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Research Article

Risk Estimation in Exchange Rate Markets Based on Stochastic Copula Approach

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Risk estimation is of great importance in financial risk management. In this study, the risk estimation of the exchange rate portfolio is performed via the stochastic copula approach. This model-based latent process has a parameter that changes over time and thus can model the dependency structure between variables in a comprehensive and dynamic way. First, the marginals of the returns are handled with ARMA-GARCH-type models. Then, the dependency between variables is modeled via the stochastic copula approach. Finally, risk estimates are carried out at 95% and 99% confidence level for the foreign exchange portfolios. It is found that the proposed risk estimation model based on the stochastic copula approach outperforms both classical methods and static copula models.

1. Introduction

Risk estimation is one of crucial issues in risk management. Investors and firms need accurate estimation risk estimations in order to make an investment planning. Value at risk is the most commonly used risk measure in literature. This measure can be defined as the potential loss of a financial position at a given significance level and over a certain period. Modeling of the dependency structure between the financial assets that create the portfolio is vital for accurate risk estimation. In classical risk estimation methods, dependency structure between financial returns is modeled with Pearson correlation coefficient. However, experimental studies in the literature suggest that financial returns cannot satisfy necessary assumption for normal distribution. Therefore, more flexible approaches are needed in order to model the dependency between returns. One of the alternative methods used for this propose is copula. Copulas are multivariate distribution functions that can flexibly model the dependency structure between variables. It has growing popularity especially in econometrics and financial literature since it does not require strict assumptions on marginal distributions and can model the dependence between variables regardless of marginal distributions. It has found that marginal distributions of returns exhibit skewed and excess kurtosis [1]. On the other hand, there are various relationships between financial assets. Longin and Solnik [2] and Ang and Chen [3] found that returns of financial assets are highly correlated during market downturns than during market upturns. Methods that can model symmetric dependency failed to overcome various dependence structures such as tail dependency, and therefore, more flexible approaches are needed in modeling the dependency. There is a great deal of literature that models the dependencies between variables in financial markets and makes risk estimations (Al Rahahleh et al. [4]; Wang and Xu [5]; Peng et al. [6]; and Yang et al. [7]). In this paper, the copula theory is utilized for portfolio risk estimation. Copulas have many applications in econometric and financial fields. Breymann et al. [8] and Cherubini et al. [9] are some of the first studies to use copula in financial risk management. They estimated the VaR of the portfolio using static copulas. Patton [10] extended the static copulas to conditional distributions and modeled the dependency between exchange rates via conditional copulas.

Huang et al. [11] used the copula-GARCH model to estimate the VaR of the stock market index portfolio composed of NASDAQ and TAIEX. Nguyen et al. [12] analysed the dependency between gold price and stock markets using mixed-copulas. Yang and Hamori [13] investigated the dynamic dependency between gold prices and exchange rates by means of time-varying copulas. Lu et al. [14] applied the time-varying copula-GARCH model to estimate the VaR of a portfolio of energy markets. Wang et al. [15] investigated the dynamic dependency using time-varying copula approach and the minimum spanning tree (MST) method. Xiao et al. [16] proposed a new multivariate skewed fat-tailed copula approach and employed this model on financial market data. Ma et al. [17] investigated risk dependency of IMF via the C-vine copula and time-varying copula model. Yu et al. [18] exhibited the dependence and risk spillover effects between traditional and emerging hedging assets: bitcoin, gold, and USD via varied copulas. Wu et al. [19] explored the effect of economic policy uncertainty on the conditional dependence between China and U.S. stock markets based on a novel Copula-mixed-data sampling (Copula-MIDAS). There are many papers in in the literature that make risk estimation using static copula and classical approaches. The main contribution of this study is that, to our knowledge, it is the first paper to estimate risk using the stochastic copula approach, a special class of the copula family.

