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Research Article

A Study on Interval-Valued Fuzzy Graph with Application in Energy Industry Management

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In this paper, we define the concept of level graphs on interval-valued fuzzy graphs (IVFGs) and introduce the notions of $[\alpha,\beta]-[\alpha,\beta]-$ homomorphism, isomorphism, weak isomorphism, and co-weak isomorphism of IVFGs by $[\alpha,\beta]-$ homomorphism. Also, an application of homomorphism of IVFGs has been presented by using coloring-FG.

1. Introduction

Zadeh [1] introduced the subject of the interval-valued fuzzy set (IVFS) in 1975 as an explanation of fuzzy sets (FSs) [2] so that the values of the membership degrees are shown through intervals of numbers. IVFS is useful in applications such as fuzzy control. One of the most extreme sections of fuzzy control in terms of calculation is defuzzification [3]. The fuzzy graph (FG) theory as a generalization of Euler's graph theory was defined by Rosenfeld [4] in 1975. For the first time, the fuzzy relations between FSs were supposed by Rosenfeld, and he extended the structure of FGs and used it to achieve various graph theoretical subjects. After that, Bhattacharya [5] presented some remarks on FGs. Mordeson and Peng [6] explained some operations on FGs. Hongmei and Lianhua gave the definition of IVFG and so they introduced the notion of IVFGs [7]. For the first time, Akram and Dudek defined IVFGs [8]. In graph theory, we define a graph homomorphism that has a mapping between two graphs that relates to their structure. Different notions of graph coloring are generalized by homomorphisms. These concepts give essential information about an important class of constraint satisfaction problems [9].

Jiang et al. [10] introduced a new notion of vertex covering in cubic graphs. Turksen [11] defined IVFSs based on normal forms. Nagoorgani and Radha [12] presented isomorphism on fuzzy graphs. Jan et al. [13] studied some root-level modifications in IVFGs. Krishna et al. [14] investigated new results in cubic graphs. Talebi et al. [15, 16] defined Cayley fuzzy graphs and some operations on level graphs of bipolar fuzzy graphs. Kosari et al. [17] introduced the vague graph (VG) structure with an application in medical diagnosis. Kou et al. [18] defined some properties of VG. Qiang et al. [19] studied new kinds of domination in VGs. Rao et al. [20–22] presented some properties of vague incidence graphs and equitable domination in VGs. Shi et al. [23, 24] investigated the domination of product VGs with an application in transportation. Xu et al. [25] studied certain concepts of interval-valued intuitionistic fuzzy graphs. Rashmanlou and Pal [26] defined antipodal interval-valued fuzzy graphs. Bera et al. [27] proposed certain types of m-polar interval-valued fuzzy graphs. Zihni et al. [28] introduced interval-valued fuzzy soft graphs. Akram et al. [29–31] introduced the concept of interval-valued fuzzy line graphs, self-centered interval-valued fuzzy graphs, and certain types of interval-valued fuzzy graphs.

IVFG, belonging to the FG family, has good capabilities when faced with problems that cannot be explained by FGs and VGs. Since the principle of membership is not clear, neutrality is a good choice that can be well protected by an IVFG. Also, an IVFG is used to illustrate real-world phenomena using interval-valued fuzzy models in a variety of fields, including technology, social networking, and biological sciences. Therefore, in this paper, we introduce the concept of level graphs on IVFGs and define the notions of $[\alpha, \beta]-$ homomorphism, isomorphism, weak isomorphism, and co-weak isomorphism of an IVFG by $[\alpha, \beta]-$ homomorphism. Finally, we have tried to find the most effective person in an electrical power department based on the performance of its staff.

2. Preliminaries

In this section, we present some basic concepts of IVFGs.

Definition 1. A fuzzy graph is a pair $G = (\zeta, \xi)$ with a set X, that ζ is a fuzzy set in X and ξ is a fuzzy relation in $X \times X$, so that

$$\xi(ab) \le \min\{\zeta(a), \zeta(b)\}, \tag{1}$$

for all ab $\in X \times X$.

Suppose X is a finite nonempty set. The set of all 2-element subsets of X is shown by $\widetilde{X}^2$. A graph G is a pair (X, E) on X, so that $E \subseteq \widetilde{X}^2$, X and E are named vertex set and edge set, respectively.

Definition 2. A pair $G = (M, N)$ on graph $G^* = (X, E)$ is called interval-valued fuzzy graph (IVFG) so that $M = [\zeta_M^l, \zeta_M^u]$ is an IVFS on X and $N = [\zeta_N^l, \zeta_N^u]$ is an IVFS on E to satisfy the following conditions:

(1) $X = \{a_1, a_2, \ldots, a_n\}$ so that $[\zeta_M^l(a_i), \zeta_M^u(a_i)]$ shows the degree of membership of the node $a_i \in X$.

(2) $\zeta_N^l\colon E \longrightarrow [0,1]$, $\zeta_N^u\colon E \longrightarrow [0,1]$ are the functions that satisfy

$$\begin{aligned} \zeta_N^l(ab) &\le \min\{(\zeta_M^l(a), \zeta_M^l(b))\},\\ \zeta_N^u(ab) &\le \min\{(\zeta_M^u(a), \zeta_M^u(b))\}, \end{aligned} \tag{2}$$

for all ab $\in E$.

Definition 3. An IVFG is called a strong interval-valued fuzzy graph (SIVFG) if

$$\begin{aligned} \zeta_N^l(ab) &= \min\{(\zeta_M^l(a), \zeta_M^l(b))\},\\ \zeta_N^u(ab) &= \min\{(\zeta_M^u(a), \zeta_M^u(b))\}. \end{aligned} \tag{3}$$

for all ab $\in E$.

Definition 4. Let $L_* = \{[\alpha, \beta] | \alpha, \beta \in [0,1], 0 \le \alpha \le \beta \le 1\}$. For any $[\alpha_1, \beta_1], [\alpha_2, \beta_2] \in L_*$, we have

$$\begin{aligned} &[\alpha_1, \beta_1] \le [\alpha_2, \beta_2] \Leftrightarrow \alpha_1 \le \beta_2\\ &\quad \beta_1 \le \beta_2,\\ &[\alpha_1, \beta_1] < [\alpha_2, \beta_2] \Leftrightarrow [\alpha_1, \beta_1] \ne [\alpha_2, \beta_2]\\ &\quad \alpha_1 < \alpha_2\, \beta_1 < \beta_2. \end{aligned} \tag{4}$$

Definition 5. Let $M \in \mathrm{IVFS}(X)$. For any $[\alpha, \beta] \in L_*$, we define

$$M_{[\alpha,\beta]} = \{a \in X\colon \zeta_M^l(a) \ge \alpha, \zeta_M^u(a) \ge \beta\}. \tag{5}$$

Then, $M_{[\alpha,\beta]}$ is said to be $[\alpha, \beta]$-level set of M.

