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Research Article

Notes on Upper and Lower Truncation of Picture Fuzzy Graphs

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Due to the absence of a neutral function, there are drawbacks to the existing definition of an intuitionistic fuzzy graph (Int-FG). In that definition of Int-FG, membership function and nonmembership function are involved. In this study, lower truncation (Low-T) and upper truncation (Up-T) are applied to picture fuzzy graphs (Pic-FGs). Pic-FG has an additional neutral membership function. Furthermore, Low-T and Up-T of subdivision of the picture fuzzy line graph (Pic-FLnG) are also discussed. The degree of an edge in both Up-T and Low-T of Pic-FG is discussed. Some related theorems related to the degree of Up-T and Low-T of Pic-FG are discussed.

1. Introduction

In 1965, Zadeh [1] considerably developed the fuzzy set (FS) idea as a generalization of the crisp set which deals with imprecise or vague data. In real-world situations, the theory of the crisp set may fail to deal with vague data. The true membership degree of FS lies in the closed interval [0, 1]. FS is widely used in the medical and biological sciences. To improve fuzzy theory, Atanassov [2] established the intuitionistic fuzzy set (Int-FS) as a generalization of FS. Int-FS is useful in various domains, having membership and nonmembership degrees whose sum does not exceed 1. It is observed that Int-FS cannot deal with neutrality. Cuong [3] proposed the Pic-FS, which gives more satisfactory results. A picture fuzzy set (Pic-FS) is an extension of Int-FS due to its neutrality degree as an extra term.

Graphs have a wide range of applications in real-world problems and are used in computer science, sociology, circuit analysis, network traffic routing, biological networks, operations research, and social networks. Graphs are excellent techniques for information transfer with features such as entity relationships. Nodes can be represented by objects, and edges can be represented by relationships. The graph cannot deal with vague situations. To fulfill this limitation, Kauffman [4] initiated the concept of FG. Mahapatra et al. [5] worked on the colouring of the COVID-19-affected region based on fuzzy directed graphs. Gani and Malarvizhi [6] discussed the concept of truncation in FG. Mahapatra et al. [7] investigated Radio FG. Nowadays, a few authors are working in fuzzy analysis [8–12]. Mahapatra et al. [13] studied the edge colouring of FGs.Shao et al. [14] discussed the application of the water supplier system in Int-FG. Ghani et al. [15] presented the subdivision and middle Int-FG. The edge degree and edge regular properties of FG truncations are discussed by Radha and Kumaravel [16]. Mahapatra et al. [17] studied extended neutrosophic planar graphs. Mahapatra et al. [18] worked on predicting links in social networks by using a neutrosophic graph to look at the way people connect with each other. Furthermore, Mahapatra et al. [19] worked on a new way of link prediction in social networks. The concept of Pic-FG is foundation of Zuo et al. [20] which is extension of Int-FG. Shoaib et al. [21] studied some properties of Pic-FG. The following are the major points of this article:

(i) In this paper, we study the Low-T and Up-T of Pic-FG.

(ii) Furthermore, Low-T and Up-T of subdivision Pic-FLnG are discussed.

(iii) Pic-FG is the extension of Int-FG. In this study, we initiate the degree of an edge in truncations of Pic-FG.

The following is the structure of this paper:

We presented some basic definitions which will help to understand the paper in Section 2. In Section 3, we study the Low-T and Up-T of Pic-FG. In Section 4, we present the Up-T and Low-T of Pic-FG. We study the degree of an edge in truncations of Pic-FLnG in Section 5. At the end, we write conclusion and some future plans in Section 6.

The basic notations of this paper are shown in Table 1.

Table 1: Some basic notations.

Notation	Meaning
Fuzzy graph	FG
Intuitionistic fuzzy graph	Int-FG
Intuitionistic fuzzy set	Int-FS
Picture fuzzy graph	Pic-FG
Picture fuzzy line graph	Pic-FLnG
Picture fuzzy set	Pic-FS
Lower truncation	Low-T
Upper truncation	Up-T
Fuzzy set	FS
Picture fuzzy set	Pic-FS
Underlying crisp graph	Und-CG
Vertex set	V-set
Edge set	E-set
Subdivision picture fuzzy graph	Sud(G)
Upper truncation picture fuzzy graph	$G^{(m)}$
Lower truncation picture fuzzy graph	$G_{(m)}$
Upper truncation picture fuzzy line graph	$(Ln(G))^{(m)}$
Lower truncation picture fuzzy line graph	$(Ln(G))_{(m)}$
Subdivision of upper truncation picture fuzzy graph	$Sud(G^{(m)})$
Subdivision of lower truncation picture fuzzy graph	$Sud(G_{(m)})$

2. Preliminaries

Definition 1. [20] Let X be a nonempty universal set, then, FS is defined as

$\{\langle b: \varrho_1(b)\rangle, \varrho_1: V \longrightarrow [0,1], b \in X\}$.

Definition 2. [20] Let X be a nonempty universal set, then, Int-FS is defined as follows:

$<b: \varrho_1(x), \tau_1(b)>, b \in X$ is called Int-FS, where $\varrho_1: V \longrightarrow [0,1]$ and $\tau_1: V \longrightarrow [0,1]$, and ϱ_1 and τ_1 satisfy the axiom

$\varrho_1(b) + \tau_1(b) \leq 1$ (for all $b \in$ X).

Definition 3. [20] Let X be a nonempty universal set, then, Pic-FS is defined as:

$<b: \varrho_1(x), \sigma_1(b), \tau_1(b)>, b \in X$ is called Pic-FS, where $\varrho_1: V \longrightarrow [0,1], \sigma_1: V \longrightarrow [0,1]$, and $\tau_1: V \longrightarrow [0,1]$ and are known as degree of truth, neutral, and falsity membership of b, respectively.

ϱ_1, σ_1, and τ_1 satisfy the axiom

$$\varrho_1(b) + \sigma_1(b) + \tau_1(b) \leq 1. \quad (1)$$

The refusal membership degree $\pi_1(b) = 1 - (\varrho_1(b) + \sigma_1(b) + \tau_1(b))$.

Definition 4. [20] A Pic-FG $G = (\varrho, \sigma, \tau)$, which is of crisp graph $G^* = (\varrho^*, \sigma^*, \tau^*)$ with $(\varrho_1, \sigma_1, \tau_1)$ be a picture fuzzy subset on V and $(\varrho_2, \sigma_2, \tau_2)$ be a picture fuzzy relation on V is defined as

(i) ϱ_1, τ_1, and $\sigma_1: V \longrightarrow [0,1]$, where $0 \leq \varrho_1$ (b) $+ \sigma_1$ (b) $+ \tau_1$ (b) $\leq 1 \forall b \varepsilon V$.