In this paper, the risk of the exchange rate portfolio is estimated via the stochastic copula approach. This approach allows modeling of the dynamic dependence structure between exchange rates and also takes into account the latent process as well as observations. Thus, it models dependency structure in a comprehensive and flexible way. In this study, three separate portfolios of exchange rates are created. These portfolios are considered in pairs due to the nature of the stochastic copula approach. First, the marginals of exchange rate returns are dealt with the ARMA-GARCH approach. Then, the dependency structure between returns is modeled by means of the stochastic copula approach. Finally, conditional means and volatilities are combined with dependency structures, and risk estimations are made for each portfolio at different significance levels by simulation. We found that stochastic copula approach is superior to classical methods and static copula approach in terms of risk estimation performance on three different datasets. It has been observed that the dependence between exchange rates has changed over time. There is a lower tail dependency between exchange rates. This indicates that exchange rate prices are significantly affected by extreme events.

The rest of the paper is organized as follows: Section 2 introduces the stochastic copula theory and modeling marginal distributions. In Section 3, the calculation of risk measures based on the stochastic copula approach is described. Empirical findings are presented in Section 4. Section 5 discusses the results.

2. Dependence Modeling

Modeling the dependency structure between variables has great importance in many applications, especially in finance and econometrics. Standard approaches require normality assumption to model the dependency between variables. However, experimental studies have found out that financial and econometric variables are far from normal distribution [1]. Therefore, alternative approaches are needed in modeling dependency. Copulas are flexible tools that can model the whole dependence structure between variables. It attracts great attention to applications especially in finance and econometrics since it can model the dependency structure between variables independent of marginals. Static copulas assume that the dependency structure between variables does not change over time. However, it is found out that the dependency structure between financial variables is time-varying [20]. Time-varying (conditional) copulas are based on the assumption that the dependency structure between variables is dynamic. These models offer more effective results in modeling the dependency compared to static copulas. On the other hand, time-varying copula can model dependence structure based on observations and do not consider the latent process. The stochastic copula can model time-varying dependency structure between variables and also takes into account the latent process as well as observations in the dependency modeling process. For this reason, the stochastic copula approach can handle the dependency structure between variables in a more flexible and comprehensive way. The stochastic copula model based on the latent process is nonlinear and has a time-varying parameter. This parameter follows the AR latent process, and this parameter is estimated by ML-EIS method due to the high-dimensional integral problem.

2.1. Stochastic Copula Approach.

2.1. Stochastic Copula Approach. Let $(v_{1,t}, v_{2,t})$ for $t = 1, \ldots, T$ be bivariate time series with dynamic parameters, and these are distributed as follows:

$$\left(v_{1,t}, v_{2,t}\right) \sim C\left(v_1, v_2 | \theta_t\right). \qquad (1)$$

Here, $\theta_t \in \Theta R^M$ and is assumed to be obtained by λ_t latent stochastic process. In this paper, it is determined as $M = 1$, and one parameter families of copula are discussed, where $\theta_t = \psi(\lambda_t)$ and ψ is a transformation. It ensures that the copula parameter remains within its own domain. Convenient ψ transform depends on the selected copula. λ_t latent process follows the first-order Gaussian autoregressive process:

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \delta\varepsilon_t, \quad |\beta| < 1, \delta > 0, \qquad (2)$$

where ε_t is a Gaussian innovation process. The time-varying parameters in the stochastic copula autoregressive (SCAR) model are estimated by an independent stochastic process. The nonlinear SCAR copula model can be written in its own state space representation. State equation and transition equation are given in (3) and (4), respectively.

$$\left(v_{1,t}, v_{2,t}\right)|\lambda_t \sim C\left(v_1, v_2|\psi\left(\lambda_t\right)\right), \quad (3)$$

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \delta\varepsilon_t. \quad (4)$$

For detailed information about the parameter estimation of the SCAR model, Hafner and Manner [21] and Yıldırım and Cengiz [22] can be viewed.