The set $M^* = \{a | a \in X, \zeta_M^l(a) > 0 \text{ or } \zeta_M^u(a) > 1\}$ is named the support M.

Definition 6. Suppose $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ are two IVFGs. Then,

(1) A homomorphism (HM) ψ from G_1 to G_2 is a mapping $\psi\colon X_1 \longrightarrow X_2$ with the following conditions:

(i) $\zeta_{M_1}^l(a) \le \zeta_{M_2}^l(\psi(a))$, $\zeta_{M_1}^u(a) \le \zeta_{M_2}^u(\psi(a))$

(ii) $\zeta_{N_1}^l(ab) \le \zeta_{N_2}^l(\psi(a)\psi(b))$, $\zeta_{N_1}^u(ab) \le \zeta_{N_2}^u(\psi(a)\psi(b))$, for all $a \in X$, ab $\in E$

(2) An isomorphism ψ from G_1 to G_2 is a bijective mapping (BM) $\psi\colon X_1 \longrightarrow X_2$ with the following conditions:

(i) $\zeta_{M_1}^l(a) = \zeta_{M_2}^l(\psi(a))$, $\zeta_{M_1}^u(a) = \zeta_{M_2}^u(\psi(a))$

(ii) $\zeta_{N_1}^l(\mathrm{ab}) = \zeta_{N_2}^l(\psi(a)\psi(b))$, $\zeta_{N_1}^u(\mathrm{ab}) = \zeta_{N_2}^u(\psi(a)\psi(b))$, for all $a \in X$, ab $\in E$

(3) A weak isomorphism (WI) ψ from G_1 to G_2 is a BM $\psi\colon X_1 \longrightarrow X_2$ with the following conditions:

(i) ψ is HM

(ii) $\zeta_{M_1}^l(a) = \zeta_{M_2}^l(\psi(a))$,$\zeta_{M_1}^u(a) = \zeta_{M_2}^u(\psi((a)))$, $a \in X$

(4) A co-weak isomorphism (CWI) ψ from G_1 to G_2 is a BM $\psi\colon X_1 \longrightarrow X_2$ with the following conditions:

(i) ψ is HM

(ii) $\zeta_{N_1}^l(ab) = \zeta_{N_2}^l(\psi(a)\psi(b))$, $\zeta_{N_1}^u(ab) = \zeta_{N_2}^u(\psi(a)\psi(b))$, for all $ab \in E$

Definition 7. Suppose $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ are two IVFGs. Then, G_1 is an interval-valued subgraph (IVSG) of G_2, if $M_1 \subseteq M_2$ and $N_1 \subseteq N_2$.

Definition 8. A family $\Gamma = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ of IVFSs on X is called a k-coloring of IVFG $G = (M, N)$ if

(i) $\vee\Gamma = M$.

(ii) $\lambda_i \wedge \lambda_j = 0$, for $1 \le i, j \le k$.

(iii) For each strong edge ab of X, $\min\{\lambda_i(a), \lambda_i(b)\} = 0$, for $1 \le i \le k$. We consider that a graph is k-colorable if it can be colored by k colors.

Table 1: Some essential notations.

Notation	Meaning
FG	Fuzzy graph
IVFS	Interval-valued fuzzy set
FS	Fuzzy set
IVFG	Interval-valued fuzzy graph
SIVFG	Strong interval-valued fuzzy graph
BM	Bijective mapping
HM	Homomorphism
WI	Weak isomorphism
IH	Injective homomorphism
CWI	Co-weak isomorphism
BH	Bijective homomorphism
SG	Subgraph
IVSG	Interval-valued subgraph
CIVFG	Complete interval-valued fuzzy graph
CG	Complete graph
IV	Isolated vertex

All the essential notations are denoted in Table 1.

3. Homomorphisms and Isomorphisms of Interval-Valued Fuzzy Graphs

In this part, we explain HM and isomorphisms on IVFGs by the HM of level graphs in IVFGs.

Theorem 1. *Let X be a finite nonempty set, $M \in IVFS(X)$, and $N \in IVFS(\widetilde{X}^2)$. Then, $G = (M, N) \in IVFG(X)$ if and only if $G_{[\alpha,\beta]} = (M_{[\alpha,\beta]}, N_{[\alpha,\beta]})$ is a graph, for all $[\alpha,\beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$.*

Proof. Suppose $G = (M, N) \in \text{IVFG}(X)$ is an IVFG. For any $[\alpha,\beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$, consider that $\text{ab} \in N_{[\alpha,\beta]}$. Then, $\zeta_N^l(\text{ab}) \geq \alpha$ and $\zeta_N^u(\text{ab}) \geq \beta$. Because G is an IVFG,

$$\begin{aligned}\alpha &\leq \zeta_N^l(ab) \leq \min\left\{\left(\zeta_M^l(a), \zeta_M^l(b)\right)\right\},\\ \beta &\leq \zeta_N^u(ab) \leq \min\left\{\left(\zeta_M^u(a), \zeta_M^u(b)\right)\right\}.\end{aligned} \tag{6}$$

Also, we have

$$\begin{aligned}\alpha \leq \zeta_M^l(a), \quad \beta \leq \zeta_M^u(a) &\Rightarrow a \in M_{[\alpha,\beta]},\\ \alpha \leq \zeta_M^l(b), \quad \beta \leq \zeta_M^u(b) &\Rightarrow b \in M_{[\alpha,\beta]}.\end{aligned} \tag{7}$$

Therefore, $x, y \in A_{[\alpha,\beta]}$ and $(A_{[\alpha,\beta]}, B_{[\alpha,\beta]})$ is a graph.

