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Research Article

Simulation of Tennis Spinning Ball Flight Path Based on Fuzzy Reasoning Algorithm

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Topspin is one of the most widely used hitting techniques in a tennis match and it is an effective tool to win over the opponent. Hence, flight path simulation of a spinning ball can be a tremendous analysis tool to help tennis players perfect their game. This article proposes a fuzzy logic model based on the principles of kinematics and mechanics. This study analyzes the physical characteristics of a spinning ball during the flight process, which are divided into two categories: the characteristics of the ball on impact (including the floating and rotating it causes) and the landing rebound characteristics. These two characteristics are considered as the constraints of the flight path simulation and the inputs of the fuzzy logic model. Fuzzy logic is used to fuzzify the impact and landing rebound information of the ball based on the knowledge base, solve the problem, and finally defuzzify the results into crisp outputs, that is, accurate flight trajectory. The simulation results show that the estimation error of the proposed model is lower than 3.7 cm/s and 0.9°, and the success rate of accurate topspin execution is 100%, indicating that the proposed model is effective to train tennis players.

1. Introduction

Tennis is known as the fifth most popular sport in the world [1, 2]. It is usually played between two single players (Singles) or two teams of two players (Doubles). Each player uses a racket to play tennis across the tennis court against their opponent who is on the other side of the net. The origin and development of tennis can be summed up in four words: it was bred in France, born in England, began to popularize and form a climax in the United States, and now it is popular all over the world.

The trajectory of a tennis ball's flight is extensively studied in the literature [3]. Topspin is a common technique resulting from a low to high swing path in the forehand groundstroke while keeping the racket surface vertical [4–6]. Accurate execution of this technique requires the tennis player to hone their technique and reduce their error. Developing a model to simulate the ball's trajectory in the case of a topspin hit would help coaches and players to better analyze their technique and improve their accuracy by training based on the analysis results [7]. Topspin is known for its high flying range, fast descent, small bounce angle, and large forward momentum [8–10]. The racket swings and rubs the whole ball from the lower back to the upper front and rotates it in the same direction. When the racket is hitting the ball, it will increase the amplitude of the upward-lifting and swinging, so that the ball will spin up sharply.

The rotation of a tennis ball in topspin has a very complex mechanical principle. Using the aerodynamic principle to analyze the topspin ball is a continuous action under different kinds of forces in the flight process [11–14]. Topspin creates translation and rotation kinetic energy [15, 16], making it difficult for the opponent to return to the ball. During the match, topspin not only bypasses the players at the net, but also gives the players in the baseline a false impression of going out of the boundaries. It is not easy to master topspin. Some players overemphasize the topspin of the ball which results in the lack of speed. Some players hit the ball with speed, but the topspin of the ball is not strong enough which may not be as accurate. Hence, it is very important to study the flight path simulation of topspin to improve the accuracy of players in executing this technique.

Although there are studies that investigate the ball trajectory simulation in some sports such as golf [17], table tennis [18–21], football [22–25], basketball [26], baseball [27], and handball [28], due to the difference between the physical characteristics of the ball that is used in these games, the simulation models are not applicable from one study to the other. There are multiple studies that strictly focus on investigating the physical behavior and characteristics of a tennis ball such as its impact with a tennis racket [29, 30] or a court surface [31], aerodynamics [32, 33], bounce physics [34], or simulating and predicting its impact behavior [35] and flight trajectory [36, 37] using mathematical models and computer algorithms. Among the most prominent studies on tennis ball simulation, Glynn et al. [35, 38] proposed to use forward dynamic computer simulation to simulate the physical characteristics of the impact between a racket and a tennis ball. And in another study, which is much closer in subject to this study, Kwon et al. [3] proposed to use correlational statistics to model and confirm the relationship between topspin angular velocity and racket kinematics. These studies account for parameters such as racket speed and impact angle. In a practical scenario, there are many more known and unknown parameters such as temperature or environmental pressure that impact the physical behavior of a ball and consequently its flight trajectory. However, such studies only rely on models which work with crisp values and do not account for these extra values. Methods such as fuzzy logic can account for these uncertainties to some extent.

Based on the concept of fuzzy reasoning and using the method of fuzzy logic to simulate human thinking and reasoning, the fuzzy information is comprehensively analyzed, and the fuzzy rules are deduced by using the fuzzy mathematical method [39–42]. A fuzzy reasoning algorithm is an advanced calculation framework based on the concepts of fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning. By using the reasoning mechanism, the high-precision output or conclusion can be obtained according to the known rules and facts. The fuzzy method has a good real-time performance and does not need to know the precise mathematical model of the controlled object [10, 43–45], which is suitable for the real-time requirements of tennis ball flight trajectory simulation with fast motion. Therefore, this article studies the flight path simulation of tennis topspin based on the fuzzy inference algorithm and discusses the flight motion law of tennis topspin starting from principles of mechanics. This study helps the players or practitioners to gain a more in-depth understanding of the trajectory of the topspin ball and continuously improve their skill in topspin the ball at a professional competitive level.

The contributions of this study can be summarized as follows:

(1) Proposing a mathematical model for tennis spinning and bouncing behavior

(2) Incorporating the fuzzy inference concept in the proposed mathematical model to account for uncertainties

The rest of this article is organized as follows: Section 2 reviews the most relevant studies in this field and discusses the research gap in the literature. Section 3 defines the target problem in this article. Section 4 reports the simulation and experimental setup and the obtained results. Section 5 discusses the practical implications and use cases of the proposed model. And finally, Section 6 concludes this article.

2. Literature Review

In this section, the researches that focus on tennis simulations and modeling, especially tennis ball physics modeling, are reviewed and the research gap that this study intends to fill is discussed.

2.1. Tennis Modeling. There have been many studies on the aerodynamics of tennis balls and how they behave in the environment in presence of forces such as gravity [46, 47]. Also, there have been some experimental studies that focus on gathering data on practical scenarios for different parameters and how a tennis ball reacts in a specific environment [7, 29]. Furthermore, there are more recent studies that have been conducted in the wake of technologies such as virtual reality and examine the accuracy and effectiveness of such environments in simulating sports such as tennis [37, 48]. However, researches that focus on a specific technique and try to use the obtained model both in computer simulations and practically are scarce. In this section, all the studies that focus on tennis ball movement modeling or simulation are reviewed.

Brody [34] modeled the bouncing action of a tennis ball from the court's surface. He made some simplified assumptions about the physical characteristics of the ball. He assumed that the ball is a hollow rigid object with no deformation and the court's surface is also hard and will not be deformed upon the ball's impact to the surface. Additionally, he used the simple law of friction to account for friction force on the court's surface and assumed that the horizontal and vertical motions of the ball are independent.

In another study, Jafri and Vance [31] modeled the impact of a tennis ball on a flat surface. They used a more complex two-mass model which accounts for the vertical translational motion with a spring and a damper in the vertical direction while using a torsional spring and a damper for the rotational motion of the masses.

Alam et al. [32] focused on the spinning motion of a tennis ball and modeled the effect of the ball's aerodynamics on the motion. As opposed to previous studies that considered the tennis ball to be a simple object, they accounted for the surface complexities of the ball in their model.

Glynn et al. [35] simulated the impact effects between a racket and a tennis ball using forward dynamic simulation. They proposed a detailed viscoelastic model for both the racket surface and the tennis ball. Their experiments indicate a less than 3% root mean squared error for the simulated rebound velocity in the range of 16 m/s to 27 m/s.