2.2. Marginal Distribution Modeling. Returns of financial assets can exhibit some characteristics such as asymmetric volatility and leverage effect. Generalized autoregressive conditional heteroscedasticity (GARCH) type models proposed by Bollerslev [23] can handle these features. In this study, ARMA-GARCH type models defined as in (5) are used to model the conditional mean and volatility of the return series.

$$\begin{aligned} y_t &= \mu + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t, \\ \epsilon_t &= \sigma_t \eta_t, \\ \sigma_t^2 &= \omega_0 + \sum_{i=1}^{k} \gamma_i \sigma_{t-i}^2 + \sum_{i=1}^{l} \varsigma_j \epsilon_{t-j}^2, \end{aligned} \quad (5)$$

Here, μ and ω_0 are constant terms. φ_i and θ_j represent ith AR and jth MA parameter, respectively. γ_i is the ARCH parameter, while ς_j is the GARCH parameter. Errors are assumed to be normal, Student t, and skewed Student t distributions. The best fitted marginal distribution is chosen depending on the AIC and BIC information criteria.

3. Evaluation of Risk Measure

Value at risk (VaR) and conditional value at risk (CoVaR) are the most commonly used risk measure in risk management. VaR can be defined as the maximum potential loss of a financial position with given p probability over t time period. On the other hand, CoVaR can be expressed as the average of losses exceeds of value at risk.

VaR and CoVaR are mathematically described as follows:

$$\begin{aligned} VaR_{1-p}(x) &= \inf\{x|F_t(x) \geq 1-p\}, \\ CoVaR_{1-p}(x) &= E\left(x|x > VaR_{1-p}(x)\right), \end{aligned} \quad (6)$$

where $F_t(x)$ is the cumulative distribution function of the portfolio at time t. In this paper, the distribution function is created with the stochastic copula-GARCH model.

3.1. VaR Calculation Based on the Stochastic Copula Approach. One-day-ahead VaR estimation based on the the proposed stochastic copula can be performed by the following steps:

Step 1 : ARMA-GARCH models are estimated using T observations.

Step 2 : average return and volatility are forecasted for $T+1$ time. Let these values be defined as $\hat{r}_{T+1}^{x_i}$ and $\hat{\sigma}_{T+1}^{x_i}$, respectively.

Step 3 : $\eta_t^{x_i}$ $(i = 1, 2)$ standardized residuals from ARMA-GARCH type models are transformed into $u_t^{x_i}$ $(i = 1, 2)$ uniform variables using probability integral transform, and thus, the input variables required for stochastic copula estimation are obtained.

Step 4 : simulation is carried out from the estimated stochastic copula model for time $T+1$. The simulated standardized residuals are obtained using inverse functions of the estimated marginal distributions:

$$\left(\eta_{T+1}^{x_1}, \eta_{T+1}^{x_1}\right) = \left(F_{x_1,T+1}^{-1}\left(u_{T+1}^{x_1}; \hat{\delta}_{x_1}\right), F_{x_2,T+1}^{-1}\left(u_{T+1}^{x_2}; \hat{\delta}_{x_2}\right)\right). \quad (7)$$

Step 5 : the simulated logarithmic returns using the simulated standardized residuals obtained in step 4 with the $\hat{r}_{T+1}^{x_i}$ returns and $\hat{\sigma}_{T+1}^{x_i}$ volatilities forecasted in step 2 and are computed as follows:

$$\left(r_{T+1}^{x_1}, r_{T+1}^{x_2}\right) = \underline{w}\left(\hat{r}_{T+1}^{x_1} + \eta_{T+1}^{x_1}\sqrt{\hat{\sigma}_{T+1}^{x_1}}, \hat{r}_{T+1}^{x_2} + \eta_{T+1}^{x_2}\sqrt{\hat{\sigma}_{T+1}^{x_2}}\right), \quad (8)$$

where $\underline{w} = (1/2, 1/2)$ since the equally weighted portfolio is created.

Step 6 : Step 4 and step 5 are repeated N times.