Conversely, suppose $G_{[\alpha,\beta]} = (M_{[\alpha,\beta]}, N_{[\alpha,\beta]})$ is a graph, for $[\alpha,\beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$. For any $\text{ab} \in \widetilde{X}^2$, let $\zeta_N^l(\text{ab}) = \alpha$, $\zeta_N^u(\text{ab}) = \beta$. Then, $\text{ab} \in N_{[\alpha,\beta]}$. Hence, $a, b \in M_{[\alpha,\beta]}$. Thus, $\zeta_M^l(a) \geq \alpha$, $\zeta_M^l(b) \geq \alpha$, $\zeta_M^u(a) \geq \beta$, $\zeta_M^u(a) \geq \beta$. This results that

$$\min\left\{\left(\zeta_M^l(a), \zeta_M^l(b)\right)\right\} \geq \alpha = \zeta_N^l(\text{ab}), \tag{8}$$

and

$$\min\left\{\left(\zeta_M^u(a), \zeta_M^u(b)\right)\right\} \geq \beta = \zeta_N^u(\text{ab}). \tag{9}$$

Thus, $G = (M, N)$ is an IVFG. □

Definition 9. Suppose $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ are two IVFGs on X_1 and X_2, respectively, $\psi\colon X_1 \longrightarrow X_2$ a mapping. For any $[\alpha,\beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$, if ψ is a HM from $G_{1[\alpha,\beta]} = (M_{1[\alpha,\beta]}, N_{1[\alpha,\beta]})$ to $G_{2[\alpha,\beta]} = (M_{2[\alpha,\beta]}, N_{2[\alpha,\beta]})$, then, ψ is called $[\alpha,\beta]-$ HM mapping from G_1 to G_2.

Theorem 2. *Suppose $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ are two IVFGs. Then, $\psi\colon G_1 \longrightarrow G_2$ is a HM from G_1 to G_2 if and only if ψ is $[\alpha,\beta]-$ HM from G_1 to G_2 .*

Proof. Let $\psi\colon G_1 \longrightarrow G_2$ be a HM from G_1 to G_2. Assume, $[\alpha,\beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$. If $a \in M_{[\alpha,\beta]}$, then,

$$\alpha \leq \zeta_{M_1}^l(a) \leq \zeta_{M_2}^l(\psi(a)), \quad \beta \leq \zeta_{M_1}^u(a) \leq \zeta_{M_2}^u(\psi(a)). \tag{10}$$

Therefore $\psi(a) \in M_{2[\alpha,\beta]}$ implying ψ is a mapping from $M_{1[\alpha,\beta]}$ to $M_{2[\alpha,\beta]}$. For $a, b \in M_{1[\alpha,\beta]}$, let $\text{ab} \in N_{1[\alpha,\beta]}$. Then,

$$\alpha \leq \zeta_{N_1}^l(\text{ab}), \quad \beta \leq \zeta_{N_1}^u(\text{ab}). \tag{11}$$

Hence,

$$\begin{aligned}\alpha &\leq \zeta_{N_1}^l(\text{ab}) \leq \zeta_{N_2}^l(\psi(a)\psi(b)),\\ \beta &\leq \zeta_{N_1}^u(\text{ab}) \leq \zeta_{N_2}^u(\psi(a)\psi(b)).\end{aligned} \tag{12}$$

That implies $\psi(a)\psi(b) \in N_{2[\alpha,\beta]}$. Thus, ψ is a HM from $G_{1[\alpha,\beta]}$ to $G_{2[\alpha,\beta]}$.

Conversely, suppose $\psi\colon X_1 \longrightarrow X_2$ is a $[\alpha,\beta]-$ HM from G_1 to G_2. For the arbitrary element of $a \in G_1$, let $\zeta_{M_1}^l(a) = w_1$, $\zeta_{M_1}^u(a) = w_2$. Then, $a \in M_{1[w_1,w_2]}$, so that, $\psi(a) \in M_{2[w_1,w_2]}$, because ψ is a HM from $(M_{1[w_1,w_2]}, N_{1[w_1,w_2]})$ to $(M_{2[w_1,w_2]}, N_{2[w_1,w_2]})$. We have

$$w_1 \leq \zeta_{M_2}^l(\psi(a)), w_2 \leq \zeta_{M_2}^u(\psi(a)). \tag{13}$$

That is,

$$\zeta_{M_1}^l(a) \leq \zeta_{M_2}^l(\psi(a)), \zeta_{M_1}^u(a) \leq \zeta_{M_2}^u(\psi(a)). \tag{14}$$

Now for arbitrary of $a, b \in X_1$, let $\zeta_{N_1}^l(\text{ab}) = t_1$, $\zeta_{N_1}^u(\text{ab}) = t_2$. Then,

$$\begin{aligned}t_1 &= \zeta_{N_1}^l(\text{ab}) \leq \min\left\{\zeta_{M_1}^l(a), \zeta_{M_1}^l(b)\right\},\\ t_2 &= \zeta_{N_1}^u(\text{ab}) \leq \min\left\{\zeta_{M_1}^u(a), \zeta_{M_1}^u(b)\right\}.\end{aligned} \tag{15}$$

Hence, $a, b \in M_{1[t_1,t_2]}$ and $a, b \in M_{1[t_1,t_2]}$. Because ψ is a HM from $G_{1[t_1,t_2]} = (M_{1[t_1,t_2]}, N_{1[t_1,t_2]})$ to $G_{2[t_1,t_2]} = (M_{2[t_1,t_2]}, N_{2[t_1,t_2]})$, we conclude that $\psi(a), \psi(b) \in M_{2[t_1,t_2]}$ and $\psi(a)\psi(b) \in N_{2[t_1,t_2]}$. Therefore,

$$\begin{aligned}\zeta_{N_2}^l(\psi(a)\psi(b)) \geq t_1 = \zeta_{N_1}^l(ab)\zeta_{N_2}^u(\psi(a)\psi(b))\\ \geq t_2 = \zeta_{N_1}^u(xy).\end{aligned} \tag{16}$$

□

Theorem 3. *Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be IVFGs on X_1 and X_2 , respectively. Then, $\psi\colon X_1 \longrightarrow X_2$ is a WI from G_1 to G_2 , if and only if, ψ is a bijective $[\alpha,\beta]-$ HM from G_1 to G_2 and*

$$\zeta_{M_1}^l(a) = \zeta_{M_2}^l(\psi(a)), \zeta_{M_1}^u(a) = \zeta_{M_2}^u(\psi(a))a \in X. \tag{17}$$