(ii) The functions ϱ_2, σ_2, and $\tau_2 : E \subseteq V \times V \longrightarrow [0, 1]$ are defined by

$$\begin{aligned} &\varrho_2(rk) \leq \min\{\varrho_1(a), \varrho_1(b)\},\\ &\sigma_2(rk) \leq \min\{\sigma_1(a), \phi_1(b)\},\\ &\tau_2(rk) \geq \max\{\tau_1(a), \tau_1(b)\},\\ &0 \leq \varrho_2(rk) + \sigma_2(rk) + \tau_2(rk) \end{aligned} \quad (2)$$

Example 1. Suppose a Pic-FG shown as in Figure 1 having four vertices z, w, x, and y and four edges zy, wx, xz, and wy such that

$U = \lt(w/0.2, x/0.2, y/0.3, z/0.3), (w/0.1, x/0.3, y/0.4, z/0.3), (w/0.4, x/0.3, y/0.1, z/0.2) >$ be the picture fuzzy V-set and

$W = \lt(wx/0.1, xz/0.1, zy/0.1, wy/0.1), (wx/0.1, xz/0.2, zy/0.1), (wy/0.1), (wx/0.4, xz/0.5, zy/0.4, wy/0.4) >$ be the picture fuzzy E-set.

3. Truncation and Subdivision Pic-FG

Definition 5. Let $(\varrho_1, \sigma_1, \tau_1)$ be a picture fuzzy subset of a set V. Low-Ts and Up-Ts of $(\varrho_1, \sigma_1, \tau_1)$ at the level m, $0 \leq m \leq 1$, are the picture fuzzy subsets $(\varrho_1, \sigma_1, \tau_1)_{(m)}$ and $(\varrho_1, \sigma_1, \tau_1)^{(m)}$ defined by:

$$\begin{aligned} (\varrho_1, \sigma_1, \tau_1)_{(m)}(b) &= \begin{cases} (\varrho_1, \sigma_1, \tau_1)(b) & \text{if } b \in (\varrho_1)^m, b \in (\tau_1)^m, \text{and } b \in (\sigma_1)^m,\\ 0 & \text{if } b \notin (\varrho_1)^m, b \notin (\sigma_1)^m \text{or } b \notin (\tau_1)^m, \end{cases}\\ (\varrho_1, \sigma_1, \tau_1)^{(m)}(b) &= \begin{cases} m & \text{if } b \in (\varrho_1)^m, b \in (\sigma_1)^m \text{and } b \in (\tau_1)^m,\\ (\varrho_1, \sigma_1, \tau_1)(b) & \text{if } b \notin (\varrho_1)^m, b \notin (\varrho_1)^m, \text{or } b \notin (\tau_1)^m, \end{cases} \end{aligned} \quad (3)$$

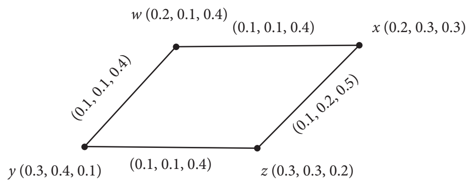

Figure 1: Pic-FG.

where

$$(\varrho_1)^m = \left\{\frac{b \in V}{\varrho_1(b) \geq m}\right\},$$
$$(\sigma_1)^m = \left\{\frac{b \in V}{\sigma_1(b) \geq m}\right\}, \quad (4)$$
$$(\tau_1)^m = \left\{\frac{b \in V}{\tau_1(b) \leq m}\right\}.$$

Consider $V_{(m)} = (\varrho_1, \sigma_1, \tau_1)^{(m)}$ and $E_{(m)} = (\varrho_2, \sigma_2, \tau_2)^{(m)}$.

Let $G_{(m)} = (\varrho_m, \sigma_m, \tau_m)$ be Pic-FG with underlying crisp graph (Und-CG) G^*: $(V_{(m)}, E_{(m)})$. It is known as Low-T of the Pic-FG G at level m, where $V_{(m)}$ and $E_{(m)}$ can be a proper subset of V and E, respectively. Take $V^{(m)} = V$ and $E^{(m)} = E$, then, $G^{(m)}$: $(\varrho^{(m)}, \sigma^{(m)}, \tau^{(m)})$ is an Pic-FG with Und-CG $G^{(m)*}$: $(V^{(m)}, E^{(m)})$. It is called as Up-T of the Pic-FG G at level m.

Theorem 1. *For any level m, $0 < m \leq 1$, the Low-T of $Sud(G)$ is equal as the subdivision of $G_{(m)}$, i.e., $(Sud(G))_{(m)} = Sud(G_{(m)})$.*

Proof. Suppose G: (ϱ, σ, τ) is a Pic-FG with its underlying V-set V and crisp graph G^*: $(\varrho^*, \sigma^*, \tau^*)$ as (V, E).

We claim that V-set $(Sud(G))_{(m)}$ = V-set $Sud(G_{(m)})$. Let b is a node in Pic-FG. Then, by using the definition of Low-T of $Sud(G)$, we have

$$\begin{aligned}&((\varrho_1, \sigma_1, \tau_1)_{Sud})_{(m)}(b) = (\varrho_1, \sigma_1, \tau_1)_{Sud})(b), \text{if } b \in (\varrho_1)^m_{Sud}, b \in (\sigma_1)^m_{Sud} \text{and } b \in (\tau_1)^m_{Sud},\\ &((\varrho_1, \sigma_1, \tau_1)_{Sud})_{(m)}(b) = (\varrho_1, \sigma_1, \tau_1)_{Sud})(b), \text{if } b \notin (\varrho_1)^m_{Sud}, b \notin (\sigma_1)^m_{Sud} \text{and } b \notin (\tau_1)^m_{Sud}.\end{aligned} \quad (5)$$

By using the definition of Sud(G),

$$\varrho_{1Sud}(b) = \begin{cases}\varrho_1(b), \text{if} & b \in V,\\ \varrho_2(b), \text{if} & b \in E,\end{cases} \quad (6)$$

$$\sigma_{1Sud}(b) = \begin{cases}\sigma_1(b), \text{if} & b \in V,\\ \sigma_2(b), \text{if} & b \in E,\end{cases} \quad (7)$$

$$\tau_{1Sud}(b) = \begin{cases}\tau_1(b), \text{if} & b \in V,\\ \tau_2(b), \text{if} & b \in E.\end{cases} \quad (8)$$

□

Case 1. Consider the node b in $(Sud(G))_{(m)}$, such that $b \in (\varrho_1)^m_{Sud}$, $b \in (\sigma_1)^m_{Sud}$, and $b \in (\tau_1)^m_{Sud}$.

$$(\varrho_1)_{Sud}(b) \geq m, (\sigma_1)_{Sud}(b) \geq m \text{ and } (\tau_1)_{Sud}(b) \leq m. \quad (9)$$