In one of the most prominent studies in this field, Kwon et al. [3] proposed to use racket impact angle, horizontal and vertical racket velocity before impact, racket trajectory, and hitting zone length as the effective parameters in modeling a topspin's angular velocity and the player's forehand accuracy. Their experiments and simulation results indicate that a racket impact angle of 70° to 85° is the most suitable angle for the proper execution of topspin technique. They also realized that increasing the racket's vertical velocity before impact is correlated with topspin angular velocity. However, the results show that parameters such as hitting zone length, racket trajectory, and racket horizontal velocity do not impact the angular velocity of the ball or the accuracy of the player. Hence, these parameters are not covered in this study.

Nadar et al. [27] proposed a model for tennis balls and baseballs based on computational fluid dynamics [49, 50]. They compared the two types of balls and defined certain characteristics such as surface roughness to differentiate between the two. They also propose to use free stream velocity and tangential direction of the ball movement to model the flight trajectory of the ball but fail to provide enough experimental results to thoroughly demonstrate the applicability of their proposed model in practical scenarios.

Studies such as [36, 51] mostly rely on image processing techniques to determine or predict the physical behavior of tennis balls or their flight trajectory. Ke et al. [36] proposed to use machine learning and neural networks to determine the impact coordinates of a tennis ball under certain conditions. Their proposed method requires a training dataset to train the model and video cameras to record each player's training sessions and analyze their training effectiveness and efficiency post-training.

2.2. Research Gap. As discussed in the previous subsection, there are a limited number of studies that strictly focus the topspin technique in tennis. However, multiple studies attempt to model different physical attributes and behavior in a tennis match, especially the physical interaction between the racket and the ball, and between the ball and the court surface.

Table 1 tabulates all the studies that have been discussed in the previous subsection. It is evident from this table that the topspin technique has the lowest number of researches dedicated to it. Also, all the other studies use deterministic models that do not account for data noise and unknown parameters that might impact the results and are not considered in the model. The proposed model in this article utilizes fuzzy inference while taking advantage of previously proposed models in the literature. Taking advantage of fuzzy inference reduces the negative impact of uncertainties in input data and can handle the impact of other unknown and ignored parameters, such as temperature, to some extent.

3. Problem Statement

This section defines the main problem. But before getting into the problem statement and discussing the proposed fuzzy inference model, basic physics principles of a tennis ball and its impact characteristics need to be discussed. Hence, the next subsection reports all the considered parameters and their notation. Then, the physics principles of the problem are discussed (Table2).

3.1. Notation List. 3.2. Kinematic Analysis of Tennis. Topspin has two stages in the flight process, namely the impact and the landing rebound of the ball [52]. Ideally, a tennis ball does not rotate in the air. However, most of the time it is affected by gravity, buoyancy, additional mass force, air resistance, and hitting height [53], which would lead to rotation during flight. The air resistance is opposite of the direction of tennis movement as shown in Figure 1. The impact of other parameters (gravity, buoyancy, additional mass force, and hitting height) is also depicted in this figure. When the speed of the ball changes very little, the additional mass force can be ignored.

3.2.1. Physical Characteristics of the Impact of a Tennis Ball. Let V be the speed of the ball. Then the air resistance, F, is calculated as:

$$F = \mu V. \tag{1}$$

If the tennis ball is impacted in the front, the loss function ΔT after the collision is related to the coefficient of recovery K [54], and the gravity on the tennis ball in flight m. If m^t is the mass of the arm, V_1 and V_2 represent the speed of the ball before and after the collision, respectively, then ΔT is defined as:

$$\begin{aligned}\Delta T &= \frac{m^t m}{2\left(m^t + m\right)} \\ &= \left(1 - K^2\right)\left(V_1 - V_2\right)^2.\end{aligned} \tag{2}$$

The change in tennis ball velocity V_2 after the collision can be expressed as:

$$V_2 = V_2 + (1 + K)\frac{m^t}{m^t + m}\left(V_1 - V_2\right). \tag{3}$$

If $J_z = (2/3)mR^2$ and w and R are the angular velocity of tennis and the resistance, respectively, then the kinetic energy of a tennis ball after a collision is expressed as:

$$T_2 = \frac{1}{2}mV_2^{t2} + \frac{1}{2}J_z w^2. \tag{4}$$

The trajectory of a spinning tennis ball is always subject to resistance R [55], which is the opposite of the direction of motion. Its range is usually in the high Reynolds range. If C_D is the resistance coefficient, then it is calculated as:

$$R = C_D\frac{1}{8}\pi d^2 \rho_0 V^2. \tag{5}$$

It has been shown that C_D can be expressed in three different formulas according to different Reynolds number ranges as:

Table 1: Survey on related works.

Reference	Research field	Method
[34]	Court impact physics	Simplified ball and surface model and simple law of friction
[31]	Court impact physics	Two-mass model, spring and damper in vertical direction, and torsional spring and damper for rotational motion
[32]	Spinning motion	Complex ball model
[35]	Racket impact physics	Viscoelastic model
[3]	Racket impact physics in topspin	Complex racket model
[27]	Spinning motion	Computational fluid dynamics
[36]	Racket impact physics and spinning motion	Machine learning image processing
This study	Racket impact physics, court impact physics, and spinning motion flight trajectory	Court surface and ball model, air and court friction model, and fuzzy inference

Table 2: The parameters, description, and related units of the problem.

Parameter	Description	Unit
V	Speed of the ball	Meter per second (m/s)
F	Air resistance	Newton (N)
K	Coefficient of recovery	-
m	Gravity on the ball	Meter per second squared (m/s^2)
m^t	Mass of the arm	Gram (g)
V_1	Speed of the ball before the collision	Meter per second (m/s)
V_2	Speed of the ball after the collision	Meter per second (m/s)
ΔT	Loss function after the collision	(m/s)^2
w	Angular velocity of the tennis ball	Radian per second (r/s)
T_2	Kinetic energy of the ball after collision	Joules (J)
C_D	Resistance coefficient	—
R_r	Reynolds number	—
F_c	Centripetal force	Newton (N)
v	Fluid velocity	Meter^3 per second (m^3/s)
p	Pressure	Dyne per square meter (D/m^3)
ρ	Air density	Gram per liter (g/l)
g	Gravity	Meter per second squared (m/s^2)
y	Potential	Joule per coulomb (J/c)
L	Lift	Newton (N)
C_L	Lift coefficient	—
λ	Threshold value of the fuzzy rule	—
d	Distance	Meter (m)
C_n'	Inference result	—
r^{COA}	Center of gravity of membership function	—

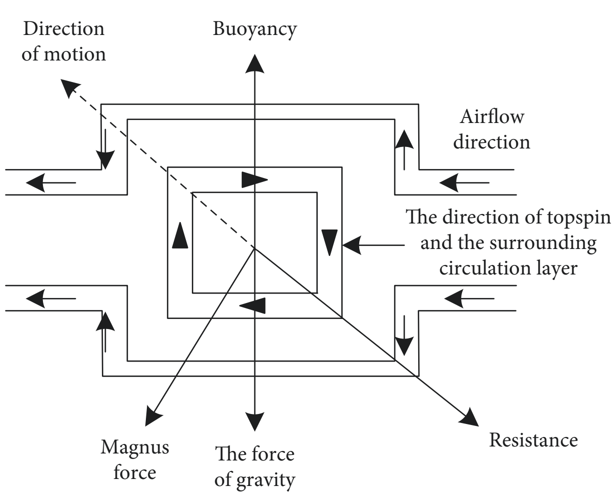

Figure 1: Aerodynamic analysis of the flight of a spinning tennis ball.

$$C_D = \frac{24}{R_r}\,(R_r < 1),$$
$$C_D = \frac{24}{R_r}\left(1 + \frac{R_t^{(2/3)}}{6}\right)(1 < R_r < 1000), \quad (6)$$
$$C_D \approx 0.44\left(1000 < R_r < 2 \times 10^5\right),$$

where R_r is the Reynolds number, and the resistance is proportional to the square of the velocity V.