Step 7 : N simulated logarithmic returns are sorted in the ascending order and for $T+1$ time VaR estimations at 95% and 99% confidence level which are calculated as follows:

$\text{VaR}_{95} = N(1-0.95)$ th value of sorted simulated logarithmic returns
$\text{VaR}_{95} = N(1-0.99)$ th value of sorted simulated logarithmic returns
$\text{CoVaR}_{95} = \text{mean}[\text{first } N(1-0.95)-1]$ values of sorted simulated logarithmic returns
$\text{CoVaR}_{99} = \text{mean}[\text{first } N(1-0.99)-1]$ values of sorted simulated logarithmic returns

The choice of N simulation numbers is crucial. The number of simulations of 100,000 suggested by Fantazzini [24] was applied for this study.

3.2. Backtesting. The performances of VaR models are compared using backtesting techniques. The unconditional convergence of Kupiec [25] is one of these tests. Unconditional convergence tests whether the number of losses exceeding the VaR is acceptable and the risk estimation model is correct under null hypothesis. Including the independence of losses exceeding VaR in Kupiec's [25] test, Christoffersen [26] proposed the conditional convergence test. Acceptance of the null hypothesis indicates that the VaR model is correct.

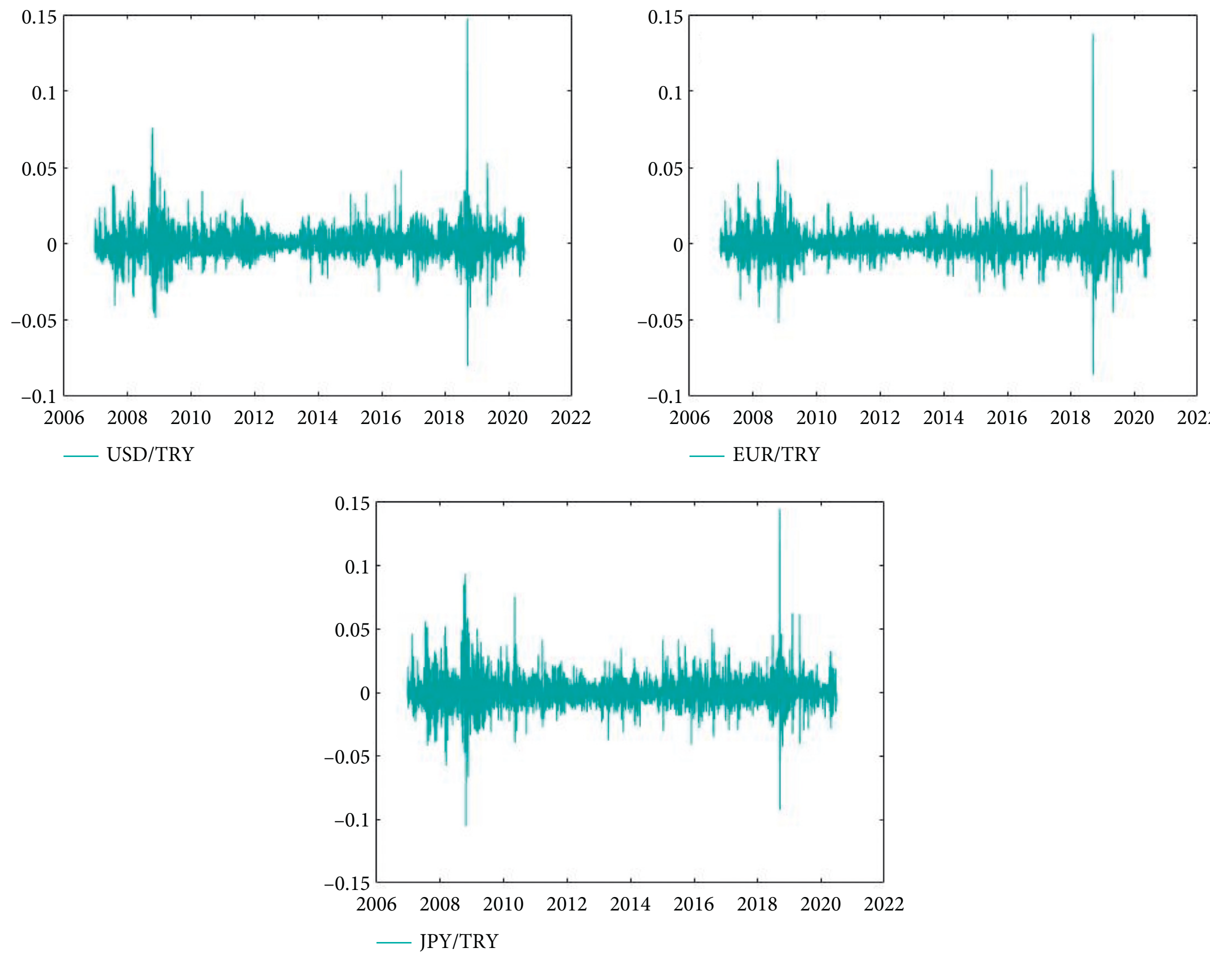

Figure 1: Return series of the exchange rates.

Table 1: Descriptive statistics of the exchange rate returns.

	USD/TRY	EUR/TRY	JPY/TRY
Mean	0.00045	0.00039	0.00048
Std. dev.	0.00975	0.00921	0.01238
Skewness	1.34919	1.04213	0.80030
Kurtosis	21.71060	20.34435	12.47604
JB stat.	69671	60881	23033
P value	<0.001	<0.001	<0.001

The magnitude of the losses is calculated to determine the power of the VaR models that passed the tests aforementioned. The loss function proposed by Blanco and Ihle [27] is defined as follows:

$$C_t^{\mathrm{BI}} = \begin{cases} \dfrac{L_t - \mathrm{VaR}_t}{\mathrm{VaR}_t}, & \text{if } L_t > \mathrm{VaR}_t, \\ 0, & \text{if } L_t \le \mathrm{VaR}_t, \end{cases} \quad (9)$$

where L_t is described as the loss at time t. Backtesting is performed via the mean of loss functions.

Table 2: Statistical tests for the exchange rate returns.

	USD/TRY	EUR/TRY	JPY/TRY
ADF stat.	−20.4	−21.1	−21.5
P value	<0.001	<0.001	<0.001
Ljung box *Q* stat.	58.101	68.059	25.734
P value	<0.001	<0.001	<0.001
ARCH LM stat.	570.97	644.33	596.06
P value	<0.001	<0.001	<0.001

$$\widehat{C}_{\mathrm{BI}}^{L} = \frac{1}{T} \sum_{i=1}^{T} C_t^{\mathrm{BI}}. \quad (10)$$

The backtesting performances of different VaR models are evaluated in two stages. First, model accuracy is determined by statistical tests, and then loss functions are used to compare VaR models that passed these tests.

4. Empirical Findings

In this study, using USD/TRY, EUR/TRY, and JPY/TRY exchange rates, the risk estimation of three equally weighted portfolios, each consisting of two exchange rates, is

Table 3: Parameter estimations for marginal distributions and statistical test.

	USD/TRY: TGARCH-ST (skewed)			EUR/TRY: TGARCH-ST (skewed)		
Parameter	Value	Std. error	*P* value	Value	Std. error	*P* value
φ_1	−1.60457	0.05474	<0.001	0.76112	0.03235	<0.001
φ_2	−0.83524	0.03072	<0.001	—	—	—
φ_3	−0.04556	0.01180	<0.001	—	—	—
θ_1	1.58162	0.05503	<0.001	−0.77093	0.03189	<0.001
θ_2	0.79146	0.02581	<0.001	—	—	—
ω	0.00011	0.00004	<0.001	0.00018	0.00006	<0.001
γ_1	0.07649	0.01576	<0.001	0.09120	0.01740	<0.001
ς_1	0.92689	0.01642	<0.001	0.90527	0.01949	<0.001
η	−0.37096	0.10895	<0.001	−0.44036	0.10233	<0.001
ν	5.80374	0.60787	<0.001	6.26857	0.69364	<0.001
ξ	1.14864	0.02975	<0.001	1.08954	0.02812	<0.001
Ljung box Q stat.		3.3947			8.9556	
P value		0.8462			0.2559	
ARCH LM stat.		10.964			8.1992	
P value		0.1402			0.3154	