Subcase 1. If $b \in$ V, by equations (6)–(9), $\varrho_1(b) \geq m$, $\sigma_1(b) \geq m$, and $\tau_1(b) \leq m$. Hence, $b \in \varrho_1^m$, $b \in \sigma_1^m$, and $b \in \tau_1^m$. By the definition of $G_{(m)}$, $(\varrho_1, \sigma_1, \tau_1)_{(m)}(b) = (\varrho_1, \sigma_1, \tau_1)(b)$, if $b \in \varrho_1^m$, $b \in \sigma_1^m$, and $b \in \tau_1^m$, i.e., b is vertex in $G_{(m)}$ such that

$$(\varrho_1, \sigma_1, \tau_1)_{(m)}(b) = (\varrho_1, \sigma_1, \tau_1)(b). \quad (10)$$

Subcase 2. If $b \in E$, let $b = f$; equations (6) and (7) $\Rightarrow (\varrho_1, \sigma_1, \tau_1)_{Sud}(f) = (\varrho_2, \sigma_2, \tau_2)(f)$, by equation (8), $\varrho_2(f) \geq m, \sigma_2(f) \geq m, \tau_2(f) \leq m$. Hence, $f \in (\varrho_2, \sigma_2, \tau_2)^m$. In G_m, $(\varrho_2, \sigma_2, \tau_2)_{(m)}(f) = (\varrho_2, \sigma_2, \tau_2)(f)$ as $f \in (\varrho_2, \sigma_2, \tau_2)^m$, that is, $b = f \in E$ is an edge in $G_{(m)}$ which taken together as the nodes of $Sud(G_{(m)})$, by Subcases 1 and 2, b is node of $Sud(G_{(m)})$, $(\varrho_1, \sigma_1, \tau_1)_{Sud}(m) = (\varrho_1, \sigma_1, \tau_1)_{Sud}(b) = ((\varrho_1, \sigma_1, \tau_1)_{(m)})_{Sud}(m)$ if $b \in (\varrho_1)^m_{Sud}$, $(\sigma_1)^m_{Sud}$ and $b \in (\tau_1)^m_{Sud}$.

Case 2. Suppose the node b in $Sud(G_{(m)})$ in such a way $u \notin (\varrho_1)^m_{Sud}$, $u \notin (\sigma_1)^m_{Sud}$, and $b \notin (\tau_1)^m_{Sud}$; equation (5) $\Rightarrow ((\varrho_1, \sigma_1, \tau_1)_{Sud})_{(m)}(b) = 0$, i.e., $((\varrho_1, \sigma_1, \tau_1)_{Sud})_{(m)}(b) = 0$ as $(\varrho_1, \sigma_1, \tau_1)_{Sud}(b) < m$.

Subcase 3. Let $u \in V$; equations (6) and (7) $\Rightarrow (\varrho_1, \sigma_1, \tau_1)_{Sud}(b) = (\varrho_1, \sigma_1, \tau_1)(b)$. Hence, $(\varrho_1, \sigma_1, \tau_1)(b) < m$. $u \notin (\varrho_1^m)$, $u \notin (\sigma_1^m)$ (or) $b \notin (\tau_1^m)$, by definition of $(G_{(m)})(b)$ is a node of $(G_{(m)})$, $(\varrho_1, \tau_1)(b) = 0$; i.e., u is node in $G_{(m)}$ such that $(\varrho_1, \tau_1)_{(m)}(b) = 0$.

Subcase 4. Consider $b \in E$, $b = f$; equations (6) and (7) $\Rightarrow (\varrho_1, \sigma_1, \tau_1)(b) = (\varrho_2, \sigma_2, \tau_2)(b)$. Hence, $(\varrho_1, \sigma_1, \tau_1)(b) < m$, i.e., $f \in (\varrho_2, \sigma_2, \tau_2)^m$. So, by definition of $G_{(m)}$, $(\varrho_2, \sigma_2, \tau_2)(f) = 0$; i.e., f is edge in $G_{(m)}$ such that $(\varrho_2, \sigma_2, \tau_2)_{(m)}(f) = 0$. Therefore, from Subcases 3 and 4 and by definition of $Sud(G_{(m)})u$ is vertex of $Sud(G_{(m)})$ such that $((\varrho_1, \sigma_1, \tau_1)_{(m)})_{Sud}(b) = 0$, so, by Cases 1 and 2, $((\varrho_1, \sigma_1, \tau_1)_{Sud})_{(m)}$
$(m) = ((\varrho_1, \sigma_1, \tau_1)_{(m)})_{Sud}(b) \in b \in V \cup E$.

Claim 1. Now, our claim is E-set $Sud(G)_{(m)}$ is equal E-set $Sud(G_{(m)})$.

Take (b, f) to be edge in $Sud(G)_{(m)}$. By using the definition of $Sud(G)_{(m)}$,

$$((\varrho_2,\sigma_2,\tau_2)_{Sud})_{(m)}(b,f) = (\varrho_1,\sigma_1,\tau_1)_{Sud}(b,f)\text{if}\,(b,f)\in(\varrho_2,\sigma_2,\tau_2)^m_{Sud}, \tag{11}$$

$$((\varrho_2,\sigma_2,\tau_2)_{Sud})_{(m)}(b,f) = 0\text{if}\,(b,f)\notin(\varrho_2)^m_{Sud},\,(b,f)\notin(\sigma_2)^m_{Sud}\text{or}\,(b,f)(\tau_2)^m_{Sud}. \tag{12}$$

By using the definition of $Sud(G)$,

$$\varrho_{2Sud}(f,b) = \varrho_{1Sud}(b)\wedge\varrho_{1Sud}(f),\ \text{if}\ u\in V,\ f\in E\ \text{and}\ b\ \text{which lies on}\ f, \tag{13}$$

$$\varrho_{2Sud}(f,b) = 0,\ \text{otherwise}, \tag{14}$$

$$\sigma_{2Sud}(f,b) = \sigma_{1Sud}(b)\wedge\sigma_{1Sud}(f),\ \text{if}\ b\in V,\ f\in E\ \text{and}\ b\ \text{lies on}\ f, \tag{15}$$

$$\sigma_{2Sud}(f,b) = 0,\ \text{otherwise}, \tag{16}$$

$$\tau_{2Sud}(f,b) = \tau_{1Sud}(b)\vee\tau_{1Sud}(f),\ \text{if}\ b\in V,\ f\in E\ \text{and}\ b\ \text{lies on}\ f, \tag{17}$$

$$\tau_{2Sud}(f,b) = 0,\ \text{otherwise}. \tag{18}$$

Case 3. Let $b\in V, f\in E$ be such that $(b,f)\in(\varrho_2,\tau_2)^{(m)}_{Sud}$.