Proof. Suppose ψ is a WI from G_1 to G_2. From the definition of HM, ψ is a BH from G_1 to G_2. By Theorem 2, ψ is a bijective $[\alpha,\beta]$– HM from G_1 to G_2 and also, by the definition of WI, we have

$$\zeta_{M_1}^l(a) = \zeta_{M_2}^l(\psi(a)), \zeta_{M_1}^u(a) = \zeta_{M_2}^u(\psi(a))a \in X. \tag{18}$$

Conversely, by hypothesis, ψ: $M_{1[0,1]} = X_1 \longrightarrow M_{2[0,1]} = X_2$ is a BM and

$$\zeta_{M_1}^l(a) = \zeta_{M_2}^l(\psi(a)), \zeta_{M_1}^u(a) = \zeta_{M_2}^u(\psi(a))a \in X. \tag{19}$$

For $a, b \in X_1$, let $\zeta_{N_1}^l(\text{ab}) = t_1$, $\zeta_{N_1}^u(\text{ab}) = t_2$. Then,

$$\begin{aligned} t_1 &= \zeta_{N_1}^l(\text{ab}) \le \min\left\{\zeta_{M_1}^l(a), \zeta_{M_1}^l(b)\right\},\\ t_2 &= \zeta_{N_1}^u(\text{ab}) \le \min\left\{\zeta_{M_1}^u(a), \zeta_{M_1}^u(b)\right\}. \end{aligned} \tag{20}$$

That implies $a, b \in M_{1[t_1,t_2]}$ and $\text{ab} \in N_{1[t_1,t_2]}$. Because ψ is a HM from $G_{1[t_1,t_2]} = (M_{1[t_1,t_2]}, N_{1[t_1,t_2]})$ to $G_{2[t_1,t_2]} = (M_{2[t_1,t_2]}, N_{2[t_1,t_2]})$, we have $\psi(a), \psi(b) \in M_2$ and $\psi(a)\psi(b) \in N_{2[t_1,t_2]}$. Thus,

$$\begin{aligned} \zeta_{N_2}^l(\psi(a)\psi(b)) \ge t_1 = \mu_{N_1}^l(\text{ab}), \zeta_{N_2}^u(\psi(a)\psi(b))\\ \ge t_2 = \zeta_{N_1}^u(\text{ab}). \end{aligned} \tag{21}$$

□

Theorem 4. *Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be IVFGs on X_1 and X_2 , respectively. Then, ψ: $X_1 \longrightarrow X_2$ is a CWI from G_1 to G_2 , if and only if, ψ is a bijective $[\alpha,\beta]$– HM from G_1 to G_2 and*

$$\begin{aligned} \zeta_{N_1}^l(\text{ab}) = \zeta_{N_2}^l(\psi(a)\psi(b)), \zeta_{N_1}^u(\text{ab})\\ = \zeta_{N_2}^u(\psi(a)\psi(b))\text{ab} \in \widetilde{X}^2. \end{aligned} \tag{22}$$

Proof. Let ψ: $X_1 \longrightarrow X_2$ be a CWI from G_1 to G_2. Then, ψ is a BH from G_1 to G_2. By Theorem 2, ψ is a bijective $[\alpha,\beta]$– HM from G_1 to G_2. Also, by the description of CWI, we write

$$\begin{aligned} \zeta_{M_1}^l(\text{ab}) = \zeta_{N_2}^l(\psi(a)\psi(b)), \zeta_{N_1}^u(\text{ab})\\ = \zeta_{N_2}^u(\psi(a)\psi(b))\text{ab} \in \widetilde{X}^2. \end{aligned} \tag{23}$$

Conversely, by hypothesis, we see that ψ: $M_{1[0,1]} = X_1 \longrightarrow M_{2[0,1]} = X_2$ is a BM and

$$\zeta_{N_1}^l(\text{ab}) = \zeta_{N_2}^l(\psi(a)\psi(b)), \zeta_{N_1}^u(\text{ab}) = \zeta_{N_2}^u(\psi(a)\psi(b)). \tag{24}$$

For arbitrary principle $a \in X_1$, consider $\zeta_{M_1}^l(a) = s_1$, $\zeta_{M_1}^u(a) = s_2$. Then, we have $a \in M_{1[s_1,s_2d]}$. Now, because ψ is a HM from $G_{1[s_1,s_2]} = (A_{1[s_1,s_2]}, B_{1[s_1,s_2]})$ to $G_{2[s_1,s_2]} = (M_{2[s_1,s_2]}, N_{2[s_1,s_2]})$, $\psi(a) \in M_{2[s_1,s_2]}$. Therefore,

$$\zeta_{M_2}^l(\psi(a)) \ge s_1 = \zeta_{M_1}^l(a), \zeta_{M_2}^u(\psi(a)) \ge s_2 = \zeta_{M_1}^u(a). \tag{25}$$

That implies ψ is a CWI from G_1 to G_2. □

Corollary 1. *Let $G_1 = (M_1, N_1) \in IVFG(X_1)$ and $G_2 = (M_2, N_2) \in IVFG(X_2)$. If ψ: $X_1 \longrightarrow X_2$ is a CWI from G_1 to G_2, then, ψ is an IH from $G_{1[\alpha,\beta]}$ to $G_{2[G_{2(\alpha,\beta)}]}$, for every $[\alpha,\beta] \in L_*$, $M_{1[\alpha,\beta]} \ne \varnothing$.*

Example 1. Suppose $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ are two IVFGs, as in Figure 1. Consider the mapping of ψ: $X_1 \longrightarrow X_2$, defined by $\psi(x_{1i}) = x_{2i}$, $1 \le i \le 3$. $[\alpha,\beta]$– level graphs of G_1 and G_2 in Figure 1, if $M_{[\alpha,\beta]} \ne \varnothing$, therefore, ψ is an IH from $G_{1[\alpha,\beta]}$ to $G_{2[\alpha,\beta]}$, but ψ is not a CWI.

$$\begin{aligned} G_{1[\alpha,\beta]}, [0.1, 0.2] < [\alpha,\beta] \le [0.1.0.3], G_{2[\alpha,\beta]}, [0.1, 0.2]\\ < [\alpha,\beta] \le [0.1.0.3]. \end{aligned} \tag{26}$$