		JPY/TRY: TGARCH-ST (skewed)		
Parameter		Value	Std. error	*P*-value
φ_1		0.53027	0.04661	<0.001
θ_1		−0.56260	0.04552	<0.001
ω		0.00039	0.00009	<0.001
γ_1		0.10113	0.01413	<0.001
ς_1		0.88710	0.01625	<0.001
η		−0.46406	0.09934	<0.001
ν		5.58052	0.59365	<0.001
ξ		1.11510	0.02753	<0.001
Ljung box Q stat.	8.7512			
P value	0.2710			
ARCH LM stat.	5.8363			
P value	0.5590			

performed via the stochastic copula approach. The data set ranges from January 1, 2007, to May 15, 2020 and consists of 3490 daily exchange rate prices.

Turkey and the world economy have experienced significant economic and financial crises in considered period. Sharp movements can be realized in prices in such periods. The relevant period is chosen in order to show that the proposed approach is a powerful tool in modeling the price movements during this period. The sample period is divided into two subperiods to compare the performance of risk estimation models. The first subperiod is utilized for the model estimations, and the second subperiod is used to evaluate the performance of the risk estimation models. The first subsample period includes between January 1, 2007, and May 31, 2015, and consists of 2718 observations. The second subsample period is between June 1, 2017, and May 15, 2020, and contains 772 daily exchange rate prices. Returns series are plotted in Figure 1. For analysis, logarithmic return series from price series are calculated as follows:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right), \tag{11}$$

where p_t and r_t are defined as price and return at time t, respectively. Descriptive statistics of the return series are presented in Table 1.

The average of all returns is positive, and JPY/TRY exchange provides the highest return. Considering the variances, JPY/TRY has the highest variability. On the other hand, the variances for all series are greater than the means of returns, indicating that all series have high volatility.

All exchange rate returns tend to exhibit high positive returns since the skewness coefficient is positive for all series. The kurtosis coefficients for all returns are greater than the normal distribution, and thus all returns demonstrate excess kurtosis. These results point that the series are far from the normal distribution. Null hypothesis of normal distribution for all series is rejected by the Jarque–Bera test. The results of the statistical tests for log return series are displayed in Table 2. According to the ADF test, all return series are stationary at 1% significance level. Ljung–Box test demonstrates that there is autocorrelation in return series. The presence of ARCH effect in all series is confirmed by Engle's ARCH LM test. These findings indicate that ARMA-GARCH type approaches are required to model the marginals of the return series.

ARMA-GARCH models estimated for return series are presented in Table 3. Marginal models are estimated as follows: ARMA(3,2)-TGARCH(1,1) model for USD/TRY and ARMA(1,1)-TGARCH(1,1) models for EUR/TRY and JPY/TRY. It was found out that there is the effect of asymmetric volatility in all return series. Skewed Student t is determined as the best fitted distribution for the errors of the

Table 4: Backtesting of VaR forecasts at 95% confidence level.