After the tennis ball is hit, it encounters certain air resistance in the flight, which increases the pressure difference resistance. The vibration caused by the tennis ball being hit will produce simple harmonic vibration F_v to the air [56]. If H is the force amplitude, F_c is the centripetal force, and ϕ is the initial phase, then F_v is formulated as:

$$F_v = H \sin\left(F_c + \phi\right). \quad (7)$$

For a tennis ball in flight, the above parameters are all fixed values. Let v be the fluid velocity, p be the pressure, ρ be the air density, g be the acceleration of gravity, y be related to potential, and $c_1(4)$ be the constant, then the following equation would be obtained from the Bernoulli equation:

$$\frac{v^2}{2} + \frac{p}{\rho} = gy = c_1(4). \tag{8}$$

The interaction between the circulation and the airflow changes the streamline distribution when a rotating sphere moves in the environment, forming a certain pressure difference [57]. According to Bernoulli's law, the lift $L = (1/8) C_1 \pi d^2 \rho V^2$ and the lift coefficient $C_L = (8L/(\pi d^2 \rho V^2))$ are generated, where d is the diameter of the tennis sphere.

The difference between topspin and flat attack is that there is an upward and forward lifting action in topspin. Therefore, topspin causes the ball to have a movement of upward rotation around itself in the first stage of its flight. In topspin, the air pressure above the ball is large. Therefore, it will fall to the ground quickly when it drifts in the first stage after passing the net, which makes it hard to hit.

Compared to the flat shot, the rotation speed of the tennis ball is larger than the increase in V_1, so the rotation speed of the ball which is hit under the same conditions is larger than that of the flat shot in the second stage of flight. The pressure difference between the up and down airflow of the ball will further expand, resulting in its downward trend to increase simultaneously.

3.2.2. The Rebound Characteristics of Topspin. In the process of landing rebound after the ball is hit, its physical changes should be carried out from the horizontal and vertical force angles [58]. The force after the ball is hit includes different force changes in the horizontal and vertical directions. Hence, the speed of the ball is composed of horizontal and vertical components after being hit. In such an environment, the resultant force which changes the ball movement should be considered to analyze the final characteristics more rationally.

When a tennis ball moves forward in a horizontal throw, its velocity V is composed of horizontal velocity V_P and vertical velocity V_C. That is to say, the size of V after the rebound of the tennis ball is determined by the size of V_P and V_C, while the size of incident angle α_1 and reflection angle α_2 of the tennis ball is determined by the ratio of V_P and V_C at the moment of the landing of the ball. The above characteristic analysis is carried out in the simulation of the standing on state, excluding the external influence factors. This analysis result can explain the main relationship that causes the change in the speed of the ball being hit. The flatness of the side surface, the change of airflow, the gravity of the Earth, and the resistance of the controller will affect the impact and the rebound angle, but only to a lesser extent. The physical characteristics of the spinning ball when it is hit and the rebound characteristics of the landing of the spinning ball are regarded as the constraints of the simulation of the flight path of the ball.

3.3. Fuzzy Reasoning Algorithm

3.3.1. Principle of Fuzzy Reasoning. There are three basic reasoning modes in fuzzy reasoning [59], that is, fuzzy hypothetical reasoning, fuzzy refusal reasoning, and fuzzy syllogism reasoning. In this article, fuzzy hypothetical reasoning is used to simulate the tennis ball flight trajectory based on a fuzzy logic algorithm. Fuzzy hypothetical reasoning can be directly expressed as:

Rules: if x is A then y is B
Fact: x is A'
Conclusion: y is B'

In the fuzzy hypothetical reasoning of known facts, it is necessary to construct the corresponding fuzzy relation E [60] according to the fuzzy set in the fuzzy rules and then get the conclusion through a combination of known facts E.

If λ represents the threshold value of the fuzzy rule f, which is used to represent the application conditions of f, then the rule is defined as:

$$f x \text{ is } A \text{ then } y \text{ is } B(\lambda). \tag{9}$$

If the fact is known to be "x is A'," A' and A can be fuzzy matched, the conclusion "y is B'" can be obtained from the fuzzy hypothetical reasoning. If E represents the fuzzy relation between A and B, the fuzzy set B' can be obtained from the following composite operation:

$$B' = A' \circ E. \tag{10}$$

There are many ways to construct the fuzzy relation between two fuzzy sets. Here, the minimax rule of a conditional proposition is adopted, and its construction form is defined as:

$$A = \int_U \mu_A \frac{(u)}{u}, \quad B = \int_U \mu_B \frac{(v)}{v}. \tag{11}$$

Then the fuzzy relationship between A and B can be defined as:

$$E(A,B) = (A \times B) \cup (\overline{A} \times V) = \int_{U\times V} \frac{(\mu_A(u)\,\mu_B(v)) \vee (1-\mu_A(u))}{(u,v)}. \tag{12}$$

3.3.2. Establishing the Fuzzy Reasoning Algorithm. According to the kinematic characteristics of a spinning tennis ball, the flight path is determined by two factors, namely the impact of the racket and the landing rebound of the ball. As shown in Figure 2, the two inputs of the fuzzy reasoning algorithm are the impact of the tennis ball and the landing rebound of the tennis ball [61].

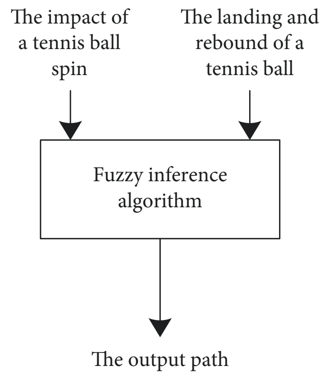

Figure 2: Fuzzy inference algorithm.

(1). Fuzzy sets and membership functions. The linguistic variable of the impact speed of the tennis ball is represented by d, the domain is $X = [0, 600]$, the fuzzy subset of its response is $A_i\,(i = 1, 2, 3, 4, 5)$, and the corresponding linguistic value is {"minimum," "small," "medium," "large," and "maximum"}, using trapezoid membership function [62], as shown in Figure 3.

For the landing rebound of the tennis ball, the proportion of the linear distance between two endpoints in the flight path and their projected distance in the x direction is determined [63]. The linguistic variable of the signal is t, the domain is $Y = [0, 1]$, the corresponding fuzzy set is $B_i\,(i = 1, 2, 3, 4, 5)$, and the corresponding linguistic value is {"minimum," "small," "medium," "large," and "maximum"}. The trapezoidal membership function is adopted as shown in Figure 4.

For the output, the linguistic variable is g, which represents the scenario in which the line deviating from the starting point is close to the x-axis [64], the domain is $Z = [-180, 180]$, the corresponding fuzzy subset is $C_i\,(i = 1, 2, 3, 4, 5)$, the corresponding linguistic weight is {"minimum," "small," "medium," "large," and "maximum"}. The trapezoid membership function is adopted as shown in Figure 5.