$$((\varrho_2,\sigma_2,\tau_2)_{Sud})(m)(b,f) = \begin{cases} (\varrho_2)_{Sud}(b,f)\geq m, \\ \sigma_2)_{Sud}(b,f)\geq m, \\ (\tau_2)_{Sud}(b,f)\leq m. \end{cases} \tag{19}$$

As $m>0$, equation (12) $\Rightarrow\varrho_{1Sud}(b)\wedge\varrho_{1Sud}(f)\geq m$, equation (12) $\Rightarrow\sigma_{1Sud}(b)\wedge\sigma_{1Sud}(f)\geq m$, and equation (14) $\Rightarrow$

$$\begin{aligned}\tau_{1Sud}(b)\vee\tau_{1Sud}(f)\leq m &\Rightarrow\varrho_1(b)\wedge\varrho_1(f)\geq m,\sigma_1(b)\wedge\sigma_1(f)\geq m,\tau_1(b)\vee\tau_1(f)\leq m,\\ &\Rightarrow\varrho_1(b),\varrho_2(b)\geq m,\sigma_1(b),\sigma_2(b)\geq m, b\ \text{lie on}\ f,\tau_1(b),\tau_2(b)\leq m\ b\ \text{lie on f},\\ &\Rightarrow b\in(\varrho_1,\sigma_1,\tau_1)^m, f\in(\tau_2,\sigma_2,\tau_2)^m\ \text{and}\ b\ \text{lies on f}.\end{aligned} \tag{20}$$

By definition of $G_{(m)}$,

$$\begin{aligned}(\rangle_1,\sigma_1,\tau_1)_m(b) &= (\rangle_1,\sigma_1,\tau_1)(b), (\rangle_2,\sigma_2,\tau_2)_m(f)\\ &= (\rangle_2,\sigma_2,\tau_2)(f) as\, b\in(\rangle_1,\sigma_1,\tau_1)^m, f\in\\ &\quad(\rangle_2,\sigma_2,\tau_2)^m.\end{aligned} \tag{21}$$

By equations (12), (14), and (16), we have,

$$\begin{aligned}(\varrho_2)_{Sud}(b,f) &= (\varrho_1)_m(b)\wedge(\varrho_2)_m(b)\text{as}\ b\ \text{lie on}\ f\\ &= ((\varrho_1)_m)_{Sud}(b)\wedge((\varrho_2)_m)_{Sud}(b)\text{as}\ b\ \text{lie on}\ f\\ &= ((\varrho_2)_m)_{Sud}(b,f),\\ (\sigma_2)_{Sud}(b,f) &= (\sigma_1)_m(b)\wedge(\sigma_2)_m(b)\text{as}\ b\ \text{lie on}\ f\\ &= ((\sigma_1)_m)_{Sud}(b)\wedge((\sigma_2)_m)_{Sud}(b)\text{as}\ b\ \text{lie on}\ f\\ &= ((\sigma_2)_m)_{Sud}(b,f),\\ (\tau_2)_{Sud}(b,f) &= (\tau_1)_m(b)\wedge(\tau_2)_m(b)\text{as}\ b\ \text{lie on}\ f\\ &= ((\tau_1)_m)_{Sud}(b)\wedge((\tau_2)_m)_{Sud}(b)\text{as}\ b\ \text{lie on}\ f\\ &= ((\tau_2)_m)_{Sud}(b,f).\end{aligned} \tag{22}$$

So, (b,f) is an edge in $Sud(G_{(m)})$ and

$$((\varrho_2,\sigma_2,\tau_2)_{Sud})_{(m)}(b,f) = ((\varrho_1,\sigma_1,\tau_1)_m)_{Sud}(b,f)\text{if}\,(b,f)\in(\varrho_2,\sigma_2,\tau_2)^{(m)}_{Sud}. \tag{23}$$

Case 4. Suppose (b,f) is an edge in $Sud(G)_{(m)}$ be such that

$$((\varrho_2,\sigma_2,\tau_2)_{Sud})_{(m)}(b,f)=0,\Rightarrow \text{by}\,(7)\,(b,f)\notin \varrho^m_{Sud},\ (b,f)\notin(\sigma_2)^m_{Sud},$$
$$(b,f)\notin(\tau_2)^m_{Sud}\Rightarrow(\varrho_2)_{Sud}(b,f)<m,\ (\sigma_2)_{Sud}(b,f)<m\,(or)\,(\tau_2)_{Sud}(b,f)>m. \quad (24)$$

Subcase 5. If b lies on f, equations (12) and (14)

$$\Rightarrow(\varrho_1)_{Sud}(b)\wedge(\varrho_1)_{Sud}(f)<m,\ (\sigma_1)_{Sud}(b)\wedge(\sigma_1)_{Sud}(f)<m,\ (\tau_1)_{Sud}(b)\vee(\tau_1)_{Sud}(f)>m,$$
$$\Rightarrow\varrho_1(b)\wedge\varrho_2(f)<m,\sigma_1(b)\wedge\sigma_2(f)<m,\tau_1(b)\vee\tau_2(f)>m, \quad (25)$$
$$\Rightarrow\varrho_2(f)<m\,(or)\tau_2(f)>m.$$

So, $f\notin(\varrho_2)^m, f\notin(\varrho_2)\quad {}^m\,(or)f\notin(\tau_2)^m$, i.e., $(\varrho_2),\sigma_2),\tau_2)_{(m)}(f)=0$. Considering

$$\begin{aligned}((\varrho_2)_{(m)})_{Sud}(b,f)&=((\varrho_1)_m)_{Sud}(b)\wedge((\varrho_2)_m)_{Sud}(b)\\&=((\varrho_1)_m)(b)\wedge((\varrho_2)_m)(b)\\&=0\,(\text{since}\,(\varrho_2)_m)(f),\text{and}\,b\,\text{lies on}\,f,\\((\sigma_2)_{(m)})_{Sud}(b,f)&=((\sigma_1)_m)_{Sud}(b)\wedge((\sigma_2)_m)_{Sud}(b)\\&=((\sigma_1)_m)(b)\wedge((\sigma_2)_m)(b)\\&=0\,(\text{since}\,(\sigma_2)_m)(f),\text{and}\,b\,\text{lies on}\,f,\\((\tau_2)_{(m)})_{Sud}(b,f)&=((\tau_1)_m)_{Sud}(b)\wedge((\tau_2)_m)_{Sud}(b)\\&=((\tau_1)_m)(b)\wedge((\tau_2)_m)(b)\\&=0\,(\text{since}\,(\tau_2)_m)(f),\text{and}\,b\,\text{lies on}\,f.\end{aligned} \quad (26)$$