Theorem 5. *Let $G_1 = (M_1, N_1) \in IVFG(X_1)$ and $G_2 = (M_2, N_2) \in IVFG(X_2)$, and ψ: $X_1 \longrightarrow X_2$ is a mapping. For any $[\alpha,\beta] \in L_*$, $M_{1[\alpha,\beta]} \ne \varnothing$, if ψ is an isomorphism from $G_{1[\alpha,\beta]}$ to a subgraph (SG) of $G_{2[\alpha,\beta]}$, then, ψ is a CWI from $G_{1[\alpha,\beta]}$ to an induced IVSG of $G_{2[\alpha,\beta]}$.*

Proof. The mapping ψ from $G_{1[0,1]} = (X, M_{[0,1]})$ to a SG $G_{2[0,1]} = (X, N_{[0,1]})$ is an isomorphism, so ψ: $X_1 \longrightarrow X_2$ is an IM. For arbitrary $a \in X_1$, we consider $\zeta_{M_1}^l(a) = \alpha$, $\zeta_{M_1}^u(a) = \beta$. Then, $a \in M_{1[\alpha,\beta]}$ and so $\psi(a) \in M_{2[\alpha,\beta]}$. Therefore,

$$\zeta_{M_2}^l(\psi(a)) \ge \alpha = \zeta_{M_1}^l(a), \zeta_{M_2}^u(\psi(a)) \ge \beta = \zeta_{M_1}^u(a). \tag{27}$$

For $a, b \in X_1$, let $\zeta_{N_1}^l(\text{ab}) = \alpha$, $\zeta_{N_1}^u(\text{ab}) = \beta$ that

$$\alpha \le \zeta_{M_1}^l(a), \alpha \le \zeta_{M_1}^l(b), \beta \le \zeta_{M_1}^u(a), \beta \le \zeta_{M_1}^u(b). \tag{28}$$

and $\text{ab} \in N_{1[\alpha,\beta]}$. Thus, $a, b \in M_{1[\alpha,\beta]}$ and $\text{ab} \in N_{1[\alpha,\beta]}$.

Since ψ is an isomorphism from $G_{1[\alpha,\beta]}$ to $G_{1[\alpha,\beta]}$, we get $\psi(a), \psi(b) \in M_{2[\alpha,\beta]}$ and $\psi(a)\psi(b) \in N_{2[\alpha,\beta]}$.

Therefore,

$$\begin{aligned} \zeta_{N_2}^l(\psi(a)\psi(b)) \ge \alpha = \zeta_{N_1}^l(\text{ab}), \zeta_{N_2}^u(\psi(a)\psi(b))\\ \ge \beta = \zeta_{N_1}^u(\text{ab}). (I). \end{aligned} \tag{29}$$

Now suppose $\zeta_{N_2}^l(\psi(a)\psi(b)) = z_1$, $\zeta_{N_2}^u(\psi(a)\psi(b)) = z_2$. Then $\psi(a)\psi(a) \in M_{2[z_1,z_2]}$. Because ψ is injective and an isomorphism from $G_{1[z_1,z_2]}$ to a SG of $G_{2[z_1,z_2]}$, we have $a, b \in M_{1[z_1,z_2]}$ and $\text{ab} \in N_{1[z_1,z_2]}$. Therefore,

$$\begin{aligned} \zeta_{N_2}^l(\psi(a)\psi(b)) = z_1 \le \zeta_{N_1}^l(\text{ab}), \zeta_{N_2}^u(\psi(a)\psi(b))\\ = z_2 \le \zeta_{N_1}^u(\text{ab}). (II). \end{aligned} \tag{30}$$

Now by (I) and (II), we conclude the following statement:

$$\zeta_{N_2}^l(\psi(a)\psi(b)) = \zeta_{N_1}^l(\text{ab}), \zeta_{N_2}^u(\psi(a)\psi(b)) = \zeta_{N_1}^u(\text{ab}). \tag{31}$$

□

Corollary 2. *Let $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ be two IVFGs with $|X_1| = |X_2|$, and ψ: $X_1 \longrightarrow X_2$ a mapping. For $[\alpha,\beta] \in L_*, M_{[\alpha,\beta]} \ne \varnothing$, if, ψ is an isomorphism from $G_{1(\alpha,\beta)}$ to a SG of $G_{2[\alpha,\beta]}$, then, ψ is a CWI from G_1 to G_2 .*

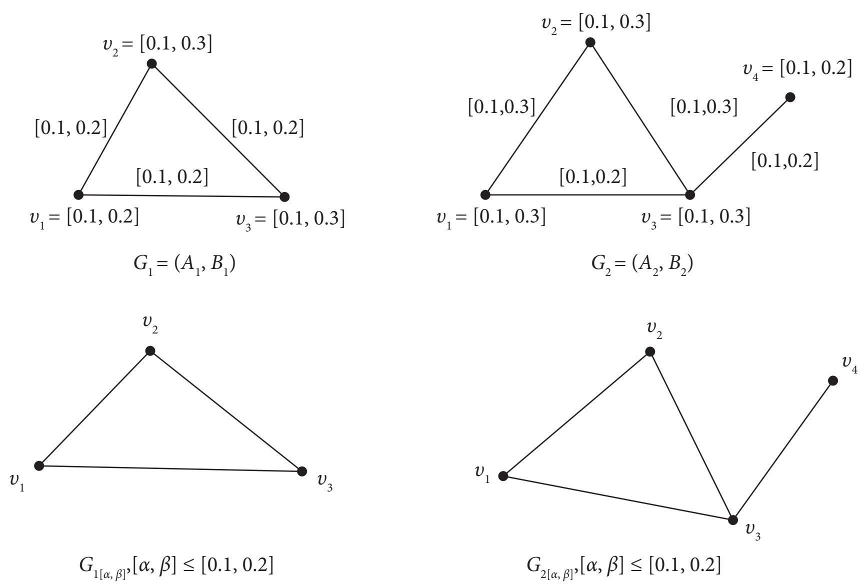

Figure 1: IVFGs and $[\alpha, \beta]$– level graphs.

Theorem 6. *Suppose we have two graphs $G_1 = (M_1, N_1)$ and $G_2 = (M_2, N_2)$ that they are IVFGs, and ψ: $X_1 \longrightarrow X_2$ is a BM. If for any $[\alpha, \beta] \in L_*$, ψ is an isomorphism from $G_{1[\alpha,\beta]}$ to $G_{2[\alpha,\beta]}$, then, ψ is an isomorphism from G_1 to G_2.*

Proof. By hypothesis, ψ^{-1}: $X_2 \longrightarrow X_1$ is a BM and an isomorphism from $G_{2[\alpha,\beta]}$ to $G_{1[\alpha,\beta]}$.