(2) Fuzzy inference rules. According to the starting point in the flight path and the requirements of the fast and stable motion, a series of fuzzy inference rules are developed as shown in Table 3.

In this article, Mamdani fuzzy inference [65, 66], namely the min-max inference method, is used for fuzzy logic and defuzzification. Considering that the distance is d, the proportion is t, that is, each fuzzy rule is represented as A_i and $B_i \Rightarrow C_k$, and $\wedge$ is min, that is, the minimum value, the inference result C_n' can be obtained as:

$$u' c_n(r) = u_{A_i}(d_0) \wedge u_{B_i}(h_0) \wedge u_{C_k}(r). \tag{13}$$

Hence, C' is the result of comprehensive reasoning on C_n'. Let $\vee$ be Max, that is, the maximum value, then:

$$u_{c'}(r) = u_{c_1'}(r) \vee u_{c_2'}(r) \vee \cdots \vee u_{c_i'}(r). \tag{14}$$

The rotation direction and angle of the moving object corresponding to the fuzzy set C' are obtained as:

$$r^{\mathrm{COA}} = \frac{\int_z r u c'(r) \mathrm{d}r}{\int_z u c'(r) \mathrm{d}r}. \tag{15}$$

The value of r^{COA} is the center of gravity of the membership function of the fuzzy set C'. The crisp defuzzied output results are the final obtained flight path of the tennis ball.

4. Results

The practice of tennis spinning shows that for high-level players, the hitting speed is the primary factor affecting their playing quality. For the same player, the hitting height is relatively fixed, but the hitting position and the swinging speed can be changed. Therefore, during the simulation, the tennis spinning track with different hitting speeds and fixed height is simulated. Experiments are performed on the flight path of tennis spinning ball with different hitting angles under the conditions of fixed hitting height and hitting speed. To simulate the flight path of a tennis ball, the proper coordinate system is established, and the effective fall point range is calculated to find the average speed error and the average method error.

4.1. Simulation Results at Different Impact Speeds. This experiment is used to determine the flight path parameters of a spinning ball, including the height from the racket to the ground, the impact speed, the angle between the initial speed and each axis, and the rotation speed. The simulation is divided into two parts: the first part studies the impact speed of tennis on the flight path under the same hitting height and hitting angle; the second part studies the impact of hitting angle on the flight path under the same hitting height and hitting speed. Table 4 shows the simulation parameters at different impact speeds.

Assuming that the impact coordinates are (1, 0, 2.8), Figure 6(a) is to simulate the relationship between the impact speed of the tennis ball and the x-coordinate of the landing point by using the method in this article. The effective range of x is (−4.115,0). Figure 6(b) shows the relationship between the impact speed of the ball and the y-coordinate of the landing point. The effective range of y is (11.885,18.285).

It can be seen from Figure 6 that in the case of considering air resistance and Magnus force, there is an error between the falling point of a spinning ball and the ideal falling point, and the faster the ball speed is, the greater the error is. It can be seen from Figure 6(a) that under the same impact height and angle, the larger the initial speed of the spinning ball is, the farther the x-axis falling point is from the centerline. Considering that the air resistance is not considered compared with Magnus's force, the falling point of the tennis ball on the x-axis is far from the centerline. It can be seen from Figure 6(b) that under the same impact height and angle, the larger the impact speed of tennis is, the farther the y-axis falling point becomes. Considering that the air resistance is not considered compared with Magnus's force, the falling point of the tennis ball on the y-axis is closer. The

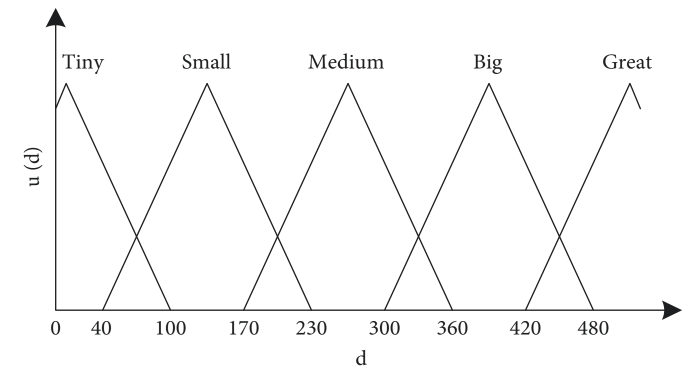

Figure 3: Membership function of the fuzzy subset A_i.

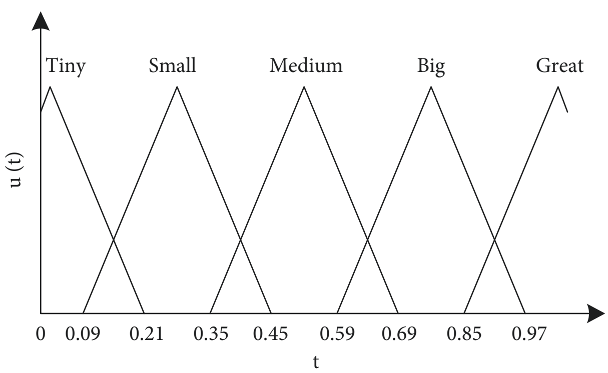

Figure 4: Membership function of the fuzzy subset B_i.

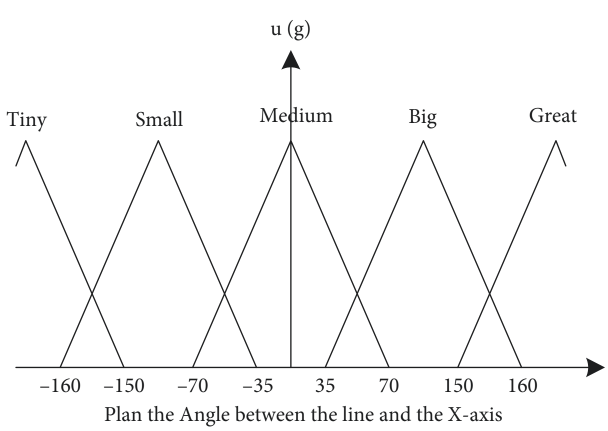

Figure 5: Membership function of the fuzzy subset C_i.

Table 3: Fuzzy inference rules.

Speed	Speed				
	Tiny	Small	Medium	Big	Great
Tiny	Tiny	Tiny	Tiny	Small	Medium
Small	Tiny	Small	Small	Medium	Big
Medium	Tiny	Small	Medium	Big	Great
Big	Small	Medium	Big	Big	Great
Great	Medium	Big	Great	Great	Great

Table 4: Simulation parameters at different impact speeds.

Project	Parameter value
Gravitational acceleration	9.7 m/s^2
Quality of tennis	0.056 kg
Air density	1.104 kg/m^3
Tennis ball diameter	0.0643 m
The height at which a tennis ball spins off the racket	2.7 m
The angle between the velocity of impact and the x-axis	97.77°
The angle between the velocity of impact and the y-axis	8.77°
The angle between the velocity of impact and the direction of the z-axis	99°
Angular velocity of rotation	49 rad/s
Lift coefficient	0.4
Resistance index	0.3

simulation verifies the influence of air resistance and Magnus force on the flight trajectory of the tennis ball.