Subcase 6. If b does not lie on f, $((\varrho_2,\sigma_2,\tau_2)_{(m)})_{Sud}(b,f)=0$. It yields, by using Subcases 5 and 6, if (b,f) is an arc in $(Sud(G))_{(m)}$ such that $((\varrho_2,\sigma_2,\tau_2)_{Sud})_{(m)}(b,f)=0$, then, $((\varrho_2,\sigma_2,\tau_2)_{(m)})_{Sud}(b,f)=0$.

$$((\varrho_2,\sigma_2,\tau_2)_{Sud})_{(m)}(b,f)=((\varrho_2,\sigma_2,\tau_2)_{(m)})_{Sud}(b,f). \quad (27)$$

If $(b,f)\notin(\varrho_2)^m_{Sud}$, $(b,f)\notin(\sigma_2)^m_{Sud}\,(or)\,(b,f)\notin(\tau_2)^m_{Sud}$. Hence, by Cases 3 and 4, $((\varrho_2,\sigma_2,\tau_2)_{Sud})_{(m)}(b,f)=((\varrho_2,\sigma_2,\tau_2)_{(m)})_{Sud}(b,f)$. So, $(Sud(G))_{(m)}=(Sud(G)_{(m)})$.

Theorem 2. *For any level* $0<m\leq1$, $((Sud(G))^{(m)}=(Sud(G)^{(m)}))$.

4. Truncation and Pic-FLnG

Theorem 3. *For any level set* $0<m\leq1$, $((Ln(G))_{(m)}=(Ln(G)_{(m)}))$.

Proof. Let G: (ϱ,σ,τ) is a Pic-FG with its underlying set V and crisp graph G^*: $(\varrho^*,\sigma^*,\tau^*)$ as (V,E). □

Case 5. Take $Ln(G)_{(m)}$: $((\alpha_1,\tau_1)_{(m)},(\alpha_2,\tau_2)_{(m)}),(\alpha_3,\tau_3)_{(m)}$ by applying the definition of equations (6) and (7),

$$(\alpha_1,\tau_1)_{(m)}(S_a)=\begin{cases}(\alpha_1,\tau_1)_{(m)}(S_a)\text{if}\,S_a\in\alpha_1^m,S_a\in\tau_1^m,\\0,\,\text{if}\,S_a\notin\alpha_1^m\,(or)S_a\notin\tau_1^m.\end{cases} \quad (28)$$

Hence, $S_a\in\alpha_1^m\Rightarrow\alpha_1(S_a)\geq t$ and $S_a\in\tau_1^m\Rightarrow\tau_1(S_a)\leq t$.

$$\Rightarrow\varrho_2(a)\geq m,\sigma_2(a)\geq m\,\text{and}\,\tau_2(a)\leq m\Rightarrow a\in\varrho_2^m,\sigma_2^m,\text{and}\,a\in\tau_2^m,$$
$$\Rightarrow(\varrho_2,\sigma_2,\tau_2)_m(a)=(\varrho_2,\sigma_2,\tau_2)(a),$$
$$S_a\notin\alpha_1^m\Rightarrow\alpha_1(S_a)\leq m\,(or)S_a\notin\tau_1^m\Rightarrow\tau_1(S_a)\geq m, \quad (29)$$
$$\Rightarrow\varrho_2(a)\leq m,\sigma_2(a)\leq m\,(or)\tau_2(a)\geq m\Rightarrow a\notin\varrho_2^m,a\notin\sigma_2^m\text{and}\,a\notin\tau_2^m,$$
$$\Rightarrow(\varrho_2,\sigma_2,\tau_2)_m(a)=0.$$

So, a node in $Ln(G)_{(m)}$ gives an edge in $(G)_{(m)}$ which in turn in form of a node in $Ln((G)_{(m)})$.

$$\Rightarrow V-set\,Ln(G)_{(m)}\subseteq V-set(LnG_{(m))}). \quad (30)$$

Similarly, it can be proved that

$$\Rightarrow\text{vertex set}\,Ln(G)_{(m)}\supseteq\text{vertex set}\,(LnG)_{(m)},$$
$$(\alpha_2,\tau_2)_{(m)}(S_a)=\begin{cases}(\alpha_2,\tau_2)_{(m)}(S_a)\text{if}\,S_a\in\alpha_2^m,S_a\in\tau_2^m,\\0,\,\text{if}\,S_a\notin\alpha_2^m\,(\text{or})S_a\notin\tau_2^m.\end{cases} \quad (31)$$

Hence, $S_a\in\alpha_2^m\Rightarrow\alpha_2(S_a)\geq t$ and $S_a\in\tau_2^m\Rightarrow\tau_2(S_a)\leq m$.

$$\begin{aligned}
&\Rightarrow \varrho_2(a) \geq m, \sigma_2(a) \geq m \text{ and} \tau_2(a) \leq m \Rightarrow a \in \varrho_2^m, \sigma_2^m, \text{and } a \in \tau_2^m,\\
&\Rightarrow (\varrho_2, \sigma_2, \tau_2)_m(a) = (\varrho_2, \sigma_2, \tau_2)(a),\\
&S_a \notin \alpha_2^m \Rightarrow \alpha_2(S_a) \leq m \,(\text{or}) S_a \notin \tau_2^m \Rightarrow \tau_2(S_a) \geq m,\\
&\Rightarrow \varrho_2(a) \leq m, \sigma_2(a) \leq m \,(\text{or}) \tau_2(a) \geq m \Rightarrow a \notin \varrho_2^m, a \notin \sigma_2^m \text{and } a \notin \tau_2^m,\\
&\Rightarrow (\varrho_2, \sigma_2, \tau_2)_m(a) = 0.
\end{aligned} \tag{32}$$

So, a node in $Ln(G)_{(m)}$ gives an edge in $(G)_{(m)}$ which in turn in form of a node in $Ln((G)_{(m)})$.

$$\Rightarrow V - \text{set } Ln(G)_{(m)} \subseteq V - \text{set } (LnG)_{(m)}. \tag{33}$$

Similarly, it can be proven that

$$\Rightarrow V - \text{set } Ln(G)_{(m)} \supseteq V - \text{set } (LnG)_{(m)}. \tag{34}$$

Case 6. Let S_a, S_s be node in $Ln(G)_{(m)}$, where $a, s \in E$ and they consists common node in G.

$$(\alpha_2, \tau_2)_{(m)}(S_a, S_s) = \begin{cases} (\alpha_2, \tau_2)(S_a, S_s) \text{if } (S_a, S_s) \in (\alpha_2, \tau_2)^m, \\ 0, \text{otherwise.} \end{cases} \tag{35}$$

Subcase 7. If $(S_a, S_s) \in (\alpha_2, \tau_2)^{(m)} \Rightarrow \alpha_2(S_a, S_s) \geq m$.