By Theorem 3.7, ψ is a CWI from G_1 to G_2 and ψ^{-1} is a CWI from G_2 to G_1. Therefore, ψ is an isomorphism from G_1 to G_2. □

Theorem 7. *Suppose $G = (M, N)$ is an IVFG. Then, G is a complete interval-valued fuzzy graph (CIVFG), if and only if, $G_{[\alpha,\beta]} = (M_{[\alpha,\beta]}, N_{[\alpha,\beta]})$ is a complete graph (CG) for $[\alpha, \beta] \in L_*$.*

Proof. If $G = (M, N)$ is a CIVFG and for $[\alpha, \beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$, $a, b \in M_{[\alpha,\beta]}$, then

$$\alpha \le \zeta^l_M(a), \alpha \le \zeta^l_M(b), \beta \le \zeta^u_M(a), \beta \le \zeta^u_M(b) \tag{32}$$

and so

$$\begin{aligned}\zeta^l_N(\text{ab}) &= \min\left\{\zeta^l_M(a), \zeta^l_M(b)\right\} \ge \alpha, \zeta^u_N(\text{ab}) \\ &= \min\{\zeta^u_M(a), \zeta^u_M(b)\} \ge \beta.\end{aligned} \tag{33}$$

Hence, $a, b \in N_{[\alpha,\beta]}$. It shows that $G_{[\alpha,\beta]}$ is a CG.

Conversely, consider $G = (M, N)$ is not a CIVFG. Then, there are $a, b \in X$ so that

$$\begin{aligned}\zeta^l_N(ab) &< \min\left\{\zeta^l_M(a), \zeta^l_M(b)\right\} \text{or} \zeta^u_N(ab) \\ &< \min\{\zeta^u_M(a), \zeta^u_M(b)\}.\end{aligned} \tag{34}$$

Let $\zeta^l_N(ab) < \min\left\{\zeta^l_M(a), \zeta^l_M(b)\right\}$, and/or $\alpha = \min\left\{\zeta^l_M(a), \zeta^l_M(b)\right\}$, for $\alpha \in (0, 1]$. Then, $\zeta^l_M(a) \ge \alpha$, $\zeta^l_M(b) \ge \alpha$. Hence, $a, b \in M_{[\alpha,\beta]}$, for a $\beta \in [0, 1]$, but $ab \notin N_{[\alpha,\beta]}$. This implies that $G_{[\alpha,\beta]}$ is not a CG. For the case $\zeta^u_N(ab) < \min\{\zeta^u_M(a), \zeta^u_M(b)\}$, it follows similarly. □

Theorem 8. *Let $G = (M, N) \in IVFG(X)$. Then, $G_{[\alpha,\beta]}$ does not have an isolated vertex (IV), for any $[\alpha, \beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$ if and only if for each $a \in X$, there is $b \in X$ so that $\zeta^l_N(ab) = \zeta^l_M(a)$, $\zeta^u_N(ab) = \zeta^u_M(b)$.*

Proof. Suppose that for each $[\alpha, \beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$, graph $G_{[\alpha,\beta]}$ does not have an IV and there is a vertex $a \in X$ so that for each $b \in X$, $\zeta^l_N(ab) < \zeta^l_M(a)$ or $\zeta^u_N(ab) < \zeta^u_M(a)$. Let $\zeta^l_N(ab) < \zeta^l_M(a)$ and $\zeta^l_M(a) = \alpha$, $\zeta^u_M(b) = \beta$, for $[\alpha, \beta] \in L_*$.

Then, $a \in M_{[\alpha,\beta]}$ and for each $a \in X$, $a \neq b$, $ab \notin N_{[\alpha,\beta]}$. Thus, a is an IV in the graph $G_{[\alpha,\beta]} = (M_{[\alpha,\beta]}, N_{[\alpha,\beta]})$, which is a contradiction.

Now let that for $[\alpha, \beta] \in L_*$, $M_{[\alpha,\beta]} \neq \varnothing$, vertex $a \in M_{[\alpha,\beta]}$ is an IV in $G_{(\alpha,\beta)}$. If $y \notin M_{[\alpha,\beta]}$, then,

$$\zeta^l_N(ab) \le \zeta^l_M(b) < \alpha \le \zeta^l_M(a) \text{or} \zeta^u_N(ab) \le \zeta^u_M(b) < \beta \le \zeta^u_M(a). \tag{35}$$

and if $b \in M_{[\alpha,\beta]}$, it is trivial that $ab \notin N_{[\alpha,\beta]}$, hence,

$$\left.\zeta^l_N(ab)\right) < \alpha \le \zeta^l_M(a) \text{or} \zeta^u_N(ab) < \beta \le \zeta^u_M(a). \tag{36}$$

Thus, for any $b \in X$, $\zeta^l_N(ab) \neq \zeta^l_M(a)$, $\zeta^u_N(ab) \neq \zeta^u_M(a)$. □

Theorem 9. *A VG $G = (M, N)$ is r -colorable $\Leftrightarrow$ there exists a HM from X to the $K_{r,M'}$.*

Proof. Let G be r-colorable with r colors, so that $\Gamma = \{\lambda_1, \lambda_2, \ldots, \lambda_r\}$. Let $V_i = \{v \in V | \lambda_i(v) \neq 0\}$. We denote CIVFG $K_{r,M'}$ with vertices set $\{1, 2, \ldots, r\}$, so that the degree of membership vertex i is $\zeta^l_{M'}(i) = \min\left\{\zeta^l_M(v) | v \in V_i\right\}$ and the degree of nonmembership vertex i is $\zeta^u_{M'}(i) =$

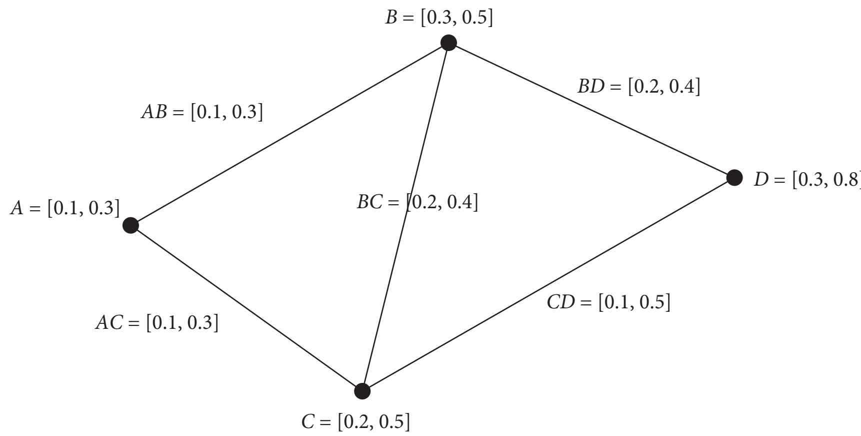

Figure 2: Interval-valued fuzzy graph $G_1 = (A_1, B_1)$.