Estimation method	Exceeds of VaR	Exceeds of CoVaR	LL_{UC} P value	LL_{CC} P value	BI_{LOSS}
Portfolio 1: USD/TRY–EUR/TRY					
Static t copula	44	17	0.76234 (0.38259)	4.85892 (0.08808)	0.02470
Stochastic-normal	38	16	0.00986 (0.92087)	0.65313 (0.72061)	0.01934
Stochastic-Gumbel	39	15	0.00434 (0.94741)	0.52398 (0.76951)	0.02160
Stochastic-Frank	37	15	0.07074 (0.79025)	0.85703 (0.65147)	0.01934
Stochastic-rotGumbel	34	12	0.60015 (0.43851)	0.76717 (0.68141)	0.01859
Historical simulation	61	23	11.72106 (<0.001)	16.96855 (<0.001)	0.07096
Variance-covariance	42	19	0.30684 (0.57962)	3.09041 (0.21326)	0.04274
Portfolio 2: USD/TRY–JPY/TRY					
Static t copula	37	14	0.07074 (0.79025)	2.41326 (0.29920)	0.01748
Stochastic-normal	32	14	1.25816 (0.26199)	2.99303 (0.22390)	0.01522
Stochastic-Gumbel	33	13	0.89750 (0.34345)	2.40800 (0.29999)	0.01617
Stochastic-Clayton	32	11	1.25816 (0.26199)	2.99303 (0.22390)	0.01444
Stochastic-Frank	32	13	1.25816 (0.26199)	2.99303 (0.22390)	0.01501
Stochastic-rotGumbel	32	10	1.25816 (0.26199)	2.99303 (0.22390)	0.01462
Stochastic-rotClayton	39	15	2.73964 (0.09788)	3.40725 (0.18202)	0.01933
Historical simulation	52	21	4.43744 (0.03515)	6.16530 (0.04583)	0.04879
Variance-covariance	38	18	0.00986 (0.92087)	4.14689 (0.12575)	0.02890
Portfolio 3: EUR/TRY–JPY/TRY					
Static t copula	41	15	0.15408 (0.69466)	0.46543 (0.79237)	0.02012
Stochastic-normal	29	13	2.73964 (0.09788)	5.26395 (0.07193)	0.01550
Stochastic-Gumbel	38	15	0.00986 (0.92087)	0.65531 (0.07206)	0.01751
Stochastic-Frank	29	11	2.73964 (0.09788)	5.26395 (0.07193)	0.01546
Stochastic-rotGumbel	29	9	2.73964 (0.09788)	5.26395 (0.07193)	0.01503
Historical simulation	51	22	3.82532 (0.05048)	4.62346 (0.09908)	0.05065
Variance-covariance	39	18	0.00434 (0.94741)	3.77213 (0.15166)	0.03483

Table 5: Backtesting of VaR forecasts at 99% confidence level.

Estimation method	Exceeds of VaR	Exceeds of CoVaR	LL_{UC} (P-value)	LL_{CC} (P-value)	BI_{LOSS}
Portfolio 1: USD/TRY–EUR/TRY					
Static t copula	9	7	0.20352 (0.65188)	3.10763 (0.21143)	0.00525
Stochastic-normal	9	6	0.20352 (0.64188)	3.10763 (0.21143)	0.00414
Stochastic-gumbel	10	7	0.62222 (0.43022)	3.12586 (0.20952)	0.00494
Stochastic-frank	10	7	0.62222 (0.43022)	3.12586 (0.20952)	0.00548
Stochastic-rotGumbel	8	5	0.01013 (0.91980)	3.37601 (0.18488)	0.00390
Historical simulation	19	8	11.83104 (0.00058)	18.64638 (<0.001)	0.01370
Variance-covariance	25	10	24.58674 (<0.001)	28.47479 (<0.001)	0.01686
Portfolio 2: USD/TRY–JPY/TRY					
Static t copula	8	6	0.01013 (0.91980)	3.37601 (0.18488)	0.00288
Stochastic-normal	8	7	0.01013 (0.91980)	3.37601 (0.18488)	0.02439
Stochastic-gumbel	8	7	0.01013 (0.91980)	3.37601 (0.18488)	0.00293
Stochastic-clayton	7	4	0.07001 (0.79130)	3.97508 (0.13703)	0.00223
Stochastic-frank	10	7	0.62222 (0.43022)	3.12586 (0.20952)	0.00343
Stochastic-rotgumbel	7	4	0.07001 (0.79130)	3.97508 (0.13703)	0.00225
Stochastic-rotclayton	11	6	2.19791 (0.13819)	2.23963 (0.32633)	0.00475
Historical simulation	14	8	4.15854 (0.04142)	14.46645 (<0.001)	0.01031
Variance-covariance	15	9	5.43664 (0.01971)	15.07623 (<0.001)	0.01066
Portfolio 3: EUR/TRY–JPY/TRY					
Static t copula	8	6	0.01013 (0.91980)	3.37601 (0.18488)	0.00355
Stochastic-normal	8	6	0.01013 (0.91980)	3.37601 (0.18488)	0.00273
Stochastic-gumbel	8	7	0.01013 (0.91980)	3.37601 (0.18488)	0.00349
Stochastic-frank	8	7	0.01013 (0.91980)	3.37601 (0.18488)	0.00373
Stochastic-rotgumbel	8	5	0.01013 (0.91980)	3.37601 (0.18488)	0.00250
Historical simulation	13	8	3.02607 (0.08193)	14.48208 (<0.001)	0.01086
Variance-covariance	18	10	10.05496 (<0.001)	17.49497 (<0.001)	0.01305