$$\begin{aligned}
&\tau_2(S_a, S_s) \leq m \Rightarrow \alpha_1(S_a) \wedge \alpha_1(S_s) \geq m, \tau_1(S_a) \vee \tau_1(S_s) \leq m,\\
&\Rightarrow \varrho_2(a) \wedge \varrho_2(s) \geq m, \sigma_2(a) \wedge \sigma_2(s) \geq m, \tau_2(a) \vee \tau_2(s) \leq m,\\
&\text{i.e., } a, s \in \varrho_2^m, a, s \in \sigma_2^m, \text{and } a, s \in \tau_2^m. \text{ If are the end vertices of } a \text{ and } s, \text{ respectively, as } a, s \in (\varrho_2, \sigma_2, \tau_2)^m,\\
&\Rightarrow (\varrho_1, \sigma_1, \tau_1)(b_a), (\varrho_1, \sigma_1, \tau_1)(v_a), (\varrho_1, \sigma_1, \tau_1)(b_s), (\varrho_1, \sigma_1, \tau_1)(v_s) \geq t,\\
&\Rightarrow b_a, v_a, b_s, v_s \in (\varrho_1, \sigma_1, \tau_1)^m, b_a, v_a, b_s, v_s \in (\varrho_1, \sigma_1, \tau_1)_m.
\end{aligned} \tag{36}$$

We know that by using equations (6) as union of S_a and S_s is nonempty, $a = (b_a, w_a, v_a)$ and $s = (b_s, w_s v_s)$ have a common node, where $a, s \in (\varrho_1, \sigma_1, \tau_1)_m$. So, the respective S_a and S_s will be node in $Ln(G_{(m)})$ and union of S_a and S_s is nonempty. Therefore, (S_a, S_s) will be arc in $Ln(G_{(m)})$ such that the membership value of (S_a, S_s) is $Ln(G_{(m)})_{\alpha_2} = \varrho_2(a) \wedge \varrho_2(s) = (\alpha_2)_m(S_a, S_s)$, indeterminate value of (S_a, S_s) is $Ln(G_{(m)})_{\alpha_2} = \sigma_2(a) \wedge \sigma_2(s) = (\alpha_2)_m(S_a, S_s)$, and the nonmembership value of (S_a, S_s) is $Ln(G_{(m)})_{\sigma_2} = \tau_2(a) \wedge \tau_2(s) = (\sigma_2)_m(S_a, S_s)$.

Subcase 8. Let

$$\begin{aligned}
&(\alpha_2, \tau_2)_{(m)}(S_a, S_s) = 0,\\
&\text{i.e} (S_a, S_s) \notin \alpha_2^m \,(\text{or}) (S_a, S_s) \notin \tau_2^m,\\
&\Rightarrow \alpha_1(S_a, S_s) < m \,(\text{or}) \alpha_2(S_a, S_s) > m,\\
&\Rightarrow \alpha_1(S_a) \wedge \alpha_1(S_s < m \,(\text{or}) \alpha_1(S_a) \vee \alpha_1(S_s > m,\\
&\Rightarrow \varrho_2(a) \wedge \varrho_2(s) < m, \sigma_2(a) \wedge \sigma_2(s) < m, \text{or} \tau_2(a) \vee \tau_2(s) > m,\\
&\Rightarrow \text{The edges } a, s \in E \text{ have a node in common, but at least either a (or)} s \notin (\varrho_2, \sigma_2, \tau_2)_{(m)}.
\end{aligned} \tag{37}$$

So, the respective S_a and S_s will not become node in $Ln(G_{(m)})$, and (S_a, S_s) will not become an arc in $Ln(G_{(m)})$. So, by using Subcases 7 and 8,

$$\Rightarrow E - \text{set } Ln(G)_{(m)} \subseteq E - \text{set } (LnG)_{(m)}. \tag{38}$$

Similarly, it can be proven that

$$\Rightarrow E - \text{set } Ln(G)_{(m)} \supseteq E - \text{set } (LnG)_{(m)}. \tag{39}$$

By using equations (18), (21), and (27), $Ln(G)_{(m)} = (LnG)_{(m)}$

Theorem 4. *For any level* $0 < m \leq 1$, $((Ln(G))^{(m)} = (Ln(G)^{(m)}))$.

5. Degree of an Edge in Truncations of Pic-FG

5.1. Degree of an Edge in Lower Truncation of Pic-FG

$$\begin{aligned}
d_{(G)_{(m)}}(ba) &= \left(d_{\varrho(G)_{(m)}}(ba), d_{\sigma(G)_{(m)}}(ba), d_{\tau(G)_{(m)}}(ba)\right),\\
d_{\varrho(G)_{(m)}}(ba) &= \sum_{bw\in E_{(m)}, w\neq a} \varrho_{2_{(m)}}(bw) + \sum_{\wr\in E_{(m)}, w\neq b} \varrho_{2_{(m)}}(\wr), \quad \forall ba\in E_{(m)},\\
&= \sum_{bw\in E_{(m)}, w\neq a} \varrho_2(bw) - \sum_{bw\in E_{(m)}, w\neq v \text{ and}_{\varrho_2(bw)<m}} \varrho_2(bw) + \sum_{\wr\in E_{(m)}, w\neq b} \varrho_2(\wr) - \sum_{\wr\in E_{(m)}, w\neq b \text{ and}_{\varrho_2(\wr)<m}} \varrho_2(\wr), \quad \forall ba\in E_{(m)},\\
&= d_{\varrho(G)}(ba) - \sum_{bw\in E_{(m)}, w\neq v \text{ and}_{\varrho_2(bw)<m}} \varrho_2(bw) - \sum_{\wr\in E_{(m)}, w\neq b \text{ and}_{\varrho_2(\wr)<m}} \varrho_2(\wr), \quad \forall ba\in E_{(m)},\\
d_{\sigma(G)_{(m)}}(ba) &= \sum_{bw\in E_{(m)}, w\neq a} \sigma_{2_{(m)}}(bw) + \sum_{\wr\in E_{(m)}, w\neq b} \sigma_{2_{(m)}}(\wr), \quad \forall ba\in E_{(m)},\\
&= \sum_{bw\in E_{(m)}, w\neq a} \sigma_2(bw) - \sum_{bw\in E_{(m)}, w\neq v \text{ and}_{\sigma_2(bw)<m}} \sigma_2(bw) + \sum_{\wr\in E_{(m)}, w\neq b} \sigma_2(\wr) - \sum_{\wr\in E_{(m)}, w\neq b \text{ and}_{\sigma_2(\wr)<m}} \sigma_2(\wr), \forall ba\in E_{(m)},\\
&= d_{\sigma(G)}(ba) - \sum_{bw\in E_{(m)}, w\neq v \text{ and}_{\sigma_2(bw)<m}} \sigma_2(bw) - \sum_{\wr\in E_{(m)}, w\neq b \text{ and}_{\sigma_2(\wr)<m}} \sigma_2(\wr), \quad \forall ba\in E_{(m)},\\
d_{\tau(G)_{(m)}}(ba) &= \sum_{bw\in E_{(m)}, w\neq a} \tau_{2_{(m)}}(bw) + \sum_{\wr\in E_{(m)}, w\neq b} \tau_{2_{(m)}}(\wr), \quad \forall ba\in E_{(m)},\\
&= \sum_{bw\in E_{(m)}, w\neq a} \tau_2(bw) - \sum_{bw\in E_{(m)}, w\neq v \text{ and}_{\tau_2(bw)>m}} \tau_2(bw) + \sum_{\wr\in E_{(m)}, w\neq b} \tau_2(\wr) - \sum_{\wr\in E_{(m)}, w\neq b \text{ and}_{\tau_2(\wr)>m}} \tau_2(\wr), \quad \forall ba\in E_{(m)},\\
&= d_{\tau(G)}(ba) - \sum_{bw\in E_{(m)}, w\neq v \text{ and}_{\tau_2(bw)>m}} \tau_2(bw) - \sum_{\wr\in E_{(m)}, w\neq b \text{ and}_{\tau_2(\wr)>m}} \tau_2(\wr), \quad \forall ba\in E_{(m)}.
\end{aligned} \tag{40}$$