$\min\{\zeta^u_M(v)|v \in V_i\}$. Now the mapping $\psi\colon G \longrightarrow K_{r,M'}$ defined by $\psi(v) = i$ is a graph homomorphism, because

$$\begin{aligned}\zeta^l_M(v) &\leq \min\left\{\zeta^l_M(w)|w \in V_i\right\} = \zeta^l_{M'}(i) = \zeta^l_{M'}(\psi(v)),\\ \zeta^u_M(v) &\leq \min\{\zeta^u_M(w)|w \in V_i\} = \zeta^u_{M'}(i) = \zeta^u_{M'}(\psi(v)),\end{aligned} \tag{37}$$

and by the definition of CIVFG, for $u \in V_i$ and $v \in V_j$, we have

$$\begin{aligned}\zeta^l_N(uv) &\leq \min\left\{\zeta^l_M(u), \zeta^l_M(v)\right\} \leq \min\left\{\zeta^l_{M'}(i), \zeta^l_{M'}(j)\right\} = \min\left\{\zeta^l_{M'}(\psi(u)), \zeta^l_{M'}(\psi(v))\right\},\\ \zeta^u_N(uv) &\leq \min\{\zeta^u_M(v), \zeta^u_M(v)\} \leq \min\{\zeta^u_{M'}(i), \zeta^u_{M'}(j)\} = \min\{\zeta^u_{M'}(\psi(u)), \zeta^u_{M'}(\psi(v))\}.\end{aligned} \tag{38}$$

Then, $\zeta^l_N(uv) \leq \zeta^l_{N'}(\psi(u)\psi(v))$, $\zeta^u_N(uv) \leq \zeta^u_{N'}(\psi(u)\psi(v))$, for all $uv \in \widetilde{V}^2$.

Conversely, let $\phi\colon G \longrightarrow K_{r,M'}$ is a HM. For a given $k \in V(K_{r,M'})$, we define the set $\psi^{-1}(k) \subseteq V$ to be

$$\psi^{-1}(k) = \{x \in V|\psi(x) = k\}. \tag{39}$$

If $v \in \psi^{-1}(k)$, let $\lambda_k(v) = [\zeta^l_{\lambda_k}(v), \zeta^u_{\lambda_k}(v)] = [\zeta^l_A(v), \zeta^u_M(v)]$, otherwise $\lambda_k(v) = 0$. Thus, the IVFG G is r-colorable with coloring set $\{\lambda_1, \lambda_2, \ldots, \lambda_r\}$. □

4. Application

In graph theory, the subject of coloring is an important case of graph labeling, and there are more examples of coloring graph. In this part, we have expressed an application of the coloring of vertices in an IVFG, and we present this concept in the form of an example.

Example 2. Let $G = (M_1, N_1)$ be an IVFG (see Figure 2). In this example, we consider four sport groups such as A, B, C, D so that they are vertices of the graph. We show each one of the sports groups with interval-valued. The interval-valued of the vertices are the good activity rate in the world cup games, so that are

$$\begin{aligned}\left[\zeta^l_{M_1}(A), \zeta^u_{M_1}(A)\right] &= [0.1, 0.3], \left[\zeta^l_{M_1}(B), \zeta^u_{M_1}(B)\right]\\ &= [0.3, 0.5],\\ \left[\zeta^l_{M_1}(C), \zeta^u_{M_1}(C)\right] &= [0.2, 0.5], \left[\zeta_{M_1 l}(D), \zeta^u_{M_1}(D)\right]\\ &= [0.3, 0.8].\end{aligned} \tag{40}$$

Suppose that AB, BC, AC, CD, and BD are the edges of graph G_1.

$$\begin{aligned}\left[\zeta^l_{N_1}(AB), \zeta^u_{N_1}(AB)\right] &= [0.1, 0.3], \left[\zeta^l_{N_1}(BC), \zeta^u_{N_1}(BC)\right]\\ &= [0.2, 0.4],\\ \left[\zeta^l_{N_1}(AC), \zeta_{N^u_1}(AC)\right] &= [0.2, 0.4], \left[\zeta^l_{N_1}(CD), \zeta^u_{N_1}(CD)\right]\\ &= [0.3, 0.5],\\ \left[\zeta^l_{N_1}(BD), \zeta^u_{N_1}(BD)\right] &= [0.1, 0.3].\end{aligned} \tag{41}$$

Here we want to see how many days they will need to exercise between these sport groups. Let X be a set of sport groups, $X = \{A, B, C, D\}$ and E = \{AB, BC, AC, CD, BD \}. Now, form graph G_1 with vertices set E, where $a, b \in E$ are neighbors, if and only if, $X(a) \cap X(b) \neq \varnothing$. For example, $X(AB) = \{A, B\}$ and $X(BD) = \{B, D\}$. So $X(AB) \cap$

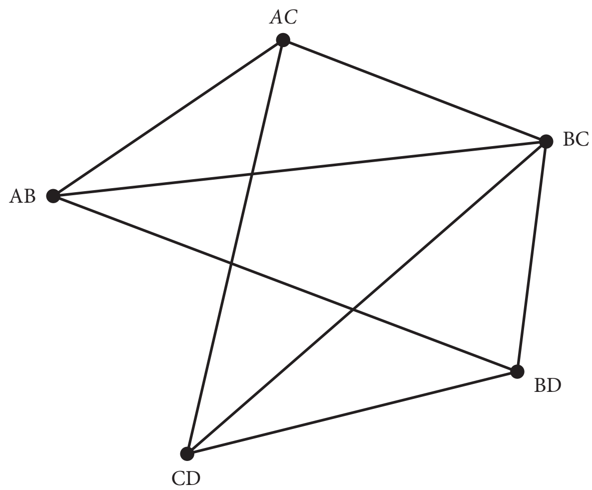

Figure 3: The colored graph of example 2.