4.2. Simulation Results under Different Impact Angles. Table 5 shows the simulation parameters of the impact angle's influence on the flight path of a spinning ball under the same impact height and speed. The angle here mainly refers to the angle between the racket surface and the z-axis.

Suppose that the impact point coordinates of the tennis ball are (1, 0, 2.8), Figure 7 (a) depicts the relationship between the impact angle and the x-coordinate of the impact point. The effective range of x is (−4.115,0). Figure 7 (b) shows the relationship between the impact angle of the tennis ball and the falling point. The effective range of y is (11.885,18.285).

In Figure 7, the impact angle of the x-axis is the angle between the tennis ball speed direction and the negative z-axis direction. It can be seen from Figure 7 that the faster the speed of the ball is, the larger the error between the ideal landing point and the actual landing point is when considering air resistance and Magnus force. It can be seen from Figure 7 (a) that under the same impact height and speed, the larger the impact angle of the tennis ball is, the closer the impact angle is to the horizontal direction and the farther the x-axis drop point would be. Considering the air resistance and Magnus force, the x-axis drop point of the tennis ball is

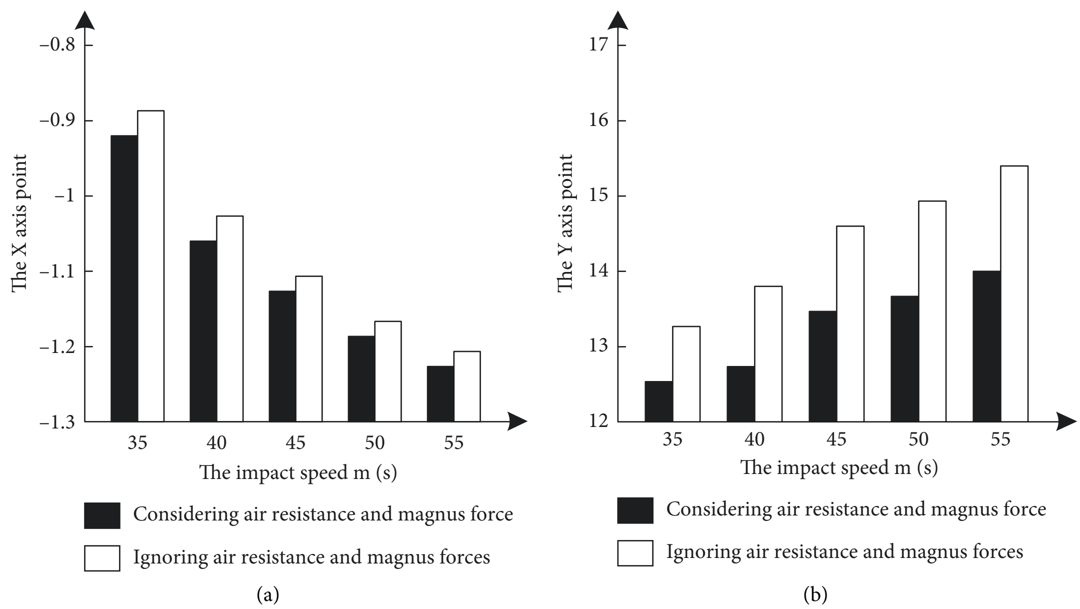

Figure 6: Relationship between the impact velocity of a tennis ball and the coordinates of its landing point: (a) x-coordinate and (b) y-coordinate.

Table 5: Simulation parameters without impact angle.

Project	Parameter value
Gravitational acceleration	9.7 m/s^2
Quality of tennis	0.056 kg
Air density	1.104 kg/m^3
Tennis ball diameter	0.0643 m
The height at which a tennis ball spins off the racket	2.7 m
The angle between the velocity of impact and the x-axis	97.77°
The speed with which a tennis ball spins	44 m/s
Angular velocity of rotation	49 rad/s
Lift coefficient	0.4
Resistance index	0.3

far away from the centerline than ignoring the air resistance and Magnus force. It can be seen from Figure 7(b) that under the same impact height and angle, the larger the impact angle of the tennis ball, the closer the impact angle is to the horizontal direction and the farther the y-axis drop point would be. Considering the air resistance and Magnus force, the distance of the tennis ball is closer when the y-axis drop point is less than when the air resistance and Magnus force are ignored. This simulation shows that in the tennis ball impact process, it is necessary to choose the right impact angle; otherwise, it would be easy to make mistakes in a topspin move.

4.3. Comparison of Simulation Results of Different Methods. To verify the simulation performance of this method, the experiment compares the speed error, the direction error, and the success rate of the spinning action executed by tennis players after training based on the current method and two flight path simulation methods based on multi-process state equation and data mining, respectively. The results are reported in Tables 6–8 . From Tables 6–8, it can be seen that the speed error, direction error, and spinning success rate of the players in the training process are significantly better than those of the two compared methods after using the method that is proposed in this article. The average speed error and the average method error are lower than 3.7 cm/s and 0.9°, respectively, and the spinning success rate of 10 players is 100%. When the multi-process state is used, the average speed error and the average method error of the athletes trained by the equation simulation method are lower than 6.4 cm/s and 1.7%, respectively, and the success rate of spinning is between 91.67% and 97.56%. Also, the average speed error and the average method error of the athletes trained by the data mining simulation method are lower than 6.1 cm/s and 2.8%, respectively, and the success rate of spinning is between 89.19% and 97.50%. The experimental results fully demonstrate that the proposed method positively impacts the tennis players' spinning training.

4.4. Discussion. This article proposes a simulation model for the flight trajectory of a topspin spinning ball based on the fuzzy logic algorithm. According to the kinematic characteristics of tennis spinning ball flight, it is found that the tennis spinning ball flight path is determined by two factors: the impact height, angle, and speed of the ball; and the landing rebound of the tennis spinning ball. By using the fuzzy inference algorithm, the information of these two factors is fuzzified by the fuzzy rules that are established previously and the defuzzification process is carried out to obtain crisp results. These results present an accurate output path obtained by the simulations. The results show that the

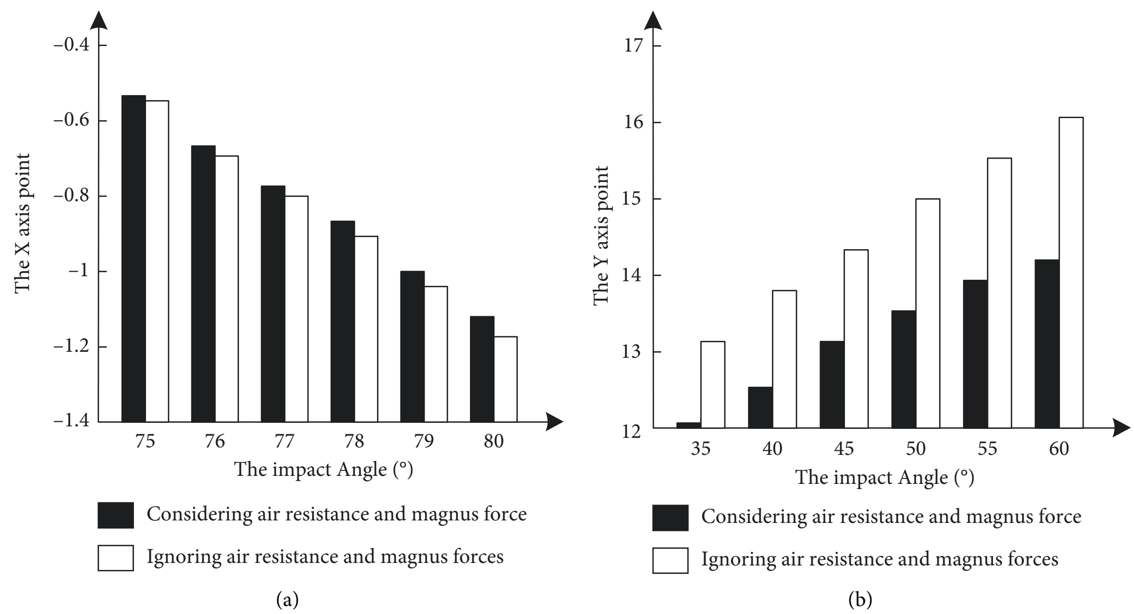

Figure 7: Relationship between the impact angle and the coordinates of its landing point: (a) x-coordinate and (b) y-coordinate.