returns. Thus, independent and identically distributed series are obtained for the estimation of copula. Due to the nature of the stochastic copula model, portfolios are analysed in pairs: USD/TRY and EUR/TRY, USD/TRY and JPY/TRY, and EUR/TRY and JPY/TRY.

Backtesting results of equally weighted portfolios at 95% confidence level are demonstrated in Table 4. Stochastic rotated Gumbel copula provides the best risk estimations for USD/TRY–EUR/TRY and EUR/TRY–JPY/TRY portfolios. The best results for an equally weighted portfolio of USD/TRY and JPY/TRY are obtained by means of the stochastic Clayton copula. The historical simulation method failed to pass the model accuracy tests for portfolios, except for the EUR/TRY and JPY/TRY portfolio. It is proven that risk estimation models based on stochastic copula approach outperform static copula models and classical methods at 95% confidence level.

Table 5 shows backtesting results of equally weighted portfolios at 99% confidence level. The best results for all portfolios were found using the stochastic rotated Gumbel copula. While the variance-covariance and historical simulation methods did not pass the model accuracy tests at the 5% significance level, model accuracy of both the static and stochastic copula approaches was confirmed by unconditional and conditional convergence tests. Considering all portfolios, it is proposed that modeling the dependency via stochastic copulas provides better performance than static approaches.

The dependence between exchange rates is best modeled in pairs through stochastic rotated Gumbel copula and the stochastic Clayton copula for USD/TRY and JPY/TRY. This suggests that there is a lower tail dependency between exchange rates. Since classical approaches can model symmetrical dependence, it is observed that they are not sufficient to estimate the risk of portfolio.

5. Conclusion

In this study, the risk estimations of equally weighted portfolios consisting of exchange rates are performed via the stochastic copula approach. Three different portfolios are created in pairs using USD/TRY, EUR/TRY, and JPY/TRY exchange rates. First, the marginals of exchange rate returns are estimated with ARMA-GARCH type models. It was found that asymmetric volatility exists in all return series, and skewed Student t is the best fitted distribution for errors. Risk estimation was carried out by means of stochastic copula approach for three different equally weighted portfolios, and the results were compared with the static copula and classical methods. It is concluded that the proposed risk estimation model based on the stochastic copula is superior to both static copula and classical approaches at both 95% and 99% confidence level. Except for portfolio consisting of the USD/TRY and JPY/TRY, stochastic rotated Gumbel copula presents the best risk estimation performance for all portfolios at 95% confidence level. For the portfolio of USD/TRY and JPY/TRY, Clayton copula is determined as the most appropriate risk estimation model. This indicates that there is a lower tail dependency between related exchange rates returns. It means that the relevant exchange rates tend to comovement during the downturn of general markets. For the future research, the risk estimation for a portfolio consisting of different markets such as the energy market can be investigated by the proposed risk estimation model based on the stochastic copula.

Data Availability

The data used in this study are available upon request from the author via email (emre.yildirim@omu.edu.tr).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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