$X(BD) = \{B\} \neq \varnothing$, and so AB, BD are neighbors. By Theorem 9, there is a HM from G_1 to complete graph with $n = 3$. Therefore, these sport groups are needed to play 3 days, $\{\{AB, CD\}, \{BC\}, \{AC, BD\}\}$ (see Figure 3).

In the next example, we want to identify the most effective employee of an electrical power department with the help of an interval-valued fuzzy influence digraph.

Example 3. Electricity is one of the discoveries that has changed the daily lives of everyone on the planet. Electricity is a staple of modern technology, and without it, most of the things we use every day will work. Other major areas where electricity is used are the production, supply, and distribution of food. Farmers use electricity for almost everything, including lights, pumps, electric motors, and so on. Electricity is very useful, especially in homes, offices, shopping malls, schools, supermarkets, and any other place we can think of. Today, it has become such a vital element of a society that depends on electricity that it is difficult for human beings to even imagine life without electricity. Access to electricity has become the most important thing in human society and has created great economic, political, and cultural changes in the world. Perhaps what was more important than electricity itself, and multiplied the importance of electricity, was the emergence of devices that received their driving forces from electricity and raised the level of well-being and quality of human life in society. It can be boldly said that the largest volume of living items in today's society are electrical appliances. Most of these devices in human life play the role of converting electrical power into mechanical, thermal, and cooling power. Therefore, considering the importance of electricity in everyday human life, we have tried to consider an electrical power department and find the most efficient person in it with the help of an interval-valued fuzzy influence. Because worthy people should be considered to lead this organization, and with their leadership, electricity consumption should be saved and this very valuable blessing should not be wasted. Therefore, we consider the vertices of the interval-valued influence graph as the heads of each ward of the electrical power department and the edges of the graph as the degree of interaction and influence of each other. For this department, the set of staff is M = {Maleki, Eskandari, Farhadi, Tahmasbi, Ashkani, Bozorgi}.

Table 2: Name of employees in an electrical department and their services.

Name	Services
Farhadi	Warehouse keeper
Maleki	Security guard
Bozorgi	Financial affairs manager
Eskandari	President of the company
Tahmasbi	Managing director
Ashkani	Board of directors

(1) Eskandari has been working with Tahmasbi for 8 years and values his views on issues.

(2) Farhadi has been responsible for maintaining the electrical department's property for a long time, and everyone is satisfied with his performance.

(3) In an electrical power department, it is very important to control the staff and check their treatment of clients. Maleki is the best person to do this.

(4) Tahmasbi and Ashkani have a long history of conflict.

(5) Due to Bozorgi's long experience in finance, he is the best person for the job.

According to the above values, we consider an interval-valued influence graph. The nodes show each of the department's staff that the lower bound indicates the ability of staff to perform related tasks and the upper bound shows the weakness and inability of each staff member. But the edges show the level of friendship and interest among employees. If this friendship is more, then the amount of influence on each other will naturally be more. For the weight of the edges, the lower bound means the amount of friendship and the upper bound means the amount of conflict. Names of

Table 3: The level of staff capability.

	Farhadi	Maleki	Bozorgi	Eskandari	Tahmasbi	Ashkani
ζ_M^l	0.2	0.5	0.5	0.4	0.3	0.3
ζ_M^u	0.4	0.5	0.5	0.5	0.3	0.4

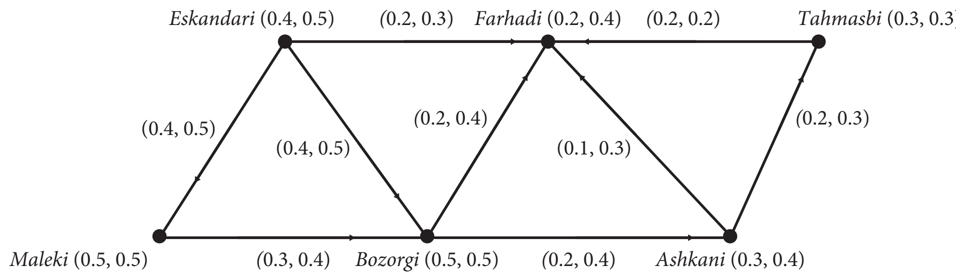

Figure 4: Interval-valued fuzzy influence digraph.

Table 4: Adjacency matrix corresponding to Figure 4.

	Farhadi	Maleki	Bozorgi	Eskandari	Tahmasbi	Ashkani
Farhadi	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
Maleki	[0, 0]	[0, 0]	[0.3, 0.4]	[0, 0]	[0, 0]	[0, 0]
Bozorgi	[0.2, 0.4]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.4]
Eskandari	[0.2, 0.3]	[0.4, 0.5]	[0.4, 0.5]	[0, 0]	[0, 0]	[0, 0]
Tahmasbi	[0.2, 0.2]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
Ashkani	[0.1, 0.3]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.3]	[0, 0]

employees and level of staff capability are shown in Tables 2 and 3. The adjacency matrix corresponding to Figure 4 is shown in Table 4.

Figure 4 shows that Ashkani has 30% of the power needed to do the work of the board of directors of the electrical power department, but unfortunately he does not have 40% of the necessary power to do so. The directional edge of Eskandari-Farhadi shows that there is 20% friendship among these two employees, but unfortunately, they have 30% conflict. Clearly, Eskandari has dominion over both Maleki and Bozorgi, and his dominance over both is 40%. It is clear that Eskandari is the most influential employee of the electrical power department because he controls both the security guard and the financial affairs manager, who have 50% of the power in the department.

5. Conclusion

IVFG, belonging to FG family, has good abilities when faced with problems that cannot be explained by FGs. It has a wide range of applications in the field of psychological sciences as well as in the reconnoiter of individuals founded on oncological behaviors. So, in this survey, we tried to show the new notion of level graph in IVFG. Also, we introduced the notion of $[\alpha, \beta]-$ HM, isomorphism, WI, and CWI of IVFG by $[\alpha, \beta]-$ HM. Finally, an application of HM of IVFGs has been presented by using coloring-FG. In our future work, we will introduce the new concepts of connectivity in IVFGs and investigate some of their properties. Also, we will study the new results of the complete dominating set, regular complete dominating set, and global dominating set on IVFGs.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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