Table 6: Training results of athletes after using this method.

Player number	Spin ball practice times (times)	Average error of flat velocity (cm/s)	Average directional error (°)	Spin success rate (%)
1	42	3.31	0.72	100
2	38	3.42	0.88	100
3	37	3.49	0.85	100
4	40	3.53	0.69	100
5	38	3.30	0.78	100
6	39	3.66	0.58	100
7	36	3.41	0.62	100
8	32	3.58	0.53	100
9	40	3.35	0.39	100
10	32	3.31	0.87	100

Table 7: Training results of athletes after using the method of multi-party process equation of state.

Player number	Spin ball practice times (times)	Average error of flat velocity (cm/s)	Average directional error (°)	Spin success rate (%)
1	40	5.82	0.93	95.00
2	41	5.44	1.68	97.56
3	38	6.22	1.44	94.74
4	42	5.76	1.39	95.24
5	36	5.29	1.41	91.67
6	35	5.81	0.99	97.14
7	38	6.06	1.06	94.74
8	36	6.33	1.35	97.22
9	35	5.74	1.60	94.29
10	40	5.90	1.28	95.00

x-axis deviation is not significant, and the y-axis change is more obvious. When the impact height and impact angle are the same, the faster the impact speed is, the larger the deviation of the ball's falling point is. Also, when the impact height and speed are the same, as the impact angle increases, the more obvious the ball's falling point deviation would be, especially at 79° to 81°. Comparing the results of the three different simulation methods shows that the average speed error, the average direction error, and the success rate of the tennis players accurately executing a topspin move are significantly better than those of the two other methods. The main reason for this method to achieve such good results is

Table 8: Training results of athletes after using data mining method.

Player number	Spin ball practice times (times)	Average error of flat velocity (cm/s)	Average directional error (°)	Spin success rate (%)
1	38	5.12	2.83	92.11
2	40	6.09	2.65	95.00
3	40	5.95	2.52	97.50
4	36	5.32	2.74	91.67
5	37	5.61	2.09	94.59
6	41	5.40	2.28	92.68
7	37	5.13	2.69	89.19
8	38	4.99	2.06	94.74
9	34	5.46	2.76	94.12
10	36	5.77	2.13	91.67

that this method adopts a fuzzy reasoning algorithm. As a branch of approximate reasoning, the fuzzy reasoning algorithm is the theoretical basis of fuzzy control and also a very important intelligent algorithm. The fuzzy reasoning algorithm has a strong ability in dealing with complex process control with uncertainty (e.g., input sensor or environment noise) and nonlinearity which is difficult to be modeled by traditional mathematical tools such as differential equation and is highly effective and complements other technologies and models perfectly.

To improve the scientific training level of the tennis topspin technique, the following three suggestions are presented:

(1) Tennis players should pay attention to the dynamic basic theory of ball flight to master the changing characteristics of topspin flight trajectory and improve their execution quality.

(2) Tennis players have a great difference in ball speed when hitting the ball. Hence, they should pay attention to the training of impact speed and adjust the impact angle properly when performing topspin to improve their success rate.

(3) In the relatively fixed period, when the players spin the ball, the ball speed of the players will not change much. Hence the players should pay attention to the adjustment of the impact angle, which should not fluctuate largely. They should gradually change the hitting angle in the training to find a more suitable angle.

5. Managerial Insights and Practical Implications

Simulating a tennis match with all the physical contributing parameters can be an invaluable tool in the hands of professional tennis players and their coaches. However, such a simulated environment requires very complex algorithms and powerful computer systems to process the computational load of such an environment. Hence, many separate studies and researches are required to be able to cover all the aspects of simulating such an environment. One of the aspects of simulating a tennis training environment is simulating different physical attributes of the related objects in the game. As discussed in Section 2, many studies focus on modeling the physics of rackets, balls, and court surfaces to determine the flight trajectory or impact coordinates of a tennis ball. However, instead of trying to model the whole behavior of the tennis ball in different situations and scenarios, this study only focuses on a single technique (topspin) and simulates the behavior of the ball of the court surface only with regards to this technique. Such an approach is much more fine-grained compared to other studies and as demonstrated in the previous section, outperforms them from an accuracy and efficiency perspective. This indicates that further studies in this field need to be focusing on individual techniques to achieve a certain level of accuracy. After covering a significant number of necessary tennis techniques, it would be possible to put all the obtained models together to develop an effective and accurate tennis training environment. This would help sport managers, coaches, and players tremendously and pushes the professionalism of the sport to levels that were believed to be impossible before the advent of such training tools.

6. Conclusions

The flight path of the tennis ball is an arc that causes the ball to drop sharply after passing the net. The player can hit a short diagonal ball and force the opponent to run out of the court to get the initiative. A topspin is also a good technique to disrupt the opponent's quick access to the balls that rebound from the surface of the court near the net. The lower topspin ball falls at the foot of the other side of the net, making it difficult to fight back. It is not hard to see that the spinning ball can attack and defend, has great power and high safety coefficient, and is a powerful weapon to defeat the enemy. This article proposes a simulation model for a spinning ball flight path based on the fuzzy logic algorithm in conjunction with the basic kinematic characteristics and mechanical principles of spinning balls. The proposed model uses fuzzy inference to simulate the high-precision spinning ball flight trajectory. The experiment results show that

(i) The average speed error is lower than 3.7 cm/s

(ii) The average method error is lower than 0.9°

(iii) The execution success rate of 10 players is 100%

The last observation is the direct result of improvement in the level of scientific training. In our future studies, we need to improve the fuzzy rule base and consider more relevant parameters such as temperature and more complex court surface properties to decrease the average speed error and average method error of the athletes. Also, different techniques such as serving are also a viable field for similar studies to develop a fully functioning tennis simulation environment step by step with accurate physical modeling.

