameter, which plays an essential role in this proposed method.

7.2. Advantages and Limitations of the Proposed Method

(1) After this scientific discussion mentioned above, we have noticed that the existing uncertain parameters such as FS, IFS, PFS can be easily obtained from PIVPNs, and hence PIVPNs are a more well-organized representation to grip a problem more effectively where the existing theories fail. Moreover, PIVPNs can grab the inconsistent condition in a more compatible way rather than other existing methods. Henceforth, the complete studies under the PIVPN environment are effective in solving MCGDM problems.

(2) Through sensitivity analysis, numerical simulation, and graphical representation of the result, we have noticed that if we vary the weights of the attributes within a certain range, then the best alternatives may change their positions within the given range. That is, it is basically case dependent which shows the limitation of this proposed technique.

8. Conclusion

The concept of PIVPN is logical and pragmatic and it has practical usefulness in the current research arena. In this research article, we have been extended the conception of the Pythagorean fuzzy set into PIVPN and developed some logical operators related to it, namely PIVPWAMO and PIVPWGMO. We also developed a score and accuracy function in the PIVPN environment and solved a critical path problem and performed a comparison with the other existing method to observe the advantage of this proposed model. Also, marketing based real life MCGDM problem is described in this article to find out the best alternatives whenever all the decision matrices members are in the PIVPN form. Applying the concept of developed operators, we solved this problem and we utilized the concept of De-Pythagorean value to short out the ranking of the alternatives. Finally, we performed a rigorous sensitivity analysis, numerical simulation, and comparison of work to check the utility, reliability, usefulness, and advantages of this proposed novel work.

In a future study, we can extend some theoretical ideas of PIVPN and can form the structure of exponential operational law, and logarithmic operational law in the PIVPN environment. Further, we can apply it to different fields like cloud computing, image segmentation, pattern recognition, medical diagnosis, banking policy making problems, economic and social science based several real-life problems, etc.

Data Availability

The data used to support the findings of this study are included in this article. However, the reader may contact the corresponding author for more details on the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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