5.2. Degree of an Edge in Upper Truncation of Pic-FG.

$$\begin{aligned}
d_G^{(m)}(ba) &= \left(d_{\varrho(G)}^{(m)}(ba), d_{\sigma(G)}^{(m)}(ba), d_{\tau(G)}^{(m)}(ba)\right),\\
d_{\varrho(G)}^{(m)}(ba) &= \sum_{bw\in E^{(m)}, w\neq a} \varrho_2^{(m)}(bw) + \sum_{\wr\in E^{(m)}, w\neq b} \varrho_2^{(m)}(\wr), \quad \forall ba\in E^{(m)},\\
&= \sum_{bw\in E^{(m)}, w\neq a} \varrho_2(bw) - \sum_{bw\in E^{(m)}, w\neq v \text{ and}_{\varrho_2(bw)\geq m}} (\varrho_2(bw)-m) + \sum_{\wr\in E_{(m)}, w\neq b} \varrho_2(\wr) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and}_{\varrho_2(\wr)\geq m}} (\varrho_2(\wr)-m), \forall ba\in E^{(m)},\\
&= d_{\varrho(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and}_{\varrho_2(bw)>m}} (\varrho_2(bw)-m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and}_{\varrho_2(\wr)>m}} (\varrho_2(\wr)-m), \quad \forall ba\in E^{(m)},\\
d_{\sigma(G)}^{(m)}(ba) &= \sum_{bw\in E^{(m)}, w\neq a} \sigma_2^{(m)}(bw) + \sum_{\wr\in E^{(m)}, w\neq b} \sigma_2^{(m)}(\wr), \forall ba\in E^{(m)}\\
&= \sum_{bw\in E^{(m)}, w\neq a} \sigma_2(bw) - \sum_{bw\in E^{(m)}, w\neq v \text{ and}_{\sigma_2(bw)\geq m}} (\sigma_2(bw)-m) + \sum_{\wr\in E_{(m)}, w\neq b} \sigma_2(\wr) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and}_{\sigma_2(\wr)\geq m}} (\sigma_2(\wr)-m), \forall ba\in E^{(m)},\\
&= d_{\sigma(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and}_{\sigma_2(bw)>m}} (\sigma_2(bw)-m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and}_{\sigma_2(\wr)>m}} (\sigma_2(\wr)-m), \quad \forall ba\in E^{(m)},
\end{aligned}$$

$$d^{(m)}_{\tau(G)}(ba) = \sum_{bw \in E^{(m)}, w \neq a} {\tau_2}^{(m)}(bw) + \sum_{\wr \in E^{(m)}, w \neq b} {\tau_2}^{(m)}(\wr), \quad \forall ba \in E^{(m)}$$
$$= \sum_{bw \in E^{(m)}, w \neq a} \tau_2(bw) - \sum_{bw \in E^{(m)}, w \neq v \text{ and}_{\tau_2(bw) \leq m}} (\tau_2(bw) - m) + \sum_{\wr \in E_{(m)}, w \neq b} \tau_2(\wr) - \sum_{\wr \in E^{(m)}, w \neq b \text{ and}_{\tau_2(\wr) \leq m}} (\tau_2(\wr) - m), \quad \forall ba \in E^{(m)},$$
$$= d_{\tau(G)}(ba) - \sum_{bw \in E^{(m)}, w \neq v \text{ and}_{\tau_2(bw) < m}} (\tau_2(bw) - m) - \sum_{\wr \in E^{(m)}, w \neq b \text{ and}_{\tau_2(\wr) < m}} (\tau_2(\wr) - m), \quad \forall ba \in E^{(m)}. \tag{41}$$

Theorem 5. *Let G be Pic-FG such that $\varrho_2(ba) \geq m$, $\sigma_2(ba) \geq m$ $\tau_2(ba) \leq m$, $\forall$ $ba \in E$, where $0 < m \leq 1$. Then, for any $ba \in\in E_{(t)}$, $d_{G_{(t)}}(ba) = d_G(ba)$.*

Proof. Since

$$\begin{aligned}
&d_{\varrho(G)_{(m)}}(ba) = d_{\varrho(G)}(ba) - \sum_{bw \in E_{(m)}, w \neq v \text{ and}_{\varrho_2(bw) < m}} \varrho_2(bw) - \sum_{\wr \in E_{(m)}, w \neq b \text{ and}_{\varrho_2(\wr) < m}} \varrho_2(\wr), \forall ba \in E_{(m)},\\
&\Rightarrow d_{\varrho(G)_{(m)}}(ba) = d_{\varrho(G)}(ba),\\
&d_{\sigma(G)_{(m)}}(ba) = d_{\sigma(G)}(ba) - \sum_{bw \in E_{(m)}, w \neq v \text{ and}_{\sigma_2(bw) < m}} \sigma_2(bw) - \sum_{\wr \in E_{(m)}, w \neq b \text{ and}_{\sigma_2(\wr) < m}} \sigma_2(\wr), \forall ba \in E_{(m)},\\
&\Rightarrow d_{\sigma(G)_{(m)}}(ba) = d_{\sigma(G)}(ba),\\
&d_{\tau(G)_{(m)}}(ba) = d_{\tau(G)}(ba) - \sum_{bw \in E_{(m)}, w \neq v \text{ and}_{\tau_2(bw) > m}} \tau_2(bw) - \sum_{\wr \in E_{(m)}, w \neq b \text{ and}_{\tau_2(\wr) > m}} \tau_2(\wr), \forall ba \in E_{(m)},\\
&\Rightarrow d_{\tau(G)_{(m)}}(ba) = d_{\tau(G)}(ba).
\end{aligned} \tag{42}$$