Data Availability

There are no available data for this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] S. Das, "Top 10 Most Popular Sports in the World July 2022. Sports Browser," 2022, https://sportsbrowser.net/most-popular-sports/.
[2] B. Cao, W. Zhang, X. Wang, J. Zhao, Y. Gu, and Y. Zhang, "A memetic algorithm based on two_Arch2 for multi-depot heterogeneous-vehicle capacitated arc routing problem," *Swarm and Evolutionary Computation*, vol. 63, Article ID 100864, 2021.
[3] S. Kwon, R. Pfister, R. L. Hager, I. Hunter, and M. K. Seeley, "Influence of tennis racquet kinematics on ball topspin angular velocity and accuracy during the forehand groundstroke," *Journal of Sports Science and Medicine*, vol. 16, no. 4, pp. 505–513, 2017.
[4] S. A. Abbas, "Fusion of halo nucleus 6 He on 238 U: evidence for tennis-ball (bubble) structure of the core of the halo (even the giant-halo) nucleus," *Modern Physics Letters A*, vol. 34, no. 27, Article ID 1950221, 2019.
[5] L. Zhong, Z. Fang, F. Liu, B. Yuan, G. Zhang, and J. Lu, "Bridging the Theoretical Bound and Deep Algorithms for Open Set Domain Adaptation," *IEEE transaction on neural networks and learning systems*, pp. 1–15, 2021.
[6] X. Qin, L. Zhang, L. Yang, and S. Cao, "Heuristics to sift extraneous factors in Dixon resultants," *Journal of Symbolic Computation*, vol. 112, pp. 105–121, 2022.
[7] Z. Bańkosz and S. Winiarski, "The kinematics of table tennis racquet: differences between topspin strokes," *The Journal of Sports Medicine and Physical Fitness*, vol. 57, no. 3, pp. 202–213, 2017.
[8] Topspin, "Wikipedia," 2022, https://en.wikipedia.org/wiki/Topspin.
[9] Y. Wang, H. Wang, B. Zhou, and H. Fu, "Multi-dimensional prediction method based on Bi-LSTMC for ship roll," *Ocean Engineering*, vol. 242, Article ID 110106, 2021.
[10] J. Wang, J. Tian, X. Zhang et al., "Control of time delay force feedback teleoperation system with finite time convergence," *Frontiers in Neurorobotics*, vol. 16, Article ID 877069, 2022.
[11] V. Leonov, A. Bagrov, G. Attia, A. Abdelaziz, and R. Ghoneim, "Study the combustion processes of space debris in the Earth's atmosphere by meteoric TV camera," *Applied Mathematics and Nonlinear Sciences*, vol. 2, no. 2, pp. 473–478, 2017.
[12] H. Zhu, M. Xue, Y. Wang, G. Yuan, and X. Li, "Fast visual tracking with Siamese oriented region proposal network," *IEEE Signal Processing Letters*, vol. 29, pp. 1437–1441, 2022.
[13] B. Cao, M. Li, X. Liu, J. Zhao, W. Cao, and Z. Lv, "Many-objective deployment optimization for a drone-assisted camera network," *IEEE transactions on network science and engineering*, vol. 8, no. 4, pp. 2756–2764, 2021.
[14] H. S. Katesari and S. Zarodi, "Effects of coverage choice by predictive modeling on frequency of accidents," *Caspian Journal of Applied Sciences Research*, vol. 5, no. 3, pp. 28–33, 2016.
[15] C. López de Subijana and E. Navarro Cabello, "Kinetic energy transfer during the serve," *Journal of Human Sport and Exercise*, vol. 4, no. 2, pp. 114–128, 2009.
[16] H. Safari-Katesari and S. Zaroudi, "Count copula regression model using generalized beta distribution of the second kind," *Statistics in Transition New Series*, vol. 21, no. 2, pp. 1–12, 2020.
[17] S. Baek and M. Kim, "Flight trajectory of a golf ball for a realistic game," *International Journal of Innovation, Management and Technology*, vol. 4, no. 3, p. 346, 2013.
[18] Y. Hayakawa, A. Nakashima, S. Itoh, and Y. Nakai, "Ball trajectory planning in serving task for table tennis robot," *SICE Journal of Control, Measurement, and System Integration*, vol. 9, no. 2, pp. 50–59, 2016.
[19] Y. Inaba, S. Tamaki, H. Ikebukuro, K. Yamada, H. Ozaki, and K. Yoshida, "Effect of changing table tennis ball material from celluloid to plastic on the post-collision ball trajectory," *Journal of Human Kinetics*, vol. 55, no. 1, pp. 29–38, 2017.
[20] J. Nonomura, A. Nakashima, and Y. Hayakawa, "Analysis of effects of rebounds and aerodynamics for trajectory of table tennis ball," in *Proceedings of the SICE Annual Conference*, Taipei, Taiwan, August 2010.
[21] Z. Zhang, D. Xu, and M. Tan, "Visual measurement and prediction of ball trajectory for table tennis robot," *IEEE Transactions on Instrumentation and Measurement*, vol. 59, no. 12, pp. 3195–3205, 2010.
[22] M. Carré, S. Haake, T. Asai, and T. Akatsuka, "The curve kick of a football," in *Proceedings of the International Conference on the Engineering of Sport*, East Midlands, UK, 2002.
[23] S. Choppin, "Calculating football drag profiles from simulated trajectories," *Sports Engineering*, vol. 16, no. 3, pp. 189–194, 2013.
[24] S. Hörzer, C. Fuchs, R. Gastinger et al., "Simulation of spinning soccer ball trajectories influenced by altitude," *Procedia Engineering*, vol. 2, no. 2, pp. 2461–2466, 2010.
[25] S. Tuplin, M. Passmore, D. Rogers, A. R. Harland, T. Lucas, and C. Holmes, "The application of simulation to the understanding of football flight," *Proceedings of the Institution of Mechanical Engineers - Part P: Journal of Sports Engineering and Technology*, vol. 226, no. 2, pp. 134–142, 2012.
[26] G. R. Hamilton and C. Reinschmidt, "Optimal trajectory for the basketball free throw," *Journal of Sports Sciences*, vol. 15, no. 5, pp. 491–504, 1997.
[27] S. Keerthekesh Nadar, T. Amrit, S. Sanjay Srinivas, and K. Balaji, "Aerodynamic characteristics of baseball and tennis ball using cfd analysis," *IOP Conference Series: Materials Science and Engineering*, vol. 1132, no. 1, Article ID 12023, 2021.
[28] L. Fradet, R. Kulpa, B. Bideau, F. Multon, and P. Delamarche, "Kinematic simulation of handball throwing," in *Proceedings of the European Simulation Multiconference*, Nottingham, UK, June 2003.

[29] H. Ghaednia, O. Cermik, and D. B. Marghitu, “Experimental and theoretical study of the oblique impact of a tennis ball with a racket,” *Proceedings of the Institution of Mechanical Engineers - Part P: Journal of Sports Engineering and Technology*, vol. 229, no. 3, pp. 149–158, 2015.

[30] Y. Kawazoe and Y. Kanda, “Analysis of impact phenomena in a tennis ball-racket system: effects of frame vibrations and optimum rocket design,” *JSME International Journal Series C: Dynamics Control Robotics Design & Manufacturing*, vol. 40, no. 1, pp. 9–16, 1997.

[31] S. M. Mohsin Jafri and J. M. Vance, “Modeling of impact dynamics of a tennis ball with a flat surface,” *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, vol. 47438, 2005.

[32] F. Alam, A. Subic, J. Naser, M. Rasul, and M. Khan, “A Study of Spin Effects on Tennis ball Aerodynamics,” *WSEAS Transactions on Fluid Mechanics*, vol. 3, 2008.

[33] A. J. Cooke, “An overview of tennis ball aerodynamics,” *Sports Engineering*, vol. 3, no. 2, pp. 123–129, 2000.

[34] H. Brody, “Bounce of a tennis ball,” *Journal of Science and Medicine in Sport*, vol. 6, no. 1, pp. 113–119, 2003.

[35] J. A. Glynn, M. A. King, and S. R. Mitchell, “A computer simulation model of tennis racket/ball impacts,” *Sports Engineering*, vol. 13, no. 2, pp. 65–72, 2011.