□

Theorem 6. *Let G be a Pic-FG such that $\varrho_2(ba) = i_1$, $\sigma_2(ba) = i_2$, and $\tau_2(ba) = i_3$, $\forall$ $ba \in E$, where i_1, i_2 and i_3 are constants. Then, for any $ba \in\in E^{(m)}$.*

$$d^{(m)}_G(ba) = \begin{cases} d_G(ba), & \text{if } i_1 < m, i_2 < m, \text{and } i_3 > m,\\ d_G(ba) - (i - m) d_{G*}(ba), & \text{if } i_1 \geq m, i_2 \geq m, \text{and } i_3 \leq m. \end{cases} \tag{43}$$

Proof. Let $i_1 < m$.

$$\begin{aligned}
d^{(m)}_{\varrho(G)}(ba) &= d_{\varrho(G)}(ba) - \sum_{bw \in E^{(m)}, w \neq v \text{ and}_{\varrho_2(bw) > m}} (\varrho_2(bw) - m) - \sum_{\wr \in E^{(m)}, w \neq b \text{ and}_{\varrho_2(\wr) > m}} (\varrho_2(\wr) - m),\\
&= d_{\varrho(G)}(ba) - \sum_{bw \in E^{(m)}, w \neq v \text{ and } i_1 > m} (i_1 - m) - \sum_{\wr \in E^{(m)}, w \neq b \text{ and } i_1 > m} (i_1 - m),\\
\Rightarrow d^{(m)}_{\varrho(G)}(ba) &= d_{\varrho(G)}(ba).
\end{aligned} \tag{44}$$

Let $i_2 < m$.

$$\begin{aligned} d^{(m)}_{\sigma(G)}(ba) &= d_{\sigma(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and}_{\sigma_2(bw)>m}} (\sigma_2(bw) - m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and}_{\sigma_2(\wr)>m}} (\sigma_2(\wr) - m), \\ &= d_{\sigma(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and } i_1>m} (i_1 - m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and } i_1>m} (i_1 - m), \\ \Rightarrow d^{(m)}_{\sigma(G)}(ba) &= d_{\sigma(G)}(ba). \end{aligned} \tag{45}$$

Let $i_3 > m$.

$$\begin{aligned} d^{(m)}_{\tau(G)}(ba) &= d_{\tau(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and}_{\tau_2(bw)<m}} (\tau_2(bw) - m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and}_{\tau_2(\wr)<m}} (\tau_2(\wr) - m), \\ &= d_{\tau(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and } i_1<m} (i_1 - m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and } i_1<m} (i_1 - m), \\ \Rightarrow d^{(m)}_{\tau(G)}(ba) &= d_{\tau(G)}(ba). \end{aligned} \tag{46}$$

Hence, $d^{(m)}_G(ba) = d_G(ba)$.

Let $i_1 \geq m$.

$$\begin{aligned} d^{(m)}_{\varrho(G)}(ba) &= d_{\varrho(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{and}_{\varrho_2(bw)>m}} (\varrho_2(bw) - m) - \sum_{\wr\in E^{(m)} w\neq b\text{and}_{\varrho_2(\wr)>m}} (\varrho_2(\wr) - m), \\ &= d_{\varrho(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v\text{and } i_1>m} (i_1 - m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and } i_1>m} (i_1 - m), \\ &= d_{\varrho(G)}(ba) - (i_1 - m)(d_G * (b) - 1) - (i_1 - m)(d_G * (a) - 1), \\ &= d_{\varrho(G)}(ba) - (i_1 - m)(d_G * (b) + d_G * (a) + 2), \\ &= d_{\varrho(G)}(ba) - (i_1 - m) d_G * (ba). \end{aligned} \tag{47}$$

Let $i_2 \geq m$.

$$\begin{aligned} d^{(m)}_{\sigma(G)}(ba) &= d_{\sigma(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and}_{\sigma_2(bw)>m}} (\sigma_2(bw) - m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and}_{\sigma_2(\wr)>m}} (\sigma_2(\wr) - m), \\ &= d_{\sigma(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and } i_1>m} (i_1 - m) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and } i_1>m} (i_1 - m), \\ &= d_{\sigma(G)}(ba) - (i_1 - m)(d_G * (b) - 1) - (i_1 - m)(d_G * (a) - 1), \\ &= d_{\sigma(G)}(ba) - (i_1 - m)(d_G * (b) + d_G * (a) + 2), \\ &= d_{\sigma(G)}(ba) - (i_1 - m) d_G * (ba). \end{aligned} \tag{48}$$

Let $i_3 \leq m$.

$$\begin{aligned} d^{(m)}_{\tau(G)}(ba) &= d_{\tau(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and}_{\tau_2(bw)<m}} \left(\tau_2(bw) - m\right) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and}_{\tau_2(\wr)<m}} \left(\tau_2(\wr) - m\right) \\ &= d_{\tau(G)}(ba) - \sum_{bw\in E^{(m)}, w\neq v \text{ and } i_1<m} \left(i_1 - m\right) - \sum_{\wr\in E^{(m)}, w\neq b \text{ and } i_1<m} \left(i_1 - m\right) \\ &= d_{\tau(G)}(ba) - \left(i_1 - m\right)\left(d_G * (b) - 1\right) - \left(i_1 - m\right)\left(d_G * (a) - 1\right) \\ &= d_{\tau(G)}(ba) - \left(i_1 - m\right)\left(d_G * (b) + d_G * (a) + 2\right) \\ &= d_{\tau(G)}(ba) - \left(i_1 - m\right) d_G * (ba). \end{aligned} \tag{49}$$

Hence, $d^{(m)}_G(ba) = d_G(ba) - (i - m)d_G * (ba)$. □

6. Conclusion

Graph theory has many applications in other fields of mathematics. Int-FGs are the backbone of many real systems, such as scheduling, networks, and image segmentation. However, due to the missing neutrality function, Int-FG is limited to representing some systems. To improve real systems, Pic-FG works better. Pic-FG is a generalization of Int-FG due to the extra neutrality membership function. To improve the fuzzy theory, we have introduced the Up-T and Low-T of Pic-FG and Pic-FLnG. In this paper, the degree of an edge in truncations of Pic-FG is also discussed. We have noticed some limitations to this work. The focus of this research was only on Pic-FGs and their related network systems. It is not always possible to collect the real data. In the future, we will apply some operations to bipolar Pic-FG.

Data Availability

The data used to support the findings of the study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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