[36] Y. Ke, Z. Liu, and S. Liu, “Prediction algorithm and simulation of tennis impact area based on semantic analysis of prior knowledge,” *Soft Computing*, pp. 1–8, 2022.

[37] P. Le Noury, T. Buszard, M. Reid, and D. Farrow, “Examining the representativeness of a virtual reality environment for simulation of tennis performance,” *Journal of Sports Sciences*, vol. 39, no. 4, pp. 412–420, 2021.

[38] H. Safari-Katesari and S. Zaroudi, “Analysing the impact of dependency on conditional survival functions using copulas,” *Statistics in Transition New Series*, vol. 22, no. 1, pp. 217–226, 2021.

[39] H. G. Çitil, “Important notes for a fuzzy boundary value problem,” *Applied Mathematics and Nonlinear Sciences*, vol. 4, no. 2, pp. 305–314, 2019.

[40] J. Li, K. Xu, S. Chaudhuri, E. Yumer, H. Zhang, and L. Guibas, “GRASS: generative recursive autoencoders for shape structures,” *ACM Transactions on Graphics*, vol. 36, no. 4, pp. 1–14, 2017.

[41] J. Zhang, C. Zhu, L. Zheng, and K. Xu, “ROSEFusion: random optimization for online dense reconstruction under fast camera motion,” *ACM Transactions on Graphics*, vol. 40, no. 4, pp. 1–17, 2021.

[42] P. Chen, J. Pei, W. Lu, and M. Li, “A deep reinforcement learning based method for real-time path planning and dynamic obstacle avoidance,” *Neurocomputing*, vol. 497, pp. 64–75, 2022.

[43] Y. Guo, S. Yu, J. Guo, and H. Li, “Simulation analysis on hybrid energy storage management strategy in warship medium voltage DC power system,” *Chinese Journal of Ship Research*, vol. 14, no. 2, pp. 126–136, 2019.

[44] X. Gong, L. Wang, Y. Mou et al., “Improved four-channel PBTDPA control strategy using force feedback bilateral teleoperation system,” *International Journal of Control, Automation and Systems*, vol. 20, no. 3, pp. 1002–1017, 2022.

[45] W. Zheng and L. Yin, “Characterization inference based on joint-optimization of multi-layer semantics and deep fusion matching network,” *PeerJ Computer Science*, vol. 8, 2022.

[46] R. Mehta, F. Alam, and A. Subic, “Review of tennis ball aerodynamics,” *Sports Technology*, vol. 1, no. 1, pp. 7–16, 2008.

[47] R. D. Mehta and J. M. Pallis, “The aerodynamics of a tennis ball,” *Sports Engineering*, vol. 4, no. 4, pp. 177–189, 2001.

[48] S. C. Michalski, A. Szpak, D. Saredakis, T. J. Ross, M. Billinghurst, and T. Loetscher, “Getting your game on: using virtual reality to improve real table tennis skills,” *PLoS One*, vol. 14, no. 9, Article ID e0222351, 2019.

[49] J. D. Anderson and J. Wendt, *Computational fluid dynamics*, Vol. 206, Springer, , McGraw-Hill, NY, USA, 1995.

[50] T. Chung, *Computational Fluid Dynamics*, Cambridge University Press, Cambridge, UK, 2002.

[51] P. Wang, Q. Zhang, Y. Jin, and F. Ru, “Studies and simulations on the flight trajectories of spinning table tennis ball via high-speed camera vision tracking system,” *Proceedings of the Institution of Mechanical Engineers - Part P: Journal of Sports Engineering and Technology*, vol. 233, no. 2, pp. 210–226, 2019.

[52] A. Cantero, F. Crespo, and S. Ferrer, “The triaxiality role in the spin-orbit dynamics of a rigid body,” *Applied Mathematics and Nonlinear Sciences*, vol. 3, no. 1, pp. 187–208, 2018.

[53] Y. Liu, D. Liu, and S. Chang, “Fast identification of aerodynamic coefficients of spin-stabilized projectile based on improved particle swarm optimization,” *Dandao Xuebao/Journal of Ballistics*, vol. 30, pp. 19–24, 2018.

[54] B. Zha, Z. Hui, and Q. Su, “Numerical simulation of tail flow field of four - nozzle solid rocket engine,” *Journal of Physics: Conference Series*, vol. 1064, Article ID 12009, 2018.

[55] M. A. Hannan, J. A. Ali, A. Mohamed, U. A. U. Amirulddin, N. M. L. Tan, and M. N. Uddin, “Quantum-behaved lightning search algorithm to improve indirect field-oriented Fuzzy-PI control for IM drive,” *IEEE Transactions on Industry Applications*, vol. 54, no. 4, pp. 3793–3805, 2018.

[56] H. G. Çitil, “Investigation of a fuzzy problem by the fuzzy Laplace transform,” *Applied Mathematics and Nonlinear Sciences*, vol. 4, no. 2, pp. 407–416, 2019.

[57] M. Kaushal and B. S. Khehra, “BBBCO and fuzzy entropy based modified background subtraction algorithm for object detection in videos,” *Applied Intelligence*, vol. 47, no. 4, pp. 1008–1021, 2017.

[58] S. Arjunan and P. Sujatha, “Lifetime maximization of wireless sensor network using fuzzy based unequal clustering and ACO based routing hybrid protocol,” *Applied Intelligence*, vol. 48, no. 8, pp. 2229–2246, 2018.

[59] G. Nie, “Analysis of landing trajectory tracking method for fully autonomous dual-fluid four-rotor UAV,” *Journal of China Academy of Electronics and Information Technology*, vol. 14, no. 2, pp. 134–138, 2019.

[60] S. K. Gharghan, R. Nordin, A. M. Jawad, H. M. Jawad, and M. Ismail, “Adaptive neural fuzzy inference system for accurate localization of wireless sensor network in outdoor and indoor cycling applications,” *IEEE Access*, vol. 6, Article ID 38475, 2018.

[61] A. T. Dosdoğru, “Improving weather forecasting using de-noising with maximal overlap discrete wavelet transform and ga based neuro-fuzzy controller,” *The International Journal on Artificial Intelligence Tools*, vol. 28, no. 3, Article ID 1950012, 2019.

[62] R. Usiskin and J. Maier, “Interfacial effects in lithium and sodium batteries,” *Advanced Energy Materials*, vol. 11, no. 2, Article ID 2001455, 2021.

[63] H. W. Wu and K. Y, “Reflective subcategory of category of fuzzy Z-quantale,” *Journal of Jilin University (Science Edition)*, vol. 56, no. 4, pp. 805–811, 2018.

[64] K. Abhishek, S. Datta, and S. S. Mahapatra, “Optimization of MRR, surface roughness, and maximum tool-tip temperature

during machining of CFRP composites," *Materials Today Proceedings*, vol. 4, no. 2, pp. 2761–2770, 2017.

[65] Y. Chai, L. Jia, and Z. Zhang, "Mamdani model based adaptive neural fuzzy inference system and its application," *International Journal of Computer and Information Engineering*, vol. 3, no. 3, pp. 663–670, 2009.

[66] E. Pourjavad and R. V. Mayorga, "A comparative study and measuring performance of manufacturing systems with Mamdani fuzzy inference system," *Journal of Intelligent Manufacturing*, vol. 30, no. 3, pp. 1085–1097, 2019.